Word embeddings



- Word embeddings idea.
 One-hot vectors. SVD
- Word2vec model, training methods
- Word2vec training optimization: hierarchical softmax and SGNS
- GloVe model
- FastText model, Hashing Trick
- Word embeddings models' quality evaluation



Word embeddings idea. One-hot vectors. SVD



Input data may have different format



Visual content

- Images
- Video



Texts

- Unstructured documents
- HTML / XML



Structured documents

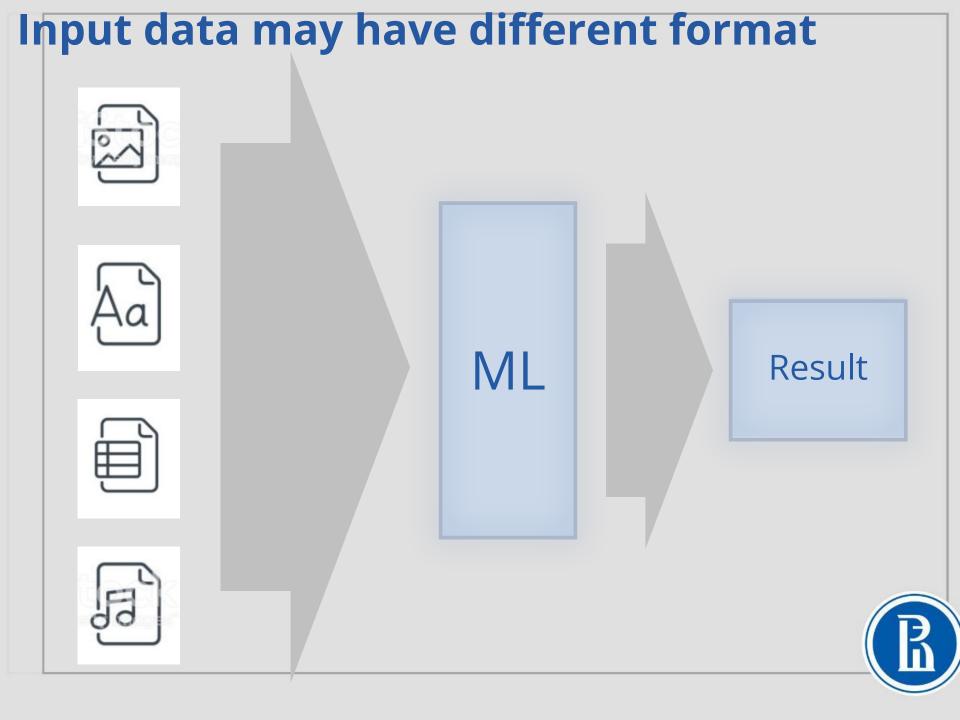
Tables with features

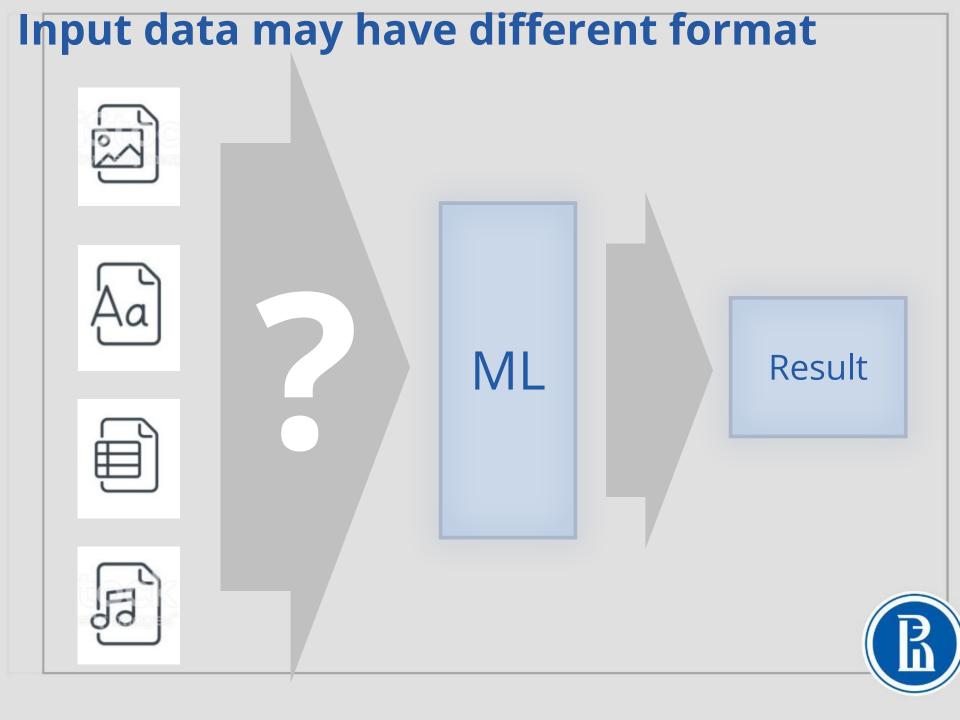


Signals

- Audio: music / speech
- Other signals







Word embeddings

"I love watching TV series"

text

'l" "watching" "series"

"love" "TV"

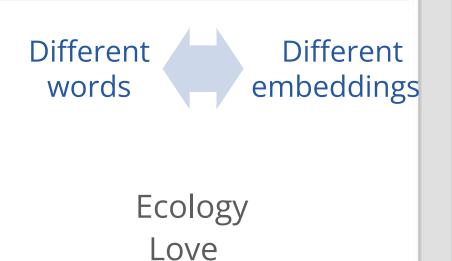
word

 $\begin{bmatrix} 123 \\ 456 \\ 12 \\ \dots \\ 89 \end{bmatrix} \begin{bmatrix} 23 \\ 372 \\ 8 \\ \dots \\ 83 \end{bmatrix} \begin{bmatrix} 16 \\ 124 \\ 76 \\ \dots \\ 29 \end{bmatrix} \begin{bmatrix} 2 \\ 12 \\ 299 \\ \dots \\ 65 \end{bmatrix} \begin{bmatrix} 177 \\ 6 \\ 504 \\ \dots \\ 304 \end{bmatrix}$

embeddings



What kind of embeddings do we want?







What does «close» mean?

