Homework : 13
Frequency Domain Filtering using Fast Discrete Fourier Transform (FDFT)

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Abstract

This experiment demonstrates the transformation of digital images from the spatial domain to the frequency domain using the **Fast Discrete Fourier Transform (FDFT)** and investigates the effects of applying **ideal low-pass, high-pass, and band-pass filters**. Five different images, each having three contrast levels (low, normal, and high), are analyzed to observe how frequency distribution and filtering behaviors vary with contrast.

Objective

- To understand how the Fast Discrete Fourier Transform (FDFT) converts images into frequency representation.
- To analyze the frequency spectrum characteristics for low-, normal-, and high-contrast images.
- To apply and compare **low-pass**, **high-pass**, and **band-pass** filters in the frequency domain and study their effects on image features.

Code Implementation

The full Python implementation used for FDFT computation and frequency-domain filtering can be accessed at:

```
GitHub Code Link: FDFT.py
```

Theoretical Background

An image can be expressed as a two-dimensional function $f(x, y)$. The 2D Discrete Fourier Transform (DFT) decomposes this image into its sinusoidal frequency components:

$$F(u, v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)\, e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$$

The magnitude $|F(u, v)|$ represents frequency strength and the phase $\arg(F(u, v))$ encodes spatial positioning. By shifting the zero-frequency (DC) component to the center using `fftshift()`, we obtain an intuitive visualization where:

- Center = low-frequency content (smooth brightness variations)
- Periphery = high-frequency content (edges and textures)

Filtering Principle: Applying a mask $H(u, v)$ to the spectrum corresponds to frequency-domain filtering:

$$G(u, v) = F(u, v) \cdot H(u, v)$$

and the filtered spatial image is recovered via the inverse transform:

$$g(x, y) = \mathcal{F}^{-1}\{G(u, v)\}$$

Methodology

1. **Image Acquisition:** Five base images were used — each with three versions (low-, normal-, and high-contrast), totaling fifteen images.
2. **Preprocessing:** Each image was converted to grayscale to simplify computation.
3. **Fourier Transformation:** The 2D FFT and its shifted form were computed to move the DC component to the center. Logarithmic magnitude $\log(1 + |F|)$ was used for display.
4. **Ideal Masks Construction:**
   - **Low-Pass Filter (LPF):** Circular mask allowing low frequencies (center).
   - **High-Pass Filter (HPF):** Complement of LPF allowing high frequencies (edges).
   - **Band-Pass Filter (BPF):** Ring-shaped mask passing mid-range frequencies.
5. **Filtering and Reconstruction:** Multiplying the DFT with each mask and applying the inverse FFT yielded filtered images.
6. **Visualization:** For each image, a 4×3 subplot was generated showing the original, its DFT, masks, filtered spectra, and corresponding reconstructed images.

Discussion of Results

Frequency Patterns in Different Contrast Levels

- **Low-Contrast Images:** Spectral energy is concentrated at the center, implying dominance of low-frequency components. Edge information (high-frequency) is weaker.
- **Normal-Contrast Images:** Moderate spread of energy across frequencies; both low and high components are visible.
- **High-Contrast Images:** Broader spectral spread, indicating enhanced high-frequency components due to stronger edges and textures.

Thus, higher contrast amplifies high-frequency details, while lower contrast compresses energy toward low frequencies.

Behavior of Frequency-Domain Filters

- **Low-Pass Filter (LPF):** Preserves smooth, large-scale variations while removing edges. Output images appear blurred and softened.
- **High-Pass Filter (HPF):** Retains edges and fine textures while suppressing smooth regions. The reconstructed images highlight boundaries but lose overall brightness.
- **Band-Pass Filter (BPF):** Retains intermediate details, often enhancing texture patterns or repetitive features while discarding both coarse and very fine information.

Interpretation

Low-contrast images show minimal change after LPF since their content is already dominated by low frequencies. High-contrast images produce stronger, sharper responses in HPF and BPF outputs. This confirms that increasing contrast shifts energy from the DC center toward outer high-frequency regions.

Implementation Notes

- Frequency spectra were visualized using $\log(1 + |F|)$ to reduce dynamic range and reveal low-magnitude regions.
- Circular ideal masks were normalized to the range $[0, 1]$ to prevent scaling effects.
- Radii were defined as fractions of the minimum image dimension for consistency across different resolutions.
- Although ideal (binary) filters provide clear frequency separation, they can cause ringing artifacts; Gaussian filters could smooth transitions.

Results: Filtered Output Figures

Below are the visual outputs for each of the 15 images (five base images with three contrast variants each). Each figure is presented inside a box with its label placed below the image.

