

Risk and Return Characteristics of Listed Private Equity Investments in Germany

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Abstract

This paper aims to investigate the risk and return characteristics for private equity investing in Germany by using the daily returns of listed private equity companies as a proxy. The authors use two different indices of LPE portfolios and Markov Switching model to characterize this relationship. The authors do not find any evidence that LPE as proxy for private equity would indicate any abnormal returns for the asset class and authors conclude that the estimation of return distribution characteristics does not fully reveal the characteristics of private equity as an asset class.

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Introduction

The importance of private equity (PE) as an alternative investment class has significantly increased over the years. According to Prequin, over \$3.2 trillion have been raised globally for PE investment (Brown, C., & Kräussl, R., 2012). Early success of the industry has led to a rapid growth of from \$20 billion in 1990 to \$500 billion in the US alone (Jegadeesh, N., Kräussl R. & Pollet, J., 2009). Even though the traditional form of PE investments through limited partnerships (LPs) is still the most common, over the last few years there has been a significant increase in the number of PE firms interested in public listing. Listed private equity (LPE) companies have existed in Europe already since 1940s, yet only recently have they surged in popularity.

Even despite the rapid growth of the industry the characteristics of risk and return for PE investments are not very well understood (Kräussl R., Jegadeesh, N. & Pollet, J., 2009). Various studies indicate opposite conclusions regarding the comparison between market and PE performance.

For the purpose of research, there are various factors that make risk and return performance estimation problematic. The most common problem for performance estimation in traditional PE investments is lack of availability of interim valuations. Since they are hardly ever traded on secondary markets, lack of market prices and limited liquidity make almost any conventional estimation methods very (Bilo et al., 2005).

For investors, LPE as alternative to traditional PE provides more liquidity, more flexible investment policy, greater accessibility. For the purpose of research, LPE provide more consistent and reliable data available for risk and return profile analysis. Even though the academic literature regarding LPE has is not very developed, a number of researchers such as Bergmann et al. (2010), Cumming, Fleming and Johan (2010), Kräussl et al. (2009), Lahr (2010) and Soderblom (2011) have addressed characteristics such as risks, returns and various evaluation methods of LPE specifically.

The research mostly indicates that even though LPE might provide some advantages, it is also not yet clear whether LPE can be considered as a good proxy for unlisted private equity in terms of return and, more significantly, in terms of risk estimation. The main problem with market prices as a proxy for interim return estimation is inconsistency in terms of information about realised investments for the company. This leads to market prices responding to changes in valuation only when significant events within the company occur.

A common question is existing literature has questioned whether including PE in investment portfolio leads to more beneficial outcomes for the investor. While some research does imply that including PE in the form of LPE leads improved performance of an investor portfolio, the results suffer from all of the problems outlined above and no clear consensus in academic literature does not exist.

All of the most commonly used methods, regardless of whether estimation occur on fund, company or stock market level, face some problems when estimating risk and return characteristics for this asset class. Even though some authors try to tackle these problems by

addressing the issues within the scope of a certain dataset, for example, Fama and French (1993) and Ljungqvist and Richardson (2003), most of existing research papers provide only compared results used across various research papers. As a result, a lack direct comparison between various methods exists.

Additionally, most of the existing researches focuses on US market exclusively, for example, Groh and Gottschalg (2009), DeLong and Magin (2009), or the worldwide market of PE and VC research. The assumptions existing in modern financial theory regarding an existence of free capital flows have mostly been used, yet there is a lack of research regarding the characteristics country specific private equity investments.

This paper aims to contribute to the existing literature in several meaningful ways. Firstly, rather than evaluating risk and return trade-off for private equity funds using a one-sided approach, this paper aims to combine several existing research methods and apply them on the same dataset. Hence, the paper is related to that of (Bilo et al., 2005). In their research, authors investigate the risk and return characteristics of 114 publicly traded private equity vehicles by combining the portfolio of these in three different ways and thus allowing for investigation in various portfolio building strategies. This also allows authors to adjust for several biases such as bid/ask spread, sample selection and thin trading. The authors show that the standard approach for estimating the volatility of LPE tends to be heavily downward biased because of the autocorrelations in LPE returns. They, do, however, find strong evidence that LPE investments earn abnormal returns.

Aigner et al. (2010) are similarly concerned mostly with autocorrelations and heavy tail risk that most research in PE does not take into account. The authors therefore use Markov switching model to capture these characteristics and investigate whether including LPE in portfolio when optimising for risk-adjusted performance is beneficial. Although not strictly conclusive, authors

find that risk averse investors can significantly benefit from using LPE as a proxy for private equity investments because of its liquidity and diversification benefits that it offers.

This paper aims to combine both of these methods and apply them to the same LPE dataset based in Germany. Doing this allows us to account for the variety that can exist when engaging in portfolio optimisation and also take into account problems that (Bilo et al., 2005) account for in their paper. The first research question we therefore aim to answer is *What is the performance of Privaste Equity investments in Germany on risk adjusted basis when using LPE as a proxy for the industry?*

At the same time, this allows us to approximate and estimate the characteristics that the more traditional underlying method of estimating risk-adjusted performance does not take into account. By applying Markov Switching model to the same dataset, we are able to estimate the risk and return characteristics such as skewness and kurtosis of the original distribution. This not only significantly improves our ability to analyse the returns from PE on risk adjusted basis, it also allows us to compare both of the approaches and estimate the effects problems associated (Bilo et al., 2005) approach has on the results. Because of that, we intend to construct the unique risk and return characteristics of PE industry in Germany and the second and answer the following question: *To what extent the performance estimates of Private Equity when using LPE as a proxy are affected by autocorrelation and heavy tail risk?*

Additionally, by focusing uniquely on the PE market in Germany, this paper also provides a valuable insight in country specific PE markets. Not only does it quantify the LPE industry in Germany, it also provides an analysis of country specific investments. Since most research

regarding private equity focuses exclusively only on the US or generally on the global markets, a lack of research concerning investing in country-specific markets exists. Although diversification beyond a specific country is preferable, because of certain biases, such as home bias, transaction costs or lack of accessibility to foreign markets, investors who prefer home-markets exclusively do exist. This does, however, indicate that our results are likely to be biased because of country-specific risk, since we use data only characterising firms and markets in Germany, the same bias influences all of our results.

Data

Because of the industry specifics, acquiring data on PE investments requires access to customized databases with a limited availability for the purpose of academic research. Focusing research on listed PE led to an extended scope of and improvement in the consistency of data available.

For the purpose of identifying listed PE companies in Germany two methods were used. Firstly, DAX subsector Private Equity and Venture Capital index which is the only publically quoted index that tracks PE companies listed on German stock exchanges was decomposed. Secondly, Orbis database was used to identify public companies whose industry was specified as either private equity or venture capital. Since not all of the companies which engage in PE investing were identified on Orbis database as a part of private equity or venture capital industries nor did the index include every single listed company in the industry, cross-listing both of the methods allowed maximizing the number of companies available (Orbis).

After identification, DataStream database, information on companies' websites and annual reports were used to drop the companies whose primary market of focus was not Germany. The final list consisted of 27 companies. To improve the availability of data and also account of survivorship bias, LPE companies which have gone bankrupt or have left public stock exchange are also included in the sample.

Both of the databases were used to acquire raw data. DataStream database was used to acquire daily return quotes for the listed PE companies. Additionally, daily quotes on DAX index returns

were acquired for the purpose of comparison, SnP Germany LM index daily return quotes were acquired as a proxy for market returns on large and medium sized companies in Germany, and Rex 1 year Bond index was used as a proxy for risk-free returns.