Semantic closeness

«Usual» word closeness

Examples:

- computer
- laptop
- PC

Closeness of embeddings

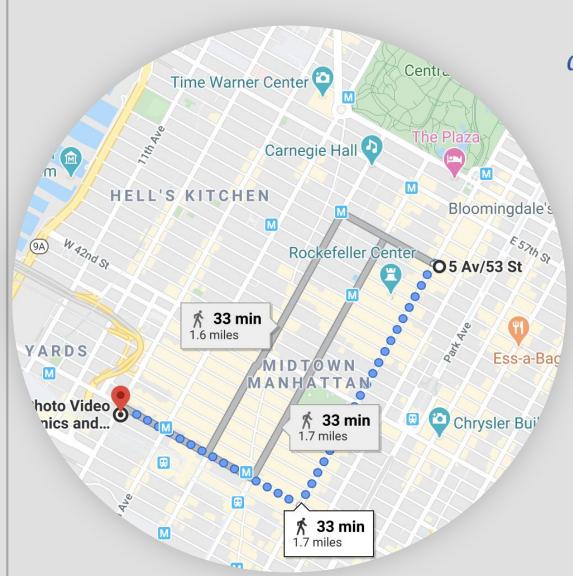
sim(w, v)

where sim() can be:

- Manhattan distance
- Euclidean distance
- Cosine distance



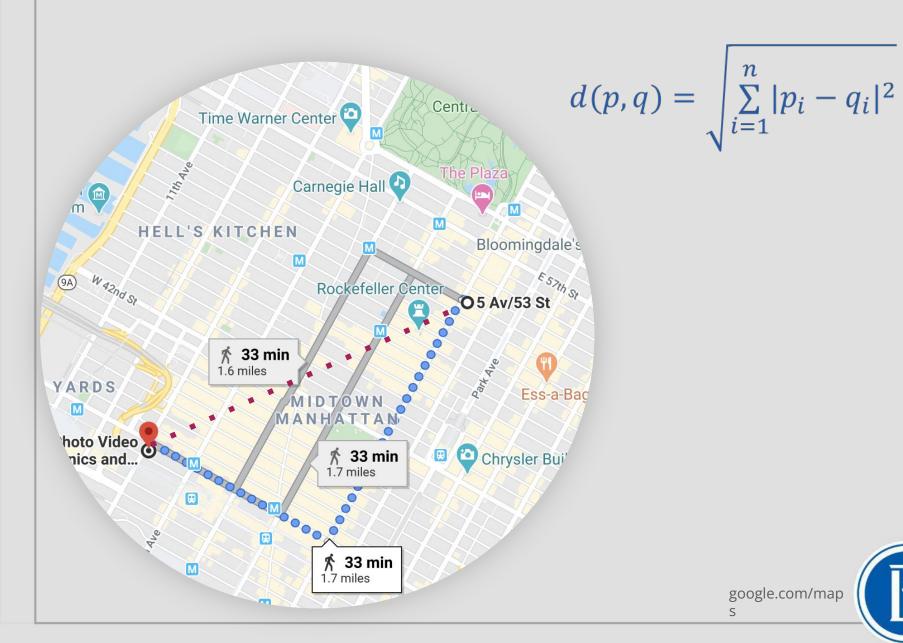
Manhattan distance



$$d(p,q) = \sum_{i=1}^{n} |p_i - q_i|$$



Euclidean distance

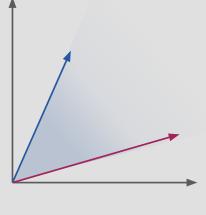




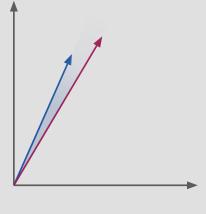
Cosine distance

$$p \cdot q = \parallel p \parallel \parallel q \parallel \cos(\theta)$$

$$\cos(\theta) = \frac{p \cdot q}{\|p\| \|q\|} = \frac{(\sum_{i=1}^{n} p_i q_i)}{\sqrt{\sum_{i=1}^{n} p_i^2} \sqrt{\sum_{i=1}^{n} q_i^2}}$$



$$\theta \approx 45^{\circ}$$
 $\cos(\theta) \approx 0.7$

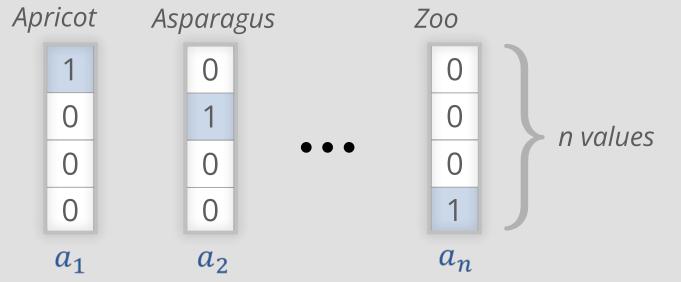


$$\theta \approx 7^{\circ}$$
 $\cos(\theta) \approx 0.99$



One-hot encoding

Suppose we have vocabulary V, |V| = n



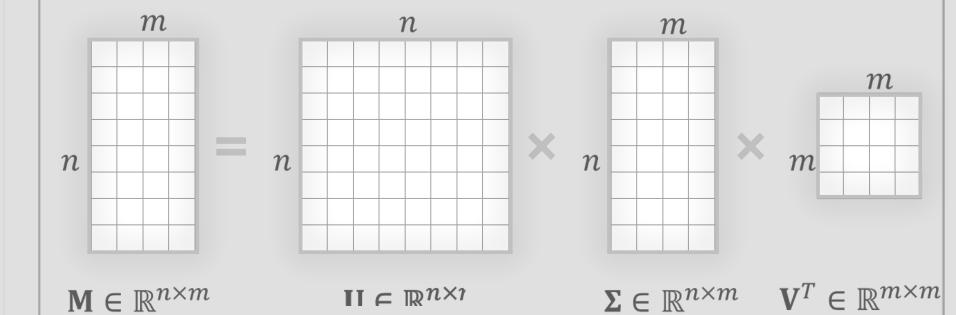
Advantages

 Simple way to obtain embeddings for a set of words

Disadvantages

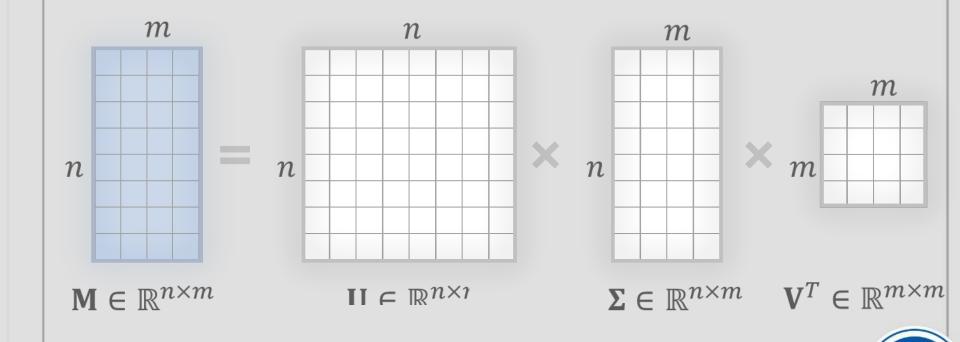
- The documents will have huge unfixed length
- Embeddings are mutually orthogonal





