Exam. Akylbek Subanbekov BA19

1.1 Simplify:

$$\frac{x^{32}}{x^{3}\cdot x^{2}}\cdot\frac{x^{7}}{x^{2}}=\frac{x^{39}}{x^{13}}=x^{26}$$

1.2 $8^{2}\cdot 4^{x}\cdot 2^{x}=8^{4}$

$(2^{3})^{2}\cdot 2^{2x}\cdot 2^{x}=(2^{3})^{4}$

$2^{6+2x+x}=2^{12}$

$3x=6$

$x=2$

1.3 if $\frac{x}{y}=3$, then $x^{-4}y^{4}=\frac{y^{4}}{x^{4}}=\left(\frac{x}{y}\right)^{-4}=\left(\frac{1}{3}\right)^{4}=\frac{1}{81}$.

1.4. $\frac{\sqrt{4^{15}}}{\sqrt{16^{7}}}=\sqrt{\frac{2^{30}}{2^{28}}}=2$.

1.5. a) $x+(y+z)=(y+x)+z$ True; b) $y(x+z)=xy+zy$

$xy+zy=xy+zy$ True

c) $x^{y+z}=x^{z}+x^{y}$

$x^{y}\cdot x^{z}\neq x^{z}+x^{y}$ False; d) $\frac{x^{z}}{x^{y}}=x^{y-z}$

$x^{z-y}\neq x^{y-z}$ False

$z-y\neq y-z$

1.6. $\ln(x)\geq e$

$e^{\ln x}\geq e^{e}$

$e^{\ln x}=x\Rightarrow x\geq e^{e}$

2.1 $\begin{cases}32=0b+a\\212=100b+a\end{cases}$; $\begin{cases}a=32\\100b=212-32\end{cases}$; $\begin{cases}a=32\\b=\frac{180}{100}=1.8\end{cases}$ $\Rightarrow$

$\Rightarrow F=1.8C+32$. $F=C$ $C=1.8C+32$;

$0.8C=-32$

$C=-40$

Therefore, $C=F=-40$

2.2. $f(x) = 3x - 12$; $f(y) = 0$; $3y - 12 = 0 \Rightarrow y = 4$

2.3. $9^{x^2-6x+2} = 81$;

~~2.4~~

$9^{x^2-6x+2} = 9^2$

$x^2 - 6x + 2 = 2$

$x(x-6) = 0$

$x = 0$; $x = 6$

2.4. $(1.03)^n = 3$; $n = \frac{\ln 3}{\ln(1 + p/100)} \approx \frac{1.09861229}{3/100} \approx \frac{110}{3} \approx 37$ years

using calculator: $n \approx 37.167$.

2.5. $\log_\pi\left(\frac{1}{\pi^5}\right) = \pi^{\log_\pi \pi^{-5}} = \pi^{-5} = -5$

2.6.

$\sum_{i=0}^{\infty}\left(\frac{1}{5^i} + 0.3^i\right) = \sum_{i=0}^{\infty}\frac{1}{5^i} + \sum_{i=0}^{\infty}\left(\frac{3}{10}\right)^i = \frac{1}{1-\frac{1}{5}} + \frac{1}{1-\frac{3}{10}} = \frac{5}{4} + \frac{10}{7} =$

$= \frac{35+40}{28} = \frac{75}{28} \approx$

3.2

$\lim_{x\to 5}\frac{x^2-25}{x-5} = \lim_{x\to 5}\frac{(x-5)(x+5)}{\cancel{(x-5)}} = \lim_{x\to 5} x+5 = 10$