Additional data will be acquired from BvK (Metzger, 2014) and EVCA (2013) annual reports, which research and examine the PE and VC industry in Germany and Europe, respectively, for the purpose of comparison between our results and their tracked returns.

Fama and French factors

For the purpose of this first part of the research, Fama and French four-factor daily factors specifically for Germany were used. (Brückner et al. 2014) provide several types of factors with different assumptions, thus allowing us to choose factors that best fit our dataset.

PE industry overview in Germany

According to European Private Equity and Venture Capital Association (EVCA, 2013), the level of private equity investments in Germany is lower than the average level in Europe (0,215 and 0,267 as % of GDP and 0,18 and 0,253 as % of GDP when viewed in terms of size of the market). However, the average investments in Germany seem to higher over the period of 2007 to 2012.

According to Guellmann (2013), a Board Member of BVK, the level of investments suffered a downfall of 70% after the crisis and in 2013 had only managed to recover to 50% of the pre-crisis level.

In one of the annual BkV supported publications German Private Equity Barometer (Metzger, 2014), authors claim that Germany has a favourable position in private equity market despite the slowdown. Even though the investment in the country has fallen, it has stayed above the historical level.

In the article by Jennen and Rahn (2014), Christian Reitberger from Wellington (partners at GmbH, a PE investment company) said that claim that the main disadvantage of PE investments in Germany is lack of a functioning stock-exchange system. In the same article, Joseph Schull (Warburg Pincus, a PE firm in London) says that the innovative business environment is the main advantage for Germany in terms of PE investing despite the investor's vague relationship to the asset class in the past. Germany can therefore be viewed as a favourable market for PE investments, however, there are there are some obstacles for this market to expand. Ross (2014)

seems to agree with this and suggests that Germany is struggling to keep up with many of its global rivals in the size of the stock market. This suggests there are still many possibilities of improvement in the investment industry.

Methodology

Estimating systematic risk using four-factor model

Risk and return is estimated by using Four-Factor Model

$$R_{i,\tau} - R_{f,\tau} = \alpha_i + \beta_{i,m}(R_{m,\tau} - R_{f,\tau}) + \varepsilon_{i,\tau} \quad (1)$$

$$R_{i,\tau} - R_{f,\tau} = \alpha_i + \beta_{i,m}(R_{m,\tau} - R_{f,\tau}) + \beta_{i,smb}SMB_{\tau} + \beta_{i,hml}HML_{\tau} + \beta_{i,wml}WML_{\tau} + \varepsilon_{i,\tau} \quad (2)$$

Where $R_{i,\tau}$ is the firm profit on asset i , $R_{m,\tau}$ is the Germany market return, and $R_{f,\tau}$ is the risk-free rate for month t , respectively. SMB_t is the difference between the returns on diversified portfolios of small stocks and big stocks; HML_t is the difference between returns on diversified portfolios of high book-to-market (value) stocks and low book-to-market (growth) stocks; and WML_t is the difference between the month t returns on diversified portfolios of the winners and losers (Fama, French, 1993) (Jegadeesh, Kräussl, Pollet, 2009)..

Construction of private equity indices

The index returns for an LPE investment is determined as the return on a portfolio that consists of out sample companies.

Market based returns

Market based daily returns are extracted using Thompson Reuters DataStream. They present the theoretical growth in the value of stock assuming that dividends are reinvested in the stock at the current price.

$$RI_t = RI_{t-1} * \frac{PI_t}{PI_{t-1}} * \left(1 + \frac{DY * f}{n}\right)$$

Here, RI_t and PI_t are return and price indices on a day t , RI_{t-1} and PI_{t-1} are return indices on the previous day, DY is the dividend yield of the price index and f is the growing factor used if the reported dividend is a net rather than gross figure, n is the number of days in the financial year * 100 (Thompson Reuters). Measurements used for Markov switching model are based on daily log returns of current period over the previous one while measurements used for four-factor model are used simply as decimal returns.

Basic risk and return characteristics of LPE

According to (Bilo et al, 2005), three factors have to be taken into account when characterizing basic risk and return properties for LPE. Firstly, since the number of listed companies increases over period, index of LPE portfolio has to be rebalanced over time. Secondly, since sample is heterogeneous regarding market capitalization of each of the firms, the performance of companies observed is also heterogeneous. Thirdly, the returns of the stocks tend to be illiquid over a small period of time in our sample. In order to account for these effects, we use different strategies for combining a portfolio of LPE similar to (Bilo et al, 2005).

Value-weighted portfolio

When constructing index based on the company value, weights in the potential portfolio are determined based on the relative market capitalization. In case of a new listing, parts of existing capital are taken out of the companies and reinvested in the new company.

$$Total\ Return\ (portfolio) = \sum_1^a \frac{Market\ value\ a}{Total\ market\ value\ (portfolio)} Return(a)$$

An equally weighted portfolio

In this case, rather than allocating weights based on market value, an equal amount of available capital is allocated to each particular company. Portfolio is rebalanced on a daily basis and in case of a new listing, the market value of company is added to the pool of available capital.

$$Total\ Return\ (portfolio) = \sum_1^a \frac{1}{total\ number\ of\ companies} Return(a)$$

Markov Switching model

To further explore our data properties, we used Markov two state autoregressive multi-asset model. This model decomposes descriptive statistics about sample data into two possible sub-states each with its own unique statistics. Under the assumption of only two possible sub-states available, the model aims to estimate the probability of change occurring between them during a specific period.

We are using the same model as P. Aigner (Modeling and managing portfolios including listed private equity, 2012). Basic bivariate markov-switching model is described by return that is received at time t using a function with a mean value μ that depends on state S at time t and an error term ϵ at time t . Function is characterized by standard deviation σ that depends on state S at time t . A bivariate indicates that both, mean and standard deviation depends on state S that the model is in at a particular time. Both of the possible states are characterized by probability matrix, where p_{ij} is a probability of being in state S_i

$$y_t = \mu_{S_t} + \sigma_{S_t} \epsilon_t \quad (1)$$

$$Prob(S_{t+1} = j | S_t = i) = p_{ij}, \quad \sum_{j=1}^k p_{ij} = 1, \quad (2)$$

When introducing simple (not state-dependant) autocorrelation with lag of one time unit, coefficient of autocorrelation (ϕ) estimated is state independent. Lag of one time unit here means that only the previous period will affect this period's returns.

$$y_t = \mu_{S_t} + \varphi(y_{t-1} - \mu_{S_{t-1}}) + \sigma_{S_t} \epsilon_t \quad (3)$$

By introducing multiple assets to our model (4) in the form Listed Private Equity Index, Stocks and Bonds, our model contains 23 unique parameters: $\mu_1, \mu_2, \sigma_1, \sigma_2, \phi$ as 3x1 vectors, p_{12}, p_{21} as scalars and $\text{Cor}_1, \text{Cor}_2$, where Cor_i is 3x3 correlation matrix.

$$y_t = \mu_{S_t} + \varphi(y_{t-1} - \mu_{S_{t-1}}) + \sigma_{S_t} \epsilon_t, \quad y_t = \begin{bmatrix} y_t^{LPE} \\ y_t^{Stocks} \\ y_t^{Bonds} \end{bmatrix}, \dots, \quad (4)$$

Conditional mean and standard deviation is a mean and variance that is specific to one of the two sub-states. Unconditional mean and standard deviation are used to characterize returns under the assumption that only one possible state exists. Therefore descriptive statistics such as population mean, standard deviation, skewness and kurtosis or return distribution are equal to unconditional statistics estimated by Markov model. Timmermann (Moments of Markov model, 2000) show how these characteristics can be calculated by moments.

