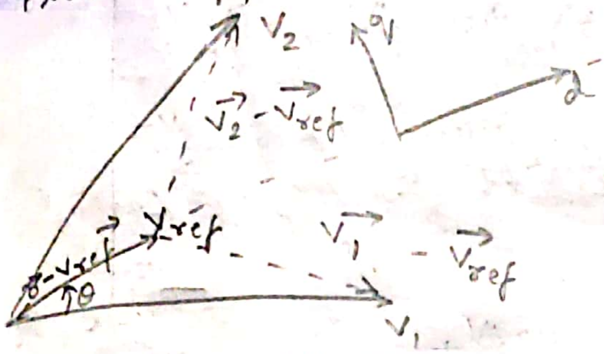
Flux - Ripple Calculation in SVPWM :-



Let $\vec{e_0} = \vec{0} - \vec{V}_{ref}$

$\vec{e_1} = \vec{V_1} - \vec{V}_{ref}$

$\vec{e_2} = \vec{V_2} - \vec{V}_{ref}$

be the 3 error vectors

$|V_1| = |V_2| = V_{dc}$

Ripple in flux (voltage) will be due ~~the~~ to the overall error function given by appropriate combination of these error vectors

Switching times of states in a single sample.

In sequence →

Vector (state) applied	$\vec{0}$	$\vec{V_1}$	$\vec{V_2}$	$\vec{0}$	$\vec{V_2}$	$\vec{V_1}$	$\vec{0}$
Time for which applied	$\frac{T_0}{4}$	$\frac{T_1}{2}$	$\frac{T_2}{2}$	$\frac{T_0}{2}$	$\frac{T_2}{2}$	$\frac{T_1}{2}$	$\frac{T_0}{4}$

time = 0 ... time = T_s

When a vector is applied, error vector corresponding to that vector is present.

Now, $V = \frac{d\phi}{dt}$ or $\phi = \int V dt$. & ϕ leads voltage by $90°$

Components of error vectors along d & q axes:-

$e_{0d} = -V_{ref}$, $e_{0q} = 0$

$e_{1d} = V_1 \cos\theta - V_{ref}$, $e_{1q} = -V_1 \sin\theta$

$= V_{dc} \cos\theta - V_{ref}$ $\quad = -V_{dc} \sin\theta$

$e_{2d} = V_2 \cos(60-\theta) - V_{ref}$, $e_{2q} = V_2 \sin(60-\theta)$

$= V_{dc} \cos(60-\theta) - V_{ref}$ $\quad = V_{dc} \sin(60-\theta)$

⇒ Voltages in +ve d-axis will produce fluxes in +ve q-axis & voltages in +ve q-axis will produce fluxes in -ve d-axis

For time $0 < t \leq T_0/4$:

Vector applied : $\vec{0}$

Corresponding error vectors : e_{0d} & e_{0q}.

Now $\Delta\phi_q = \int_0^t e_{0d}\, dt$ & $\Delta\phi_d = -\left[\int_0^t e_{0q}\, dt\right]$

$\Rightarrow$ $\boxed{\Delta\phi_q(t) = -V_{ref} \cdot t}$ & $\boxed{\Delta\phi_d(t) = 0}$

Also $\Delta\phi_q(T_0/4) = -V_{ref} T_0/4$ & $\Delta\phi_d(T_0/4) = 0$

for time $T_0/4 < t \le \frac{T_0}{4} + \frac{T_1}{2}$:

Vector applied : $\vec{V_1}$

Corresponding errors : e_{1d} & e_{1q}

$$\Rightarrow \Delta\phi_q(t) = \Delta\phi_q(T_0/4) + \int_{T_0/4}^{t} e_{1d}\,dt \;, \quad \Delta\phi_d(t) = \Delta\phi_d\left(\frac{T_0}{4}\right) - \int_{T_0/4}^{t} e_{1q}\,dt.$$

$$\Rightarrow \boxed{\Delta\phi_q(t) = -\frac{V_{ref}T_0}{4} + (V_{dc}\cos\theta - V_{ref})\left(t - \frac{T_0}{4}\right)}$$

$$\boxed{\Delta\phi_d(t) = +V_{dc}\sin\theta\left(t - \frac{T_0}{4}\right)}$$

$$\Rightarrow \Delta\phi_q\left(\frac{T_0}{4} + \frac{T_1}{2}\right) = -\frac{V_{ref}T_0}{4} + (V_{dc}\cos\theta - V_{ref})\left(\frac{T_1}{2}\right)$$

$$\Delta\phi_d\left(\frac{T_0}{4} + \frac{T_1}{2}\right) = +V_{dc}\sin\theta\left(\frac{T_1}{2}\right)$$