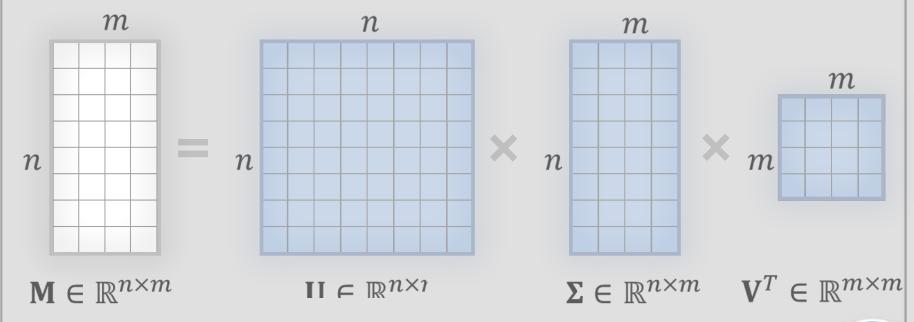


Suppose we have rectangular matrix $\mathbf{M} \in \mathbb{R}^{n \times m}$



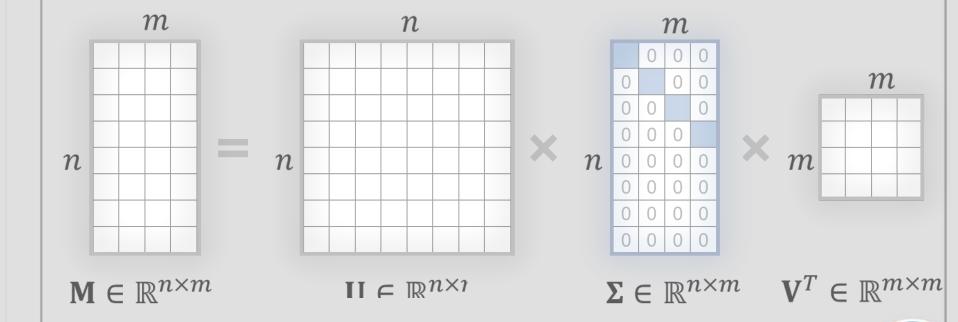
Suppose we have rectangular matrix $\mathbf{M} \in \mathbb{R}^{n \times m}$

The matrix can be represented as $\mathbf{M} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^T$

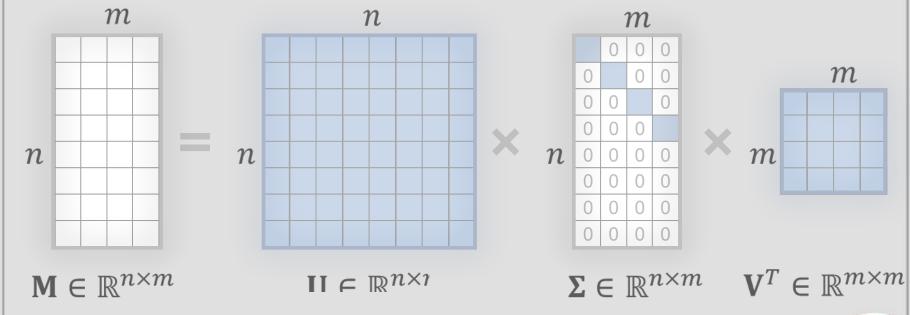




- $\Sigma \in \mathbb{R}^{n \times m}$ is a diagonal matrix with non-negative values
- ullet diagonal values of matrix Σ are singular values of matrix M



Columns U and V are left and right singular vectors of matrix M

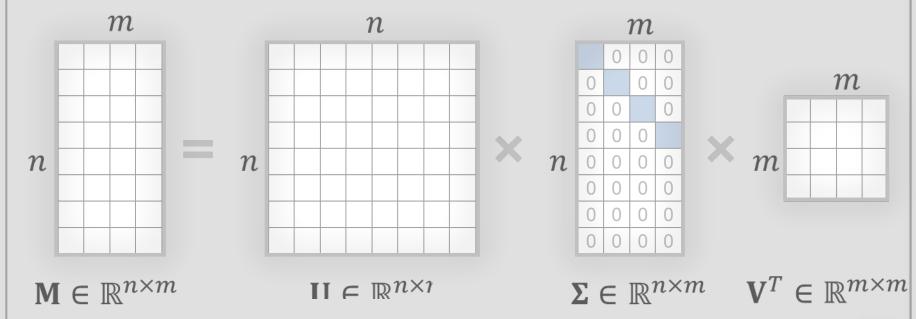




Suppose we a collection from m documents with n unique words

Then $\mathbf{M} \in \mathbb{R}^{n \times m}$ is a bag of words matrix

Let's apply SVD: $\mathbf{M} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^T$

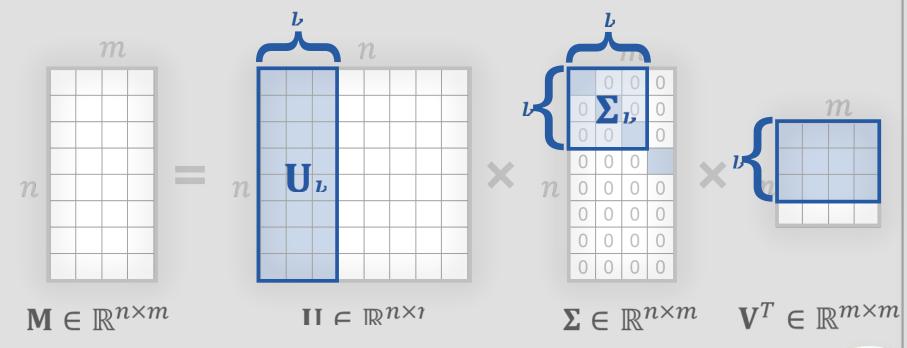




Singular Value Decomposition (reduced SVD)