First one has to calculate unconditional (ergodic) probabilities π_i to be in one of the two sub-states (S_i state).

$$\pi_1 = \frac{p_{21}}{p_{12} + p_{21}}, \quad \pi_2 = \frac{p_{12}}{p_{12} + p_{21}}, \quad (5)$$

Unconditional mean is calculated by multiplying unconditional probability vector with conditional mean vector for any single asset i ,

$$\mu_i = E[y_t] = \boldsymbol{\pi}^T \boldsymbol{\mu}_i, \quad (6)$$

Second moment is used to calculate variance-covariance matrix. For two assets i and j , variance and covariance are calculated using conditional mean vector $\boldsymbol{\mu}_i$, unconditional mean μ_i , identity vector \mathbf{I} filled with 1's, conditional standard deviation vector $\boldsymbol{\sigma}_i$, conditional correlation vector $\boldsymbol{\rho}_{ij}$ and simple autocorrelation coefficient φ_i .

$$\sigma_{ij} = E[(y_t - \mu)^2] = \boldsymbol{\pi}^T \left((\boldsymbol{\mu}_i - \mu_i \mathbf{I}) \times (\boldsymbol{\mu}_j - \mu_j \mathbf{I}) + \frac{\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j \times \boldsymbol{\rho}_{ij}}{1 - \varphi_i \varphi_j} \right), \quad (7)$$

Third moment is used to calculate unconditional skewness for any asset i .

$$Skew_i = \frac{E[(y_t - \mu)^3]}{(\sigma_i^2)^{\frac{3}{2}}}, \quad (8)$$

$$\begin{aligned} E[(y_t - \mu)^3] = & \boldsymbol{\pi}^T ((\boldsymbol{\mu}_i - \mu_i \mathbf{I}) \times (\boldsymbol{\mu}_i - \mu_i \mathbf{I}) \times (\boldsymbol{\mu}_i - \mu_i \mathbf{I})) \\ & + 3\varphi_i^2 \boldsymbol{\pi}^T \left((\mathbf{B}(\mathbf{I}_k - \varphi_i^2 \mathbf{B})^{-1} \boldsymbol{\sigma}_i^2) \times (\boldsymbol{\mu}_i - \mu_i \mathbf{I}) \right), \quad (9) \\ & + 3\boldsymbol{\pi}^T ((\boldsymbol{\mu}_i - \mu_i \mathbf{I}) \times \boldsymbol{\sigma}_i^2) \end{aligned}$$

Her, bold lowcap letters indicate vectors and bold capital letters indicate matrices. \mathbf{B} is a backward transition probability matrix that has a direct relationship to \mathbf{P} as follows:

$$b_{ij} = p_{ji} \left(\frac{\pi_j}{\pi_i} \right), \quad (10)$$

Fourth moment is used to calculate unconditional kurtosis for any asset i .

$$Kurt_i = \frac{E[(y_t - \mu)^4]}{(\sigma_i^2)^2}, \quad (11)$$

$$\begin{aligned} E[(y_t - \mu)^4] &= \boldsymbol{\pi}^T \left((\boldsymbol{\mu}_i - \mu_i \boldsymbol{\iota}) \times (\boldsymbol{\mu}_i - \mu_i \boldsymbol{\iota}) \times (\boldsymbol{\mu}_i - \mu_i \boldsymbol{\iota}) \times (\boldsymbol{\mu}_i - \mu_i \boldsymbol{\iota}) \right) \\ &+ 6\boldsymbol{\pi}^T \left((\boldsymbol{\mu}_i - \mu_i \boldsymbol{\iota}) \times (\boldsymbol{\mu}_i - \mu_i \boldsymbol{\iota}) \times \boldsymbol{\sigma}_i^2 \right) \\ &+ \boldsymbol{\pi}^T (\mathbf{I}_k - \varphi_i^2 \mathbf{B})^{-1} \left(3\boldsymbol{\sigma}_i^4 + 6\varphi_i^2 \left(\mathbf{B}(\mathbf{I}_k - \varphi_i^2 \mathbf{B})^{-1} \boldsymbol{\sigma}_i^2 \right) \times \boldsymbol{\sigma}_i^2 \right) \\ &+ 6\varphi_i^2 \boldsymbol{\pi}^T \left(\left(\mathbf{B}(\mathbf{I}_k - \varphi_i^2 \mathbf{B})^{-1} \boldsymbol{\sigma}_i^2 \right) \times (\boldsymbol{\mu}_i - \mu_i \boldsymbol{\iota}) \times (\boldsymbol{\mu}_i - \mu_i \boldsymbol{\iota}) \right) \end{aligned}$$

Note that state-independent autocorrelation coefficient does not equal to unconditional autocorrelation, because both of the sub-states have unique characteristics. Hence, Timmerman also provides a formula (12) for calculating unconditional (one existing state) autocorrelation

$$E[(y_{t+1} - \mu)(y_t - \mu)] = \boldsymbol{\pi}^T \left((\mathbf{B}(\boldsymbol{\mu}_i - \mu_i \boldsymbol{\iota})) \times (\boldsymbol{\mu}_i - \mu_i \boldsymbol{\iota}) \right) + \varphi_i \boldsymbol{\pi}^T (\mathbf{I}_k - \varphi_i^2 \mathbf{B})^{-1} \boldsymbol{\sigma}_i^2, \quad (12)$$

We are using customized ordinary least squares (COLS) to fit Markov model to our data. For fitting target we are minimizing the unconditional properties of the data descriptive statistics.

$$\begin{aligned}
COLS = \sum_{i=1}^3 & x_{mean}(\mu_{mi} - \mu_{di})^2 + x_{stdev}(\sigma_{mi} - \sigma_{di})^2 + x_{cor}(\rho_{mi} - \rho_{di})^2 + x_{ac}(\varphi_{mi} - \varphi_{di})^2 \\
& + x_{skew}(Skew_{mi} - Skew_{di})^2 + x_{kurt}(Kurt_{mi} - Kurt_{di})^2, \quad (13)
\end{aligned}$$

Because of our parameters lie in different scales (e.g means in 10^{-3} , kurtosis above 1) simple use of ordinary least squares is not feasible. Therefore, the use of RLS (relative least squares) seems to be the most appropriate. The method does have several drawbacks, such as imprecision in terms of weighing the returns of bonds, LPE and stock. As a result, we use a method very similar to both, OLS and RLS. We use several sets of constants for estimation to ensure equal representation of all 6 elements (e.g for dataset #2 we used $x_{mean}=100000$, $x_{stdev}=1000$, $x_{cor}=0.5$, $x_{skew}=1$, $x_{kurt}=0.1$).

The following constraints were applied to the model: $\mu_{mi} \in [\mu_{di} - 2\sigma_{di}; \mu_{di} + 2\sigma_{di}]$, $\sigma_{mi} \in \left[\frac{\sigma_{di}}{2}; 2\sigma_{di}\right]$, $\rho_{mi} \in [-1; +1]$, $\varphi_{mi} \in [-1; +1]$ and probability matrix constraint in (2).

