CYK Algorithm

a*nd*

Probabilistic

Context Free Grammar

Aashraya Sachdeva

Automata Theory and Computability

Dept. of Computer Science and Automation

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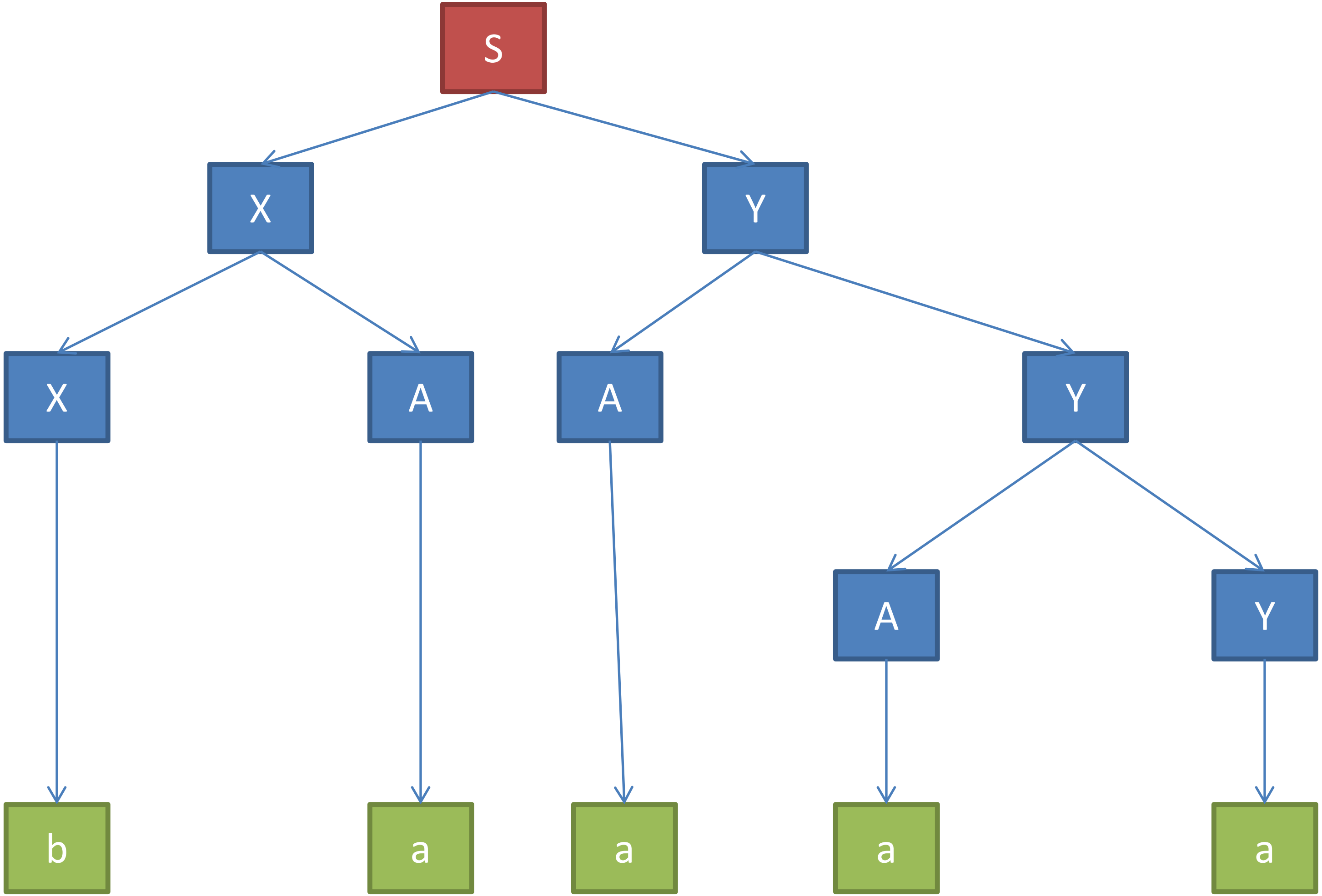
$$S \to XY$$
$$X \to XA|a|b$$
$$Y \to AY|a$$
$$A \to a$$

How can you tell if $\boldsymbol{baaaa} \in \boldsymbol{L}(\boldsymbol{G})$?

$$S \to XY$$
$$X \to XA|a|b$$
$$Y \to AY|a$$
$$A \to a$$



Can you make parse tree for

bbbbaaabaaabaaabbbaaaabaaabaaabaabbaaaaa ?

C: Cocke, J

Y: Younger, D

K: Kasami, T.

Algorithm

1. Works with Chomsky Normal Form.
2. Bottom-up parsing.
3. Dynamic Programming.
4. Polynomial time in length of input sentence.