Pick top-k largest values of ∑

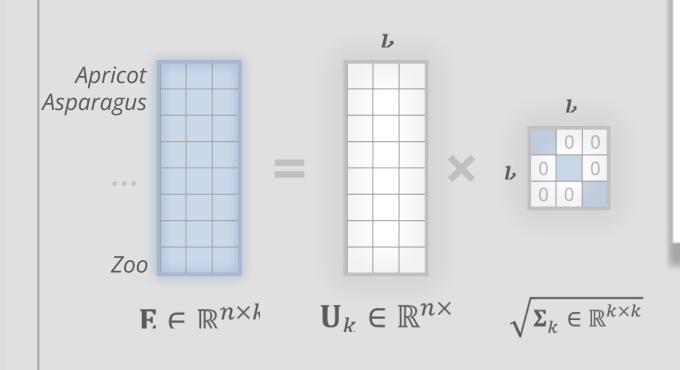
Ignore all other values and columns of matrix U - we obtain matrices U_k and Σ_k





Word embeddings with reduced SVD

We can use rows of matrix $\mathbf{U}_k \sqrt{\mathbf{\Sigma}_k}$ as word embeddings



By the way:

для получения векторов раскладывать можно не только матрицу «мешка слов»



Word embeddings with SVD

Improvements:

- Vectors have fixed size
- Vectors are no longer mutually orthogonal
- Semantic closeness is somehow taken into account

Nevertheless:

- Adding new words/documents requires new SVD calculation
- We need to operate with a huge BoW matrix
- Word embeddings are not that good



Main conclusions

- Word embeddings are used as features in NLP tasks
- Good word embeddings represent semantic closeness of words
- One-hot vectors can be useful but they are too sparse and mutually orthogonal
- SVD can produce word embeddings of fixed size that somehow represent semantics



Word2vec model



Distributive semantics hypothesis

Words with similar meaning share similar context

```
«Today I ate tasty, juicy orange»

«This apple is so sweet and juicy»

«So sweet are the apricots, so tasty»
```

• Instead of frequency counters let's train a model to predict a word by its context (and vice versa)

Harris Zellig. Distributional structure // Word. — 1954. — Vol. 10, no. 23. — Pp. 146–162.



Word2vec models: CBOW and skip-gram

Continues Bag-of-Words
 predict central word by its context



Skip-gram
 predict context by central word



Distributed Representations of Words and Phrases and their Compositionality. /
Tomas Mikolov, Ilya Sutskever, Kai Chen et al. // NeurIPS — 2013. — Pp.
3111–3119.



Suppose we have a collection with N unique words
To train a model we slide over text with a window of size 2C + 1

I share an apricot with friend



I share an apricot with friend



I share an apricot with friend





I shape an apricot with friend

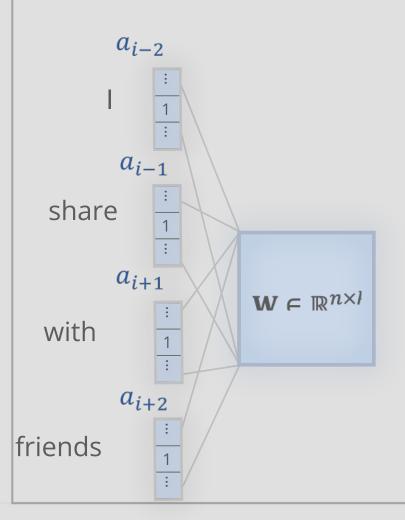


share an apricot with friend



CBOW as neural network

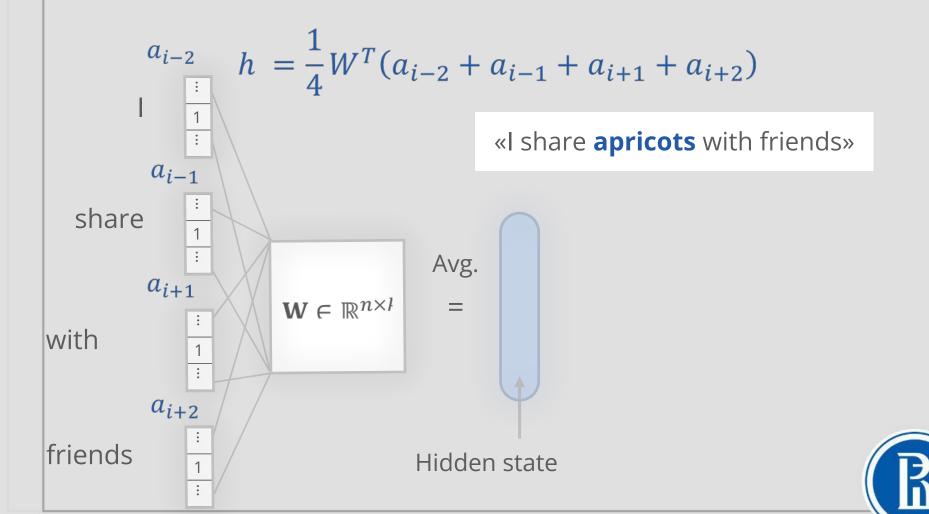
Model - two-layer neural network
Input - 2C one-hot context vectors of size n



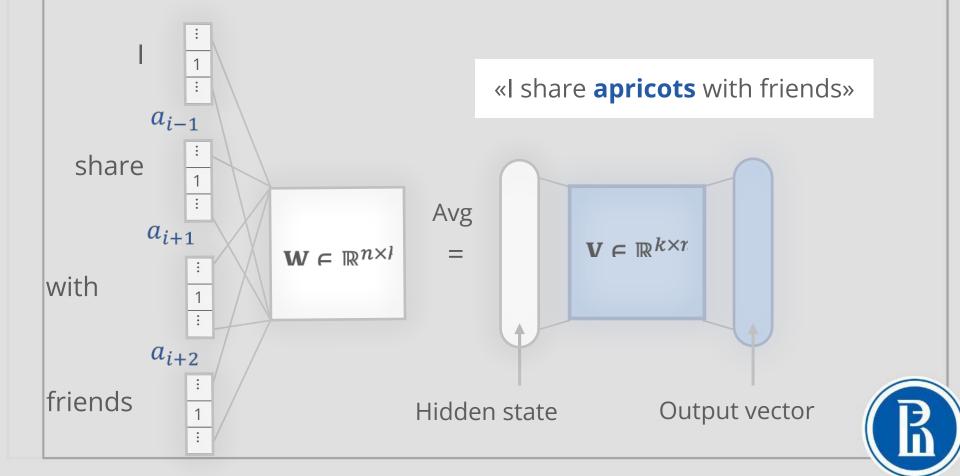
«I share **apricots** with friends»



CBOW as neural network



CBOW as neural network



Loss function

• For window with index i predict word w_i by context c_i

$$\sum_{i=1}^{N} \log p(w_i|c_i) \to \max_{W,V}$$



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Loss function

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$$\sum_{i=1}^{N} \log p(w_i|c_i) \to \max_{W,V}$$

- Therefore model's output is a vector with n probabilities
 - Therefore model's output is a vector with n probabilities

Softmax function:

$$z_j = p(w_j|c_i) = \frac{e^{b_j}}{\sum_{k=1}^n e^{b_k}}$$

And the word embeddings?

