Limit-Trajectory Single- and Two-Mode Overmodulation Technology

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***Abstract*—This paper extends the limit-trajectory two-mode overmodulation ideal [1] into single mode. The harmonic characteristics of them are analyzed and compared. Based on one TI TMS320LF 2407A DSP, the proposed method is verified with an open-loop V/F controlled diode-clamped three-level model inverter. With the test result, it is confirmed that single-mode limit-trajectory overmodulation technology is correct and effective.**

Key Words-PWM; overmodulation; three-level inverter

I. INTRODUCTION

By using Limit-trajectory two-mode overmodulation technique brought forward by N. V. Nho[1], linear control (output fundamental component has unit gain over reference voltage) in overmodulation region is successively achieved. Unlike other methods, one can gain the desired reference voltage vector, which can achieve the goal of unit gain, by analytical calculation. So, the additional look-up table operation which is needed by other methods can be avoided. In addition, the achieved analytical relationship between reference vector and output fundamental component has also great theoretical value. In this paper, on the basis of introduction of the limit- trajectory ideal, the method proposed by [1] was analyzed with method developed by Lee D. C. [2] firstly. Then the ideal of limit trajectory is extended into single mode overmodulation scheme. Finally, the output waveforms produced by single- and two-mode limit-trajectory overmodulation techniques are compared in regard to harmonic characteristics.

II. IDEAL OF LIMIT- TRAJECTORY CONTROL[1]

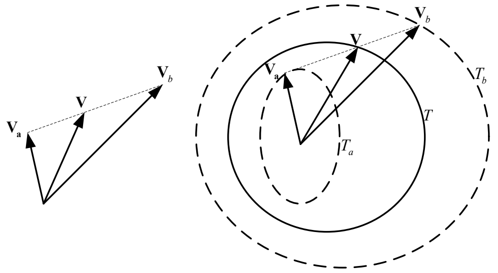

Fig.1 Ideal of limit-trajectory control [1]

Let **V**, $\mathbf{V_a}$, $\mathbf{V_b}$ be three space voltage vectors, and **V** lie on the line connecting $\mathbf{V_a}$ and $\mathbf{V_b}$ (as shown in Fig.1). Vector **V** can be expressed as linear combination of two vectors $\mathbf{V_a}$ and $\mathbf{V_b}$:

$$\mathbf{V} = (1-\eta)\mathbf{V_a} + \eta\mathbf{V_b} \tag{1}$$

Where η varies in (0, 1), when η varies from 0 to 1, **V** varies from $\mathbf{V_a}$ to $\mathbf{V_b}$.

Assuming that $\mathbf{V_a}$ and $\mathbf{V_b}$ rotate along trajectory T_a and T_b, and **V** along T, then the fundamental component of **V** can be calculated as follows:

$$\begin{aligned} V_1 &= \frac{1}{2\pi}\int_0^{2\pi}\left[(1-\eta)\cdot\mathbf{V_a} + \eta\cdot\mathbf{V_b}\right]\cdot e^{-j\theta}d\theta \\ &= \frac{(1-\eta)}{2\pi}\int_0^{2\pi}\mathbf{V_a}\cdot e^{-j\theta}d\theta + \frac{\eta}{2\pi}\int_0^{2\pi}\mathbf{V_b}\cdot e^{-j\theta}d\theta \end{aligned} \tag{2}$$

And thus we can get:

$$V_1 = (1-\eta)V_{a1} + \eta V_{b1} \tag{3}$$

In (3), V_{a1} and V_{b1} represent the corresponding fundamental component amplitude of T_a and T_b. When $\eta = 0$, $\mathbf{V}=\mathbf{V_a}$, and thus $V_1=V_{a1}$; when $\eta = 1$, $\mathbf{V}=\mathbf{V_b}$, $V_1=V_{a2}$. Through changing the η in (0, 1), we can control the fundamental component within the scope $V_{a1}<V_1<V_{b1}$.