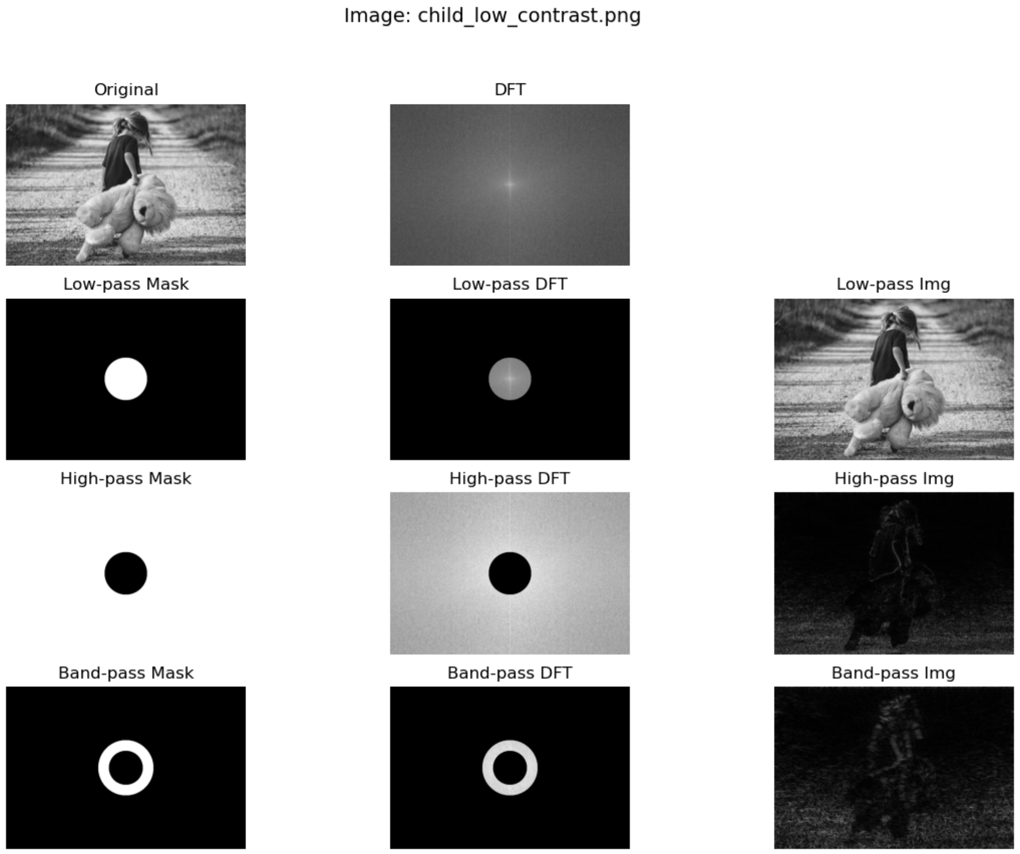

Figure 1: child_low_contrast.png

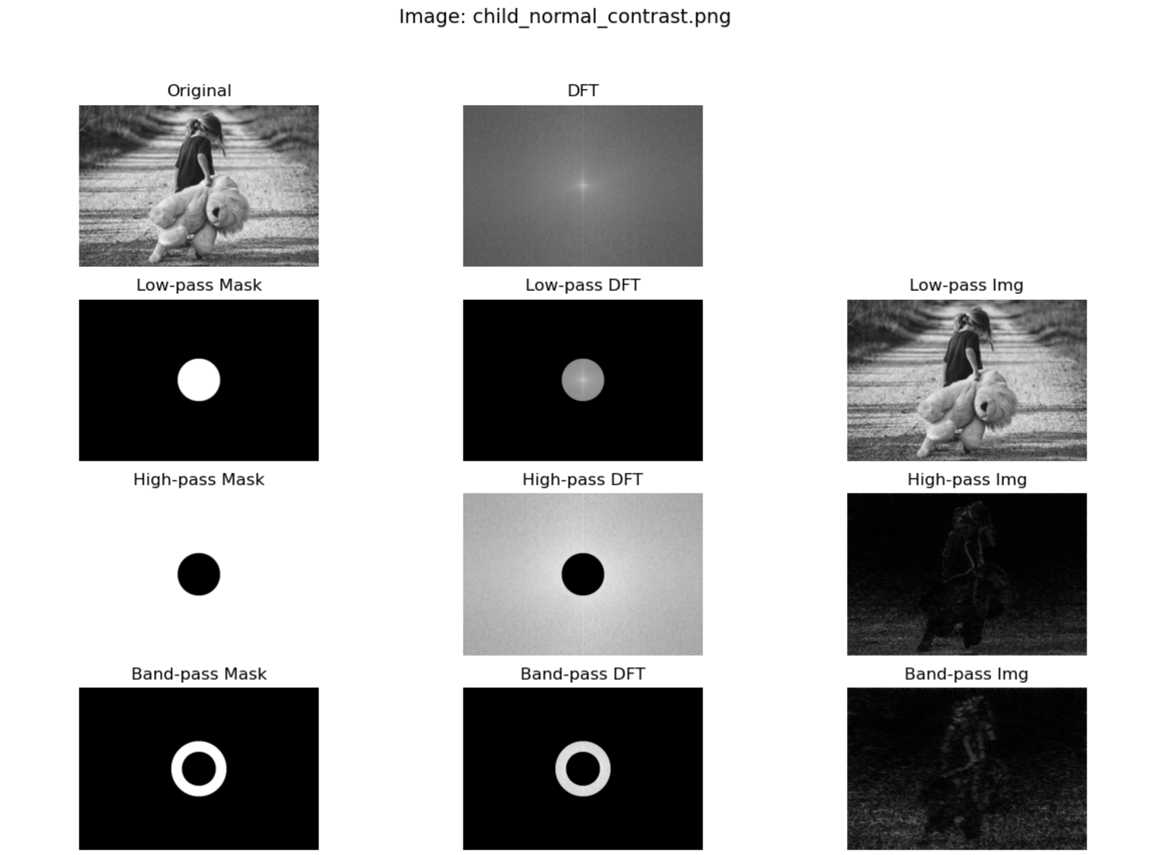

Figure 2: child_normal_contrast.png

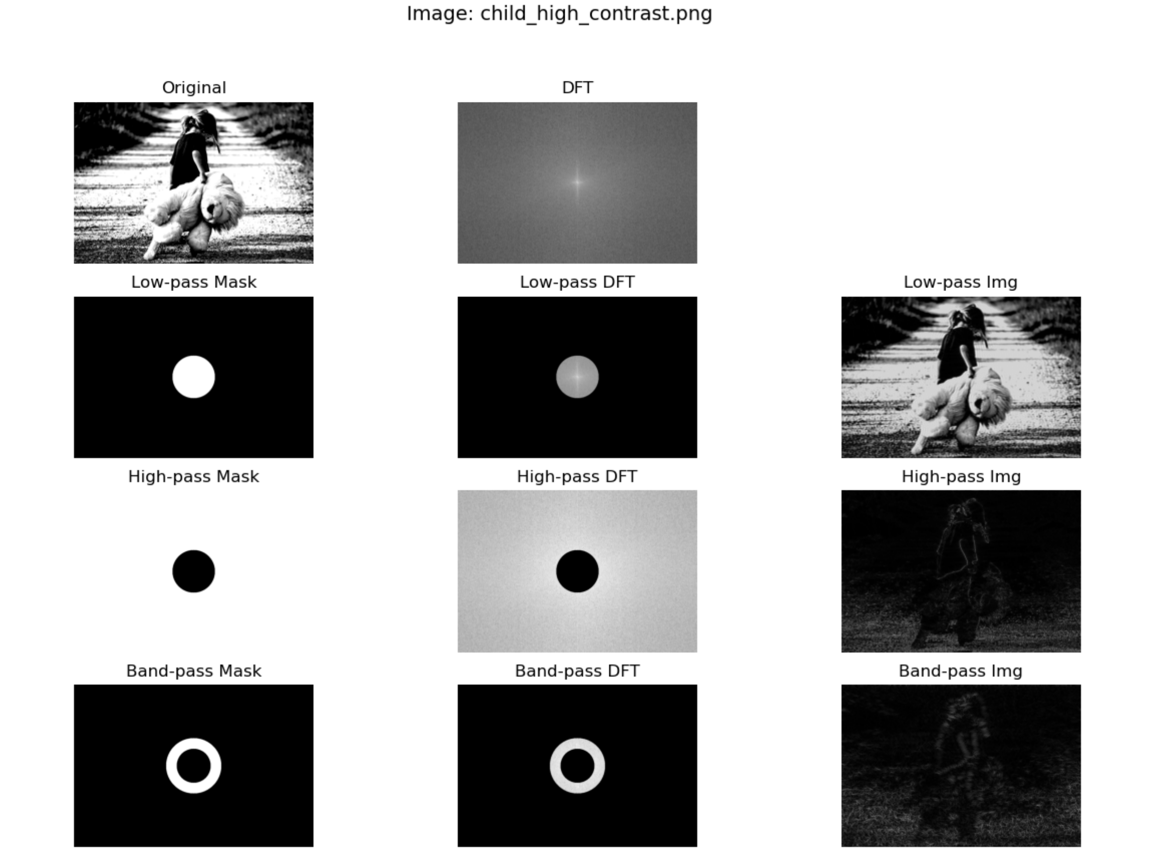

Figure 3: child_high_contrast.png

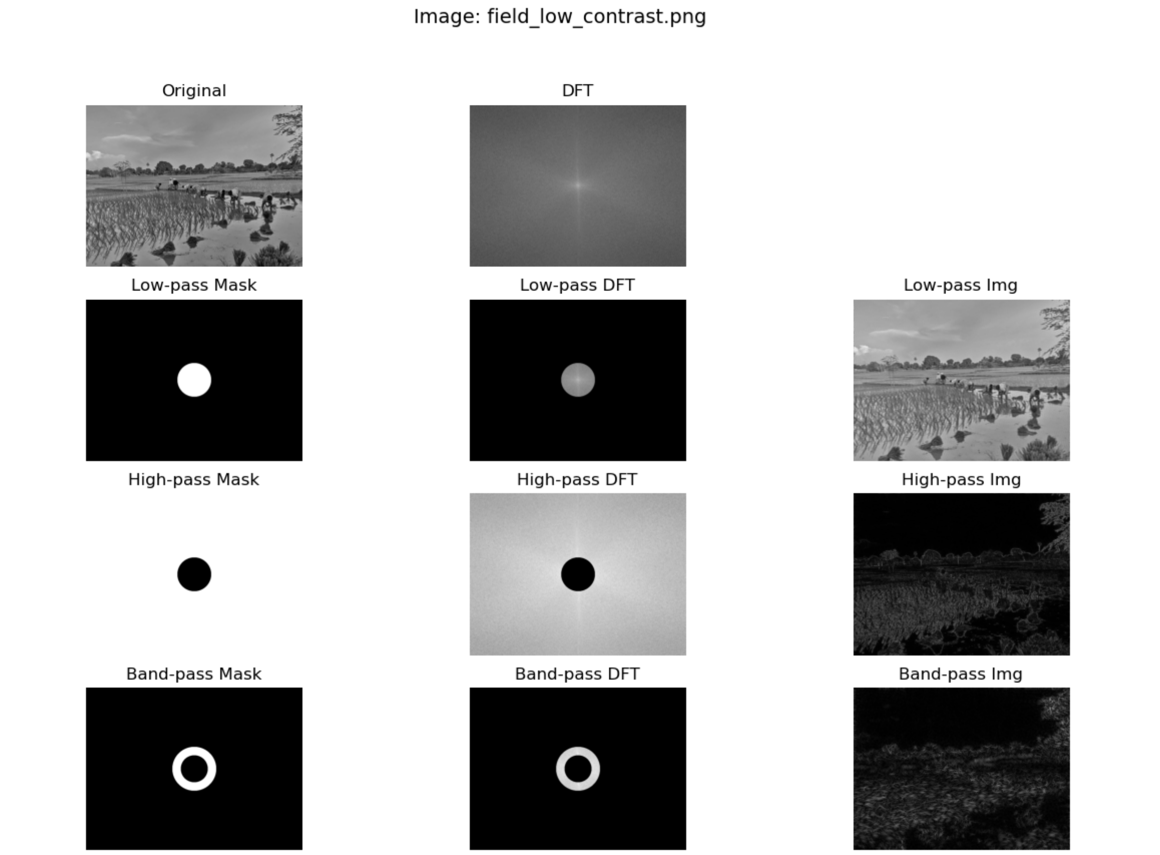