$n \times n\ matrix$

Different length Substrings

5	baaaa				
4	baaa	aaaa			
3	baa	aaa	aaa		
2	ba	aa	aa	aa	
1	b	a	a	a	a

Input Sentence

b	a	a	a	a

$$S \to XY$$
$$X \to XA|a|b$$
$$Y \to AY|a$$
$$A \to a$$

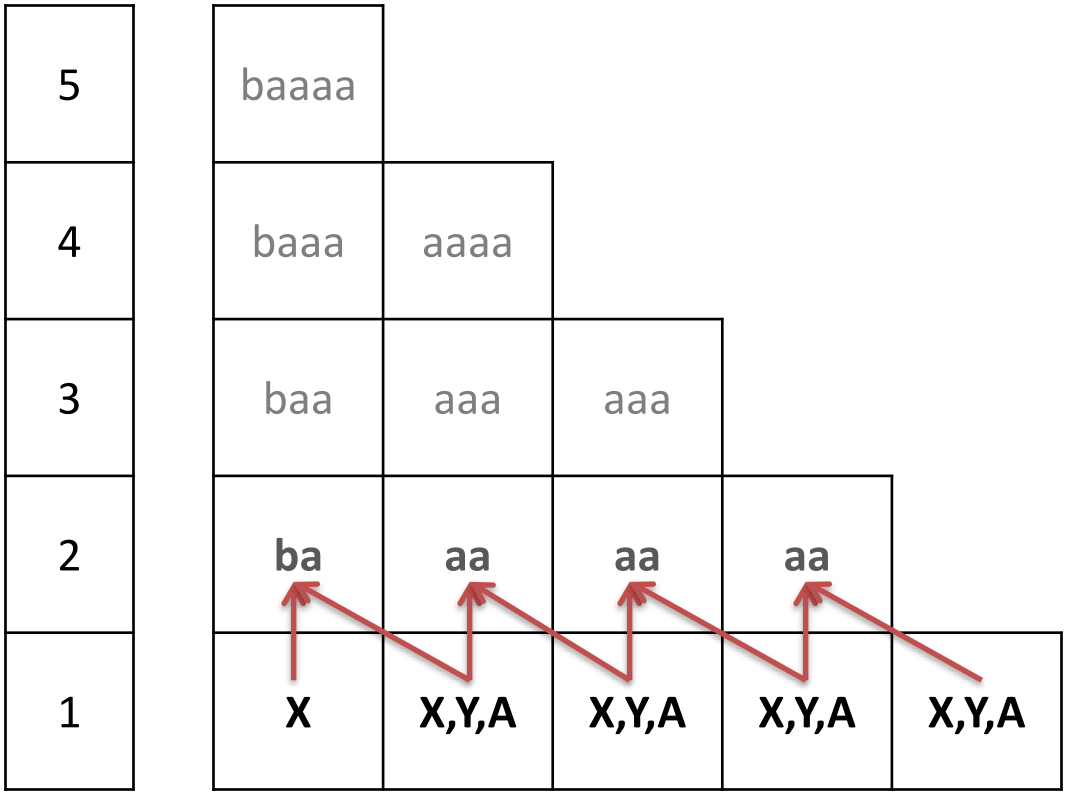
5	baaaa				
4	baaa	aaaa			
3	baa	aaa	aaa		
2	ba	aa	aa	aa	
1	**X**	**X,Y,A**	**X,Y,A**	**X,Y,A**	**X,Y,A**

Substrings of length 1

Input Sentence	b	a	a	a	a

$$S \rightarrow XY$$
$$X \rightarrow XA|a|b$$
$$Y \rightarrow AY|a$$
$$A \rightarrow a$$

Substrings of length 2



Input Sentence

b	a	a	a	a

$$X.(X, Y, A) = \{XX \cup XY \cup XA\}$$

$$S \rightarrow XY$$
$$X \rightarrow XA|a|b$$
$$Y \rightarrow AY|a$$
$$A \rightarrow a$$

Substrings of length 2

5	baaaa				
4	baaa	aaaa			
3	baa	aaa	aaa		
2	**S,X**	**aa**	**aa**	**aa**	
1	**X**	**X,Y,A**	**X,Y,A**	**X,Y,A**	**X,Y,A**

Input Sentence	b	a	a	a	a

$$(X, Y, A).(X, Y, A) = \{XX \cup XY \cup XA \cup YX \cup YY \cup YA \cup AX \cup AY \cup AA\}$$

$$S \rightarrow XY$$
$$X \rightarrow XA|a|b$$
$$Y \rightarrow AY|a$$
$$A \rightarrow a$$

Substrings of length 2

5	baaaa				
4	baaa	aaaa			
3	baa	aaa	aaa		
2	**S,X**	**S,X,Y**	**aa**	**aa**	
1	**X**	**X,Y,A**	**X,Y,A**	**X,Y,A**	**X,Y,A**
Input Sentence	b	a	a	a	a

$$S \rightarrow XY$$
$$X \rightarrow XA|a|b$$
$$Y \rightarrow AY|a$$
$$A \rightarrow a$$

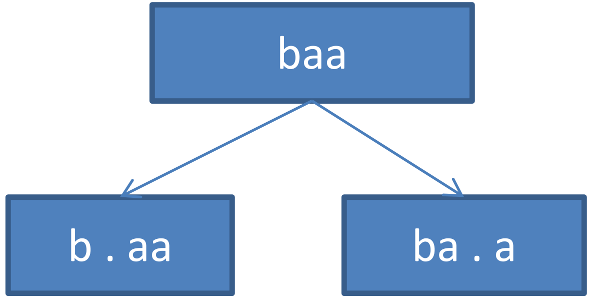
Substrings of length 2

5	baaaa				
4	baaa	aaaa			
3	baa	aaa	aaa		
2	**S,X**	**S,X,Y**	**S,X,Y**	**S,X,Y**	
1	**X**	**X,Y,A**	**X,Y,A**	**X,Y,A**	**X,Y,A**
Input Sentence	b	a	a	a	a

$$S \rightarrow XY$$
$$X \rightarrow XA|a|b$$
$$Y \rightarrow AY|a$$
$$A \rightarrow a$$

Substrings of length 3

5	baaaa				
4	baaa	aaaa			
3	baa	aaa	aaa		
2	S,X	S,X,Y	S,X,Y	S,X,Y	
1	X	X,Y,A	X,Y,A	X,Y,A	X,Y,A
Input Sentence	b	a	a	a	a



$$S \rightarrow XY$$
$$X \rightarrow XA \mid a \mid b$$
$$Y \rightarrow AY \mid a$$
$$A \rightarrow a$$

Substrings of length 3

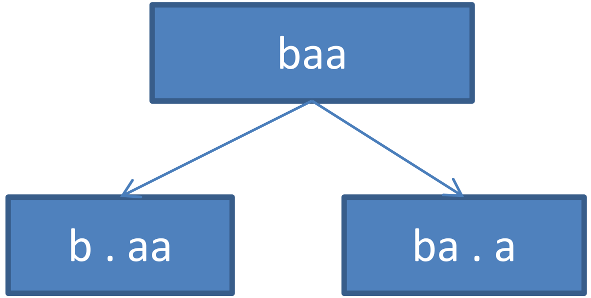
5	baaaa				
4	baaa	aaaa			
3	**S**	aaa	aaa		
2	**S,X**	**S,X,Y**	**S,X,Y**	**S,X,Y**	
1	**X**	**X,Y,A**	**X,Y,A**	**X,Y,A**	**X,Y,A**
Input Sentence	b	a	a	a	a

$$S \to XY$$
$$X \to XA \,|\, a \,|\, b$$
$$Y \to AY \,|\, a$$
$$A \to a$$

Substrings of length 3

5	baaaa				
4	baaa	aaaa			
3	**S,X**	aaa	aaa		
2	**S,X**	**S,X,Y**	**S,X,Y**	**S,X,Y**	
1	**X**	**X,Y,A**	**X,Y,A**	**X,Y,A**	**X,Y,A**
Input Sentence	b	a	a	a	a

$$S \rightarrow XY$$
$$X \rightarrow XA|a|b$$
$$Y \rightarrow AY|a$$
$$A \rightarrow a$$



