Mathematical concepts for computer science

Relations

- Relationships between elements of sets are represented using the structure called a relation, which is just a **subset of the Cartesian product** of the sets.
- Sets of ordered pairs are called **binary relations**.

DEFINITION

- **Let A and B be sets. A binary relation from A to B is a subset of $A \times B$.**

Relations

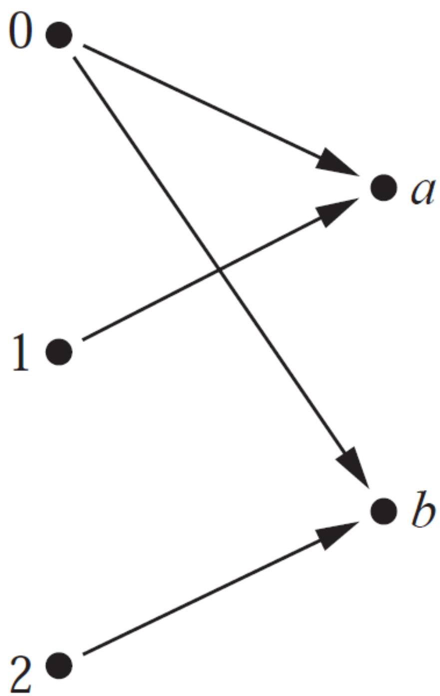
- In other words, a binary relation from A to B is a set R of ordered pairs where the **first element** of each ordered pair comes **from A** and the **second element** comes **from B**.
- **$a \; R \; b$** to denote that **$(a, b) \in R$** and $a \not R b$ to denote that **$(a, b) \notin R$**. Moreover, when (a, b) belongs to R, **a** is said to be **related to b by R.**

Relations

- Let **A** be the **set of students** in your school, and let **B** be the **set of courses**. Let **R** be the **relation** that consists of those pairs **(a, b)**, where a is a student enrolled in course b.
- For instance, if Vishnu and Arjun are enrolled in **CSC3101**, the pairs (Vishnu, **CSC3101**) and (Arjun, **CSC3101**) belong to *R.*
- However, if Vivek is not enrolled in **CSC3101**, then the pair (Vivek, **CSC3101**) is not in R.

Relations

- Let A = {0, 1, 2} and B = {a, b}. Then {(0, a), (0, b), (1, a), (2, b)} is a relation from A to B.



R	a	b
0	×	×
1	×	
2		×

Relations on a Set

- A **relation on a set A** is a **relation from A to A**.
- In other words, a **relation on a set A** is a **subset of A × A**.
- Let **A** be the set **{1, 2, 3, 4}**. Which ordered pairs are in the relation **R = {(a, b) | a divides b}**?

Relations on a Set

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- Let **A** be the set **{1, 2, 3, 4}**. Which ordered pairs are in the relation **R = {(a, b) | a divides b}**?

R = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 4), (3, 3), (4, 4)}.

Relations on a Set

- Consider these relations on the set of integers:
  - $R1 = \{(a, b) \mid a \le b\}$,
  - $R2 = \{(a, b) \mid a > b\}$,
  - $R3 = \{(a, b) \mid a = b \text{ or } a = -b\}$,
  - $R4 = \{(a, b) \mid a = b\}$,
  - $R5 = \{(a, b) \mid a = b + 1\}$,
  - $R6 = \{(a, b) \mid a + b \le 3\}$.

Which of these relations contain each of the pairs

(1, 1), (1, 2), (2, 1), (1,–1), and (2, 2)?

Relations on a Set

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  - $R4 = \{(a, b) \mid a = b\}$,
  - $R5 = \{(a, b) \mid a = b + 1\}$,
  - $R6 = \{(a, b) \mid a + b \leq 3\}$.
- (1, 1) is in R1, R3, R4, and R6;
- (1, 2) is in R1 and R6;
- (2, 1) is in R2, R5, R6;
- (1,−1) is in R2, R3, and R6;
- (2, 2) is in R1, R3, and R4.

Properties of Relations

- A relation R on a set A is called **reflexive** if (a, a) ∈ R for every element a ∈ A.
- Consider the following relations on {1*, 2, 3, 4}:*
- **R1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)},**
- **R2 = {(1, 1), (1, 2), (2, 1)},**
- **R3 = {(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)},**
- **R4 = {(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)},**
- **R5 = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)},**
- **R6 = {(3, 4)}.**

Which of these relations are reflexive?

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- Consider the following relations on {1, *2, 3, 4}:*
- **R1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)},**
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- **R4 = {(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)},**
- **R5 = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)},**
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R3 and R5 are reflexive

Properties of Relations

- Is the “divides” relation on the set of positive integers reflexive?