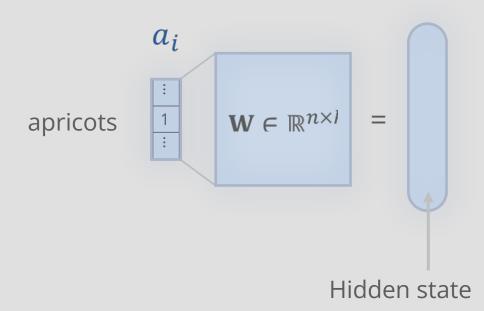
- As a result of training we obtain two matrices: W and V
- Usually, rows of matrix W are used as word embeddings
- But both columns of V and the combination of two matrices can be used



Skip-gram as neural network

Skip-gram model is arranged symmetrically

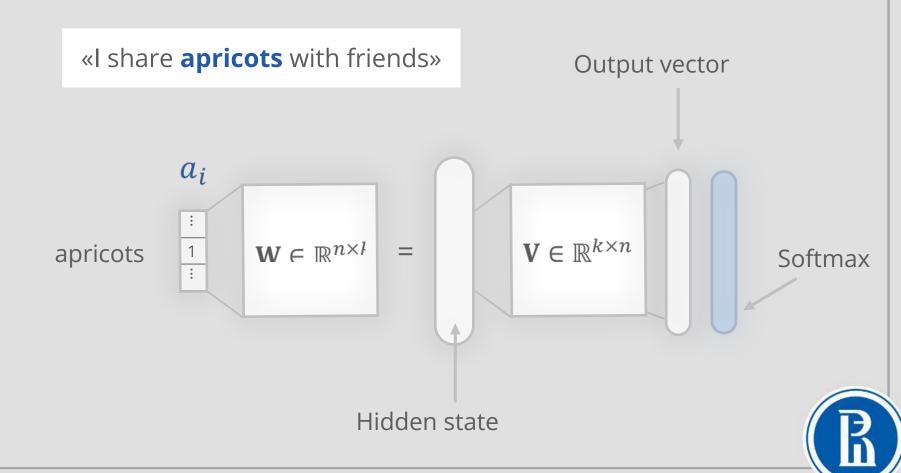
«I share **apricots** with friends»





Skip-gram as neural network

 Output — one probability distribution over words for the central word



Loss function

- Output one probability distribution over words for the central word
- 2C words from actual context should have maximal values
- Loss function:

$$\sum_{i=1}^{N} \sum_{j=-C, j\neq 0}^{C} \log p(w_{i+j}|w_i) \to \max_{W,V}$$



Main conclusions

- Word2vec models train word representations based on predictions, not on statistics
- There are two basic models: CBOW and Skip-gram
- In canonical implementation word2vec is a two-layer neural network, and its weights are the resulting word embeddings
- The quality of word2vec embeddings is better then SVD embeddings, and we don't need huge BoW matrices anymore



Word2vec training optimization: hierarchical softmax and SGNS



Word2vec neural approach disadvantages

- Training word2vec neural network is computationally difficult
- We need to calculate softmax (O(n)) and update a lot of parameters



Word2vec neural approach disadvantages

- Training word2vec neural network is computationally difficult
- We need to calculate softmax (O(n)) and update a lot of parameters
- In practice, word2vec is trained with optimization methods



- We still have a fully connected neural network
- The only thing that differs is softmax calculation

 Mnih Andriy, Hinton Geoffrey E. A Scalable Hierarchical Distributed Language Model // NeurIPS. — 2008.

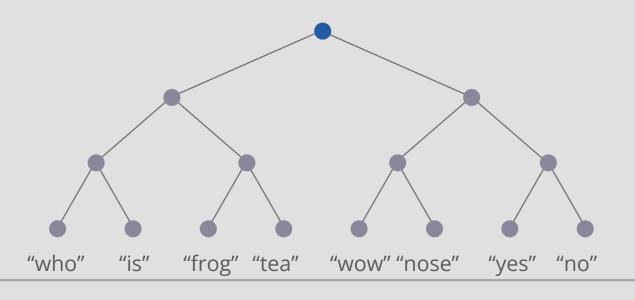


- We still have a fully connected neural network
- The only thing that differs is softmax calculation
- To calculate loss we don't need the whole vector of probabilities
- We only need the values in positions of words in the context window:

$$\sum_{i=1}^{N} \sum_{j=-C, j\neq 0}^{C} \log p(w_{i+j}|w_i) \to \max_{W,V}$$



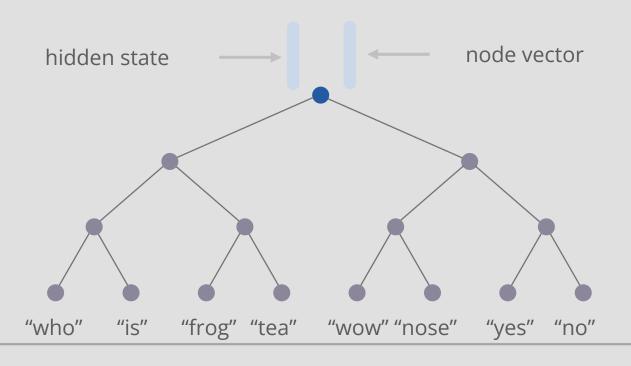
- Let's consider skip-gram model
- Hidden state after first layer $h = W^T x$
- Let's change the second layer into a binary tree (for example, Huffman tree)
- Assign every leaf one word from vocabulary
- Assign every internal node a vector of k weights





- Suppose we want to obtain a probability of a word w = "tea"
- Probability of paths (left and right) in current node:

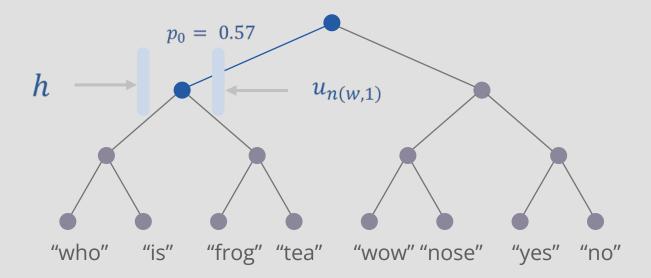
$$p_0 = \sigma(u_{n(w,0)}^T h) \qquad p_0 = \sigma(-u_{n(w,0)}^T h)$$





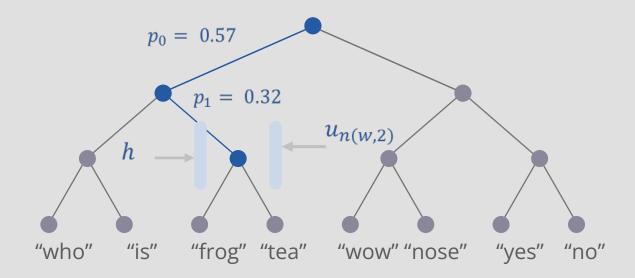
• Do the same in the next node:

$$p_1 = \sigma \left(-u_{n(w,1)}^T h \right)$$



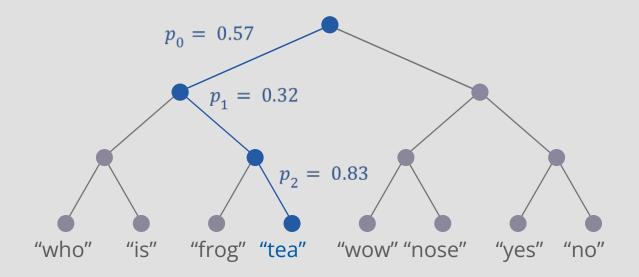


$$p_2 = \sigma \left(-u_{n(w,2)}^T h \right)$$



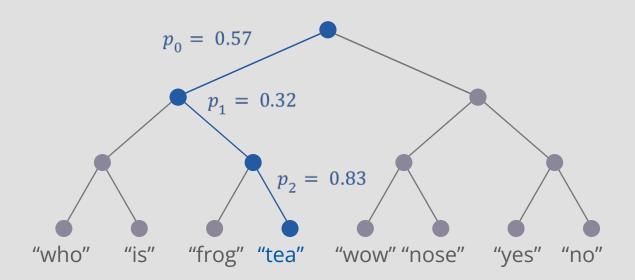


- In the end, we are at leaf with the word tea
- Every step i had a probability pi



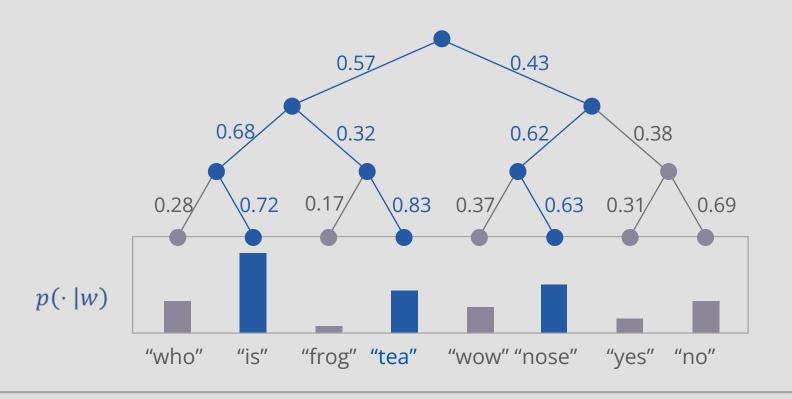


- In the end, we are at leaf with the word tea
- Every step i had a probability pi
- The final probability of path is $\prod_i p_i$



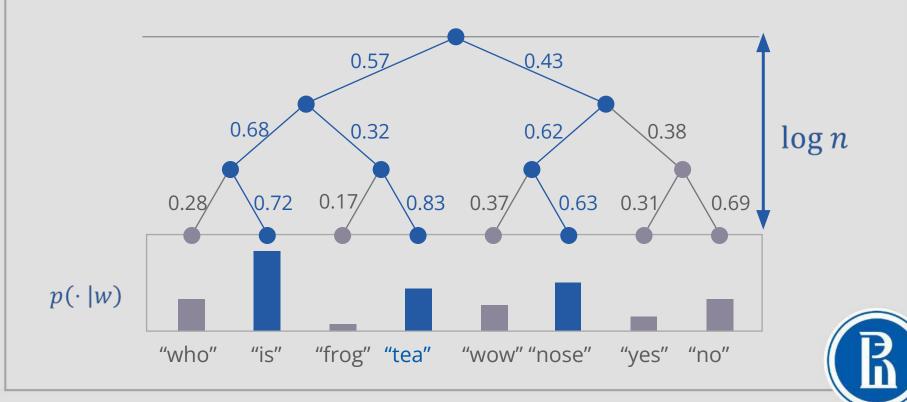


 If we do the procedure 2C times we obtain all probabilities required to calculate loss





- If we do the procedure 2C times we obtain all probabilities required to calculate loss
- The complexity of current calculation O(log n)



All in all

Before:

- Obtain h from the first layer
- Multiply h with second layer matrix V
- Apply softmax
- Use only 2C probabilities from the softmax result



All in all

Before:

- Obtain h from the first layer
- Multiply h with second layer matrix V
- Apply softmax
- Use only 2C probabilities from the softmax result

Now:

- Obtain h from the first layer
- 2C times go through the tree
- Obtain only essential probabilities



 For skip-gram model there is another popular training optimization method



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- We change the formulation of the problem and loss function
- We will solve binary classification problem



- For skip-gram model there is another popular training optimization method
- We change the formulation of the problem and loss function
- We will solve binary classification problem
- Object pair of words (w, s)
- Class 1: word s belongs to context w
- Class 2: word s doesn't belong to context w
- For each word **w** we compare trainable vector v_w which will be the sought one



What are the advantages?