Substrings of length 3

5	baaaa				
4	baaa	aaaa			
3	**S,X**	aaa	aaa		
2	**S,X**	**S,X,Y**	**S,X,Y**	**S,X,Y**	
1	**X**	**X,Y,A**	**X,Y,A**	**X,Y,A**	**X,Y,A**

Input Sentence	b	a	a	a	a

$$S \rightarrow XY$$
$$X \rightarrow XA|a|b$$
$$Y \rightarrow AY|a$$
$$A \rightarrow a$$

Substrings of length 3

5	baaaa				
4	baaa	aaaa			
3	**S,X**	aaa	aaa		
2	**S,X**	**S,X,Y**	**S,X,Y**	**S,X,Y**	
1	**X**	**X,Y,A**	**X,Y,A**	**X,Y,A**	**X,Y,A**

Input Sentence

b	a	a	a	a

$$S \rightarrow XY$$
$$X \rightarrow XA|a|b$$
$$Y \rightarrow AY|a$$
$$A \rightarrow a$$

Substrings of length 3

5	baaaa				
4	baaa	aaaa			
3	**S,X**	**S,X,Y**	aaa		
2	**S,X**	**S,X,Y**	**S,X,Y**	**S,X,Y**	
1	**X**	**X,Y,A**	**X,Y,A**	**X,Y,A**	**X,Y,A**
Input Sentence	b	a	a	a	a

$$S \rightarrow XY$$
$$X \rightarrow XA|a|b$$
$$Y \rightarrow AY|a$$
$$A \rightarrow a$$

Substrings of length 3

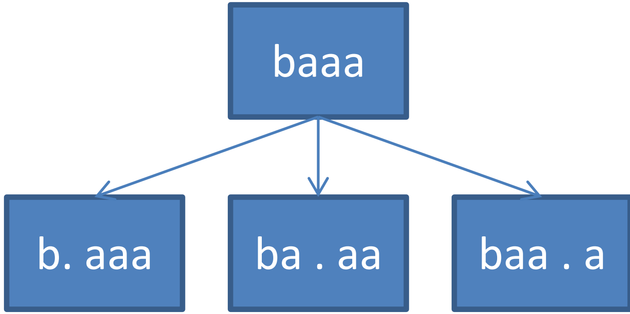
5	baaaa				
4	baaa	aaaa			
3	**S,X**	**S,X,Y**	**S,X,Y**		
2	**S,X**	**S,X,Y**	**S,X,Y**	**S,X,Y**	
1	**X**	**X,Y,A**	**X,Y,A**	**X,Y,A**	**X,Y,A**
Input Sentence	b	a	a	a	a

Substrings of length 4

$$S \rightarrow XY$$
$$X \rightarrow XA|a|b$$
$$Y \rightarrow AY|a$$
$$A \rightarrow a$$

5	baaaa				
4	**S,X,Y**	aaaa			
3	**S,X**	**S,X,Y**	**S,X,Y**		
2	**S,X**	**S,X,Y**	**S,X,Y**	**S,X,Y**	
1	**X**	**X,Y,A**	**X,Y,A**	**X,Y,A**	**X,Y,A**

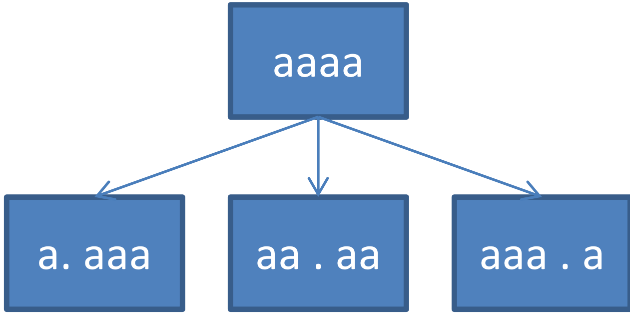
Input Sentence	b	a	a	a	a



Substrings of length 4

$$
\begin{gathered}
S \rightarrow XY \\
X \rightarrow XA|a|b \\
Y \rightarrow AY|a \\
A \rightarrow a
\end{gathered}
$$

5	baaaa				
4	S,X,Y	S,X,Y			
3	S,X	S,X,Y	S,X,Y		
2	S,X	S,X,Y	S,X,Y	S,X,Y	
1	X	X,Y,A	X,Y,A	X,Y,A	X,Y,A
Input Sentence	b	a	a	a	a



Substrings of length 5

$$S \rightarrow XY$$
$$X \rightarrow XA|a|b$$
$$Y \rightarrow AY|a$$
$$A \rightarrow a$$

5	**S,X**				
4	**S,X,Y**	**S,X,Y**			
3	**S,X**	**S,X,Y**	**S,X,Y**		
2	**S,X**	**S,X,Y**	**S,X,Y**	**S,X,Y**	
1	**X**	**X,Y,A**	**X,Y,A**	**X,Y,A**	**X,Y,A**
Input Sentence	b	a	a	a	a

Sentence can be generated from S as well as from X.

5	**S,X**				
4	**S,X,Y**	**S,X,Y**			
3	**S,X**	**S,X,Y**	**S,X,Y**		
2	**S,X**	**S,X,Y**	**S,X,Y**	**S,X,Y**	
1	**X**	**X,Y,A**	**X,Y,A**	**X,Y,A**	**X,Y,A**
	b	a	a	a	a

Complexity…?