For $\frac{T_0}{4} + \frac{T_1}{2} < t \le \frac{T_0}{4} + \frac{T_1}{2} + \frac{T_2}{2}$:-

Vector applied : $\vec{V_2}$

Corresponding errors : e_{2d} & e_{2q}

$$\Delta\phi_q(t) = \Delta\phi_q\left(\frac{T_0}{4} + \frac{T_1}{2}\right) + \int_{\frac{T_0}{4}+\frac{T_1}{2}}^{t} e_{2d}\,dt.$$

$$\Delta\phi_d(t) = \Delta\phi_d\left(\frac{T_0}{4} + \frac{T_1}{2}\right) + \int_{T_0/4+T_1/2}^{t} -e_{2q}\,dt$$

$$\Rightarrow \boxed{\Delta\phi_q(t) = V_{dc}\cos\theta\,\frac{T_1}{2} - V_{ref}\left(\frac{T_0}{4} + \frac{T_1}{2}\right) + (V_{dc}\cos(60-\theta) - V_{ref})\left(t - \left(\frac{T_0}{4} + \frac{T_1}{2}\right)\right)}$$

$$\Rightarrow \boxed{\Delta\phi_d(t) = +V_{dc}\sin\theta\,\frac{T_1}{2} - V_{dc}\sin(60-\theta)\cdot\left(t - \left(\frac{T_0}{4} + \frac{T_1}{2}\right)\right)}$$

$$\Rightarrow \Delta\phi_q\left(\frac{T_0}{4}+\frac{T_1}{2}+\frac{T_2}{2}\right) = V_{dc}\cos\theta\frac{T_1}{2} - V_{ref}\left(\frac{T_0}{4}+\frac{T_1}{2}\right) + \left(V_{dc}\cos(60-\theta) - V_{ref}\right)\left(\frac{T_2}{2}\right)$$

$$= -V_{ref}\left(\frac{T_0}{4}+\frac{T_1}{2}+\frac{T_2}{2}\right) + V_{dc}\cos\theta\frac{T_1}{2} + V_{dc}\cos(60-\theta)\frac{T_2}{2}$$

$$\Delta\phi_d\left(\frac{T_0}{4}+\frac{T_1}{2}+\frac{T_2}{2}\right) = V_{dc}\sin\theta\,\frac{T_1}{2} - V_{dc}\sin(60-\theta)\cdot\frac{T_2}{2} = 0$$

for $\frac{T_s}{2} - \frac{T_0}{4} < t \le \frac{T_s}{2} + \frac{T_0}{4}$: $(T_s = T_1 + T_2 + T_0)$

Vector applied : $\vec{0}$

Corresponding errors : e_{0d} & e_{0q}

$$\Rightarrow \Delta\phi_q(t) = \Delta\phi_q\left(\frac{T_0}{4}+\frac{T_1}{2}+\frac{T_2}{2}\right) + \int_{\frac{T_s}{2}-\frac{T_0}{4}}^{t} e_{0d}\,dt$$

$$\Delta\phi_d(t) = \Delta\phi_d\left(\frac{T_0}{4}+\frac{T_1}{2}+\frac{T_2}{2}\right) + \int_{\frac{T_s}{2}-\frac{T_0}{4}}^{t} e_{0q}\,dt$$

$$\Rightarrow \boxed{\Delta\phi_q(t) = V_{dc}\cos\theta\frac{T_1}{2} + V_{dc}\cos(60-\theta)\frac{T_2}{2} - V_{ref}\cdot t}$$

$$\boxed{\Delta\phi_d(t) = V_{dc}\sin\theta\frac{T_1}{2} - V_{dc}\sin(60-\theta)\frac{T_2}{2} = 0}$$

$$\Delta\phi_q\left(\frac{T_s}{2}+\frac{T_0}{4}\right) = V_{dc}\cos\theta\frac{T_1}{2} + V_{dc}\cos(60-\theta)\frac{T_2}{2} - V_{ref}\left(\frac{T_s}{2}+\frac{T_0}{4}\right)$$

$$\Delta\phi_d\left(\frac{T_s}{2}+\frac{T_0}{4}\right) = -V_{dc}\sin\theta\frac{T_1}{2} + V_{dc}\sin(60-\theta)\frac{T_2}{2} = 0$$

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for $\frac{T_s}{2} + \frac{T_0}{4} < t \le \frac{T_s}{2} + \frac{T_0}{4} + \frac{T_2}{2}$