Properties of Relations

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- **Because a | a whenever a is a positive integer, the “divides” relation is reflexive.**

- Is the “divides” relation on the set of all integers reflexive?

Properties of Relations

- Is the “divides” relation on the set of positive integers reflexive?
- **Because a | a whenever a is a positive integer, the “divides” relation is reflexive.**

- Is the “divides” relation on the set of all integers reflexive?
- **not reflexive because by definition 0 does not divide 0.**

Properties of Relations

- A relation R on a set A is called **symmetric** if **(b, a) ∈ R** whenever **(a, b) ∈ R**, for all a, b ∈ A.
- A relation R on a set A such that for all a, b ∈ A, if (a, b) ∈ R but (b, a) ∉ R, or vice versa is called **anti-symmetric**.

Properties of Relations

- Consider the following relations on {1*, 2, 3, 4}:*
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- **R2 = {(1, 1), (1, 2), (2, 1)},**
- **R3 = {(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)},**
- **R4 = {(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)},**
- **R5 = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)},**
- **R6 = {(3, 4)}.**

Which of these relations are **symmetric**?

Properties of Relations

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- **R6 = {(3, 4)}.**

R2 and R3 are symmetric

Properties of Relations

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- **R1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)},**
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- **R6 = {(3, 4)}.**

R2 and R3 are symmetric

Properties of Relations

- Consider these relations on the set of integers:
  - $R1 = \{(a, b) \mid a \leq b\}$,
  - $R2 = \{(a, b) \mid a > b\}$,
  - $R3 = \{(a, b) \mid a = b \text{ or } a = -b\}$,
  - $R4 = \{(a, b) \mid a = b\}$,
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  - $R6 = \{(a, b) \mid a + b \leq 3\}$.

Which of the relations are symmetric and which are anti-symmetric?

Properties of Relations

- Consider these relations on the set of integers:
- $R1 = \{(a, b) \mid a \le b\}$,
  - $R2 = \{(a, b) \mid a > b\}$,
  - **R3** $= \{(a, b) \mid a = b \text{ or } a = -b\}$, **=> $a = b$ or $a = -b$, then $b = a$ or $b = -a$.**
  - **R4** $= \{(a, b) \mid a = b\}$, **=> $a = b$ implies that $b = a$.**
  - $R5 = \{(a, b) \mid a = b + 1\}$,
  - **R6** $= \{(a, b) \mid a + b \le 3\}$.**=> $a + b \le 3$ implies that $b + a \le 3$.**

The relations **R3, R4, and R6** are **symmetric**.

Properties of Relations

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  - **R1** $= \{(a, b) \mid a \le b\}$,
  - **R2** $= \{(a, b) \mid a > b\}$,
  - R3 $= \{(a, b) \mid a = b \text{ or } a = -b\}$,
  - R4 $= \{(a, b) \mid a = b\}$,
  - **R5** $= \{(a, b) \mid a = b + 1\}$,
  - R6 $= \{(a, b) \mid a + b \le 3\}$.

The relations **R1, R2, and R5** are **antisymmetric**.

Properties of Relations

- A relation R on a set A is called **transitive** if whenever **(a, b) ∈ R and (b, c) ∈ R**, **then (a, c) ∈ R**, for all a, b, c ∈ A.

Properties of Relations

- Consider the following relations on {1*, 2, 3, 4}:*
- **R1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)},**
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Which of these relations are **transitive**?

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R4, R5, and R6 are **transitive**

Properties of Relations

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  - $R1 = \{(a, b) \mid a \leq b\}$,
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Which of the relations are **transitive**?

Properties of Relations

- Consider these relations on the set of integers:
  - $R1 = \{(a, b) \mid a \leq b\}$, **=> a ≤ b and b ≤ c imply that a ≤ c.**
  - $R2 = \{(a, b) \mid a > b\}$, **=> a > b and b > c imply that a > c.**
  - $R3 = \{(a, b) \mid a = b \text{ or } a = -b\}$,**=> a = ±b and b = ±c => a = ±c**
  - $R4 = \{(a, b) \mid a = b\}$,
  - $R5 = \{(a, b) \mid a = b + 1\}$,
  - $R6 = \{(a, b) \mid a + b \leq 3\}$.

Relations **R1, R2, R3, and R4** are **transitive**

Properties of Relations

- Is the “divides” relation on the set of positive integers transitive?