Monte Carlo methodology

Monte Carlo methodology is used for the purpose of randomising the sub-states. The method uniformly draws 1000 samples of 1000 parameter sets, chooses the best fitted parameter set for every sample, and drops the worst sample rates. In the following step, each of the samples is considered as a starting point for next 1000 uniformly drawn parameter sets, thus allowing the use of best COLS between samples.

There are several advantages to using this method for the purpose of our research. Firstly, size of the allowed space seems to matter the most. If starting constraints for mean were 3 standard deviations above statistical mean or if the allowed space for conditional standard variance was multitude of 5 or more, no reasonable results could be gained. Therefore starting constraints for mean and standard variance were introduced, and variable was used to limit and focus the searching space. These two improvements were crucial to gain reasonable results and significantly improved the accuracy of our results (2-3 magnitudes).

Secondly, although the q variable should theoretically bias the results by arbitrary filtering, it was introduced only later during tests observing no improvements in 2rd up to 4th iteration practice lower tail results. The introduction of q improved the performance of model significantly.

Thirdly, balancing weights for COLS is not only important to get a significant result, but it also allows for improvement in the calculation process by improving significance of mean values, since all the properties in the model are very sensitive to mean.

Fourthly, correct parameter choice is integral, since initial model based on variance-covariance matrix instead of standard deviation and correlation was unable to produce any significant results probably because random searching is done linearly over space and standard deviation with correlation is more linear than variance.

Fifthly, most of the time all good fittings have an unconditional probability of state 1 above 99%. That effectively cancels out the Markov model as a whole because Markov model with

significant weight in one state over the other deems the whole existence of states unnecessary. As fitting becomes more effective the limit of 99%, fits are no longer observable most likely because of the limited precision that a single state model can describe data with.

Since no reasonable alternative could be found, all the processes were implemented by team members themselves in programming language Lua during the course consulting mainly academic papers. You are welcome to consult the code itself and various raw data available online.

Literature Review

While private equity is mostly defined as investments into companies that are not quoted on stock market (EVCA & Thomson Reuters, 2014), this does not limit the scope of PE investing. PE investments can also occur in listed companies (PELE investments) by purchasing large blocks directly from investors (Stotz , 2011), in securities with “equity features” such as convertible preferred stocks or any kind of similar alternative assets with a value-adding active investment strategy (Soderblom, A., 2011). Private equity investors are perceived as successful and tend to focus on improvement of companies through corporate, operational and governance engineering (Kaplan, S. N., & Strömberg, P., 2009). Authors indicate that empirical results following increased performance of firms are attributable to PE investors bargaining well, successfully targeting boards that bargain badly and by taking advantage of market mispricing for successful investing.

According to Aigner et al. (2010), private equity can be classified mainly in two financing stages: venture capital and buyout. Venture capital funds concentrate on investments in early or expansion stage companies. The early stage includes seed, start-up and first stage financing when the companies are raising money to develop the concept of the company or business plan, and are in need for extra financing from the side. An expansion stage investing carries lower risk than the early stage and at this stage enterprises can still be in a need of additional financing to expand to new markets or further develop a production line. Buyout funds are slightly different from venture capital, as investments occur in already established companies in a later phase of financing. The companies at this stage are searching for the possibilities of a change in capital structure, change in management and possible financial turnarounds. Buyout investments are usually conducted by PE investors taking on large amount of leverage (LBOs).

Fama and French (1992) show that returns from investing in small, public companies are notably different from returns of larger companies. Empirical evidence for PE are, however, inconsistent in this manner. Ljungqvist and Richardson (2003) extend the analysis of Fama and French (1992) by comparing venture capital funds and buyout funds and find that on average buyout funds received a return of 7.7 percent higher return than venture capital funds. Alternatively, Kaplan and Scholar (2005) report a positive premium over S&P 500 for venture rather than buyout funds.

Several main forms of exiting PE investments exist. Kaplan & Strömberg (2009) indicate a sale to a specific, strategic investors (38% of cases) as the most common exit route for PE investors, followed by a sale to another PE investors or conducting a secondary leveraged buyout (24% of cases) and IPO (14% of cases). These have significantly changed over history, while IPO heavily

decrease in popularity. Additionally, historical median investment holding period is roughly six years and the term orientation of PE funds seems to have changed recently to more short-term investments.

Because of the unique characteristics that PE as an asset class has, evaluating the performance of PE funds, especially on risk-adjusted basis, tends to be challenging. Soderblom (2011) gives five reasons for this. Firstly, limited transparency requirements indicate that public data are available only by specific databases (Thompson Venture Economics, Dow Jones) and are based on voluntary reporting. Secondly, the reporting of such data is inconsistent and often confusing (e.g. unclear whether reported returns are net or gross). Thirdly, subjectivity in terms of account treatment and choice whether to report specific investments introduces potential bias in the results. This leads to systematic biases regarding reporting of interim IRRs (Cumming, D. & Walz, U., 2010). Fourthly, a short history of PE as an asset class means that a lack of useful and compare data are lacking. Lastly, return analysis is not very informative without a complete understanding of risk associated with these returns.

Most PE funds are organised in the form of limited partnerships. In this legal structure, limited partners provide capital while general partners decide on the fund's investment policy. Limited partners earn a return on their investment while managers collect investing and monitoring fees and might be entitled to payoff in case of successful fund performance. A typical fund has a life of approximately 10 years. While empirical evidence suggests that PE firms create value, it is not clear that they earn superior returns for their limited partners (Kaplan, S. N., & Strömberg, P., 2009).

There are two prevailing methods in evaluating performance of funds organised as limited partnerships. First focuses on analysing PE data on a company level by evaluating returns from each of the portfolio companies individually. There are two important advantages for using company level data. Firstly, the number of companies is larger than the number of funds thus allowing for more statistical power. Secondly, these type of investments have well defined returns if intermediate cash flows are absent. On the other hand, analysis of these returns tend to be inconsistent in terms of whether they include or exclude management fees. Additionally, these returns tend to be heavily affected by sample selection bias, since datasets for capital raising firms are dominated by better-performing firms and distressed companies tend to be left as shell companies rather than liquidated.

Empirically, PE performance using company level data tends to be inconsistent. While Gro and Gottschalg (2008) find positive and significant alphas (abnormal returns) for buyout investments in US over S&P between 1984 and 2004 (Soderblom, A., 2011). Similarly, Ljungqvist and Richardson (2003) find a 5%-6% premium for VC investments and betas of 1.08 for buyout and 1.12 for VC investments by using large public companies in the same industry as a proxy (Ang, A. & Sorensen, M., 2012). Jones and Rhodes-Kropf (2003), however, do not find any abnormal returns for 1245 US-based PE and VC funds using cash flows (Soderblom, A., 2011). Philippou and Gottschalg (2009) even show that VC and BO fund returns lag behind S&P 500 by as much as 3% (Soderblom, A., 2011).

Second method for evaluating performance of limited partnership funds focuses on data that is obtained from evaluating the funds themselves. This type of data is usually obtained from limited partners who have invested in several PE funds. In most cases it tries to capture the cash flow

streams towards the limited partners (Soderblom, A., 2011). There are two main advantages for these type of funds. Firstly, the data reported is the actual return received by limited partners or the net of fees and, secondly, the sample selection for the data is smaller. On the other hand, the fact that fund-level performance is self-reported, can cause self-selection bias to arise. Furthermore, net asset value, which is used as a proxy for market value in most methods, is a noisy substitute for the actual value at best. The alternative fund-level methods such as internal rate of return (effective compounded rate of return), total value paid-in capital multiple (total amount of capital returned over total invested amount) or public market equivalent (present value of all cash distributions over take-downs from PE investors) all are proxies for the actual return (Ang, A. & Sorensen, M., 2012).