Figure 4: field_low_contrast.png

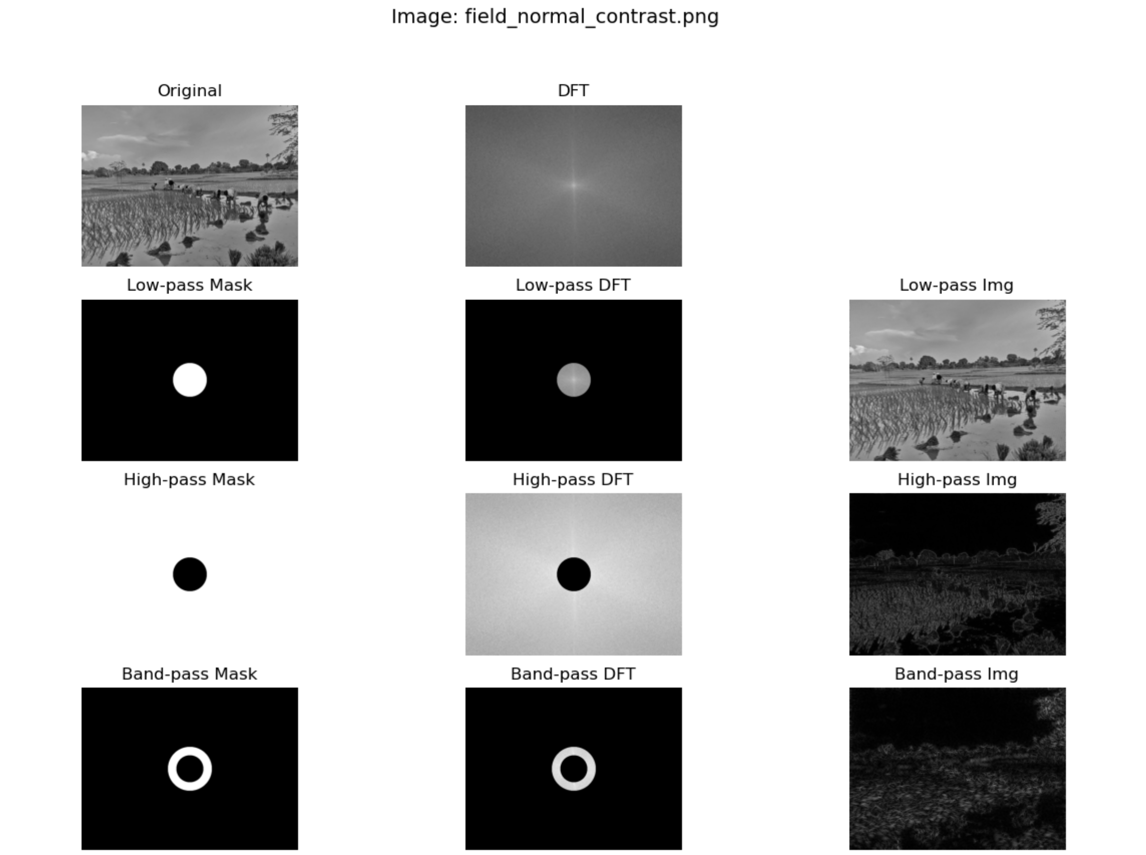

Figure 5: field_normal_contrast.png

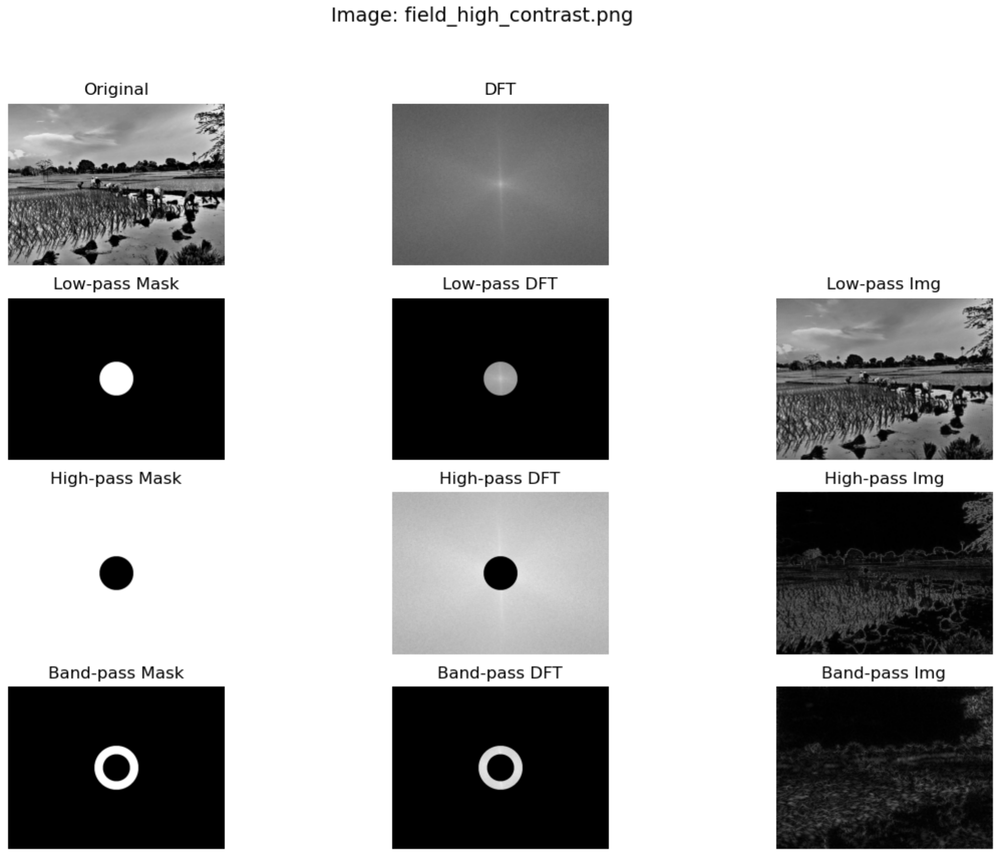

Figure 6: field_high_contrast.png

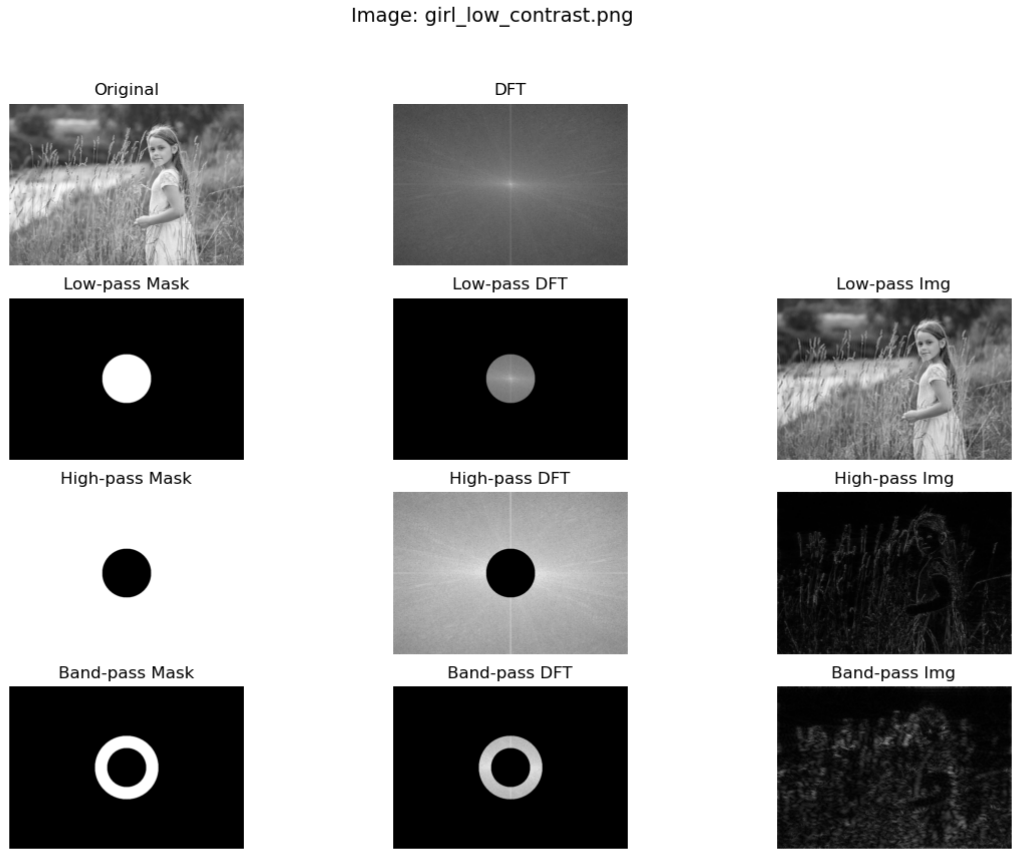

Figure 7: girl_low_contrast.png