The linear control in overmodulation region means that the fundamental component of **V** satisfies the following equation:

$$V_1 = MI\frac{2V_{dc}}{\pi} \tag{4}$$

with (3), one can get:

$$\eta = \frac{MI - M_a}{M_b - M_a} \tag{5}$$

Where M_a, M_b represents the modulation index of V_{a1}, V_{b1} respectively:

$$M_a = \frac{V_{a1}}{2V_{dc}/\pi}, \qquad M_b = \frac{V_{b1}}{2V_{dc}/\pi} \tag{6}$$

Now, an important conclusion can be achieved: if vector **V** is controlled with linear function (1) and (5), then, in the scope M_a<MI<M_b, the fundamental component (V_1) of **V** satisfies (4). In other words, the fundamental component of output voltage has unit gain

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over reference voltage.

III. Two-Mode Limit-Trajectory Overmodulation Scheme And Its Analysis

The literature [1], like [3], divides the modulation index into 3 regions: linear region (0<MI<0.907), overmodulation region I (0.907<MI<0.9514) and overmodulation region II (0.9514<MI<1). In linear region, $\mathbf{V_a}$ is set to zero vector, thus M_a=0, V_{a1}=0; the trajectory of $\mathbf{V_b}$ is set to be $\mathbf{V_b}=\frac{V_{dc}}{\sqrt{3}}\cdot e^{j\theta}$, which is the inscribed circle of vector diagram and the upper limit of linear modulation. So, $M_b=\pi/2\sqrt{3}$, $V_{b1}=V_{dc}/\sqrt{3}$. With (5), we can get $\eta=\frac{2\sqrt{3}}{\pi}MI$, and then the desired reference voltage vector $\mathbf{V}$ can be achieved by replacing this formula into (1).

In overmodulation region I, the trajectory of $\mathbf{V_a}$ is set to be $\mathbf{V_a}=\frac{V_{dc}}{\sqrt{3}}\cdot e^{j\theta}$, and thus $M_a=\pi/2\sqrt{3}$, $V_{a1}=V_{dc}/\sqrt{3}$; The trajectory of $\mathbf{V_b}$ is set to be the boundary of vector diagram, and thus $M_b=0.9514$, $V_{b1}=\frac{\sqrt{3}V_{dc}}{\pi}\ln 3$. By replacing it into (5), η can be calculated as: $\eta=\frac{2\sqrt{3}MI-\pi}{3\ln 3-\pi}$.

In overmodulation region II, the trajectory of $\mathbf{V_a}$ is set to be the boundary of vector diagram, and thus $M_a=0.9514$, $V_{a1}=\frac{\sqrt{3}V_{dc}}{\pi}\ln 3$; The trajectory of $\mathbf{V_b}$ is set to be the corresponding 6 discrete vectors of the 6 vertexes of vector diagram, and thus $M_b=1.0$, $V_{b1}=\frac{2V_{dc}}{\pi}$. By replacing it into (5), η can be calculated as: $\eta=\frac{2\sqrt{3}MI-3\ln 3}{2\ln 3-3\ln 3}$.

For analysis of the output voltage with the method introduced by Lee D. C. [2], the vector $\mathbf{V}$ need to be represented by its amplitude and phase angle. Considering the 1/6 period symmetry, only sector 1 is considered in the following. In overmodulation region I, it is easy to calculate the amplitude of reference vector as follows:

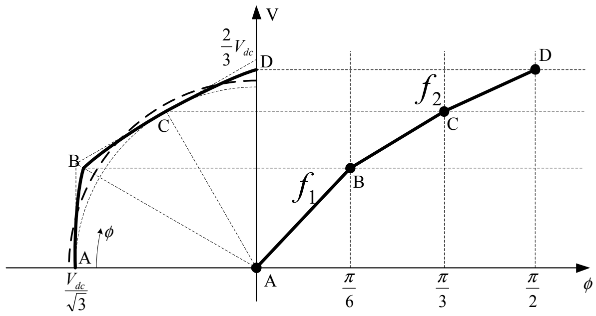
$$|\mathbf{V}|=\frac{(1-\eta_{12})\cdot V_{dc}}{\sqrt{3}}+\frac{\eta_{12}\cdot V_{dc}}{\sqrt{3}\cos(\theta-\frac{\pi}{6})} \tag{7}$$

Where, $\eta_{12}=(m-\frac{\pi}{2\sqrt{3}})\Big/(\frac{\sqrt{3}}{2}\ln 3-\frac{\pi}{2\sqrt{3}})$, $m\in[0.907,0.952]$, and m is MI indeed(Fig.2 b). θ is the vector angle of inner or outer vector.

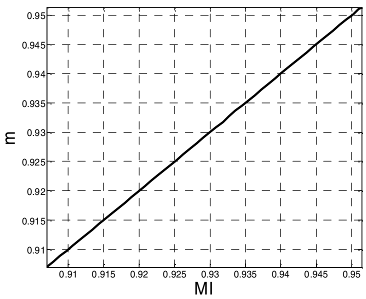
Bold real line in Fig.2a shows the trajectory of reference voltage vector gained with (7). As can be seen in Fig.2b, the phase voltage waveform can be separate into 2 parts, and their analytical expression can be as follows:

$$\begin{cases} f_1=R_1\cdot V_{dc}\cdot\sin\phi & ,0\le\phi<\pi/6 \\ f_2=R_2\cdot V_{dc}\cdot\sin\phi & ,\pi/6\le\phi<\pi/2 \end{cases} \tag{8}$$

Where, $R_1=(1-\eta_{12})/\sqrt{3}+\eta_{12}/(\sqrt{3}\cos\phi)$, $R_2=(1-\eta_{12})/\sqrt{3}+\eta_{12}/[\sqrt{3}\cos(\phi-\pi/3)]$



(a) Trajectory of voltage vector and modulated phase voltage waveform



(b) Relationship of MI and m

Fig. 2 Analysis of TMLT mode I

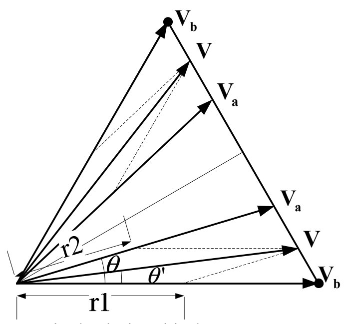


Fig. 3 Geometrical relationship between **Va**, **Vb** and **V** in overmodulation region II

In overmodulation region II, $\mathbf{V_a}$, $\mathbf{V_b}$ and $\mathbf{V}$ satisfy the relation shown in Fig.3. The following expression can be easily gain by the geometrical relation in Fig.3:

$$|\mathbf{V}|=\frac{V_{dc}}{\sqrt{3}\cos(\pi/6-\theta')} \tag{9}$$

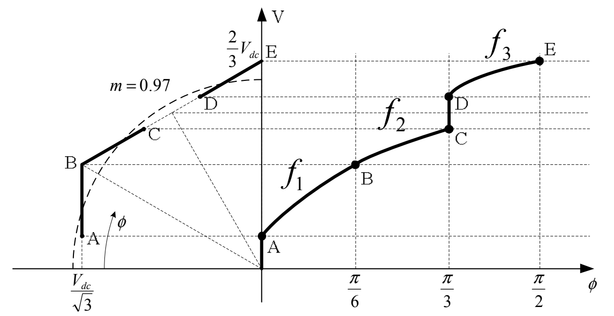
$$\theta' = \begin{cases} \arctan(\dfrac{r_2 \cdot \sin\theta}{r_2 \cdot \cos\theta + r_1}) & ,0 \le \theta < \pi/6 \\ \dfrac{\pi}{3} - \arctan[\dfrac{r_2 \cdot \sin(\pi/3-\theta)}{r_2 \cdot \cos(\pi/3-\theta) + r_1}] & ,\pi/6 \le \theta < \pi/3 \end{cases} \tag{10}$$