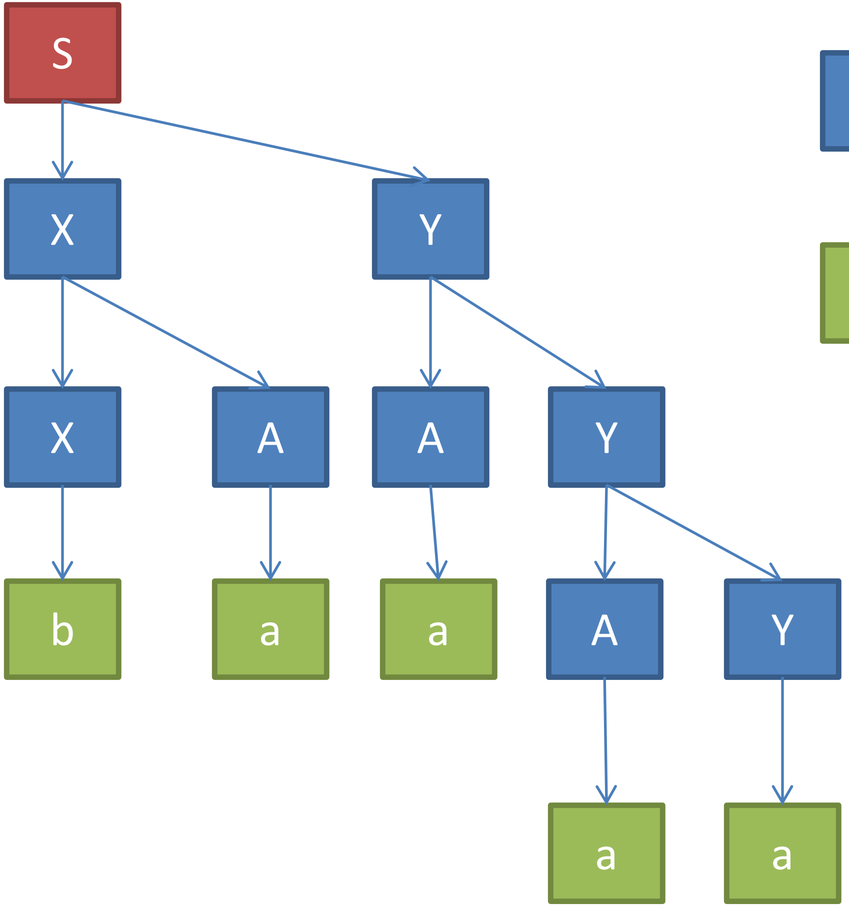
Can You generate a parse tree…?

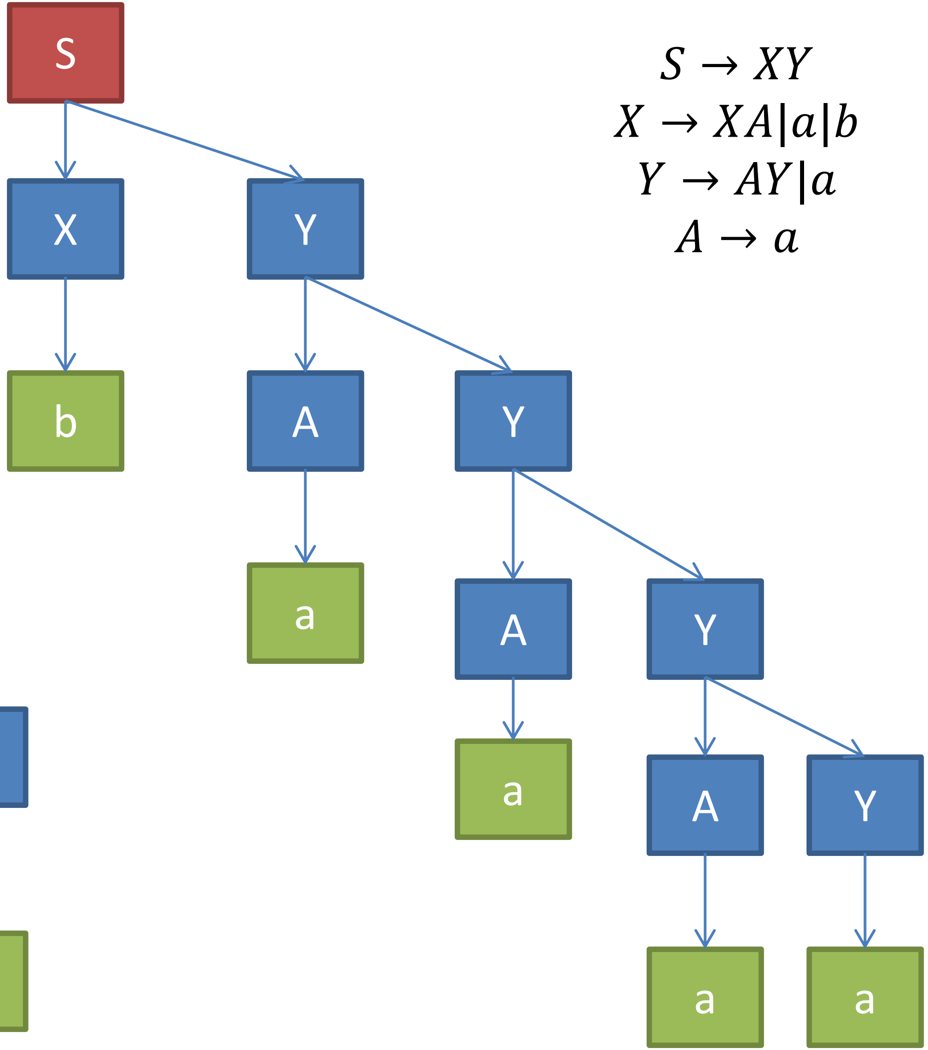
$$S \rightarrow XY$$
$$X \rightarrow XA|a|b$$
$$Y \rightarrow AY|a$$
$$A \rightarrow a$$

Maintain Back pointers!!!

5	**S**,X				
4	S,X,Y	S,X,**Y**			
3	S,X	S,X,Y	S,X,**Y**		
2	S,X	S,X,Y	S,X,Y	S,X,**Y**	
1	**X**	X,Y,**A**	X,Y,**A**	X,Y,**A**	X,Y,**A**
Input Sentence	b	a	a	a	a

Wait, did we get a different parse tree....?





$$S \rightarrow XY$$
$$X \rightarrow XA|a|b$$
$$Y \rightarrow AY|a$$
$$A \rightarrow a$$

Ambiguity

Context Free Grammar, G

T_G: set e set of all possible left-most derivations (parse trees) under the grammar G.
s: a given sentence