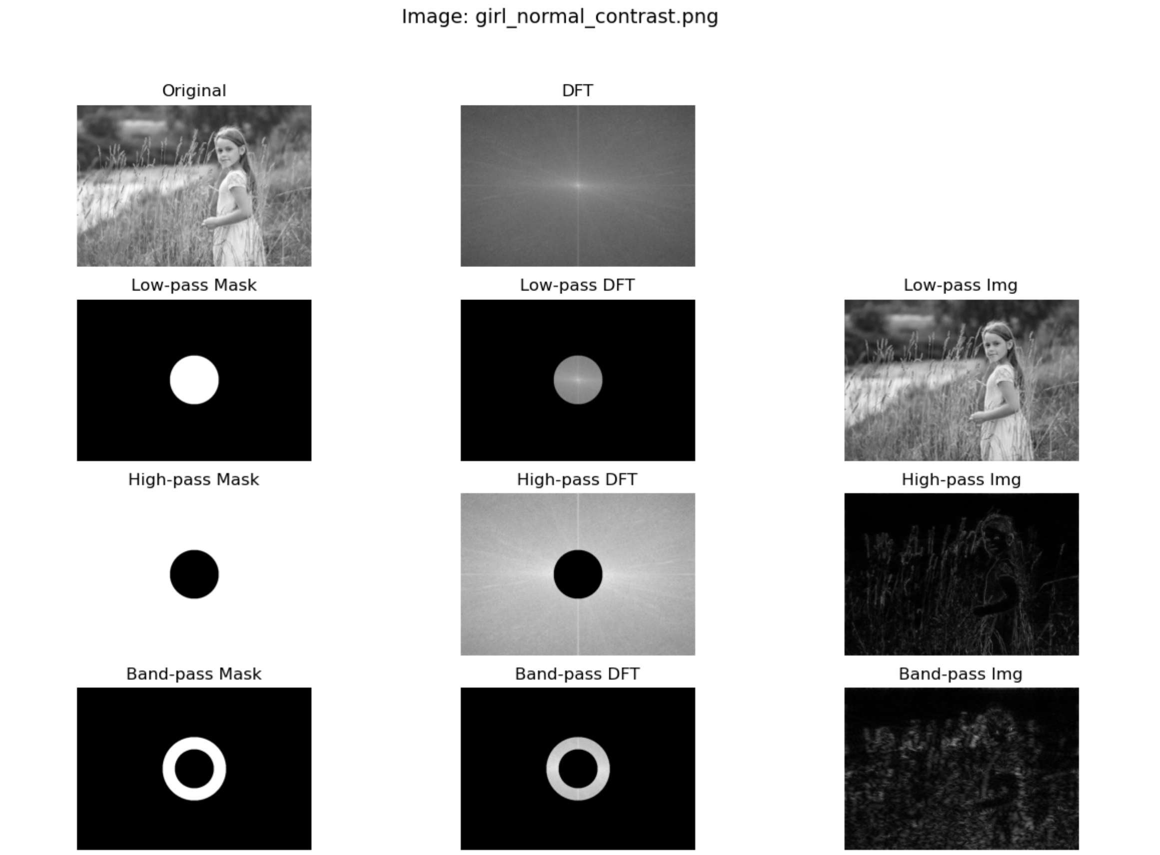

Figure 8: girl_normal_contrast.png

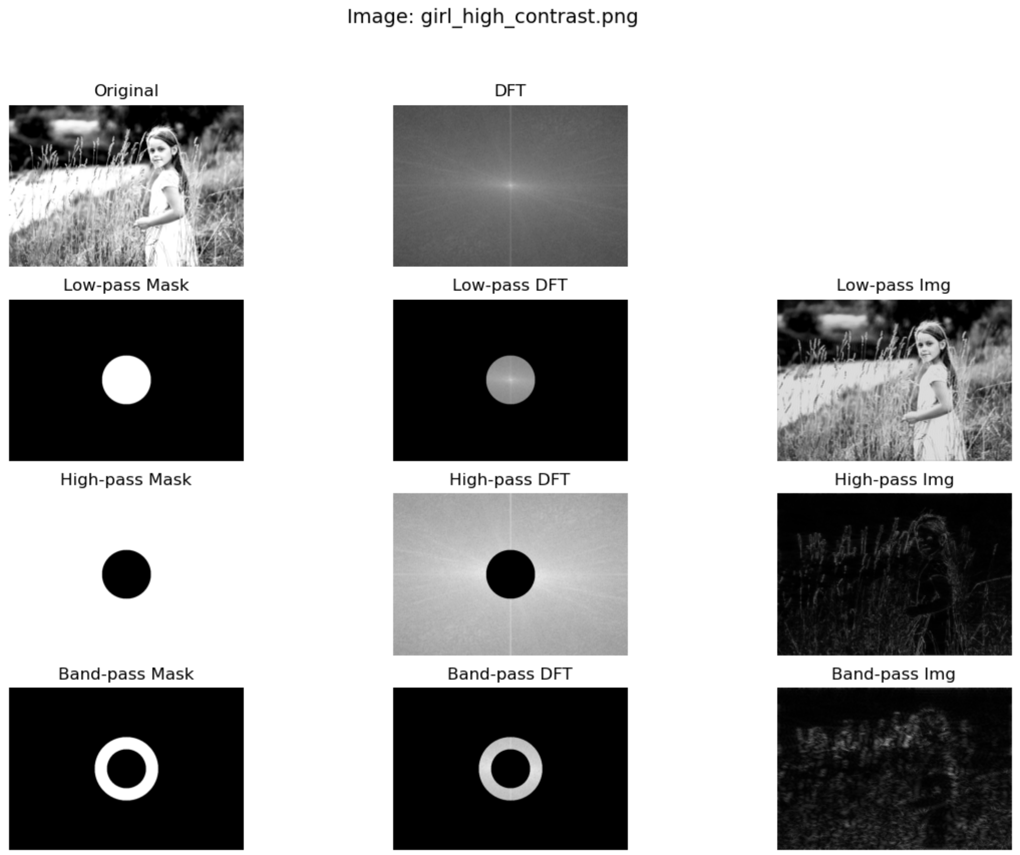

Figure 9: girl_high_contrast.png

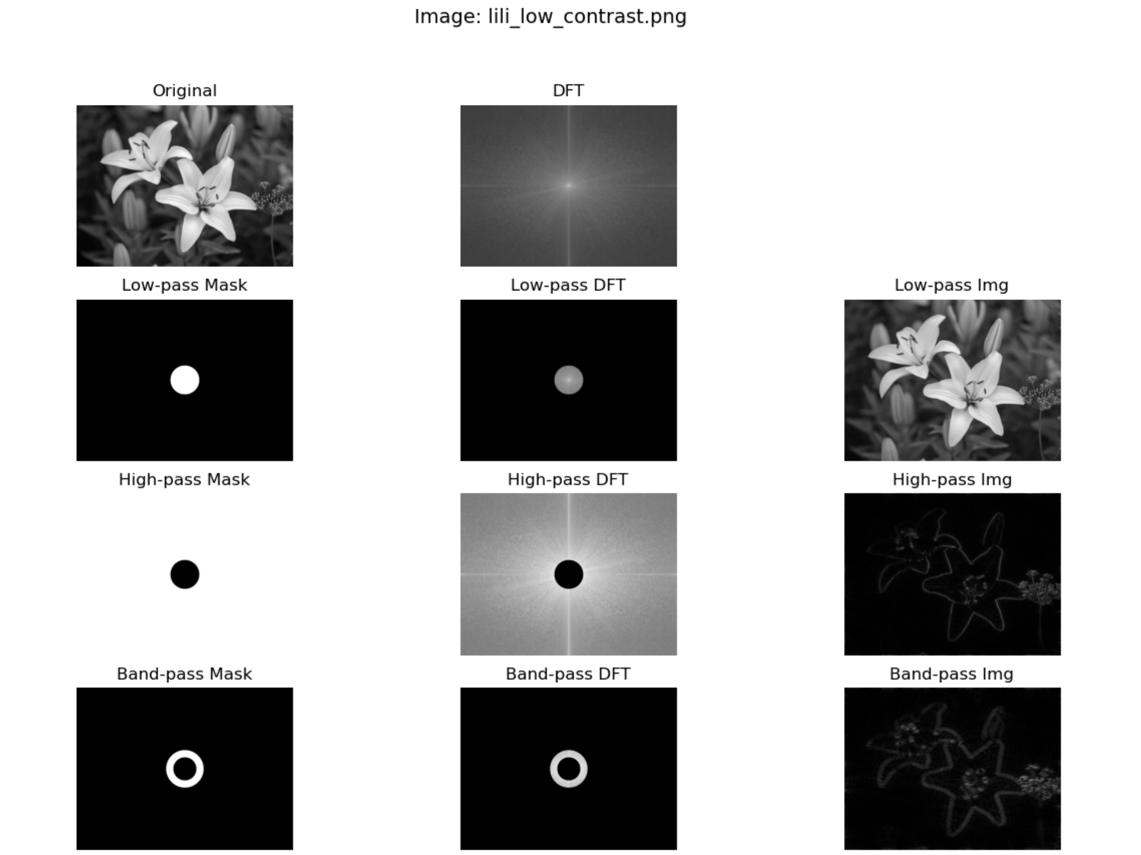

Figure 10: lili_low_contrast.png

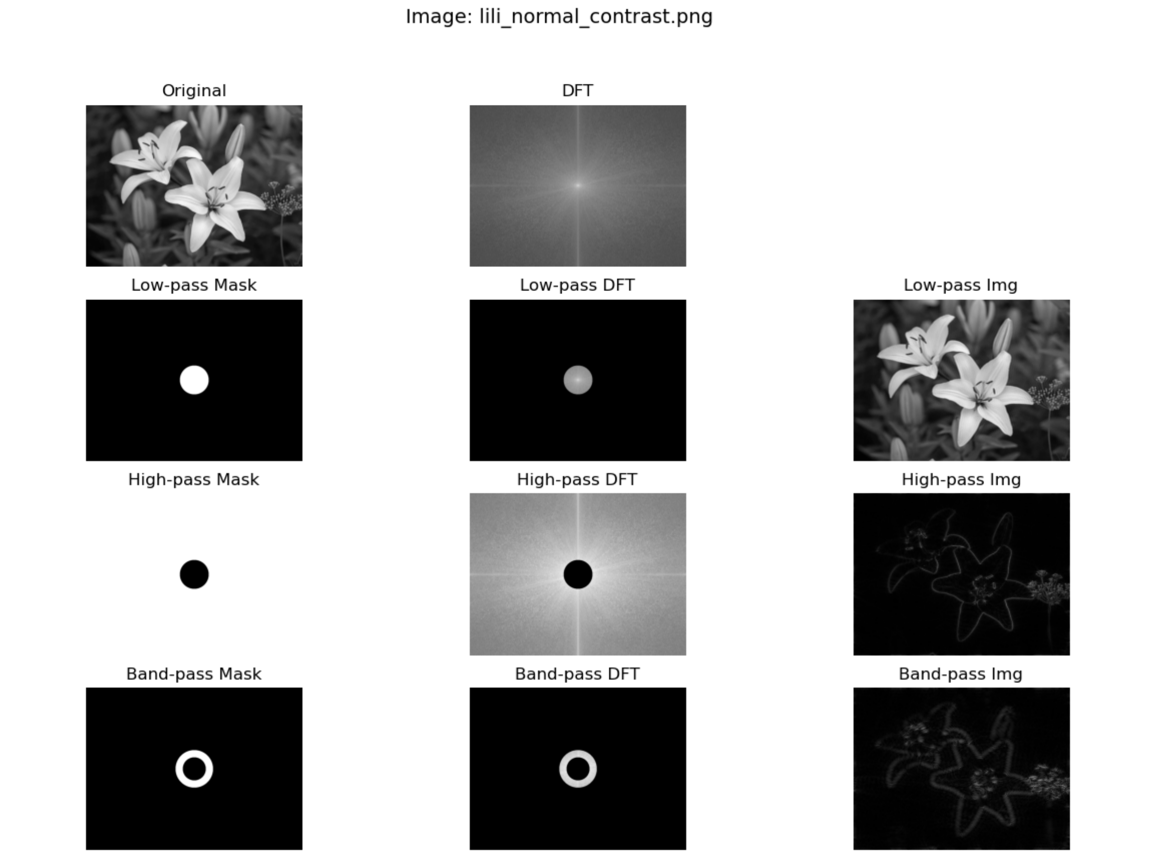