Vector applied : $\vec{V_2}$

Corresponding errors : e_{2d} & e_{2q}

$$\Delta\phi_q(t) = \Delta\phi_q\left(\frac{T_s}{2}+\frac{T_0}{4}\right) + \int_{\frac{T_s}{2}+\frac{T_0}{4}}^{t} e_{2d}\,dt \quad , \quad \Delta\phi_d(t) = \Delta\phi_d\left(\frac{T_s}{2}+\frac{T_0}{4}\right) + \int_{\frac{T_s}{2}+\frac{T_0}{4}}^{t} e_{2q}\,dt.$$

$$\boxed{\Delta\phi_q(t) = V_{dc}\cos\theta\frac{T_1}{2} + V_{dc}\cos(60-\theta)\frac{T_2}{2} + V_{dc}\cos(60-\theta)\left(t - \left(\frac{T_s}{2}+\frac{T_0}{4}\right)\right) - V_{ref}\cdot t}$$

$$\boxed{\Delta\phi_d(t) = -V_{dc}\sin(60-\theta)\left(t - \left(\frac{T_s}{2}+\frac{T_0}{4}\right)\right)}$$

$$\Delta\phi_q\left(\frac{T_0}{4}+\frac{T_2}{2}+\frac{T_s}{2}\right) = V_{dc}\cos\theta\,\frac{T_1}{2} + V_{dc}\cos(60-\theta)T_2 - V_{ref}\left(\frac{T_s}{2}+\frac{T_0}{4}+\frac{T_2}{2}\right)$$

$$\Delta\phi_d\left(\frac{T_0}{4}+\frac{T_s}{2}+\frac{T_2}{2}\right) = -V_{dc}\sin(60-\theta)\cdot\frac{T_2}{2}$$

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for $\frac{T_s}{2}+\frac{T_0}{4}+\frac{T_2}{2} < t \leq T_s - \frac{T_0}{4} \left(= \frac{T_s}{2}+\frac{T_0}{4}+\frac{T_2}{2}+\frac{T_1}{2}\right)$

Vector applied : $\vec{V_1}$

Corresponding errors : e_{1d} & e_{1q}

$$\Delta\phi_q(t) = \Delta\phi_q\left(\frac{T_0}{4}+\frac{T_2}{2}+\frac{T_s}{2}\right) + \int_{\frac{T_0}{4}+\frac{T_2}{2}+\frac{T_s}{2}}^{t} e_{1d}\,dt$$

$$\boxed{\Delta\phi_q(t) = V_{dc}\cos\theta\,\frac{T_1}{2} + V_{dc}\cos(60-\theta)T_2 + V_{dc}\cos\theta\left(t-\left(\frac{T_s}{2}+\frac{T_0}{4}+\frac{T_2}{2}\right)\right) - V_{ref}\cdot t}$$

$$\boxed{\Delta\phi_d(t) = -V_{dc}\sin(60-\theta)\frac{T_2}{2} + V_{dc}\sin\theta\left(t-\left(\frac{T_s}{2}+\frac{T_0}{4}+\frac{T_2}{2}\right)\right)}$$

$$\Delta\phi_q\left(T_s-\frac{T_0}{4}\right) = V_{dc}\cos\theta T_1 + V_{dc}\cos(60-\theta)T_2 - V_{ref}\left(T_s-\frac{T_0}{4}\right)$$

$$\Delta\phi_d\left(T_s-\frac{T_0}{4}\right) = -V_{dc}\sin(60-\theta)\frac{T_2}{2} + V_{dc}\sin\theta\cdot\frac{T_1}{2} = 0$$

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for $T_s - T_0/4 < t \leq T_s$

Vector applied : $\vec{O}$

Corresponding Errors : e_{0d} & e_{0q}

$$\Delta\phi_q(t) = \Delta\phi_q\left(T_s-\frac{T_0}{4}\right) + \int_{T_s-T_0/4}^{t} e_{0d}\,dt \quad , \quad \Delta\phi_d(t) = \Delta\phi_d\left(T_s-T_0/4\right) + \int_{T_s-T_0/4}^{t} e_{0q}\,dt$$

$$\Rightarrow \boxed{\Delta\phi_q(t) = V_{dc}\cos\theta T_1 + V_{dc}\cos(60-\theta)T_2 - V_{ref}\cdot t}$$

$$\boxed{\Delta\phi_d(t) = 0}$$

$$\boxed{\Delta\phi_q(T_s) = V_{dc}\cos\theta\, T_1 + V_{dc}\cos(60-\theta)\, T_2 - V_{ref}\, T_s \;(=0)}$$

$$\boxed{\Delta\phi_d(T_s) = 0}$$

Max. ripple in ϕ_q $= \Delta\phi_{q,max} - \Delta\phi_{q,min}$

$= \Delta\phi_q\left(T_s - \frac{T_0}{4}\right) - \Delta\phi_q\left(T_s + \frac{T_0}{4}\right)$

