L- 5 Eigenvalues and Eigenvectors

1. Find the eigenvalues and eigenvectors of the matrices.

$$(a)\begin{bmatrix}1 & 4\\ 3 & 2\end{bmatrix} \qquad (b)\begin{bmatrix}1 & -2\\ -5 & 4\end{bmatrix}$$

3. Using the Cayley-Hamilton theorem, find the inverse of

$$(a)\begin{bmatrix}1 & 1 & 2\\ 0 & -2 & 0\\ 0 & 0 & 3\end{bmatrix} \qquad (b)\begin{bmatrix}2 & 1 & 1\\ 0 & 1 & 0\\ 1 & 1 & 2\end{bmatrix} \qquad (c)\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$$

Find, by power method, the larger eigenvalue of the following matrices:

$$(a)\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix} \qquad (b)\begin{bmatrix}4 & 1\\ 1 & 3\end{bmatrix}$$

Find the largest eigenvalue and the corresponding eigenvector of the matrices:

$$(a)\begin{bmatrix}1 & 3 & -1\\ 3 & 2 & 4\\ -1 & 4 & 10\end{bmatrix} \qquad (b)\begin{bmatrix}1 & -3 & 2\\ 4 & 4 & -1\\ 6 & 3 & 5\end{bmatrix}$$

$(c)\begin{bmatrix}25 & 1 & 2\\ 1 & 3 & 0\\ 2 & 0 & -4\end{bmatrix}$ taking $[1, 0, 0]^T$ as initial eigenvector.

L- 6 EMPIRICAL LAWS AND CURVE-FITTING

. By the method of least squares, find the straight line that best fits the following data:

x:	1	2	3	4	5
y:	14	27	40	55	68

. In some determinations of the value v of carbon dioxide dissolved in a given volume of water at different temperatures θ, the following pairs of values were obtained:

$\theta = 0$	5	10	15
$v = 1.80$	1.45	1.18	1.00

Obtain by the method of least squares, a relation of the form $v = a + b\theta$ which best fits to these observations.

If V (km/hr) and R(kg/ton) are related by a relation of the type $R = a + bV^2$, find by the method of least squares a and b with the help of the following table:

$V =$	10	20	30	40	50
$R =$	8	10	15	21	30

Using the method of least squares fit the curve $y = ax + bx^2$ to following observations:

x:	1	2	3	4	5
y:	1.8	5.1	8.9	14.1	19.8

Obtain a relation of the form $y = kx^m$ for the following data by the method of least squares:

x:	1	2	3	4	5
y:	7.1	27.8	62.1	110	161

Fit the *exponential curve* $y = ae^{bx}$ to the following data:

x:	2	4	6	8
y:	25	38	56	84

L 7(1) FINITE DIFFERENCES

Write forward difference table if

x:	10	20	30	40
y:	1.1	2.0	4.4	7.9

Construct the table of differences for the data below:

x:	0	1	2	3	4
$f(x)$:	1.0	1.5	2.2	3.1	4.6

Evaluate $\Delta^3 f(2)$.

If $y = x^3 + x^2 - 2x + 1$,evaluate the values of y for $x = 0,1, 2, 3, 4, 5$ from the difference table. Find the value of y at $x = 6$ by extending the table and verify that same value is obtained by substitution.

Form a table of differences for the function $f(x) = x^3 + 5x - 7$ for $x = -1, 0, 1, 2, 3, 4, 5$.

Continue the table to obtain $f(6)$.

Express $x^3 - 2x^2 + x - 1$into factorial polynomial. Hence show that $\Delta^4 f(x) = 0$.

Express $3x^4 - 4x^3 + 6x^2 + 2x + 1$ as a factorial polynomial and find differences of all orders.

Find the first and second differences of $x^4 - 6x^3 + 11x^2 - 5x + 8$ with $h = 1$. Show that the fourth difference is constant.

Obtain the function whose first difference is (i) $2x^3 + 3x^2 - 5x + 4$.
(ii) $x^4 - 5x^3 + 3x + 4$.

Select the correct answer or fill up the blanks in the following questions:

1

. Which one is incorrect?

(a) $E = 1 + \Delta$ (b) $\Delta(5) = 0$

(c) $\Delta(f_1 + f_2) = \Delta f_1 + \Delta f_2$ (d) $\Delta(f_1 . f_2) = \Delta f_1 + \Delta f_2$.

2

. Given $x = 1\ 2\ 3$

$f(x) = 3\ 815$, then $\Delta^2 f(1) =$

(a) 3 (b) 4

(c) 2 (d) 1

3, 4

$\Delta\nabla =$

(a) $\nabla\Delta$ (b) $\nabla + \Delta$ (c) $\nabla - \Delta$.

Which one of the following results is correct:

(a) $\Delta x^n = nx^{n-1}$ (b) $\Delta x^{(n)} = nx^{(n-1)}$

(c) $\Delta^n e^x = e^x$ (d) $\Delta \cos x = -\sin x$.

5.

If $E^2 u_x = x^2$ and $h = 1$, then $u_x = \cdots$.

L-7 (2) INTERPOLATION

Using Newton 's forward formula, fin d the value of $f(1.6)$, if

x:	1	1.4	1.8	2.2
$f(x)$:	3.49	4.82	5.96	6.5

From the following table find y when $x = 1.85$ an d 2.4 by Newton's interpolation formula:

x:	1.7	1.8	1.9	2.0	2.1	2.2	2.3
$y = e^x$:	5.474	6.050	6.686	7.389	8.166	9.025	9.974

. Find $f(22)$ from the following data using Newton's backward formulae.

x:	20	25	30	35	40	45
$f(x)$:	354	332	291	260	231	204

. Find the number of men getting wages between Rs. 10 and 15 from the following data:

Wages in Rs:	0—10	10—20	20—30	30—40
Frequency:	9	30	35	42

Use Lagrange's interpolation formula to find the value of y when $x = 10$, if the following values of x and y are given:

x:	5	6	9	11
y:	12	13	14	16

The following table gives the viscosity of oil as a function of temperature. Use Lagrange's formula to find the viscosity of oil at a temperature of 140°.

Temp°:	110	130	160	190
Viscosity:	10.8	8.1	5.5	4.8