Empirical evidence using fund level data is similar to company level data. A study made by Harris, Jenkinson and Kaplan (2011) which summarises various academic studies regarding fund-level performance find the reported average IRRs for PE funds to be between 12.3% and 16.9% for buyout and 11.7% and 19.3% for VC (Ang, A. & Sorensen, M., 2012). Ljungqvist and Richardson (2003) report abnormal returns by estimating average IRRs to have a premium of 5.71% over S&P 500 for a combination of VC and buyout funds. Kaplan and Scholar (2005), however, use fund level performance estimates analysing more than 1000 VC and PE funds and find that they generate returns slightly underperform S&P 500 Index using equal-weighted basis and slightly over perform the index when using value-weighted basis for liquidated funds (Ang, A. & Sorensen, M., 2012). Chen et al. (2002) find that between 1969 and 2000 average annual IRRs of 10%, yet varying between 74% and -72% (Soderblom, A., 2011).

When using public market equivalents (PME), Kaplan and Scholar (2005) claim that private equity investments with equal betas lead to positive economic return for investors if PME for funds is greater than one. They, however, find average equal-weighted market equivalents of only 0.93 for buyout funds, yet 1.21 for VC funds. Similarly, Phalippou and Gottschalg (2009) using 852 funds find an average PME of 0.88 after introducing various adjustments. Harris, Jenkinson and Kaplan (2011) summarise several researches and report average estimated PMEs for buyout to be between 1.16 and 1.27 for buyout and 1.02 – 1.45 for VC funds (Sorensen, A. 2012).

Overall, when estimating the performance of limited partnership PE funds, studies are inconsistent regarding whether PE funds do in fact lead to increased value for investors. (NEzinu kurs article) identify several important disadvantages that investing in limited partnerships face. Firstly, portfolio diversification is extremely limited and depends on GPs capabilities as well as the market characteristics. This also indicates very high minimal amounts of investment required, significantly restricting the audience of PE investments. Secondly, inconsistencies in when capital is drawn and returns redistributed tends to create a drag on the returns, as it is unlikely that investors will have more than 50-70% of capital committed to investment actually realised. Thirdly, because no market prices on investments exist, investors experience difficulties in estimating interim returns on investments. Lastly, liquidity constraints and inability to exercise an early exit from investments is probably the main drawback from investor perspective. Problems such as lack of diversification and returns drag can partially be resolved by investing in funds-of-funds, yet the majority of them remain. Kräussl, Jegadeesh, and Pollet (2014) identify that the main problem of investment level data estimation of PE return is the difficulty of determining the intermediate market values of investments.

Listed Private Equity

Listed private equity (LPE) investments are made by purchasing the shares of an entity engaged in private capital investing. The main difference is that sales from investments are reinvested rather than redistributed to investors. There are important advantages of LPE over limited partnership PE. The most important one is liquidity, meaning that investors can always monitor the value of their investments and liquidate them, thus significantly limiting their exposure. This allows smaller investors to use PE to diversify their investments. Additionally, for LPE a limited life of investments does not exist, therefore investors can buy in the fund at any time. This also allows LPE funds to engage longer-term investments and use more flexible investment strategies. Necessity to comply with corporate regulations also means that these investments are more investor friendly and report consistent and less biased data (Brown, C. & Kräussl, R., 2012). Constantinescu (2012) shows evidence that LPE opens up investments in private equity to a variety of investors, not just limited partners. Thus, they identify characteristics, such as access, transparency, variety, costs, superior performance and discounts as advantages that confirm LPE to be an attractive investment for pension funds.

Jegadeesh et al. (2014) distinguish between listed private equity (LPE) and funds-of-funds (FoF) investments. The former are companies whose shares are publicly traded and whose main business activity is PE investing, the latter are specific funds that invest in PE and are also publicly traded. Additionally, they identify that intermediate valuation is easily observable using market prices and allows to extend the market risk to important characteristics of macro environment.

Using empirical analysis, Jegadeesh et al. (2014) find evidence that a directly observable market value of the shares is an equivalent of investing as a limited partner in an unlisted PE fund. Phalippou and Gottschalg (2009) who show that using various valuations of non-exited funds, the PE actually underperformed market by 3.83 percent Jegadeesh et al. (2009) extend their analysis to LPE. Even though their research suggests that LPE is a good proxy for unlisted private equity, they do not find significant abnormal returns.

Similarly to other companies, LPE also are typically sold at a premium during IPOs. Constantinescu (2012) finds that LPE fund premium to be -2.5% during IPOs, yet it adjusts to a long-term average of -21% after two years using net asset value estimations. Additionally, they show evidence that premium heavily depends on credit markets and systematic risk are presented, indicating a lack of information about portfolio in net asset values. On the contrary, Lahr (2010) find that on equally weighted basis, firms show excess returns of 5.8%, yet they also attribute this to liquidity risk premium imbedded in returns which is not accounted for by their CAPM estimations. Jegadeesh et al. (2009) extend the performance analysis of venture capital versus buyout to listed private equity and conclude that it looks more like a performance of small rather than large companies. Additionally, they construct a LPE index from a broad class of various LPE funds and compare it to an NVCA index used to track performance using fund level data. They find that LPE index slightly outperforms NVCA index and conclude that the latter index looks like a version of the former smoothed based on market prices.

Jegadeesh et al. (2014) use PEPI (private equity in public investments) index composed by Thompson Reuters based on quarterly cash flows to calculate the interim returns of PE investments. Since smoothed out net asset values are used, the index does not reflect the actual value of PE investments in a timely manner. When compared to index based on FoFs, Jegadeesh et al. (2014) find that estimated systematic risk using PEPI is a lot smaller than that used by FoFs or LPE because of the partial adjustment used, postulating that it underestimates the actual sensitivity of PE funds to the stock market.

Various research note that macro environment is very important determinant for PE investments. It can be directly observed in LPE, as shown by Jegadeesh et al. (2014) who find that both, GDP growth and money supply growth have positive effect on performance of LPE. In addition, they find that FoFs and LPE returns are negatively related to the credit spread, indicating that credit risk significantly affects the performance of PE.

Risk profile

Estimating risk profile for PE investments using fund-level or company-level data can be a lot more problematic and results reported by various research vary significantly. Conroy and Harris (2007) show that private equity's attractiveness is overstated and that risk-adjusted returns are significantly overstated (Soderblom, A., 2011) Estimates for betas, for example, in studies based on company-level data starting from as high as 3.6. For PE they generally tend to be above one (Ang, A. & Sorensen, M., 2012). Cochrane (2005) introduces and Korteweg and Sorensen (2010) extend on method to avoid selection bias for fund level data through using continuous time CAPM. Cochrane (2005) also estimates systematic risk for PE to be between 0.6-1.9, Korteweg and Sorensen (2010) claim it to be as high as 2.6.-2.8 (Ang, A. & Sorensen, M., 2012).

Lahr (2010) argues that systematic risk of private equity is affected by unique factors, such as acquisitions and divestments continuously changing portfolios, lack of information about companies in the portfolios and fast changes in portfolio companies. These provide challenge for strategic portfolio allocation.