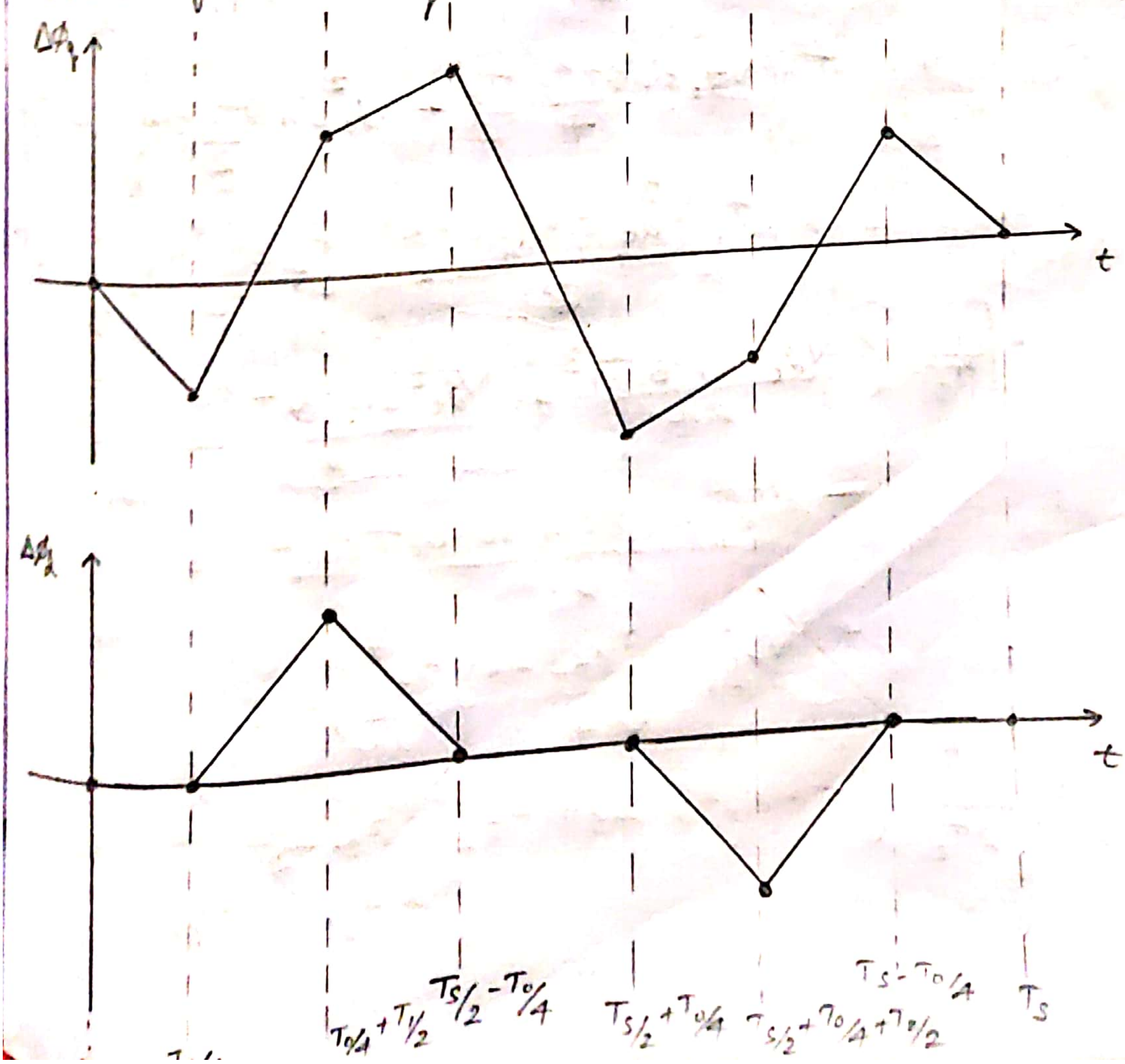
$= V_{ref}\,\frac{T_0}{2}$

Max ripple in ϕ_d $= \Delta\phi_{d,max} - \Delta\phi_{d,min}$

$= \Delta\phi_d\left(\frac{T_s}{2} + \frac{T_0}{4} + \frac{T_2}{2}\right) - \Delta\phi_d\left(\frac{T_0}{4} + \frac{T_1}{2}\right)$