Properties of Relations

- Is the “divides” relation on the set of positive integers transitive?
- Suppose that a divides b and b divides c. Then there are positive integers k and l such that b = ak and c = bl. Hence, c = a(kl), so a divides c.

It follows that this relation is transitive.

Representing Relations Using Matrices

- A relation between finite sets can be represented using a **zero–one matrix**.
- Suppose that R is a relation from A = {a1, a2, . . . , am} to B = {b1, b2, . . . , bn}. The relation R can be represented by the matrix **MR = [mij],** where

$$m_{ij} = \begin{cases} 1 \text{ if } (a_i, b_j) \in R, \\ 0 \text{ if } (a_i, b_j) \notin R. \end{cases}$$

Representing Relations Using Matrices

- Suppose that **A = {1, 2, 3}** and **B = {1, 2}**. Let R be the relation from A to B containing (a, b) if **a ∈ A, b ∈ B, and a > b**. What is the matrix representing R if a1 = 1, a2 = 2, and a3 = 3, and b1 = 1 and b2 = 2?

Representing Relations Using Matrices

- Suppose that **A = {1, 2, 3}** and **B = {1, 2}**. Let R be the relation from A to B containing (a, b) if **a ∈ A, b ∈ B, and a > b**. What is the matrix representing R if a1 = 1, a2 = 2, and a3 = 3, and b1 = 1 and b2 = 2?
- **R = {(2, 1), (3, 1), (3, 2)}**

$$\mathbf{M}_R = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

Representing Relations Using Matrices

- Suppose that the relation R on a set is represented by the matrix. Is R reflexive, symmetric, and/or anti-symmetric?

$$\mathbf{M}_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Representing Relations Using Matrices

- Suppose that the relation R on a set is represented by the matrix. Is R reflexive, symmetric, and/or anti-symmetric?

$$\mathbf{M}_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

- Because all the **diagonal elements** of this matrix are equal to **1**,R is **reflexive**. Moreover, because MR is **symmetric**, it follows that R is **symmetric**.

Representing Relations Using Matrices

- Suppose that the relations R1 and R2 on a set A are represented by the matrices. What are the matrices representing **R1 ∪ R2** and **R1 ∩ R2**?

$$\mathbf{M}_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \mathbf{M}_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Representing Relations Using Matrices

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$$\mathbf{M}_{R_1 \cup R_2} = \mathbf{M}_{R_1} \vee \mathbf{M}_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad \mathbf{M}_{R_1 \cap R_2} = \mathbf{M}_{R_1} \wedge \mathbf{M}_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Representing Relations Using Matrices

- Let the zero-one matrices for S°R, R, and S be **M (S°R)=[tij], M(R)=[rij], and M(S)=[sij], respectively** (these matrices have sizes m × p, m × n, and n × p, respectively).
- The ordered pair (ai, cj) belongs to S°R if and only if there is an element bk such that (ai, bk) belongs to R and (bk, cj) belongs to S.
- It follows that **tij = 1** if and only if **rik = skj = 1** for some **k**.

Representing Relations Using Matrices

- Find the matrix representing the relations S°R, where the matrices representing R and S are

$$\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{M}_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\mathbf{M}_{S \circ R} = \mathbf{M}_R \odot \mathbf{M}_S = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Representing Relations Using Matrices

- Find the matrix representing the relation R^2, where the matrix representing R is

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$$\mathbf{M}_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\mathbf{M}_{R^2} = \mathbf{M}_R^{[2]} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Closures of Relations

- let R be a relation on a set A. R may or may not have some property **P, such as** reflexivity, symmetry, or transitivity. If there is a relation S with property **P containing R such** that S is a subset of every relation with property **P containing R, then S is called the closure** of R with respect to **P.**

Closures of Relations

- The relation **R = {(1, 1), (1, 2), (2, 1), (3, 2)}** on the set **A = {1, 2, 3}** is not reflexive.
- How can we produce a reflexive relation containing R that is as small as possible?
- This can be done by adding **(2, 2) and (3, 3) to R**, because these are the only pairs of the form (a, a) that are not in R.
- Clearly, this new relation contains R. Furthermore, **any reflexive relation that contains R must also contain (2, 2) and (3, 3)**. Because this relation contains R, is reflexive, and is contained within every reflexive relation that contains R, it is called the **reflexive closure of R.**

Closures of Relations

- What is the reflexive closure of the relation R = {(a, b) | a < b} on the set of integers?

$$R \cup \Delta = \{(a, b) \mid a < b\} \cup \{(a, a) \mid a \in \mathbf{Z}\} = \{(a, b) \mid a \leq b\}$$

Closures of Relations

- The relation $\{(1, 1), (1, 2), (2, 2), (2, 3), (3, 1), (3, 2)\}$ on $\{1, 2, 3\}$ is not symmetric.
- How can we produce a symmetric relation that is as small as possible and contains R?