Where $r_1 = 2\eta_{23} \cdot V_{dc}/3$, $r_2 = (1-\eta_{23}) \cdot V_{dc}/[\sqrt{3}\cos(\theta - \pi/6)]$, $\eta_{23} = \dfrac{m - \sqrt{3}\ln 3/2}{1-\sqrt{3}\ln 3/2}$. θ is the phase angle of inner vector. Where control parameter $m \in [0.952, 1]$, and m is MI indeed(Fig.4b).

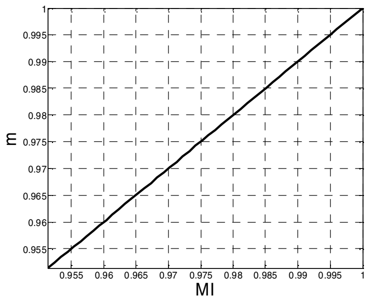
As can be seen from Fig.4a, phase voltage waveform can be divided into 3 parts, and can be as follows:

$$\begin{cases} f_1 = R_2 \cdot \sin\phi + R_1/2 & ,0 \le \phi < \pi/6 \\ f_2 = R_3 \cdot \sin\phi + R_1/2 & ,\pi/6 \le \phi < \pi/3 \\ f_3 = R_3 \cdot \sin\phi + R_1 & ,\pi/3 \le \phi < \pi/2 \end{cases} \tag{11}$$

Where $R_1 = 2\eta_{23} \cdot V_{dc}/3$, $R_2 = (1-\eta_{23}) \cdot V_{dc}/\sqrt{3}\cos\phi$, $R_3 = (1-\eta_{23}) \cdot V_{dc}/\sqrt{3}\cos(\pi/3-\phi)$.



(a) Trajectory of voltage vector and modulated phase voltage waveform



(b) Relationship of MI and m
Fig. 4 Analysis of TMLT mode II

IV. Single-Mode Limit-Trajectory Overmodulation Scheme

Indeed, the ideal of limit trajectory can also be used in single mode overmodulation scheme. In modulation region $MI \in (0.907, 1)$, the inner trajectory is set to be the inscribed circle:

$$\mathbf{V_a} = \frac{V_{dc}}{\sqrt{3}} \cdot e^{j\theta} \tag{12}$$

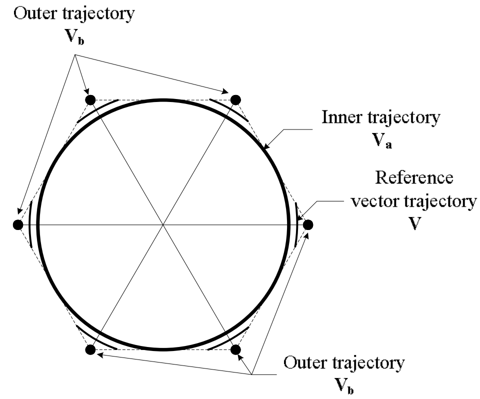


Fig.5 Single mode limit trajectory overmodulation

And the outer trajectory is set to be the corresponding 6 discrete base vectors of the 6 vertexes of vector diagram (Fig.5). Thus, we have: $M_a = \pi/2\sqrt{3}$, $V_{a1} = V_{dc}/\sqrt{3}$; and $M_b = 1.0$, $V_{b1} = \dfrac{2V_{dc}}{\pi}$. By replacing it into (5), we can get:

$$\eta = \frac{MI - \pi/2\sqrt{3}}{1-\pi/2\sqrt{3}} \tag{13}$$

By considering (1), we can get the desired vector (in sector 1):

$$\mathbf{V} = \begin{cases} (1-\eta)\dfrac{V_{dc}}{\sqrt{3}} \cdot e^{j\theta} + \eta \cdot \dfrac{2V_{dc}}{3} & , \quad 0 \le \theta < \pi/6 \\ (1-\eta)\dfrac{V_{dc}}{\sqrt{3}} \cdot e^{j\theta} + \eta \cdot \dfrac{2V_{dc}}{3} \cdot e^{j\pi/3}, & \pi/6 \le \theta \le \pi/3 \end{cases} \tag{14}$$