Figure 11: lili_normal_contrast.png

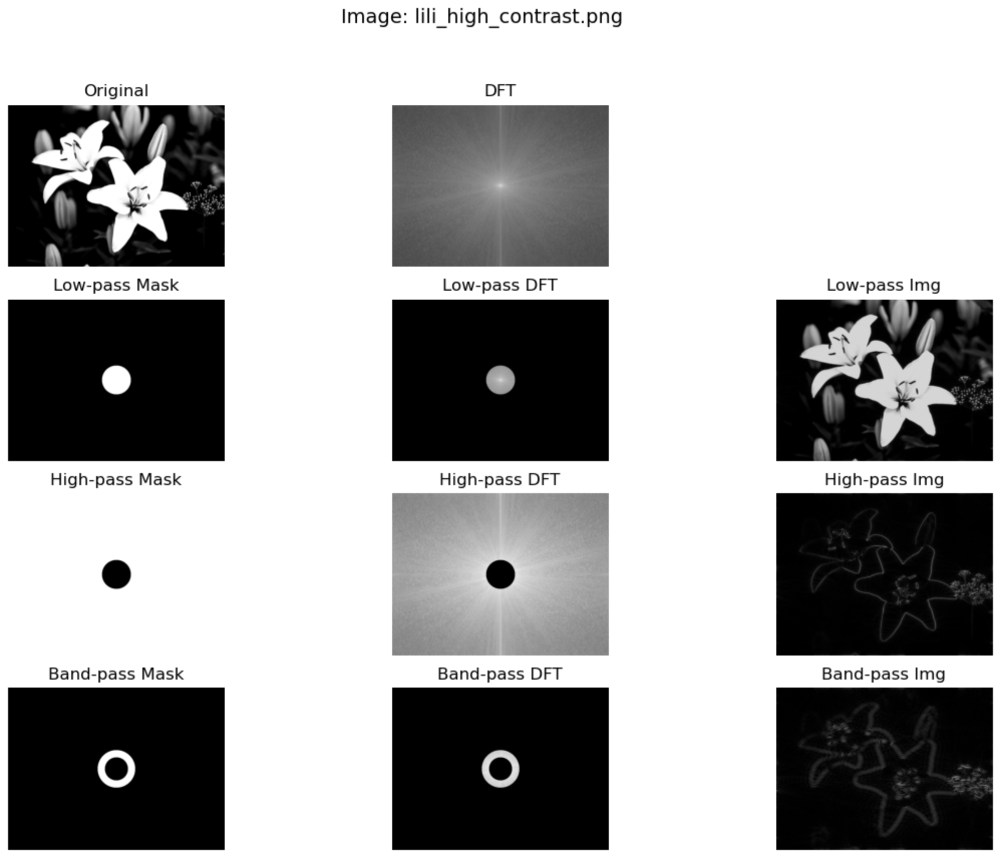

Figure 12: lili_high_contrast.png

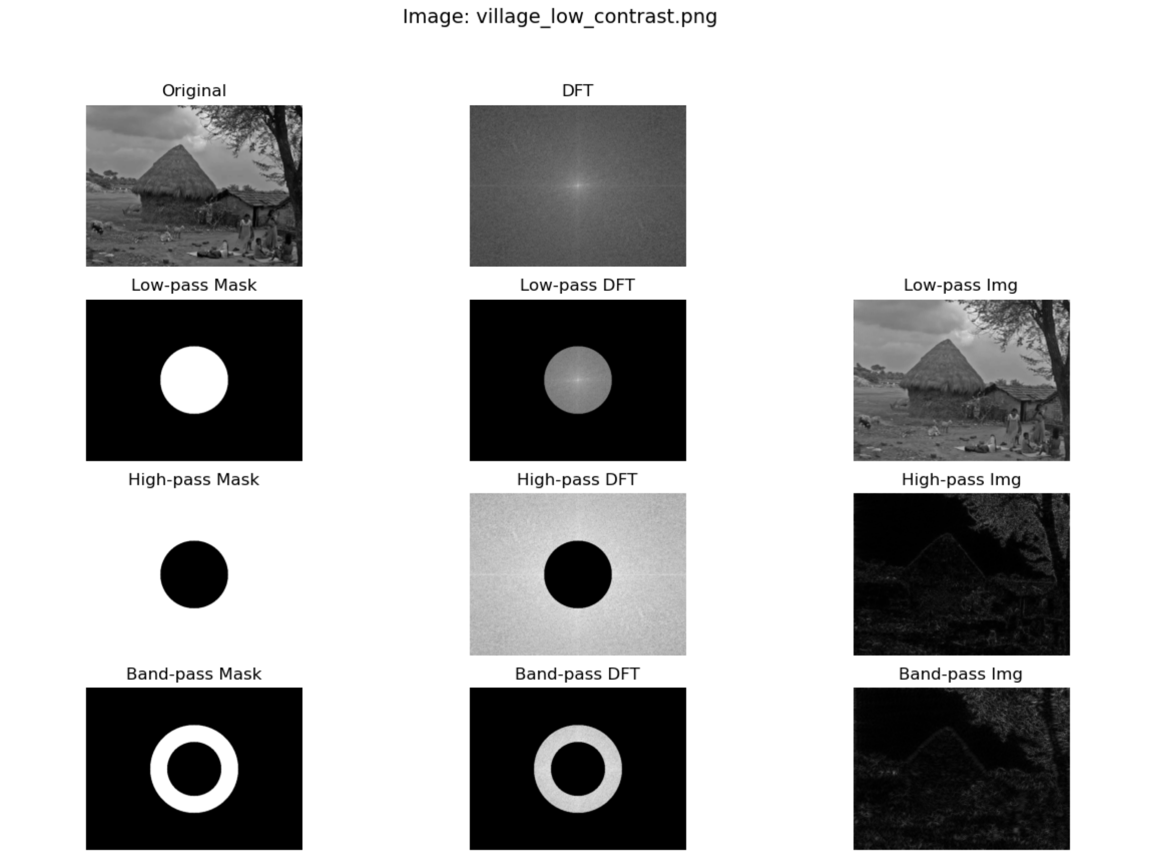

Figure 13: village_low_contrast.png

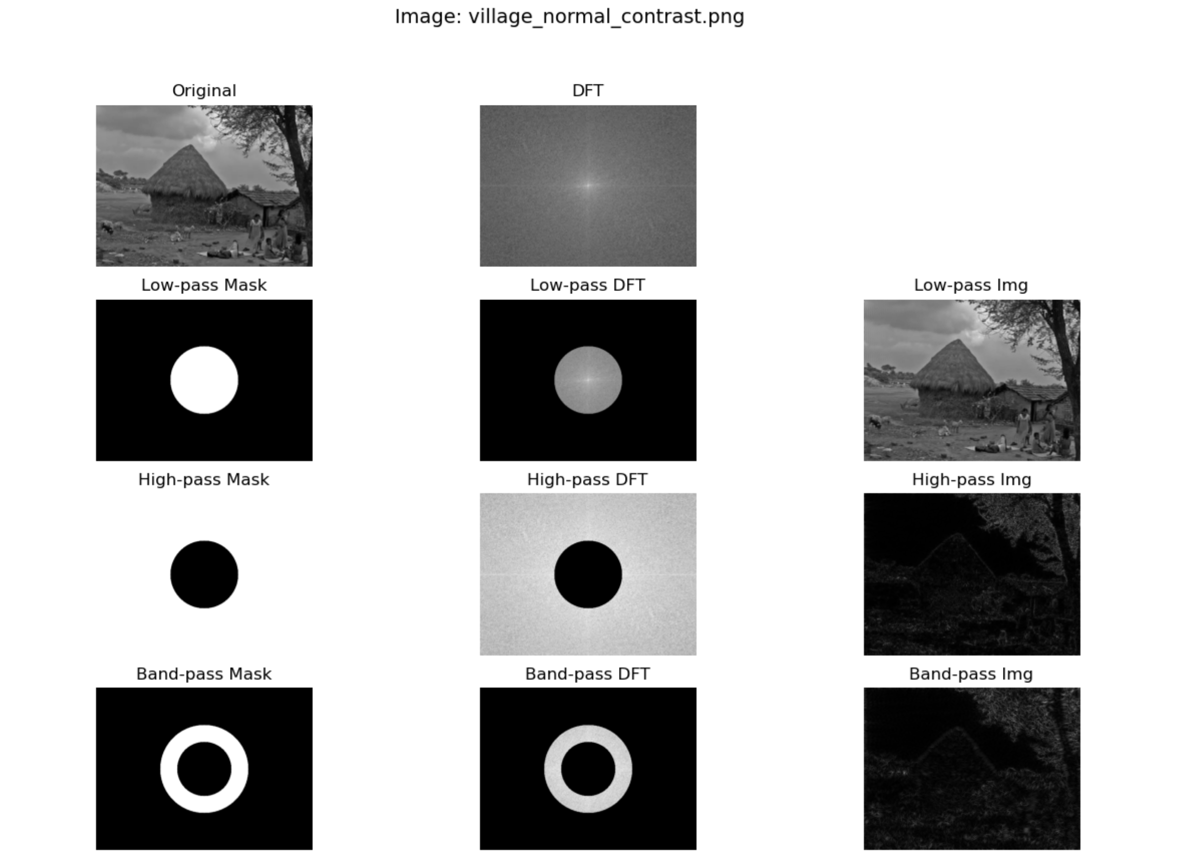

