Homework : 15
Comparing Frequency Domain Filters: Butterworth, Gaussian, and Ideal Filters on Images with Varying Contrast

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Abstract

This report demonstrates the application of various frequency domain filters on digital images with low, normal, and high contrast. Specifically, it investigates the effects of applying Butterworth and Gaussian filters—low-pass, high-pass, and band-pass filters—on images, comparing their performance with ideal low-pass, high-pass, and band-pass filters. The effect of changing the value of 'n' in Butterworth filtering is also analyzed. The results are discussed in terms of the filtering's impact on the frequency spectrum and the visual characteristics of the images.

Objective

- To apply Butterworth filters (low-pass, high-pass, band-pass) and Gaussian filters (low-pass, high-pass, band-pass) to images with varying contrast levels (low, normal, high).
- To compare the effects of Butterworth filtering and Gaussian filtering with ideal low-pass, high-pass, and band-pass filtering.
- To analyze the effect of different values of 'n' in Butterworth filtering.

Theoretical Background

An image can be represented as a two-dimensional function $f(x, y)$. The 2D Discrete Fourier Transform (DFT) decomposes this image into its sinusoidal frequency components:

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

The magnitude $|F(u,v)|$ represents the frequency strength, and the phase $\arg(F(u,v))$ encodes spatial positioning. By shifting the zero-frequency (DC) component to the center using `fftshift()`, we obtain an intuitive visualization where:

- **Center:** Low-frequency content (smooth brightness variations).
- **Periphery:** High-frequency content (edges and textures).

Filtering Principle:

Applying a mask $H(u, v)$ to the spectrum corresponds to frequency-domain filtering:

$$G(u,v) = F(u,v) \cdot H(u,v)$$

The filtered spatial image is recovered via the inverse transform:

$$g(x,y) = \mathcal{F}^{-1}\{G(u,v)\}$$

Butterworth Filters:

The **Butterworth filter** is widely used for frequency-domain filtering due to its smooth transition characteristics. It can be used for low-pass, high-pass, and band-pass filtering. The general form of the **Butterworth filter** in the frequency domain is given by:

Low-pass Butterworth Filter:

$$H_{\text{low}}(D) = \frac{1}{1 + \left(\frac{D}{D_c}\right)^{2n}}$$

- D is the distance in the frequency domain. - D_c is the cutoff frequency. - n is the order of the filter, controlling the sharpness of the cutoff.

High-pass Butterworth Filter:

$$H_{\text{high}}(D) = 1 - H_{\text{low}}(D)$$

Band-pass Butterworth Filter:

$$H_{\text{band}}(D) = H_{\text{low}}(D) \cdot H_{\text{high}}(D)$$

Gaussian Filters:

The **Gaussian filter** is based on the Gaussian distribution and provides a smoother transition between frequencies compared to the Butterworth filter. Gaussian filters are often used when a smoother cutoff is desired, as they minimize sharp transitions.

Low-pass Gaussian Filter:

$$H_{\text{low}}(D) = e^{-\frac{D^2}{2\sigma^2}}$$

- D is the distance in the frequency domain. - σ is the standard deviation of the Gaussian function, controlling the width of the filter's passband.

High-pass Gaussian Filter:

$$H_{\text{high}}(D) = 1 - H_{\text{low}}(D)$$

Band-pass Gaussian Filter:

$$H_{\text{band}}(D) = e^{-\frac{D^2}{2\sigma_{\text{high}}^2}} \cdot e^{\frac{D^2}{2\sigma_{\text{low}}^2}}$$

This filter passes a specific range of frequencies, blocking both low and high frequencies.

Code Implementation

The full Python implementation used for FDFT computation and frequency-domain filtering can be accessed at the following GitHub link:

`GitHub Code Link`

Result and Output Images

Below are the images processed with frequency-domain filters. These images show the effects of applying different filters (Butterworth, Gaussian, and ideal) to images with varying contrast levels.