$= V_{dc}\sin(60-\theta)\,\frac{T_2}{2} + V_{dc}\sin\theta\,\frac{T_1}{2}$

Plots of $\Delta\phi_d$ & $\Delta\phi_q$:-



RMS Ripple :-

$$\Delta\phi_{d,\,rms} = \left[\frac{\int_0^{T_s}(\Delta\phi_d)^2\,dt}{T_s}\right]^{1/2}$$

$$= \left[\frac{1}{T_s}\left(\int_0^{T_0/4} 0^2\,dt + \int_{T_0/4}^{T_0/4+T_1/2}\left(V_{dc}\sin\theta\left(t-\frac{T_0}{4}\right)\right)^2 dt\right.\right.$$

$$+ \int_{\frac{T_0}{4}+\frac{T_1}{2}}^{\frac{T_s}{2}-\frac{T_0}{4}}\left(V_{dc}\sin\theta\frac{T_1}{2} - V_{dc}\sin(60-\theta)\left(t-\left(\frac{T_0}{4}+\frac{T_1}{2}\right)\right)\right)^2 dt$$

$$\left.\left. + \int_{\frac{T_s}{2}-\frac{T_0}{4}}^{T_s/2} 0^2\,dt\right)\times 2\right]^{1/2}$$

$\hookrightarrow \Delta\phi_d^2$ is symm about $t = T_s/2$

$$\Rightarrow \Delta\phi_{d\,rms} = \left[\frac{1}{T_s}\left(0 + \left(\frac{V_{dc}^2\sin^2\theta}{3}\cdot\left[t^3\right]_0^{T_1/2}\right)\right.\right.$$

$$+\left(V_{dc}^2\sin^2\theta\frac{T_1^2}{4}\cdot\frac{T_2}{2} - V_{dc}^2\sin\theta\frac{T_1}{2}\sin 60-\theta\left[t^2\right]_0^{T_2/2}\right.$$

$$\left.\left.\left. + \frac{V_{dc}^2\sin^2(60-\theta)}{3}\left[t^3\right]_0^{T_2/2}\right)\right)\times 2\right]^{1/2}$$

$$\Rightarrow \Delta\phi_{d\,rms} = \left[\frac{1}{T_s}\left(\frac{V_{dc}^2\sin^2\theta\,T_1^3}{24} + \frac{V_{dc}^2\sin^2\theta\,T_1^2T_2}{8}\right.\right.$$

$$- \frac{V_{dc}^2\sin\theta\sin(60-\theta)T_1T_2^2}{8}$$

$$\left.\left. + \frac{V_{dc}^2\sin^2(60-\theta)\,T_2^3}{24}\right)\times 2\right]^{1/2}$$

$$\Rightarrow \Delta\phi_{d\,rms} = \left[\frac{V_{dc}^2}{12T_s}\left(T_1^3\sin^2\theta + 3T_1^2T_2\sin^2\theta - 3T_1T_2^2\sin\theta\sin(60-\theta)\right.\right.$$

$$\left.\left. + T_2^3\sin^2(60-\theta)\right)\right]^{1/2}$$

$$\Delta\phi_{q,rms} = \left[\frac{\int_0^{T_s} (\Delta\phi_q)^2 \cdot dt}{T_s}\right]^{1/2}$$

$$= \Bigg[\frac{1}{T_s}\Bigg(\int_0^{T_0/4} (-V_{ref}\cdot t)^2 dt + \int_{T_0/4}^{T_0/4+T_1/2} \left(-V_{ref}\frac{T_0}{4} + (V_{dc}\cos\theta - V_{ref})\left(t-\frac{T_0}{4}\right)\right)^2 dt$$

$$+ \int_{T_0/4+T_1/2}^{T_s/2-T_0/4} \left(V_{dc}\cos\theta\frac{T_1}{2} - V_{ref}\left(\frac{T_0}{4}+\frac{T_1}{2}\right) + (V_{dc}\cos(60-\theta) - V_{ref})\left(t-\left(\frac{T_0}{4}+\frac{T_1}{2}\right)\right)\right)^2 dt$$

$$+ \int_{\frac{T_s}{2}-\frac{T_0}{4}}^{\frac{T_s}{2}+T_0/4} \left(V_{dc}\cos\theta\frac{T_1}{2} + V_{dc}\cos(60-\theta)\frac{T_2}{2} - V_{ref}\,t\right)^2 dt.$$

$$+ \int_{T_s/2+T_0/4}^{\frac{T_s}{2}+T_0/4+T_2/2} \left(V_{dc}\cos\theta\frac{T_1}{2} + V_{dc}\cos(60-\theta)\frac{T_2}{2} + V_{dc}\cos(60-\theta)\left(t-\left(\frac{T_s}{2}+\frac{T_0}{4}\right)\right) - V_{ref}\,t\right)^2 dt.$$