Closures of Relations

- The relation $\{(1, 1), (1, 2), (2, 2), (2, 3), (3, 1), (3, 2)\}$ on $\{1, 2, 3\}$ is not symmetric.
- How can we produce a symmetric relation that is as small as possible and contains R?
- To do this, we need only add (2, 1) and (1, 3), because these are the only pairs of the form (b, a) with $(a, b) \in R$ that are not in R.

Symmetric Closures of Relations

- The relation {(1, 1), (1, 2), (2, 2), (2, 3), (3, 1), (3, 2)} on {1, 2, 3} is not symmetric.
- How can we produce a symmetric relation that is as small as possible and contains R?
- To do this, we need only add (2, 1) and (1, 3), because these are the only pairs of the form (b, a) with (a, b) ∈ R that are not in R.

Symmetric Closures of Relations

- The relation $\{(1, 1), (1, 2), (2, 2), (2, 3), (3, 1), (3, 2)\}$ on $\{1, 2, 3\}$ is not symmetric.
- How can we produce a symmetric relation that is as small as possible and contains R?
- To do this, we need only add (2, 1) and (1, 3), because these are the only pairs of the form (b, a) with $(a, b) \in R$ that are not in R.
- The symmetric closure of a relation **R** can be constructed by **adding all ordered pairs of the form (b, a),** where (a, b) is in the relation, that are not already present in R.

Symmetric Closures of Relations

- The symmetric closure of a relation can be constructed by taking the union of a relation with its that is, RUR^{-1} is the symmetric closure of R, where

$$R^{-1} = \{(b, a) \mid (a, b) \in R\}.$$

Symmetric
Closures of Relations

- What is the symmetric closure of the relation

$R = \{(a, b) \mid a > b\}$ on the set of positive integers?

Symmetric Closures of Relations

- What is the symmetric closure of the relation

R = {(a, b) | a > b} on the set of positive integers?

$$R \cup R^{-1} = \{(a, b) \mid a > b\} \cup \{(b, a) \mid a > b\} = \{(a, b) \mid a \neq b\}$$

Transitive Closures

Let R be a relation on a set A. The *connectivity relation* R^* consists of the pairs (a, b) such that there is a path of length at least one from a to b in R.

The transitive closure of a relation R equals the connectivity relation R^*.

Let $\mathbf{M}_R$ be the zero–one matrix of the relation R on a set with n elements. Then the zero–one matrix of the transitive closure R^* is

$$\mathbf{M}_{R^*} = \mathbf{M}_R \vee \mathbf{M}_R^{[2]} \vee \mathbf{M}_R^{[3]} \vee \cdots \vee \mathbf{M}_R^{[n]}.$$

Transitive Closures

Find the zero–one matrix of the transitive closure of the relation R where

$$\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\mathbf{M}_{R^*} = \mathbf{M}_R \vee \mathbf{M}_R^{[2]} \vee \mathbf{M}_R^{[3]}$$

Transitive Closures

$$\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\mathbf{M}_R^{[2]} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \qquad \mathbf{M}_R^{[3]} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\mathbf{M}_{R^*} = \mathbf{M}_R \vee \mathbf{M}_R^{[2]} \vee \mathbf{M}_R^{[3]}$$

Transitive Closures

$$\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\mathbf{M}_{R^*} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \vee \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \vee \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\mathbf{M}_{R^*} = \mathbf{M}_R \vee \mathbf{M}_R^{[2]} \vee \mathbf{M}_R^{[3]}$$

Transitive Closures

ALGORITHM 1 A Procedure for Computing the Transitive Closure.

procedure *transitive closure* ($\mathbf{M}_R$: zero–one $n \times n$ matrix)
$\mathbf{A} := \mathbf{M}_R$
$\mathbf{B} := \mathbf{A}$
for $i := 2$ **to** n
    $\mathbf{A} := \mathbf{A} \odot \mathbf{M}_R$
    $\mathbf{B} := \mathbf{B} \vee \mathbf{A}$
return $\mathbf{B}$ {$\mathbf{B}$ is the zero–one matrix for R^*}

Transitive Closures

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return $\mathbf{B}${$\mathbf{B}$ is the zero–one matrix for R^*}

Reference

- **Rosen, Kenneth H., and Kamala Krithivasan. Discrete mathematics and its applications: with combinatorics and graph theory. Tata McGraw-Hill Education, 2016.**