Define,

$$T_G(s) = \{t: t \in T_G, yield(t) = s\}$$

$$s \in L(G) \Leftrightarrow |T_G(s)| > 0$$

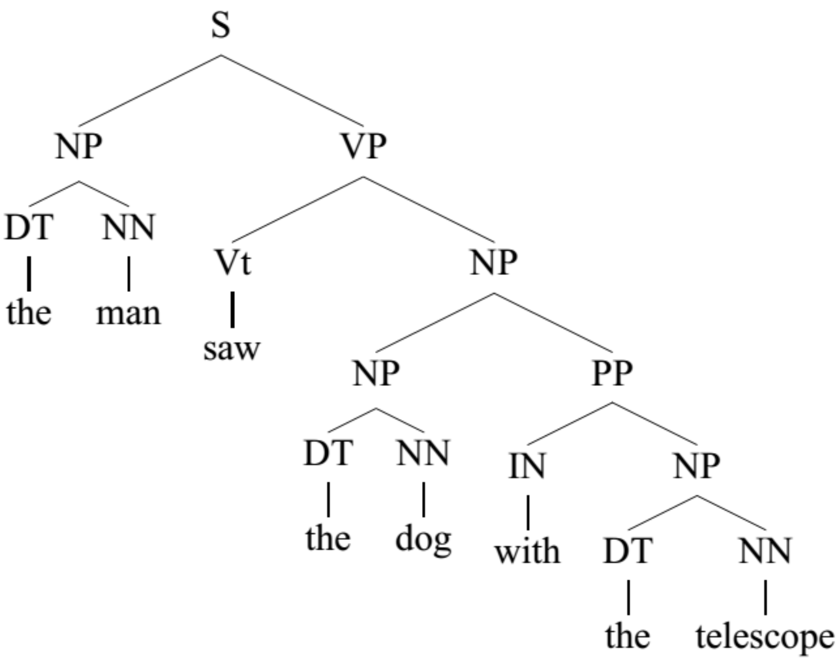
$$s \text{ is ambiguous} \Leftrightarrow |T_G(s)| > 1$$

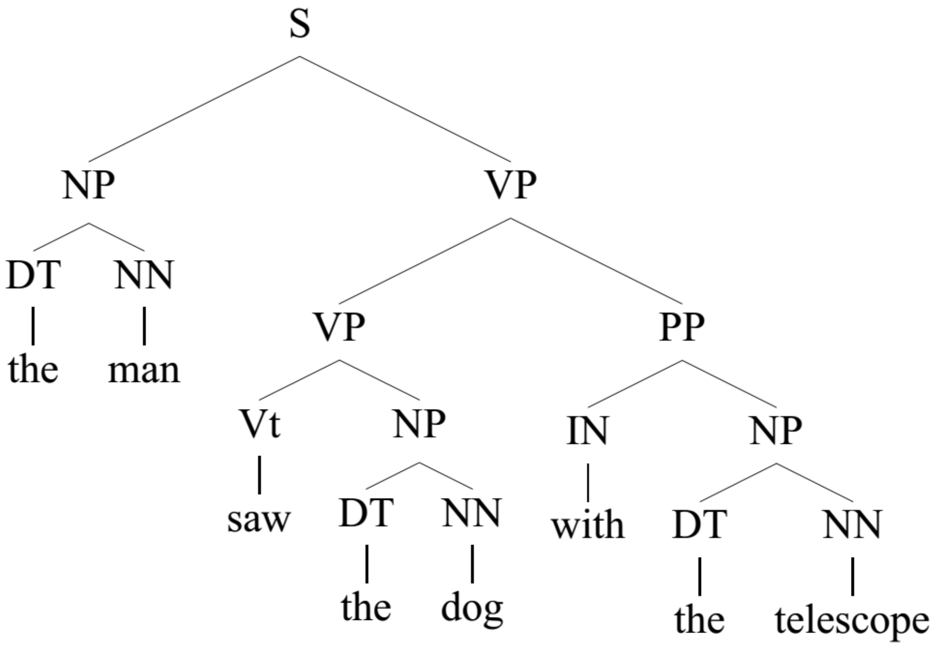
Natural Language Grammar

Non-terminals
S = sentence
VP = verb phrase
NP = noun phrase
PP = prepositional phrase
DT = determiner
Vi = intransitive verb
Vt = transitive verb
NN = noun
IN = preposition.

S	$\rightarrow$	NP	VP
VP	$\rightarrow$	Vi	
VP	$\rightarrow$	Vt	NP
VP	$\rightarrow$	VP	PP
NP	$\rightarrow$	DT	NN
NP	$\rightarrow$	NP	PP
PP	$\rightarrow$	IN	NP

Vi	$\rightarrow$	sleeps
Vt	$\rightarrow$	saw
NN	$\rightarrow$	man
NN	$\rightarrow$	woman
NN	$\rightarrow$	telescope
NN	$\rightarrow$	dog
DT	$\rightarrow$	the
IN	$\rightarrow$	with
IN	$\rightarrow$	in





the man saw the dog with the telescope

Which one is preferred over the other …?

Probabilistic Context Free Grammar

$$G_P = (N, \Sigma, S, R, \boldsymbol{q})$$

Non-terminals

Terminals

Start symbol

Rules

Something new…!

Something New

$$\forall\ \alpha \to \beta \in R$$

$$q(\alpha \to \beta) = P(\alpha \to \beta|\alpha):$$

Probability of choosing rule **α → β** *in a left-most derivation, given that the non-terminal being expanded is* α.

Let’s add probability constraints:

$$q(\alpha \to \beta) \geq 0$$

$$\sum_{\alpha \to \beta \epsilon R, \alpha = X} q(\alpha \to \beta) = 1$$

Coming back to Natural language

S	→	NP	VP	1.0
VP	→	Vi		0.3
VP	→	Vt	NP	0.5
VP	→	VP	PP	0.2
NP	→	DT	NN	0.8
NP	→	NP	PP	0.2
PP	→	IN	NP	1.0

= 1

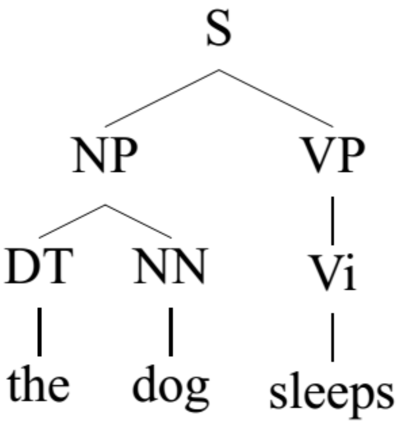
Vi	→	sleeps	1.0
Vt	→	saw	1.0
NN	→	man	0.1
NN	→	woman	0.1
NN	→	telescope	0.3
NN	→	dog	0.5
DT	→	the	1.0
IN	→	with	0.6
IN	→	in	0.4

= 1

Lets come back to ***which parse tree is better…?***

We need something to measure to compare two different parse trees.
Let’s define that measure,

$$p(t) = \prod_{i=1}^{n} q(\alpha_i \rightarrow \beta_i)$$



Assign a score

$$\begin{aligned} p(t) \quad = \quad & q(\texttt{S} \rightarrow \texttt{NP}\ \ \texttt{VP}) \times q(\texttt{NP} \rightarrow \texttt{DT}\ \ \texttt{NN}) \\ & \times q(\texttt{DT} \rightarrow \texttt{the}) \times q(\texttt{NN} \rightarrow \texttt{dog}) \times \\ & q(\texttt{VP} \rightarrow \texttt{Vi}) \times q(\texttt{Vi} \rightarrow \texttt{sleeps}) \end{aligned}$$

Let’s make a parse tree probabilistically….!