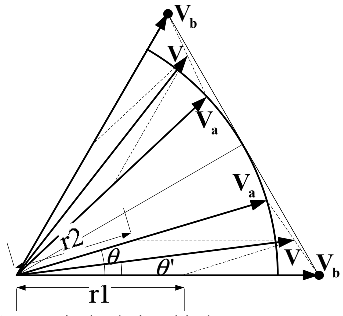


Fig.6 Geometrical relationship between **Va**,**Vb** and **V**

According to the geometrical relation of $\mathbf{V_a}$, $\mathbf{V_b}$ and $\mathbf{V}$ shown in Fig.6, the amplitude and phase angle of reference voltage vector can be expressed as follows (for the 1/6 period symmetry, only sector 1 is considered):

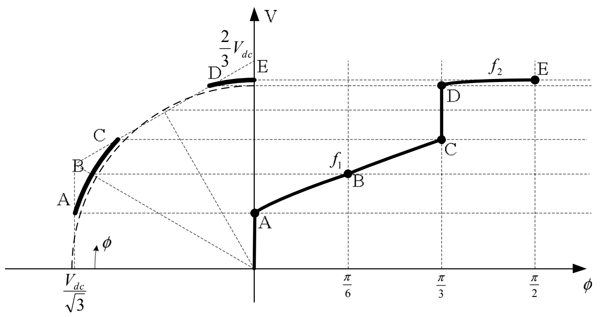
$$r' = \begin{cases} r_1 \cdot \cos\theta' + r_2 \cdot \cos(\theta - \theta') & ,0 \le \theta < \pi/6 \\ r_1 \cdot \cos(\pi/3 - \theta') + r_2 \cdot \cos(\theta - \theta') & ,\pi/6 \le \theta < \pi/3 \end{cases}$$

$$\theta' = \begin{cases} \arctan(\dfrac{r_2 \cdot \sin\theta}{r_2 \cdot \cos\theta + r_1}) & ,0 \le \theta < \pi/6 \\ \dfrac{\pi}{3} - \arctan[\dfrac{r_2 \cdot \sin(\pi/3-\theta)}{r_2 \cdot \cos(\pi/3-\theta) + r_1}] & ,\pi/6 \le \theta < \pi/3 \end{cases} \tag{15}$$

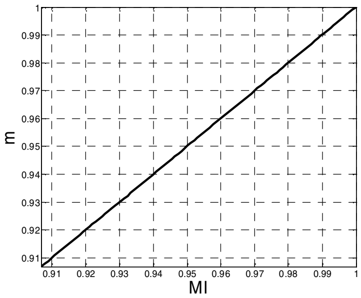
Where $r_1 = 2\eta \cdot V_{dc}/3$, $r_2 = (1-\eta) \cdot V_{dc}/\sqrt{3}$, $\eta = (MI - \pi/2\sqrt{3})/(1-\pi/2\sqrt{3})$.