$$+ \int_{T_s-\frac{T_0}{4}-T_1/2}^{T_s-T_0/4} \left(V_{dc}\cos\theta\frac{T_1}{2} + V_{dc}\cos(60-\theta)T_2 + V_{dc}\cos\theta\left(t-\left(T_s-\frac{T_0}{4}-\frac{T_1}{2}\right)\right) - V_{ref}\cdot t\right)^2 dt.$$

$$+ \int_{T_s-T_0/4}^{T_s} \left(V_{dc}\cos\theta\, T_1 + V_{dc}\cos(60-\theta)T_2 - V_{ref}\cdot t\right)^2 dt\Bigg)\Bigg]^{1/2}$$

$$\Delta\phi_{q,rms} = \Bigg[\frac{2}{T_s}\Bigg(V_{ref}^2\left[\frac{t^3}{3}\right]_0^{T_s/2} + \Bigg(V_{dc}^2\cos^2\theta\left[\frac{t^3}{3}\right]_0^{T_1/2}$$

$$-2V_{ref}V_{dc}\cos\theta\left(\frac{t^3}{3} - \frac{T_0 t^2}{8}\right)_{T_0/4}^{T_0/4+T_1/2}\Bigg)$$

$$+\Bigg(V_{dc}^2\cos^2\theta\,\frac{T_1^2}{4}\left[t\right]_0^{T_2/2} + V_{dc}^2\cos^2(60-\theta)\left[\frac{t^3}{3}\right]_0^{T_2/2}$$

$$+V_{dc}^2\cos\theta\cos(60-\theta)T_1\left[\frac{t^2}{2}\right]_0^{T_2/2}$$

$$-V_{ref}V_{dc}\cos\theta\,T_1\left[\frac{t^2}{2}\right]_{\frac{T_0}{4}+\frac{T_1}{2}}^{\frac{T_0}{4}+\frac{T_1}{2}+\frac{T_2}{2}}$$

$$-2V_{ref}V_{dc}\cos(60-\theta)\left[\frac{t^3}{3} - \left(\frac{T_0}{4}+\frac{T_1}{2}\right)\left(\frac{t^2}{2}\right)\right]_{\frac{T_0}{4}+\frac{T_1}{2}}^{\frac{T_0}{4}+\frac{T_1}{2}+\frac{T_2}{2}}\Bigg)$$

$$+\Bigg(V_{dc}^2\cos^2\theta\,\frac{T_1^2}{4}\left[t\right]_0^{T_0/4} + V_{dc}^2\cos^2(60-\theta)\frac{T_2^2}{4}\left[t\right]_0^{T_0/4}$$

$$+V_{dc}^2\cos\theta\cos(60-\theta)\frac{T_1T_2}{2}\left[t\right]_0^{T_0/4}$$

$$-V_{dc}V_{ref}\cos\theta\,T_1\left[\frac{t^2}{2}\right]_{\frac{T_0}{4}+\frac{T_1}{2}+\frac{T_2}{2}}^{\frac{T_0}{4}+\frac{T_1}{2}+\frac{T_2}{2}+\frac{T_0}{4}}$$

$$-V_{dc}V_{ref}\cos(60-\theta)\,T_2\left[\frac{t^2}{2}\right]_{\frac{T_0}{4}+\frac{T_1}{2}+\frac{T_2}{2}}^{\frac{T_0}{4}+\frac{T_1}{2}+\frac{T_2}{2}+\frac{T_0}{4}}\Bigg)\Bigg]^{1/2}$$

$$
\begin{aligned}
\Delta\phi_{q,rms} = \Bigg[\frac{2}{T_s} \Bigg(& \frac{V_{ref}^2 T_s^3}{24} + \Bigg(\frac{V_{dc}^2 \cos^2\theta\, T_1^3}{24} \\
& - \frac{V_{ref} V_{dc} \cos\theta}{8} \left(\frac{2T_1^3}{3} + \frac{T_0 T_1^2}{2} \right) \Bigg) \\
& + \Bigg(\frac{V_{dc}^2 \cos^2 T_1^2 T_2}{8} + \frac{V_{dc}^2 \cos^2(60-\theta) T_2^3}{24} \\
& + \frac{V_{dc}^2 \cos\theta \cos(60-\theta) T_1 T_2^2}{8} \\
& - V_{ref} V_{dc} \cos\theta\, T_1 \left(\frac{T_0 T_2}{8} + \frac{T_1 T_2}{4} + \frac{T_2^2}{8} \right) \\
& - V_{ref} V_{dc} \cos(60-\theta) \left(\frac{T_2^3}{12} + \frac{T_0 T_2^2}{16} + \frac{T_1 T_2^2}{8} \right) \Bigg) \\
& + \Bigg(\frac{V_{dc}^2 \cos^2\theta\, T_1^2 T_0}{16} + \frac{V_{dc}^2 \cos^2(60-\theta) T_2^2 T_0}{16} \\
& + \frac{V_{dc}^2 \cos\theta \cos(60-\theta) T_0 T_1 T_2}{8} \\
& - V_{dc} V_{ref} \cos\theta \left(\frac{3T_0^2 T_1}{32} + \frac{T_0 T_1^2}{8} + \frac{T_0 T_1 T_2}{8} \right) \\
& - V_{dc} V_{ref} \cos(60-\theta) \Bigg(\frac{3T_0^2 T_2}{32} + \frac{T_0 T_2^2}{8} \\
& + \frac{T_0 T_1 T_2}{8} \Bigg) \Bigg) \Bigg) \Bigg]^{1/2}
\end{aligned}
$$