- Define $s_1 = S, i = 1$.
- While s_i contains at least one non-terminal:
  - Find the left-most non-terminal in s_i, call this X.
  - Choose one of the rules of the form $X \rightarrow \beta$ from the distribution $q(X \rightarrow \beta)$.
  - Create s_{i+1} by replacing the left-most X in s_i by β.
  - Set $i = i + 1$.

In the end we reach a tree, ***t*** with score ***p(t).***

- Define $s_1 = S$, $i = 1$.
- While s_i contains at least one non-terminal:
  - Find the left-most non-terminal in s_i, call this X.
  - Choose one of the rules of the form $X \rightarrow \beta$ from the distribution $q(X \rightarrow \beta)$.
  - Create s_{i+1} by replacing the left-most X in s_i by β.
  - Set $i = i + 1$.

How do we choose a rule?

$$t = arg \max_{t' \in T_G(sentence)} p(t')$$

valid parse tree

Can we do it using CYK algorithm?

Obviously we can, that’s why I asked…!!!

$S \rightarrow XY$	1.0
$X \rightarrow XA$	0.5
$X \rightarrow a$	0.2
$X \rightarrow b$	0.3
$Y \rightarrow AY$	0.2
$Y \rightarrow a$	0.8
$A \rightarrow a$	1.0

Can you find ***score*** of most probable parse tree of ***baaaa***?

$$\max_{t' \in T_G(sentence)} p(t')$$

$S \rightarrow XY$	1.0
$X \rightarrow XA$	0.5
$X \rightarrow a$	0.2
$X \rightarrow b$	0.3
$Y \rightarrow AY$	0.2
$Y \rightarrow a$	0.8
$A \rightarrow a$	1.0

5	baaaa				
4	baaa	aaaa			
3	baa	aaa	aaa		
2	ba	aa	aa	aa	
1	b	a	a	a	a
	b	a	a	a	a

Keep track of probabilities also…!

$S \rightarrow XY$	1.0
$X \rightarrow XA$	0.5
$X \rightarrow a$	0.2
$X \rightarrow b$	0.3
$Y \rightarrow AY$	0.2
$Y \rightarrow a$	0.8
$A \rightarrow a$	1.0

5	baaaa				
4	baaa	aaaa			
3	baa	aaa	aaa		
2	ba	aa	aa	aa	
1	**X: 0.3**	**X:0.2 Y:0.8 A:1.0**	**X:0.2 Y:0.8 A:1.0**	**X:0.2 Y:0.8 A:1.0**	**X:0.2 Y:0.8 A:1.0**
Input Sentence	b	a	a	a	a

$$p(t|S) = q(S \to XY) * p(t_1|X) * p(t_2|Y)$$
$$= 1.0 * 0.3 * 0.8$$

$S \to XY$	**1.0**
$X \to XA$	0.5
$X \to a$	0.2
$X \to b$	0.3
$Y \to AY$	0.2
$Y \to a$	0.8
$A \to a$	1.0

5	baaaa				
4	baaa	aaaa			
3	baa	aaa	aaa		
2	**S->XY:.24** **X->XA:.15**	aa	aa	aa	
1	**X->b:0.3**	**X->a:0.2** **Y->a:0.8** **A->a:1.0**	**X->a:0.2** **Y->a:0.8** **A->a:1.0**	**X->a:0.2** **Y->a:0.8** **A->a:1.0**	**X->a:0.2** **Y->a:0.8** **A->a:1.0**

Input Sentence	b	a	a	a	a

$S \to XY$	1.0
$X \to XA$	**0.5**
$X \to a$	0.2
$X \to b$	0.3
$Y \to AY$	0.2
$Y \to a$	0.8
$A \to a$	1.0

$$p(t|X) = q(X \to XA) * p(t_1|X) * p(t_2|A)$$
$$= 0.5 * 0.3 * 1.0$$

5	baaaa				
4	baaa	aaaa			
3	baa	aaa	aaa		
2	**S->XY:0.24** **X->XA:0.15**	aa	aa	aa	
1	**X->b:0.3**	**X->a:0.2** **Y->a:0.8** **A->a:1.0**	**X->a:0.2** **Y->a:0.8** **A->a:1.0**	**X->a:0.2** **Y->a:0.8** **A->a:1.0**	**X->a:0.2** **Y->a:0.8** **A->a:1.0**
Input Sentence	b	a	a	a	a

$S \to XY$	1.0
$X \to XA$	0.5
$X \to a$	0.2
$X \to b$	0.3
$Y \to AY$	**0.2**
$Y \to a$	0.8
$A \to a$	1.0

5	baaaa				
4	baaa	aaaa			
3	baa	aaa	aaa		
2	**S->XY:0.24** **X->XA:0.15**	**S->XY:0.16** **X->XA:0.1** **Y->AY:?**	aa	aa	
1	**X->b:0.3**	**X->a:0.2** **Y->a:0.8** **A->a:1.0**	**X->a:0.2** **Y->a:0.8** **A->a:1.0**	**X->a:0.2** **Y->a:0.8** **A->a:1.0**	**X->a:0.2** **Y->a:0.8** **A->a:1.0**
Input Sentence	b	a	a	a	a

$S \rightarrow XY$	1.0
$X \rightarrow XA$	0.5
$X \rightarrow a$	0.2
$X \rightarrow b$	0.3
$Y \rightarrow AY$	0.2
$Y \rightarrow a$	0.8
$A \rightarrow a$	1.0

5	baaaa				
4	baaa	aaaa			
3	baa	aaa	aaa		
2	S->XY:0.24 X->XA:0.15	S->XY:0.16 X->XA:0.1 Y->AY:0.16	aa	aa	
1	X->b:0.3	X->a:0.2 Y->a:0.8 A->a:1.0	X->a:0.2 Y->a:0.8 A->a:1.0	X->a:0.2 Y->a:0.8 A->a:1.0	X->a:0.2 Y->a:0.8 A->a:1.0
Input Sentence	b	a	a	a	a