When evaluated for LPE, (NEZINU KAS) shows an aggregate dimension beta of 1.7 with no excess returns. The authors also differentiate between systematic risk in internally and externally managed investment vehicles and find that former are more risky than later. They also find that aggregate market risk of LPE varies over time significantly and individual betas estimated by CAPM are highly unstable and only predictable for the upcoming 2-3 years. Lastly, they find that LPE differs from traditional PE as it can be either a fund, management company, a combination of both or a fund of fund. Because of this. Lahr (2010) classifies LPE vehicles among two dimensions, participation in fees and carried interest vs degree of diversification. The extent of systematic risk varies significantly between these vehicles, with dimension beta being 0.6 for funds of funds to 1.5 for firms, 1.2 for funds and 2.0 for investment companies.

Portfolio diversification

According to the study of Humphery-Jenner (2013), portfolio diversification increases internal rates of return (IRR) in the company. According to Buchner (2012), the portfolio has to have optimal weights for the welfare effects to take place. Even though the traditional model of choosing optimal portfolio suggests a standard mean-variance framework, Buchner (2012) notes that because PE funds and public stock market investments share similar risk and return characteristics, investors might find that including PE in their portfolio leads to significant benefits because of the unique factors that allow for further diversification.

Markov Switching model

The basic Markov Switching model has finite number of states that are characterised by state dependent mean, variance and set of probabilities of change from one state into another.

While the first MS models were used in research studying business cycles already by Hamilton (1989), currently a number of different variations of Markov models exist. Allan Timmermann in his paper (2000) introduced Basic MS, autoregressive MS and state-dependent autoregressive markov MS models. Markov Switching Poisson Multifractal and Markov Switching Multifractal models were introduced by Calvet and Fisher (2004), but Hidden Markov-Switching model introduced by Rossi and Gallo (2006).

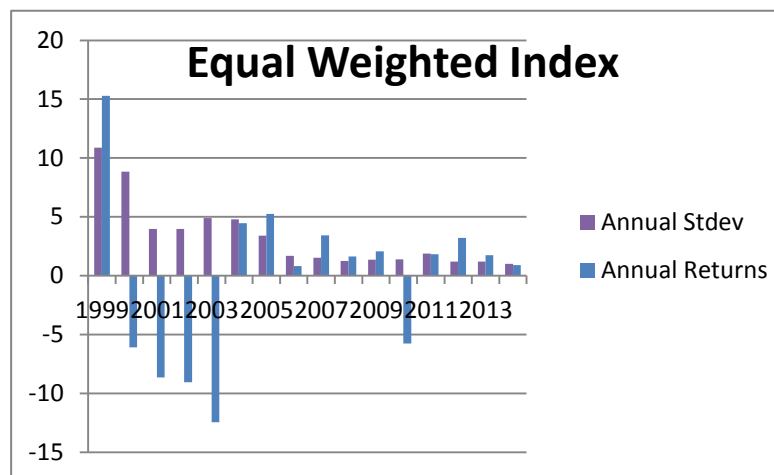
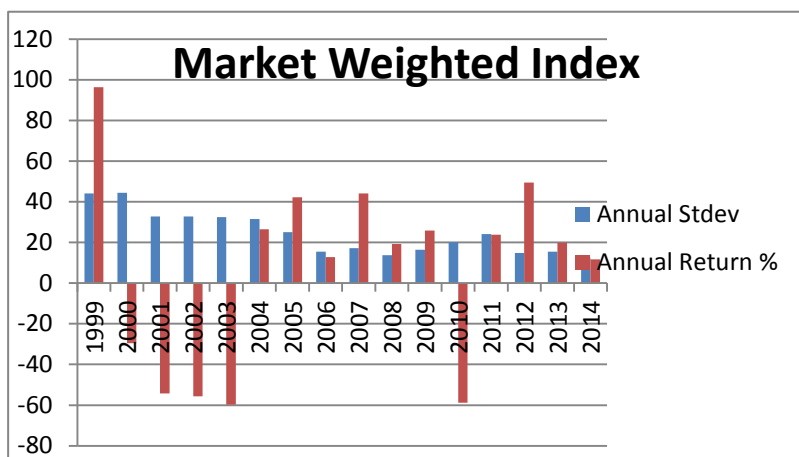
Markov models are mostly used because of their ability to better fit complex data. The models allow for generation of non-normal distributions with different coefficients of means, standard deviations, skewness, kurtosis and serial correlation (Timmermann, A., 2000), (A. Taamouti, 2012). Mean reversion, the assumption that after shock the value tends to return a theoretical mean value, is also captured by Markov Switching models according to research conducted by Abderrahim Taamouti (2012). The model can also serve as better volatility estimates, as shown by Abderrahim Taamouti (2012), who uses volatility clustering that characterises most of real world data through Markov model.

Results

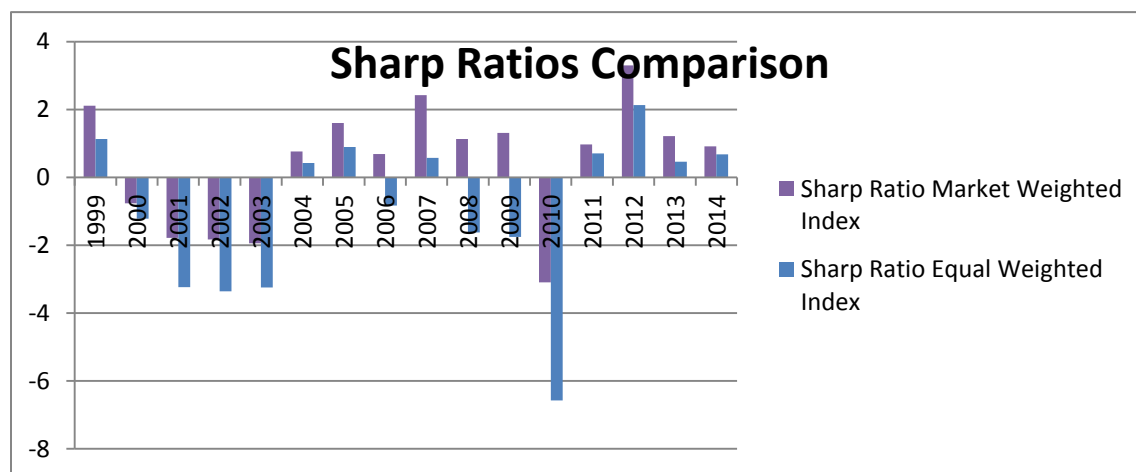
Results	Mean	Annual Stdev	Sharpe Ratio	Beta	Alpha
Market Weighted	1.867915	26.35689	-0.01829	0.07835	-0.00265
Equally Weighted	0.386151	4.354399	-0.15352	0.01181	-0.00724

The initial results of our paper concern period from 1999 till 2013. Since the number of PE firms changes considerably during this period, the portfolio is rebalanced daily to ensure that the results represent the actual performance of LPE in Germany to a sufficient extent. The historical results over the 14 year period barely suggest any kind of abnormal performance for LPE. The Standard Deviation and mean returns for market-value portfolio are both considerably higher, yet Sharpe ratio suggests that both indices considerably underperformed even risk-free returns. Both of the betas obtained are statistically significant at 95% level, yet both of the alphas, while observed to be negative, do not appear to be statistically significant for the sample (Appendix 1). Hence, when choosing the best portfolio optimization strategy, equally weighted portfolio leads to less overall exposure to systematic risk and higher Sharpe ratio.

Historical Data Annually



When looking at annual historical data, volatility of both of the indices has decreased over the years, which is most likely attributable to more LPE companies which does lead to higher diversification for the companies included in the index. While the returns for Market Weighted index tend to be higher, they also are also considerably more volatile.