- Training the model for each input object requires an update of all weights of the input layer
- Softmax in classic approach leads to an update of all weights in all layers
- In new approach we only update the weights of the layers, that are involved in the current iteration of training



Class probability is simulated with sigmoid:

$$p(1|(w,s)) = \frac{1}{1 + \exp(-v_w^T v_s)} = \sigma(v_w^T v_s)$$



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$$p(1|(w,s)) = \frac{1}{1 + \exp(-v_w^T v_s)} = \sigma(v_w^T v_s)$$

- Let D₁ be a subset of pairs (w, s) where s
 belongs to context w
- Let D_2 be a subset of all other possible pairs
- Then the likelihood function is:

$$L = \sum_{(w,s)\in D_1} \log(\sigma(v_w^T v_s)) + \sum_{(w,s)\in D_2} \log(\sigma(-v_w^T v_s))$$



If we optimize this function, we solve the task!



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But:

- We need examples of pairs
- D_1 can be obtained from data, while D_2 is not presented in data as is



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But:

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Solution: generate negative samples by sampling random pairs of words on each training step



Main conclusions

- Canonical word2vec implementation is scales poorly on dictionary and corpus volume
- Main difficulty is the second layer and softmax calculation
- There are several training optimization methods, the main ones are hierarchical softmax and negative sampling

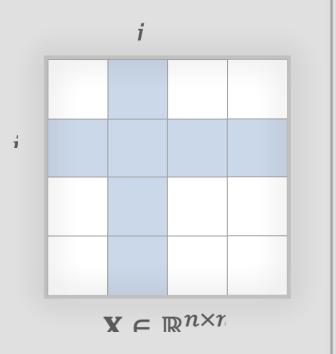


GloVe model



GloVe (Global Vectors)

- Construct a matrix $X \in \mathbb{R}^{n \times n}$
- Every column and row correspond to a word from a dictionary

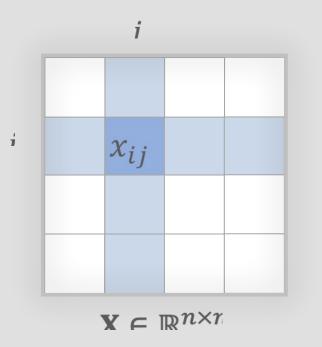


 Pennington Jeffrey, Socher Richard, Manning Christopher D. Glove: Global Vectors for Word Representation. // EMNLP. — Vol. 14. — 2014. — Pp. 1532–1543.



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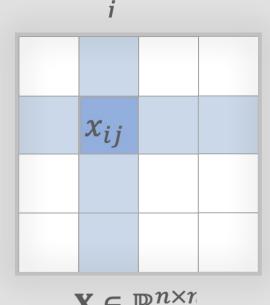




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of row i

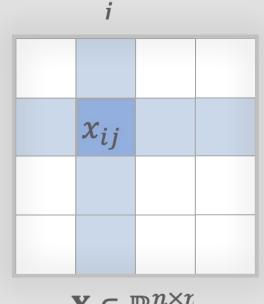


 $\mathbf{X} \in \mathbb{R}^{n \times r}$



- Construct a matrix $X \in \mathbb{R}$
- Every column and row correspond to a word from a dictionary
- x_{ij} number of times word i occurs in context of word i
- $X_i = \sum_i x_{ij}$ sum of elements of row i
- $\mathbf{P}_{ij} = \frac{x_{ij}}{x_i}$ probability of word

i to occur in context of word i



 $\mathbf{X} \in \mathbb{R}^{n \times r}$



- Assume we know vector representation v_i for every word i
- Also assume that we know all x_{ij}



- Assume we know vector representation v_i for every word i
- Also assume that we know all x_{ij}
- Define function $F(f(v_i, v_j, v_k)) = \frac{P_{ik}}{P_{jk}}$
- This function F shows which one of words i and j is more likely to occur in context of word k
- Function f is some function from input to real number



We need F to satisfy

$$F((v_i - v_j)^T v_k) = \frac{F(v_i^T v_k)}{F(v_j^T v_k)} = \frac{P_{ik}}{P_{ij}}$$

F can be exp(x)



So

$$F(x) = \exp(x)$$

$$P_{ij} = \frac{x_{ij}}{X_i}$$



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Rewrite

$$F(v_i^T v_k) = P_{ik}$$
$$v_i^T v_k = \log(x_{ik}) - \log(X_i)$$



• So

$$F(x) = \exp(x)$$

$$P_{ij} = \frac{x_{ij}}{X_i}$$

Rewrite

$$F(v_i^T v_k) = P_{ik}$$
$$v_i^T v_k = \log(x_{ik}) - \log(X_i)$$

• Now we remember that we only have x_{ij}



we want

$$v_i^T v_k = \log(x_{ik}) - \log(X_i)$$

• Therefore we rewrite

$$\sum_{i} \sum_{k} F(x_{ik}) \left(v_i^T v_k + b_i + b_k - \log(x_{ik}) \right)^2 \rightarrow \min_{v_i, b_i, i \in \{1, n\}'}$$
$$\log(X_i) = b_i + b_k$$

and obtain word embeddings.



Main conclusions

- Another example on an approach based on frequencies
- In practice, it works quite similar to word2vec
- There are many pre-trained GloVe models



FastText model, Hashing Trick



Problem 1: Out-of-Vocabulary (OOV) words



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 We train our model on corpus V
 Then we try to obtain a vector for a new word w
 But there is no embedding for it



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- Problem 2: Lack of consideration of morphology
- Suppose we train a model for a language with rich morphology (for example, russian):



- Problem 2: Lack of consideration of morphology
- Suppose we train a model for a language with rich morphology (for example, russian):
- We get a new embedding for each word
- As a result:
 - Many similar embeddings (and redundant memory)
 - Less samples for each word in training data

яблоко яблока яблоку ... яблоками яблоках



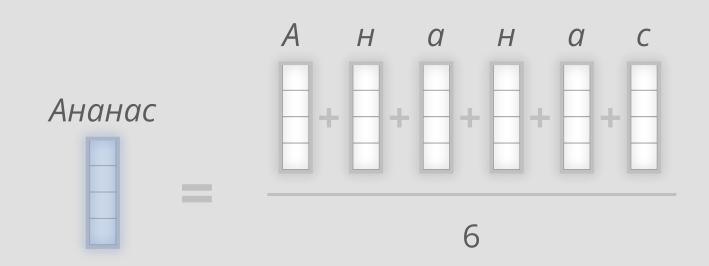
Character embeddings

- We can try to get embeddings for a smaller piece of language
- For example, for each character
- All training methods remain the same
- Word embedding can be obtained by averaging character embedding



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Character n-grams embeddings

- Instead of characters we can use character n-grams
- It improves the quality of the resulting word embeddings significantly



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- It improves the quality of the resulting word embeddings significantly

N-grams for a word «doctor»

