Mathematical concepts for computer science

Applications of Propositional Logic

Translating English Sentences

- Translating sentences into compound statements **removes the ambiguity**.
- Translated sentences from English into logical expressions can be used to **analyze the sentences** and can use **rules of inference to reason about them**.

Applications of Propositional Logic

How can this English sentence be translated into a logical expression?

- "You can access the Internet from campus only if you are a computer science major or you are not a freshman."

a => "You can access the Internet from campus"

c => "You are a computer science major"

f => "You are a freshman"

Applications of Propositional Logic

How can this English sentence be translated into a logical expression?

- "You can access the Internet from campus only if you are a computer science major or you are not a freshman."

a => "You can access the Internet from campus"

c => "You are a computer science major"

f => "You are a freshman"

$$\mathbf{a \rightarrow (c \vee \neg f)}$$

Applications of Propositional Logic

How can this English sentence be translated into a logical expression?

- “You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old.”

q => “You can ride the roller coaster”

r => “You are under 4 feet tall”

s => “You are older than 16 years old,”

Applications of Propositional Logic

How can this English sentence be translated into a logical expression?

- “You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old.”

q => “You can ride the roller coaster”

r => “You are under 4 feet tall”

s => “You are older than 16 years old,”

$$(r \land \neg s) \rightarrow \neg q$$

Applications of Propositional Logic

System Specifications

- Translating sentences in natural language (such as English) into logical expressions is an essential part of specifying both hardware and software systems.

Express the specification "**The automated reply cannot be sent when the file system is full**" using logical connectives.

Applications of Propositional Logic

System Specifications

Express the specification "**The automated reply cannot be sent when the file system is full**" using logical connectives.

p => "The automated reply can be sent"

q => "The file system is full."

Applications of Propositional Logic

System Specifications

Express the specification "**The automated reply cannot be sent when the file system is full**" using logical connectives.

p => "The automated reply can be sent"

q => "The file system is full."

$$q \rightarrow \neg p$$

Applications of Propositional Logic

Boolean Searches

- Logical connectives are used extensively in **searches of large collections of information**, such as indexes of Web pages. Because these searches **employ techniques from propositional logic**, they are called **Boolean searches.**
- **AND** is used to match records that contain **both of two** search terms
- **OR** is used to match **one or both** of two search terms
- **NOT** is used to **exclude** a particular search term.

Applications of Propositional Logic

Logic Puzzles

- Puzzles that can be solved using logical reasoning are known as **logic puzzles.**
- An island that has **two kinds of inhabitants**, **knights**, who always tell the truth, and their opposites, **knaves**, who always lie. You encounter two people **A and B**. What are A and B if **A says "B is a knight"** and **B says "The two of us are opposite types**"

Applications of Propositional Logic

Logic Puzzles

- p=> A is a knight
- q=>B is a knight
- $\neg p$ =>
- $\neg$q =>

Applications of Propositional Logic

Logic Puzzles

- p=> A is a knight
- q=>B is a knight
- ¬*p =>* A is a knave
- ¬q => B is a knave

Applications of Propositional Logic

Logic Puzzles

A says "B is a knight" and **B says "The two of us are opposite types"**

- **p=> A is a knight**
- **q=>B is a knight**
- ***¬p =>* A is a knave**
- **¬q => B is a knave**
- If **A is a knight**, then he is telling the truth when he says that B is a knight, so that q is true **(p, q=T).**
- If **B is a knight**, then B's statement that A and B are of opposite types**.**

$$(p \land \neg q) \lor (\neg p \land q)$$

Applications of Propositional Logic

Logic Puzzles

A says "B is a knight" and **B says "The two of us are opposite types"**

- **p=> A is a knight**
- **q=>B is a knight**
- **$\neg$*p =>* A is a knave**
- **$\neg$q => B is a knave**

If A is a knave, then because everything a knave says is false, **A's statement that B is a knight, that is, that q is true, is a lie**. This means that q is false and **B is also a knave (P,Q=F).**

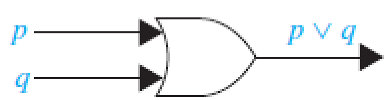
If B is a knave, then B's statement that **A and B are opposite types is a lie**, which is **consistent** with both A and B being knaves. We can conclude that **both A and B are knaves**.

Applications of Propositional Logic

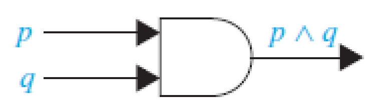
Logic Circuits

- A **logic circuit (or digital circuit)** receives input signals p1, p2, . . . , pn, each a bit [either 0 (off) or 1 (on)], and produces output signals s1, s2, . . . , sn, each a bit.

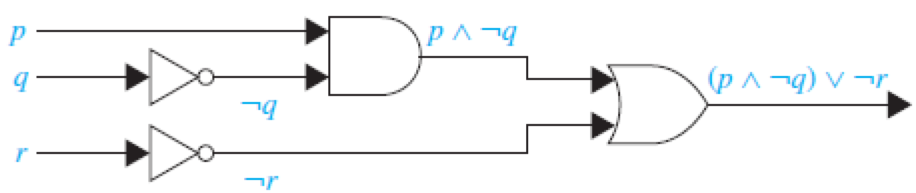
Inverter



OR gate



AND gate



Reference

- **Rosen, Kenneth H., and Kamala Krithivasan. Discrete mathematics and its applications: with combinatorics and graph theory. Tata McGraw-Hill Education, 2016.**