$S \rightarrow XY$	1.0
$X \rightarrow XA$	0.5
$X \rightarrow a$	0.2
$X \rightarrow b$	0.3
$Y \rightarrow AY$	0.2
$Y \rightarrow a$	0.8
$A \rightarrow a$	1.0

5	baaaa				
4	baaa	aaaa			
3	baa	aaa	aaa		
2	**S->XY:0.24** **X->XA:0.15**	**S->XY:0.16** **X->XA:0.1** **Y->AY:0.16**	**S->XY:0.16** **X->XA:0.1** **Y->AY:0.16**	**S->XY:0.16** **X->XA:0.1** **Y->AY:0.16**	
1	**X->b:0.3**	**X->a:0.2** **Y->a:0.8** **A->a:1.0**	**X->a:0.2** **Y->a:0.8** **A->a:1.0**	**X->a:0.2** **Y->a:0.8** **A->a:1.0**	**X->a:0.2** **Y->a:0.8** **A->a:1.0**
Input Sentence	b	a	a	a	a

A bit tricky…!

$S \rightarrow XY$	1.0
$X \rightarrow XA$	0.5
$X \rightarrow a$	0.2
$X \rightarrow b$	0.3
$Y \rightarrow AY$	0.2
$Y \rightarrow a$	0.8
$A \rightarrow a$	1.0

5	baaaa				
4	baaa	aaaa			
3	S X	aaa	aaa		
2	S->XY:0.24 X->XA:0.15	S->XY:0.16 X->XA:0.1 Y->AY:0.16	S->XY:0.16 X->XA:0.1 Y->AY:0.16	S->XY:0.16 X->XA:0.1 Y->AY:0.16	
1	X->b:0.3	X->a:0.2 Y->a:0.8 A->a:1.0	X->a:0.2 Y->a:0.8 A->a:1.0	X->a:0.2 Y->a:0.8 A->a:1.0	X->a:0.2 Y->a:0.8 A->a:1.0
Input Sentence	b	a	a	a	a

Lets see where are they coming from?

A bit tricky…!

$1*0.15*0.8=0.12$ $\geq$ $1*0.3*0.16=0.048$

$S \rightarrow XY$	1.0
$X \rightarrow XA$	0.5
$X \rightarrow a$	0.2
$X \rightarrow b$	0.3
$Y \rightarrow AY$	0.2
$Y \rightarrow a$	0.8
$A \rightarrow a$	1.0

5	baaaa				
4	baaa	aaaa			
3	S->XY X	aaa	aaa		
2	S->XY:0.24 X->XA:0.15	S->XY:0.16 X->XA:0.1 Y->AY:0.16	S->XY:0.16 X->XA:0.1 Y->AY:0.16	S->XY:0.16 X->XA:0.1 Y->AY:0.16	
1	X->b:0.3	X->a:0.2 Y->a:0.8 A->a:1.0	X->a:0.2 Y->a:0.8 A->a:1.0	X->a:0.2 Y->a:0.8 A->a:1.0	X->a:0.2 Y->a:0.8 A->a:1.0
Input Sentence	b	a	a	a	a

$S \rightarrow XY$	1.0
$X \rightarrow XA$	0.5
$X \rightarrow a$	0.2
$X \rightarrow b$	0.3
$Y \rightarrow AY$	0.2
$Y \rightarrow a$	0.8
$A \rightarrow a$	1.0

5	baaaa				
4	baaa	aaaa			
3	**S->XY:0.12 X->XA:.075**	aaa	aaa		
2	**S->XY:0.24 X->XA:0.15**	**S->XY:0.16 X->XA:0.1 Y->AY:0.16**	**S->XY:0.16 X->XA:0.1 Y->AY:0.16**	**S->XY:0.16 X->XA:0.1 Y->AY:0.16**	
1	**X->b:0.3**	**X->a:0.2 Y->a:0.8 A->a:1.0**	**X->a:0.2 Y->a:0.8 A->a:1.0**	**X->a:0.2 Y->a:0.8 A->a:1.0**	**X->a:0.2 Y->a:0.8 A->a:1.0**
Input Sentence	b	a	a	a	a

$S \rightarrow XY$	1.0
$X \rightarrow XA$	0.5
$X \rightarrow a$	0.2
$X \rightarrow b$	0.3
$Y \rightarrow AY$	0.2
$Y \rightarrow a$	0.8
$A \rightarrow a$	1.0

5	baaaa				
4	baaa	aaaa			
3	S->XY:0.12 X->XA:.075	S->XY:.08 X->XA:0.05 Y->AY:.032	S->XY:.08 X->XA:0.05 Y->AY:.032		
2	S->XY:0.24 X->XA:0.15	S->XY:0.16 X->XA:0.1 Y->AY:0.16	S->XY:0.16 X->XA:0.1 Y->AY:0.16	S->XY:0.16 X->XA:0.1 Y->AY:0.16	
1	X->b:0.3	X->a:0.2 Y->a:0.8 A->a:1.0	X->a:0.2 Y->a:0.8 A->a:1.0	X->a:0.2 Y->a:0.8 A->a:1.0	X->a:0.2 Y->a:0.8 A->a:1.0
Input Sentence	b	a	a	a	a

$S \rightarrow XY$	1.0
$X \rightarrow XA$	0.5
$X \rightarrow a$	0.2
$X \rightarrow b$	0.3
$Y \rightarrow AY$	0.2
$Y \rightarrow a$	0.8
$A \rightarrow a$	1.0

5	baaaa				
4	S->XY:0.06 X->XA: 0.0375	S->XY:0.04 X->AX:? Y->AY:?			
3	S->XY:0.12 X->XA:.075	S->XY:.08 X->XA:0.05 Y->AY:.032	S->XY:.08 X->XA:0.05 Y->AY:.032		
2	S->XY:0.24 X->XA:0.15	S->XY:0.16 X->XA:0.1 Y->AY:0.16	S->XY:0.16 X->XA:0.1 Y->AY:0.16	S->XY:0.16 X->XA:0.1 Y->AY:0.16	
1	X->b:0.3	X->a:0.2 Y->a:0.8 A->a:1.0	X->a:0.2 Y->a:0.8 A->a:1.0	X->a:0.2 Y->a:0.8 A->a:1.0	X->a:0.2 Y->a:0.8 A->a:1.0
Input Sentence	b	a	a	a	a

$S \rightarrow XY$	1.0
$X \rightarrow XA$	0.5
$X \rightarrow a$	0.2
$X \rightarrow b$	0.3
$Y \rightarrow AY$	0.2
$Y \rightarrow a$	0.8
$A \rightarrow a$	1.0