3.3 $f(x) = x^3 - 4$ at $(-2; -12)$

$f'(x) = 3x^2$

$f'(-2) = 3\cdot 4 = 12$

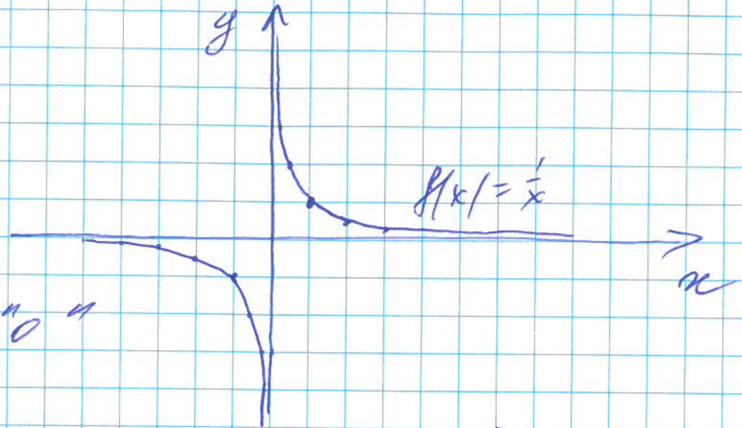
3.4 $f(x) = \frac{x^5+3}{x^2-1}$; $f'(x) = \frac{5x^4(x^2-1) - 2x(x^5+3)}{(x^2-1)^2}$.

3.5. $f(x) = x^9 + 3$;

$f'(x) = 9x^8$;

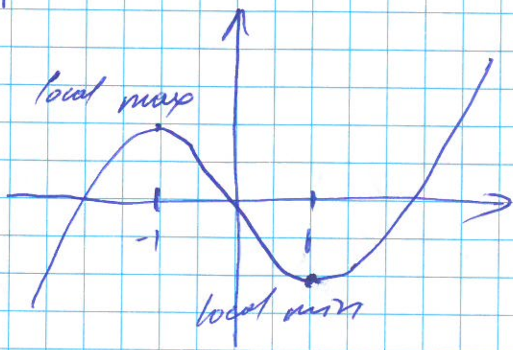
$f''(x) = 72x^7$

3.6 $f(x) = \frac{1}{x}$



$f(x)$ approaches "0" infinitely but never turns into "0"

3.7



$f(x) = 4x^3 - 12x;$

$f'(x) = 12x^2 - 12$

$f'(x) = 0 \quad ; \quad 12x^2 - 12 = 0;$

$x^2 = 1;$

$x_1 = 1$ minimum $\quad f''(x) = 24x \quad ; \quad f''(1) = 24 \cdot 1 = 24$ conv

$x_2 = -1$ maximum. $\quad f''(-1) = -24$ concave

3.8. $f(x,y) = x^3 - y^2. \quad f(2,3) = 2^3 - 3^2 = 8 - 9 = -1$

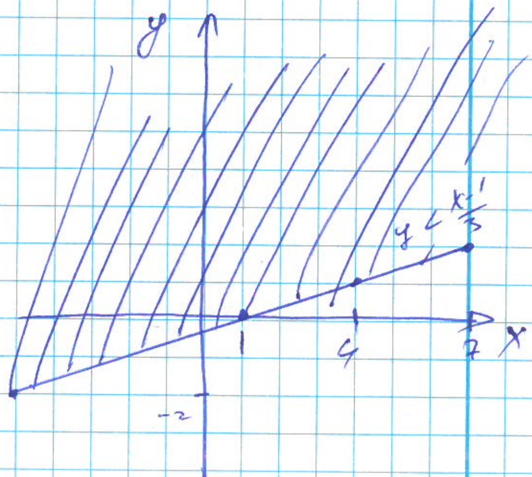
3.9. $f(x,y) = \ln(x - 3y)$

$x - 3y > 1$

$3y < x - 1$

$y < \frac{x-1}{3}$

y	x
2	7
1 2/3	6
4/3	5
1	4
1/3	3
1/3	2
0	1
-1/3	0
-2	-5
-3	-8



3.10 $\frac{\partial}{\partial x}\left(x^5 y^7 + \frac{x^2}{y^3}\right) = 5x^4 y^7 + \frac{2x}{y^3};$