As can be seen from Fig.7a, phase voltage waveform can be divided into 2 parts, and can be expressed as:

$$\begin{cases} f_1 = r_2 \cdot \sin\phi + r_1/2 & ,0 \le \phi < \pi/3 \\ f_2 = r_2 \cdot \sin\phi + r_1 & ,\pi/3 \le \phi < \pi/2 \end{cases} \tag{16}$$



(a) Trajectory of voltage vector and modulated phase voltage waveform



(b) Relationship of MI and m

Fig. 7 Analysis of SMLT

V. Performance Comparison Between Single- and Two-Mode Limit-Trajectory Overmodulation Scheme

THD is defined as follows:

$$THD = \frac{\sqrt{\left(V_r^2 - V_1^2\right)}}{V_1} \tag{17}$$

Where V_r, V_1 are rms values of output voltage and its fundamental component respectively. The THD of the two methods mentioned above is drawn in Fig.8 (method of [1] is referred as TMLT and the one introduced by this paper is referred as SMLT). For comparison purpose, THD of the method of [3] (referred as TM1) and [4] (referred as SM) are also drawn in the same diagram. It is easy to conclude that THD of two mode method TMLT, TM1 is smaller than that of single mode ones. Additionally, SMLT is better than SM in regard to THD. Index THD reflects the distortion degree of voltage. But for inductive machine, the distortion of current is the fact that we indeed care about. Accordingly, WTHD is used bellow to evaluate the output waveform in the situation of inductive load.

The full name of WTHD is weighted total harmonic distortion. It is induced from THD by weighted disposal of each harmonic component. It is defined as:

$$WTHD = \frac{1}{V_1}\sqrt{\sum_{i=2}^{\infty}\left(\frac{V_i}{i}\right)^2} \tag{18}$$

WTHD reflect the current distortion on inductive load. Fig.9 shows the WTHD of all the overmodulation mentioned above. As can be seen from it, two-mode scheme is still better than single- mode. The important truth is SM if better than SMLT in regard to WTHD, which is reverse in regard to THD. This exemplifies the necessary of WTHD.

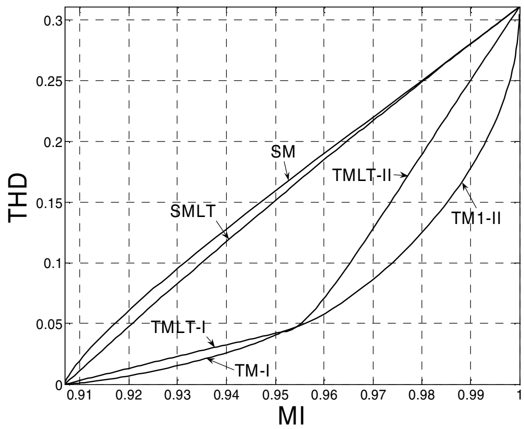


Fig.8 Comparison of THD

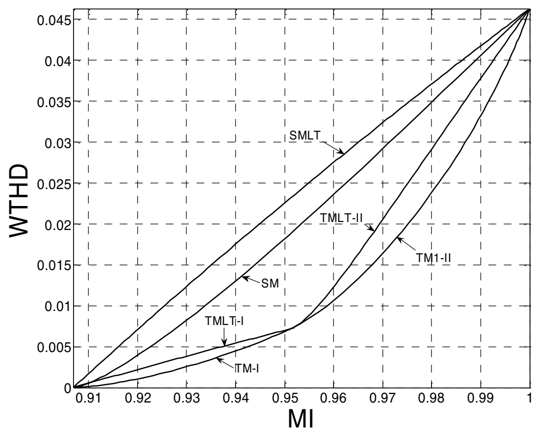


Fig.9 Comparison of WTHD

VI. Experiment

The 4 overmodulation schemes were testified on an open-loop diode-clamped V/F controlled three-level model inverter with one TI TMS320LF 2407A DSP as controller[5][6] (Fig.10, TABLE I.). The U phase voltage is calculated online and output by D/A. As can be seen from Fig.11, all the tested waveform is identical to analytical results. When MI=1, all schemes can produce the 6 step output. Fig.12 shows line voltage and current of the single-mode limit-trajectory overmodulation scheme.