$$\Delta\phi_{q,rms} = \Bigg[\frac{2}{T_s}\Bigg(\frac{V_{ref}^2 T_s^3}{24} + \frac{V_{dc}^2\cos^2\theta\, T_1^2}{8}\left(\frac{T_0}{2}+\frac{T_1}{3}+T_2\right)$$

$$+\frac{V_{dc}^2\cos^2(60-\theta)\,T_2^2}{8}\left(\frac{T_0}{2}+\frac{T_2}{3}\right)$$

$$+\frac{V_{dc}^2\cos\theta\cos(60-\theta)\,T_1T_2(T_0+T_2)}{8}$$

$$-\frac{V_{dc}V_{ref}\cos\theta}{8}\left(\frac{2T_1^3}{3}+2T_0T_1T_2+2T_1^2T_2 + T_1T_2^2+\frac{3T_0^2T_1}{4}+\frac{3T_0T_1^2}{2}\right)$$

$$-\frac{V_{ref}V_{dc}\cos(60-\theta)}{8}\left(\frac{2T_2^3}{3}+\frac{3T_0T_2^2}{2}+T_1T_2^2 + \frac{3T_0^2T_2}{4}+T_0T_1T_2\right)\Bigg)\Bigg]^{1/2}$$

Scanned with CamScanner

Peak - Peak Ripple :-

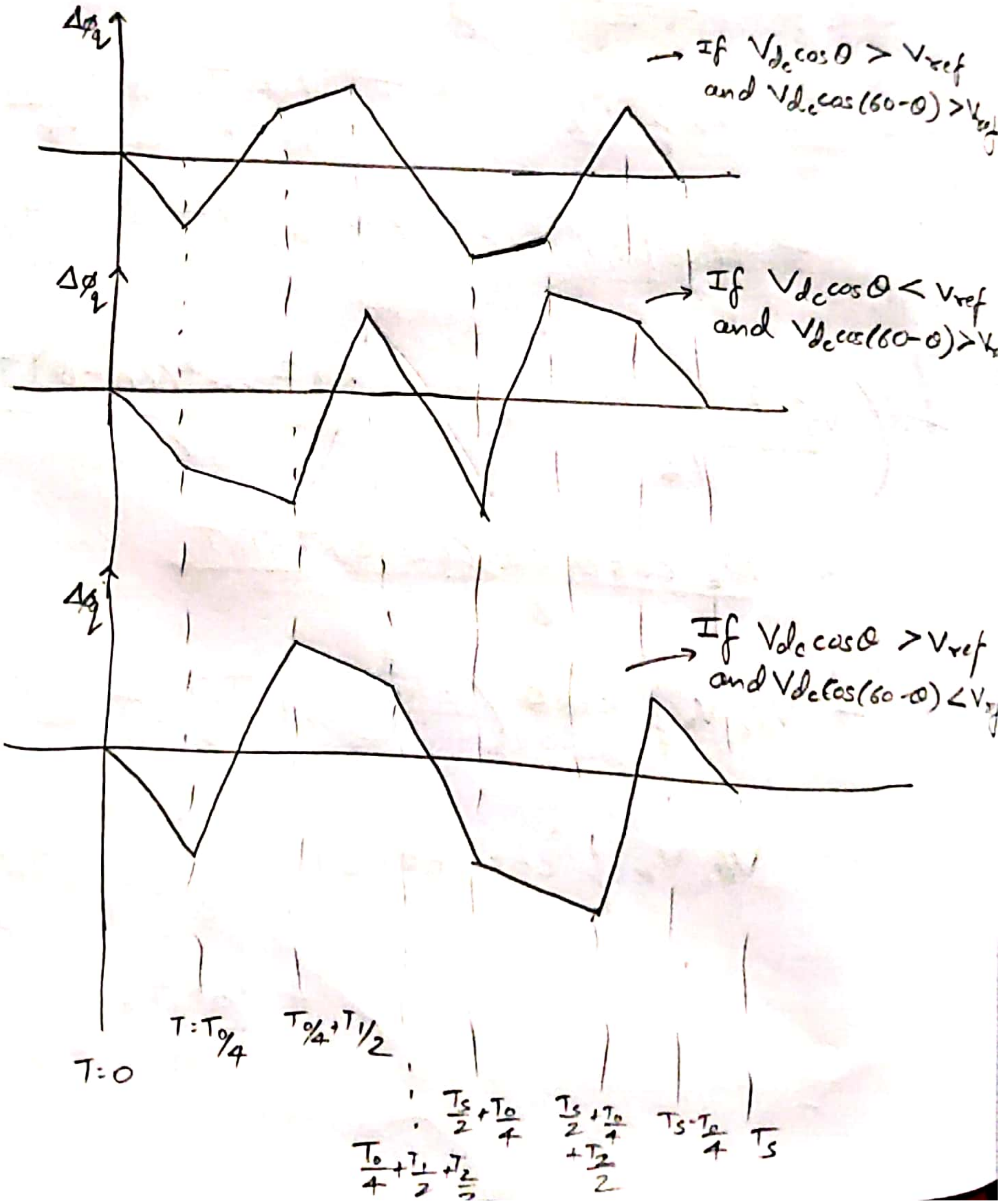
$$\Delta\phi_{d,pp} = \max(\Delta\phi_d) - \min(\Delta\phi_d)$$