3.11 $f(x,y) = \sqrt{xy} - x - y \quad ; \quad xy > 0$

$f'_x(x,y) = \frac{1}{2} x^{-\frac{1}{2}} \sqrt{y} - 1 = \frac{\sqrt{y}}{2\sqrt{x}} - 1 = \frac{y}{2\sqrt{xy}} - 1$

$f'_y(x,y) = \sqrt{x} \, \frac{1}{2} y^{-\frac{1}{2}} - 1 = \frac{x}{2\sqrt{xy}} - 1$

$$\begin{cases} \frac{y}{2\sqrt{xy}} - 1 = 0; & \frac{y}{\sqrt{xy}} = 2 \\ \frac{x}{2\sqrt{xy}} - 1 = 0; & \frac{x}{\sqrt{xy}} = 2 \end{cases}$$

$\Rightarrow \frac{y}{\sqrt{xy}} = \frac{x}{\sqrt{xy}} \Rightarrow x = y \Rightarrow$

$\frac{\sqrt{y}}{2\sqrt{y}} - 1 = -\frac{1}{2}$;

3.12 $f(x,y) = x^2y^2 \Rightarrow max$ s.t. $2x+y=9$;

$L = x^2y^2 - \lambda(2x+y-9)$; $\frac{\partial L}{x} = 2xy^2 - \lambda(2) = 2xy^2 - 2\lambda = 0$; $\lambda = xy^2$

$\frac{\partial L}{y} = 2yx^2 - \lambda = 0$; $\lambda = 2x^2y$; $\frac{\partial}{\lambda} = 2x+y-9=0$

$xy^2 = 2x^2y$; $y = 2x$; $2x+2x-9=0$; $x = 9/4 = 2.25$

$y = 2\cdot\frac{9}{4} = 4.5 \Rightarrow f(2.25, 4.5) = 102.57$

4.1

			2	5
			2	1
			7	6
1	0	1	9	11
9	1	5	55	76

$BA = \begin{bmatrix} 9 & 11 \\ 55 & 76 \end{bmatrix}$

4.2

		8	4	0
		2	1	2
5	3	46	23	6
0	1	2	1	2
1	2	12	6	4

$AB = \begin{bmatrix} 46 & 23 & 6 \\ 2 & 1 & 2 \\ 12 & 6 & 4 \end{bmatrix}$

4.3 $A^T = \begin{bmatrix} e & 2 & 4 \\ 33 & 6.1 & \pi \\ 4.7 & 4.22 & 0 \end{bmatrix}$

4.4. $A = \begin{bmatrix} 2 & 6 \\ 2 & 8 \end{bmatrix}$

$\det A = ad - bc = 2\cdot 8 - 6\cdot 2 = 4$

5.1 6 AND 6 $= 6\cdot 6 = 36$.

sample space = 36

	1	2	3	4	5	6
1	11	21	31	41	51	61
2	12	22	32	42	52	62
3	13	23	33	43	53	63
4	14	24	34	44	54	64
5	15	25	35	45	55	65
6	16	26	36	46	56	66

5.2. Assumed there are $100{,}000 \cdot 10^3$ individuals.

Out of them:

			positive		negative	
drug users	0.1%	$100 \cdot 10^3$	98	98%	2	2%
not drug users	99.9%	$99{,}990 \cdot 10^3$	299.97	0.3%		99.7%
total		$100{,}000 \cdot 10^3$	$397.97 \cdot 10^3$			

Therefore, out of 397,970 people with a positive drug test, 98,000 are drug users. The probability of a drug user with a positive drug test:

$$100 \times \frac{98}{397.97} \approx 24.62\%$$

5.3

Getting five: $E = (1)(1/6) + (E+1)(5/6)$

$E = 6$

Roll 20 times: $20 \cdot \frac{1}{6} = \frac{20}{6} = 3\frac{1}{3}$.