The historical Sharpe ratios seem to fluctuate more for Equal Weighted Index, which is contrary to what historical average might lead one to believe. Equal Weighted Index seems to perform worse during both, economic expansion and economic downturn cycles, while Market Weighted Index seems to over perform during economic growth cycles and only slightly underperform relatively to Equal Weighted Index.

Because of the limitations of available data, the comparison of alphas and betas over historical period is the most problematic, as the results are often not statistically significant at 95% level.

Limitations

There are several things that should be addressed regarding the sample used above. Firstly, as pointed out earlier, the returns during IPOs tend to be overestimate the actual returns gained by a company because companies are often listed at a discount. To take this issue into account, we also estimate the same regressions by not including the first ten days of trading into our sample. The results are indistinguishable from the results we acquired above.

Secondly, an objection about possibility of existence of several trends in our sample was addressed using Dickey Fuller test (Appendix 1). We find no indications that sample is bias in this way.

Thirdly, while several authors have suggested that PE investments tend to suffer from significant autocorrelation or cross correlation between returns, we find no proof that this is the case with our sample by using corrgram test and directly testing cross correlation between market returns and constructed indecies (Appendix 1).

Fourthly, when using four-factor model, authors of the factors offer several possibilities in terms of factors that we can use. For our results, we use the factors that provide most statistical explanation for the results.

Markov Switching Model

Dataset #1

Our best fit for dataset #1 is in the table below. As it is seen, mean value is precise only 2 digits after comma and for LPE is off by 0.6% or 30 times but from other side it is within 1.5 standard deviations. Standard deviation is rather precise by 2 digits after comma, variance-covariance matrix is very precise by 6 digits after comma. Skewness and autocorrelation is off by factor of 10 and Kurtosis is off by factor of 100. Relative precision can be controlled by coefficients in COLS and for Dataset #2 we raised relative weight for mean value.

Statistical properties, Dataset #1				Markov fitted properties, Dataset #1		
	GE LPE	GE Stocks	GE Bonds	GE LPE	GE Stocks	GE Bonds
Mean	0.000159	0.000175	0.000012	-0.006113	0.000630	-0.000277
VCV (1)	0.000023	0.000000	0.000000	0.000060	-0.000003	0.000001
VCV (2)	0.000000	0.000028	0.000000	-0.000003	0.000060	-0.000001
VCV (3)	0.000000	0.000000	0.000000	0.000001	-0.000001	0.000000
St.dev	0.004790	0.005273	0.000315	0.007726	0.007734	0.000449
Skewness	-1.581739	-0.533728	-0.333150	-0.074609	-0.030972	0.071477
Kurtosis	24.552038	6.345066	26.810331	0.222705	-0.058676	-0.011517
AutoCor	-0.072788	0.126724	-0.366573	-0.745823	-0.575322	-0.035191

At first we see that unconditional probabilities to be in states are 38.4% and 61.6% but standard deviations are very similar, as well mean for LPE and Bonds. But mean return for Stocks are inverse, and all correlation changes between states. This suggest a time period where LPE and Bonds are indifferent, but Stocks have significant growing periods or falling periods and in this falling phase both LPE and Bonds move to opposite direction. Conditional probabilities show that being in state two one is more likely to stay in state two than being in state two. To sum up, this don't look realistic but earned best COLS score.

Markow Autoregressive (lag 1) bivariate joint 3 asset model cond. Params, Dataset#1						
uProb.	0.384234	0.615766		Prob (1)	0.395610	0.604390
AutoCor	-0.755549	-0.688452	-0.040442	Prob (2)	0.377136	0.622864
	State 1			State2		
Mean	-0.007210	0.004547	-0.000447	-0.005429	-0.001814	-0.000172
Cor (1)	1.000000	0.581946	0.828801	1.000000	-0.564372	-0.181633
Cor (2)	0.581946	1.000000	-0.129448	-0.564372	1.000000	-0.397085
Cor (3)	0.828801	-0.129448	1.000000	-0.181633	-0.397085	1.000000
VCV (1)	0.000042	0.000018	0.000002	0.000015	-0.000012	0.000000
VCV (2)	0.000018	0.000023	0.000000	-0.000012	0.000029	-0.000001
VCV (3)	0.000002	0.000000	0.000000	0.000000	-0.000001	0.000000
St.dev	0.006460	0.004753	0.000399	0.003878	0.005369	0.000445

Dataset #2

Below is output of best fit for the 2nd dataset. The aim of the model is to match the unconditional characteristics of those generated by the model to the best possible extent. We can see that mean and standard deviation both match up to four decimal points, skewness and auto-correlation approximately 10 times more different, and kurtosis does match at all. As we discussed above, although weights for COLS significantly affect this relative precision, they also improve the overall fitness of the whole model, thus producing more significant results.

Statistical properties				Markov fitted properties		
	GE LPE	GE Stocks	GE Bonds	GE LPE	GE Stocks	GE Bonds
Mean	0.000104	0.000075	0.000042	0.000158	-0.000046	0.000086
VCV (1)	0.033534	-0.000018	-0.000001	0.033545	-0.000054	-0.000024
VCV (2)	-0.000018	0.000044	0.000000	-0.000054	0.000048	0.000000
VCV (3)	-0.000001	0.000000	0.000000	-0.000024	0.000000	0.000000
St.dev	0.183122	0.006658	0.000211	0.183152	0.006932	0.000221
Skewness	-0.359429	-0.013262	0.151860	-0.232509	-0.002668	0.327011
Kurtosis	20.092276	7.412959	34.388661	-0.335328	0.000595	0.193181
AutoCor	-0.271805	-0.019537	-0.152000	0.087494	0.105883	0.225368

The unconditional probabilities estimated by the model below are 53.45% and 46.55% and both states have significantly different properties. In conditional probability matrix these states have significant chances to alter and first state is more stable than second state. The greatest observable difference exists for mean return of LPE . In first state it is grows at the rate 11.47% but in second falls at the rate of -13.14%. The situations for other assets are opposite.

Second greatest inversion is correlation – this fitted model shows high inverse relationships between LPE and Stocks that flips in other states. State 2 has significantly less correlation between assets, and also has a bit greater yet not significantly different volatility.

Markow Autoregressive (lag 1) bivariate joint 3 asset model cond. Params						
uProb.	0.5344707	0.4655293		Prob (1)	0.2998600	0.7001400
AutoCor	0.5702866	0.1081146	0.3669755	Prob (2)	0.8038254	0.1961746
	State 1			State2		
	GE LPE	GE Stocks	GE Bonds	GE LPE	GE Stocks	GE Bonds
Mean	0.1147493	-0.0004368	0.0000026	-0.1314034	0.0004023	0.0001812
Cor (1)	1.0000000	-0.9946458	-0.2369431	1.0000000	0.8515266	-0.6357739
Cor (2)	-0.9946458	1.0000000	0.6927961	0.8515266	1.0000000	-0.0612679
Cor (3)	-0.2369431	0.6927961	1.0000000	-0.6357739	-0.0612679	1.0000000
VCV (1)	0.0090043	-0.0006538	-0.0000034	0.0164326	0.0007447	-0.0000182
VCV (2)	-0.0006538	0.0000480	0.0000007	0.0007447	0.0000465	-0.0000001
VCV (3)	-0.0000034	0.0000007	0.0000000	-0.0000182	-0.0000001	0.0000001
St.dev	0.0948910	0.0069275	0.0001503	0.1281898	0.0068225	0.0002237

Conclusions

This paper aims to answer two research questions. Firstly, it aims to characterize private equity risk and return characteristics by using LPE as a proxy in Germany. Secondly, it aims to compare the risk and return characteristics that are acquired using traditional method with those estimated using Markow Switching model.