N=3: ^do, doc, oct, cto, tor, or\$

N=4: ^doc, doct, octo, octor, tor\$

N=5: ^doct, docto, octor, ctor\$



Character n-grams embeddings

- Instead of characters we can use character n-grams
- It improves the quality of the resulting word embeddings significantly

N-grams for a word «doctor»

N=3: ^do, doc, oct, cto, tor, or\$

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N=5: ^doct, docto, octor, ctor\$

Word embeddings are obtained by averaging



Improvements:

- OOV problem solved
- Morphology problems also solved

But:

- There can be even more sequences of characters than there are different variants of words
- Tens of millions of vectors may simply not fit into RAM



Hash-functions and hash-tables

String hash-function converts a string into a number



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- Function requirements:
 - The range of values is limited by an interval
 - Function values are distributed approximately uniformly over the interval



Hash-functions and hash-tables

- String hash-function converts a string into a number
- Function requirements:
 - The range of values is limited by an interval
 - Function values are distributed approximately uniformly over the interval
- Hash-table an array of values + hash-function that transforms input string into array's indices



We fix the maximum number of vectors that we want to train



- We fix the maximum number of vectors that we want to train
- Match them a hash-table



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- Match them a hash-table
- All n-gramm char-embeddings are distributed over the values of array
- Several n-grams use the same embedding



FastText

- Package that trains word embeddings
- Uses CBOW / skip-gram both for words and character n-grams
- Optimizes RAM consumption by hashing trick
- Parallels training process on CPU

 Enriching Word Vectors with Subword Information / Piotr Bojanowski, Edouard Grave, Armand Joulin, Tomas Mikolov // Transactions of the Association for Computational Linguistics. — 2017. — Vol. 5. — Pp. 135–146.



Main conclusions

- Classic word2vec models work poorly with OOV words and morphology
- Models that operate with word fragments can solve these problems
- FastText package allows to train such models efficiently on CPU



Word embeddings evaluation



Types of metrics

 Word embeddings' quality can be measured by internal and external criteria



Types of metrics

- Word embeddings' quality can be measured by internal and external criteria
- Internal:
 - Quality of similar words search
 - Quality of analogies solving



Types of metrics

- Word embeddings' quality can be measured by internal and external criteria
- Internal:
 - Quality of similar words search
 - Quality of analogies solving
- External:
 - Quality of the final problem solution (the problem you use word embedding in)



Similar words search

- You have a word embedding model
- You have a corpus with human-evaluated semantic similarity between words



Similar words search

- You have a word embedding model
- You have a corpus with human-evaluated semantic similarity between words
- Calculate cosine similarity between word embeddings:

REMINDER

$$\cos(\theta) = \frac{p \cdot q}{\|p\| \|\|q\|} = \frac{\sqrt{\sum_{i=1}^{n} p_i q_i}}{\sqrt{\sum_{i=1}^{n} p_i^2} \sqrt{\sum_{i=1}^{n} q_i^2}}$$



Similar words search

- You have a word embedding model
- You have a corpus with human-evaluated semantic similarity between words
- Calculate cosine similarity between word embeddings:

REMINDER

$$\cos(\theta) = \frac{p \cdot q}{\|p\| \|\|q\|} = \frac{\sqrt{\sum_{i=1}^{n} p_i q_i}}{\sqrt{\sum_{i=1}^{n} p_i^2} \sqrt{\sum_{i=1}^{n} q_i^2}}$$

Check that it correlates with human evaluation

cos(абрикос, персик) > cos(абрикос, маска)



- You have triplets of words a, a^* , b
- Words a, a^* have some kind of relation

$$a =$$
читать, $a^* =$ чтение $b =$ петь



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- We want to find a word b* that has the same kind of relation with the word b

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a = читать, a^* = чтение b = петь, b^* = пение
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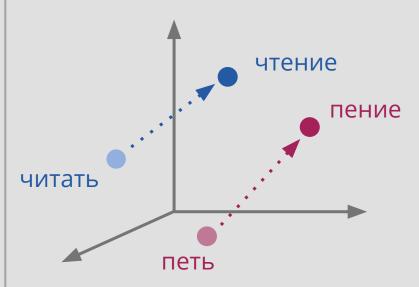


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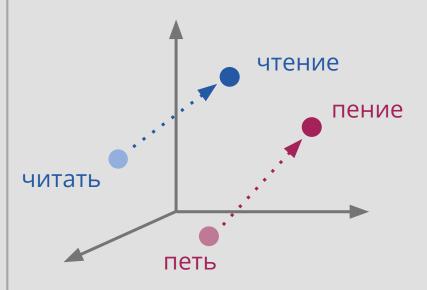
$$a^* - a + b$$

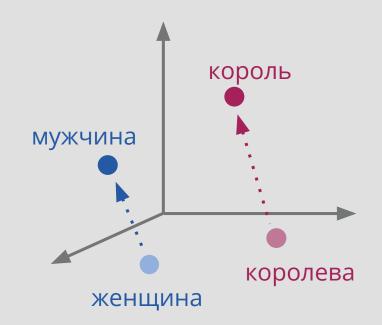
•
$$a =$$
 читать, $a^* =$ чтение $b =$ петь, $b^* =$ пение $a^* - a + b \approx b^*$ чтение — читать $+$ петь \approx пение



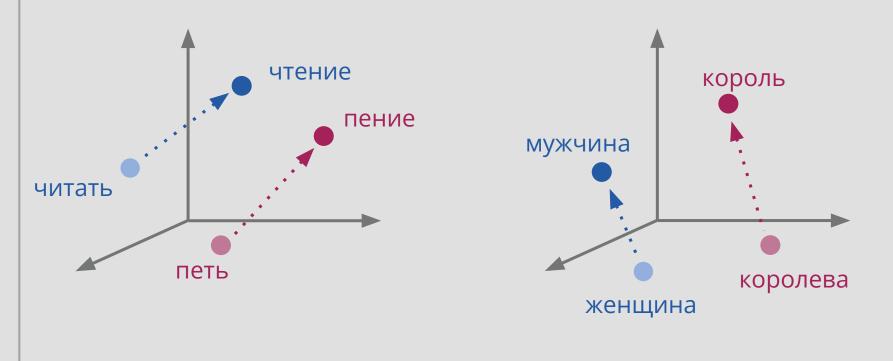














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- Mutual quality may differ for different tasks and datasets



Main conclusions

- The quality of word embeddings can be measured by different methods
- Internal criteria measure the models' quality in terms of their internal properties
- External criteria are more abstract and focus on the final problem where the word embedding model is applied