TABLE I.
EXPERIMENT CONDITION

DC-Link Voltage	DC100V
Switching Frequency	1kHz
DC-Link Capacitance	1000uF
Switching Device	MOSFET 10A/400V
AC Motor	Squirrel Cage 380V/1.1kW
Controller	TMS320LF 2407A DSP
Inverter Capacitance	1kVA
Dead Time	4us
D/A Precision	12Bit

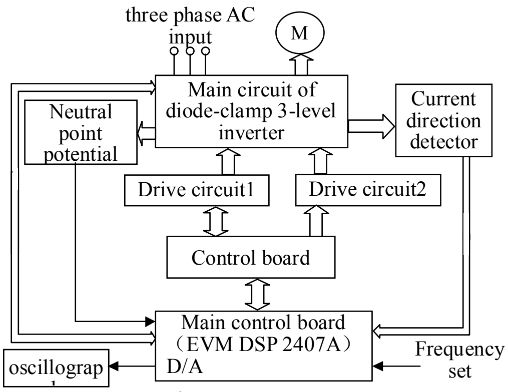
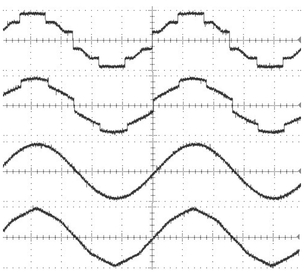
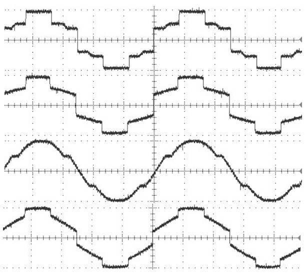


Fig.10 System setup


(a) MI=0.93


(b) MI=0.97

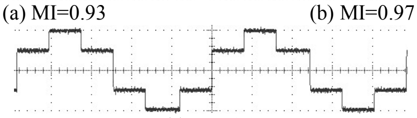
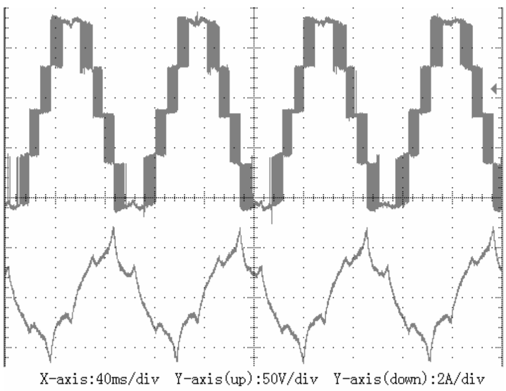
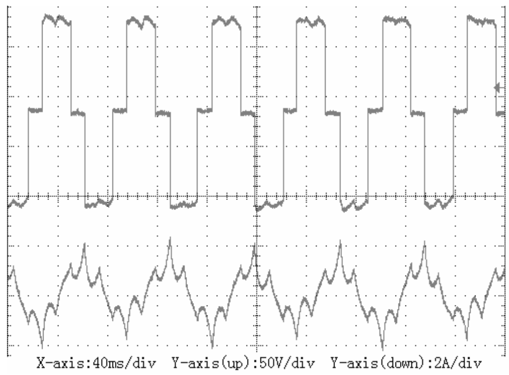

(c) MI=1.0

Fig.11 Out-put phase voltage, from top to bottom: SM, SMLT, TM1, TMLT



(a) MI=0.96



(b) MI=1.0

Fig.12 Line voltage and current of single-mode limit-trajectory overmodulation scheme.

VII. CONCLUSION

It is feasible to extend the ideal of limit trajectory control into single mode overmodulation. By doing this, single analytical formula can be achieved in the whole overmodulation region, by which the desired voltage vector can be calculated according to modulation index directly and thus the fundamental component of output voltage can be controlled accurately. Despite the degraded harmonic characteristics the single-mode limit-trajectory overmodulation scheme is valuable for calculation –resource-scarce applications such as three-level inverter.

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