This paper does not find any evidence of abnormal returns for private equity. The alphas, estimated to be negative, are not statistically significant. The sharp ratios seem to be negative when using historic data, yet they seem to be less favourable when using a portfolio with equal rather than market weights.

Markow switching model estimation provide some insight into the distribution of returns of private equity, yet these results are not conclusive. This paper shows that access to workable data about Private Equity is hard to obtain. Therefore we used different methods to our disposal and also experimented with implementation of markov model that proved itself worthy. Private Equity environment is more complicated and not so transparent as Stocks, therefore besides problems and challenges it also offer great opportunities, that in our case was to firetest known and new models.

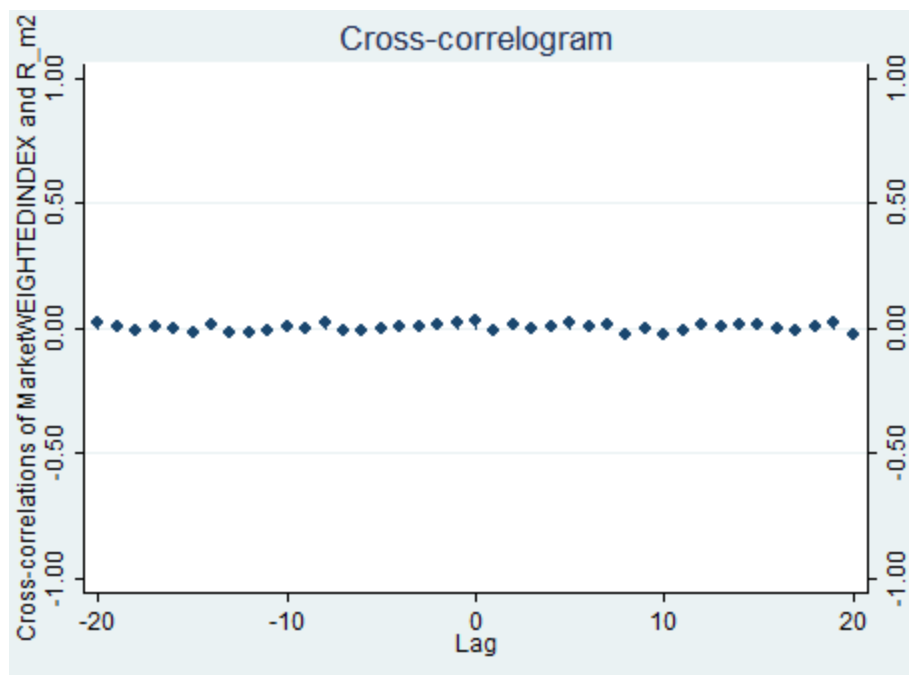

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. regress asset22 premium2 SMB2 HML2 WML2 if tin(01jan1999, 01jan2015)
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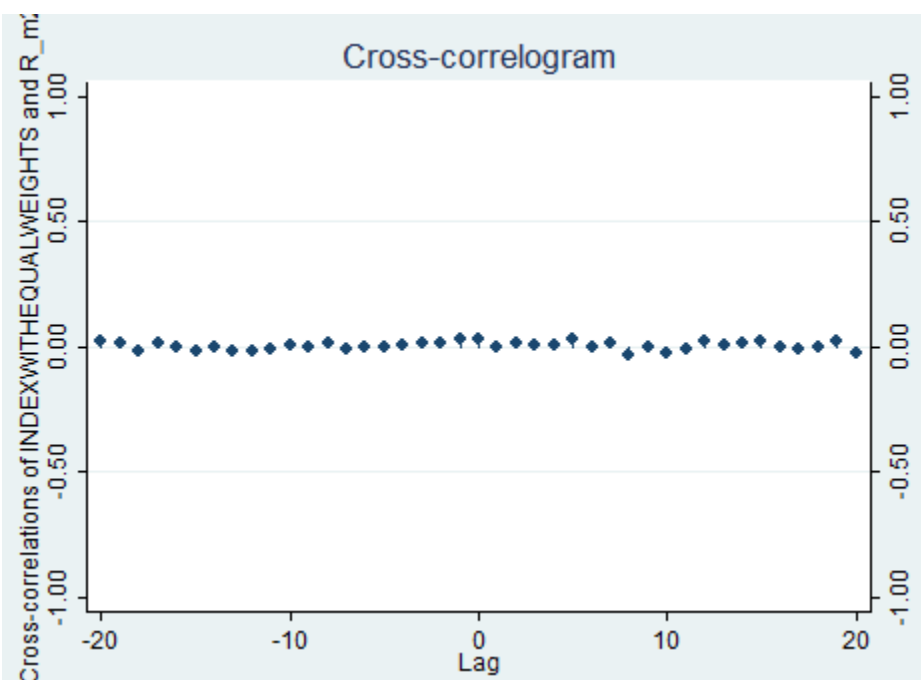
Source	SS	df	MS	Number of obs =	3896
Model	1.66362631	4	.415906576	F(4, 3891) =	5.75
Residual	281.265327	3891	.072286129	Prob > F =	0.0001
				R-squared =	0.0059
				Adj R-squared =	0.0049
Total	282.928953	3895	.072639012	Root MSE =	.26886

asset22	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
premium2	.0118167	.0058385	2.02	0.043	.0003699 .0232635
SMB2	.0029121	.0074941	0.39	0.698	-.0117806 .0176048
HML2	-.0155789	.0061957	-2.51	0.012	-.0277261 -.0034318
WML2	.009438	.0037821	2.50	0.013	.0020229 .016853
_cons	-.0072384	.0043232	-1.67	0.094	-.0157144 .0012376

LAG	AC	PAC	Q	Prob>Q	-1 0 1 -1 0 1 [Autocorrelation] [Partial Autocor]
1	0.0288	0.0368	3.2345	0.0721	
2	0.0238	0.0405	5.4455	0.0657	
3	0.0377	0.0449	10.977	0.0119	
4	0.0290	0.0600	14.256	0.0065	
5	0.0063	.	14.41	0.0132	
6	0.0291	.	17.705	0.0070	
7	0.0447	.	25.505	0.0006	
8	-0.0001	.	25.505	0.0013	
9	0.0335	.	29.89	0.0005	
10	0.0017	.	29.901	0.0009	
11	-0.0235	.	32.066	0.0007	
12	0.0321	.	36.1	0.0003	

LAG	AC	PAC	Q	Prob>Q	-1	0	1	-1	0	1
					[Autocorrelation]			[Partial Autocor]		
1	0.0358	0.0456	6.269	0.0123						
2	0.0244	0.0410	9.1853	0.0101						
3	0.0299	0.0315	13.568	0.0036						
4	0.0194	0.0384	15.418	0.0039						
5	0.0026	.	15.452	0.0086						
6	0.0297	.	19.784	0.0030						
7	0.0352	.	25.856	0.0005						
8	-0.0128	.	26.656	0.0008						
9	0.0153	.	27.809	0.0010						
10	0.0077	.	28.098	0.0017						
11	-0.0236	.	30.836	0.0012						
12	0.0286	.	34.849	0.0005						





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