5	S->XY:0.03				
4	S->XY:0.06 X->XA: 0.0375	S->XY:0.04 X->AX:? Y->AY:?			
3	S->XY:0.12 X->XA:.075	S->XY:.08 X->XA:0.05 Y->AY:.032	S->XY:.08 X->XA:0.05 Y->AY:.032		
2	S->XY:0.24 X->XA:0.15	S->XY:0.16 X->XA:0.1 Y->AY:0.16	S->XY:0.16 X->XA:0.1 Y->AY:0.16	S->XY:0.16 X->XA:0.1 Y->AY:0.16	
1	X->b:0.3	X->a:0.2 Y->a:0.8 A->a:1.0	X->a:0.2 Y->a:0.8 A->a:1.0	X->a:0.2 Y->a:0.8 A->a:1.0	X->a:0.2 Y->a:0.8 A->a:1.0
Input Sentence	b	a	a	a	a

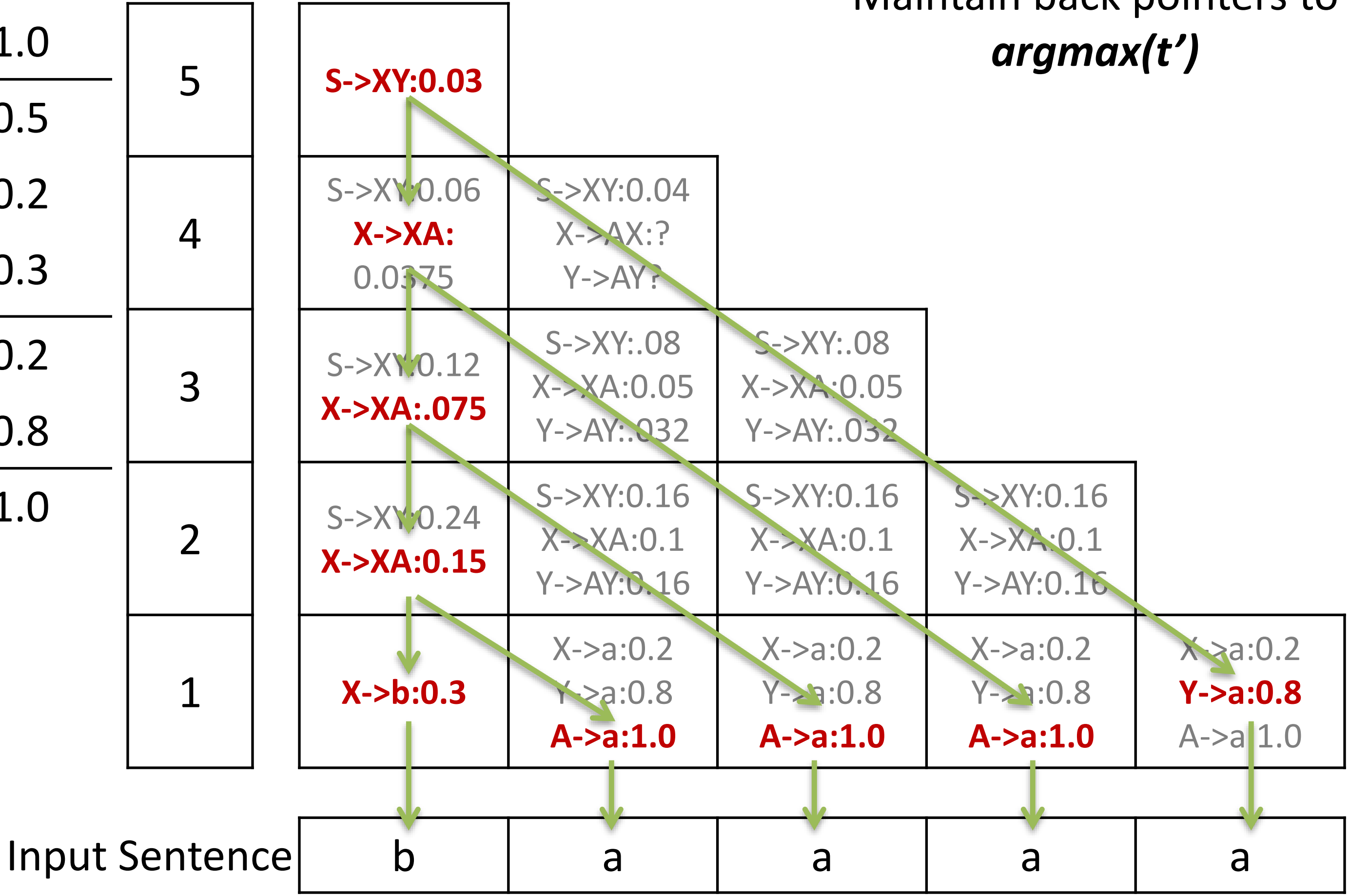
$$\max_{t' \in T_G(sentence)} p(t') = 0.03$$

Can we infer most probable parse tree?

$$\arg\max_{t' \in T_G(sentence)} p(t')$$

Maintain back pointers to ***argmax(t')***

$S \rightarrow XY$	1.0
$X \rightarrow XA$	0.5
$X \rightarrow a$	0.2
$X \rightarrow b$	0.3
$Y \rightarrow AY$	0.2
$Y \rightarrow a$	0.8
$A \rightarrow a$	1.0



Proof of correctness

$$p(t) = q(X \rightarrow YZ) * p(t_1) * p(t_2)$$

At each step, we take $\max p(t)$

It is enough to show that:

1. Tree t_1 , rooted at ***Y,*** has max probability over the words it spans.
2. Tree t_2 rooted at ***Z,*** has maximum probability over the words its spans.

$$s = \{w_1, \ldots wi, \ldots ws, \ldots wj, \ldots wn\}$$

Recursive definition of probability build up!

$$\pi(i,j,X) = \max_{\substack{X \to YZ \epsilon R \\ s \epsilon (i,j)}} q(X \to YZ) * \pi(i,s,Y) * \pi(s+1,j,Z)$$

Highest scoring tree rooted at X.

How do you get ***q…?***

Treebank: text corpus that annotates syntactic sentence structure. E.g., Penn Tree Bank[1].

$$q = \frac{Count(\alpha \to \beta)}{Count(\alpha)}$$

[1] http://www.cis.upenn.edu/~treebank/