$$= \Delta\phi_d\left(\frac{T_0}{4}+\frac{T_1}{2}\right) - \Delta\phi_d\left(\frac{T_s}{2}+\frac{T_0}{4}+\frac{T_2}{2}\right)$$

$$= V_{dc}\sin\theta\,\frac{T_1}{2} + V_{dc}\sin(60-\theta)\,\frac{T_2}{2}$$

For q-axis error:-

$\Delta\phi_q$ can have three possibilities



If $V_{dc}\cos\theta > V_{ref}$ & $V_{dc}\cos(60-\theta) > V_{ref}$

$$\Delta\phi_{q,\,p\text{-}p} = \Delta\phi_q\left(\frac{T_s}{2}-\frac{T_0}{4}\right) - \Delta\phi_q\left(\frac{T_s}{2}+\frac{T_0}{4}\right)$$

$$= V_{ref}\,\frac{T_0}{2}$$

If $V_{dc}\cos\theta < V_{ref}$ & $V_{dc}\cos(60-\theta) > V_{ref}$.

$$\Delta\phi_{q,\,p\text{-}p} = \Delta\phi_q\left(\frac{T_s}{2}+\frac{T_0}{4}+\frac{T_2}{2}\right) - \Delta\phi_q\left(\frac{T_s}{2}-\frac{T_0}{4}-\frac{T_2}{2}\right)$$

$$= -\left(\cancel{V_{dc}\cos\theta\,\frac{T_1}{2}} - V_{ref}\left(\cancel{\frac{T_0}{4}}+\frac{T_1}{2}\right)\right)$$

$$+\left(\cancel{V_{dc}\cos\theta\,\frac{T_1}{2}} + V_{dc}\cos(60-\theta)T_2 - V_{ref}\left(\frac{T_s}{2}+\cancel{\frac{T_0}{4}}+\frac{T_2}{2}\right)\right)$$

$$= V_{ref}\left(\frac{-T_1+T_2+T_s}{2}\right) + V_{dc}\cos(60-\theta)T_2$$

If $V_{dc}\cos\theta > V_{ref}$ & $V_{dc}\cos(60-\theta) < V_{ref}$

$$\Delta\phi_{q,\,p\text{-}p} = \Delta\phi_q\left(\frac{T_0}{4}+\frac{T_1}{2}\right) - \Delta\phi_q\left(T_s-\frac{T_0}{4}-\frac{T_1}{2}\right)$$

$$= \left(\cancel{V_{dc}\cos\theta\left(\frac{T_1}{2}\right)} - V_{ref}\left(\cancel{\frac{T_0}{4}}+\frac{T_1}{2}\right)\right)$$

$$-\left(\cancel{V_{dc}\cos\theta}\,\frac{T_1}{2} + V_{dc}\cos(60-\theta)T_2 - V_{ref}\left(\frac{T_s}{2}+\cancel{\frac{T_0}{4}}+\frac{T_2}{2}\right)\right)$$

$$= V_{ref}\left(\frac{T_s+T_2-T_1}{2}\right) - V_{dc}\cos(60-\theta)T_2$$