Figure 14: village_normal_contrast.png

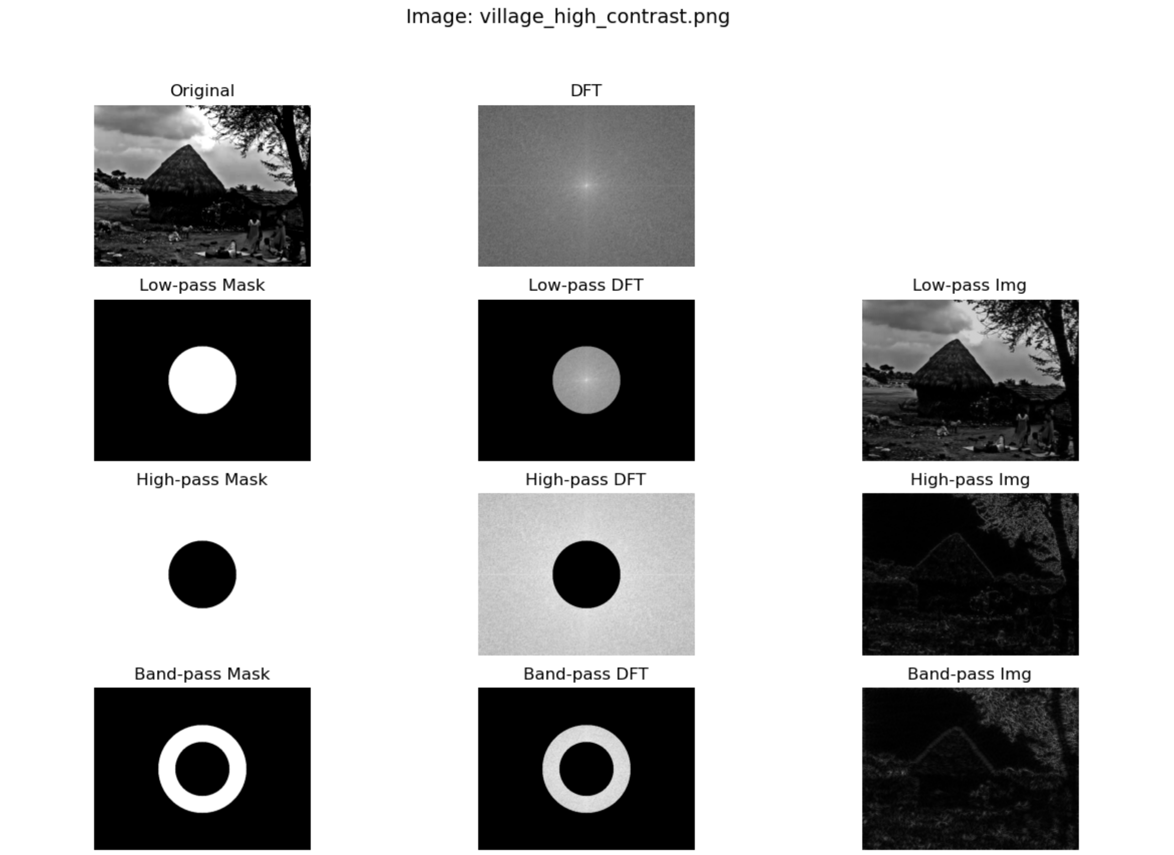

Figure 15: village_high_contrast.png

Conclusion

This experiment confirms that the Fast Discrete Fourier Transform (FDFT) effectively separates image information into distinct frequency components. Contrast variation directly influences spectral energy distribution:

- Low contrast $\Rightarrow$ energy concentrated near the center (low frequencies).
- High contrast $\Rightarrow$ broader spread of energy (strong high-frequency edges).

The low-pass, high-pass, and band-pass filters exhibit complementary behaviors—respectively isolating coarse brightness, edge details, and texture-level features. The outcomes illustrate the strong relationship between spatial contrast and its frequency-domain representation.