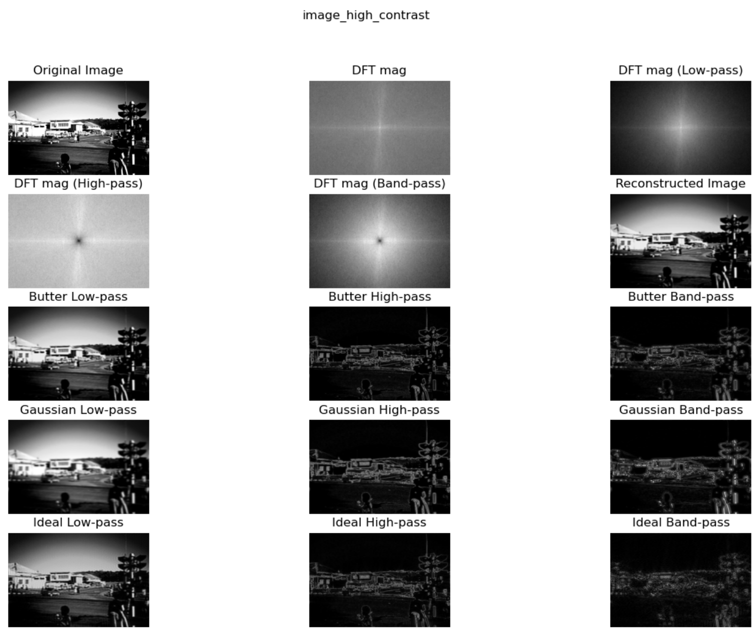

Figure 1: High Contrast Image

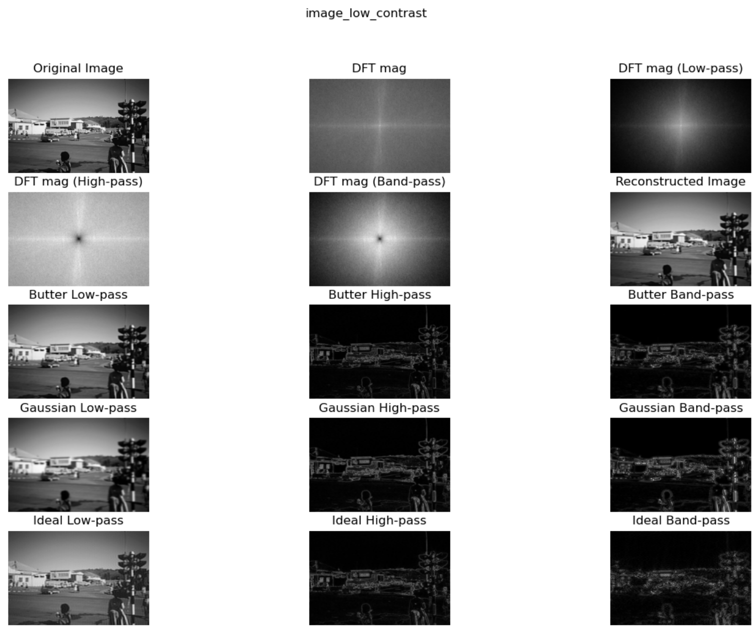

Figure 2: Low Contrast Image

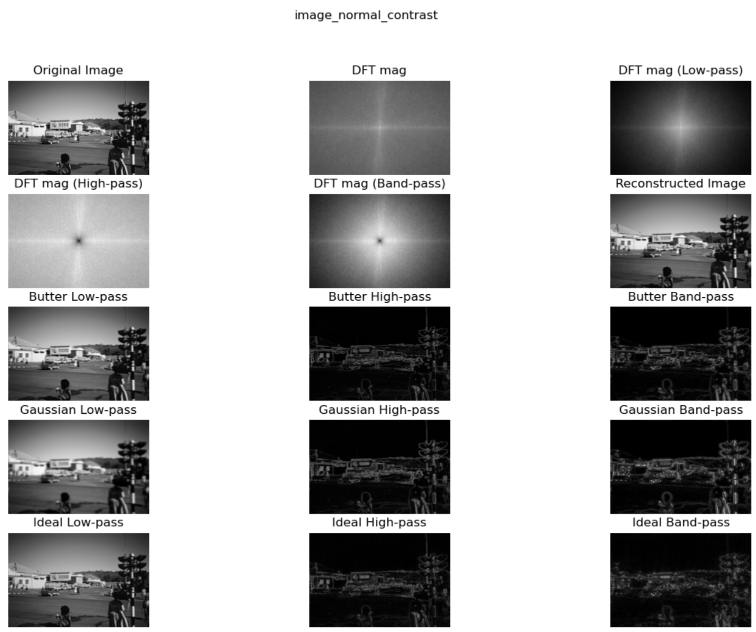

Figure 3: Normal Contrast Image

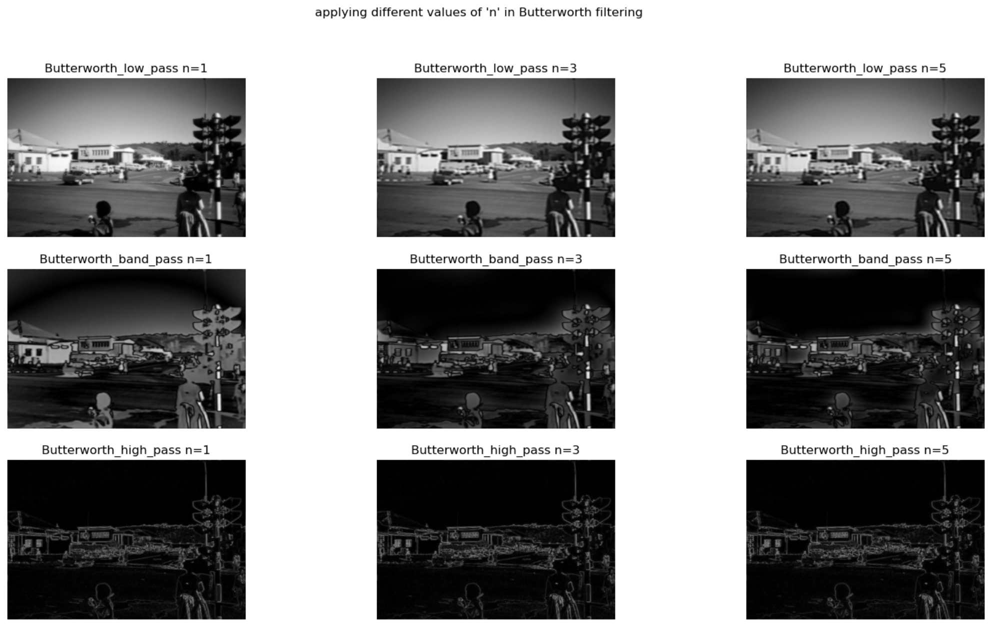

Figure 4: Applying Different Values of ‘n‘ in Butterworth Filtering

Discussion of Results

Frequency Patterns in Different Contrast Levels

The frequency patterns for images with varying contrast levels are as follows:

- **Low-Contrast Images:** Spectral energy is concentrated at the center, implying dominance of low-frequency components. Edge information (high-frequency) is weaker.
- **Normal-Contrast Images:** Moderate spread of energy across frequencies; both low and high components are visible.
- **High-Contrast Images:** Broader spectral spread, indicating enhanced high-frequency components due to stronger edges and textures.

Thus, higher contrast amplifies high-frequency details, while lower contrast compresses energy toward low frequencies.

Effect of Frequency-Domain Filters

- **Low-Pass Filter (LPF):** Smoothens the image, removing high-frequency details like edges.
- **High-Pass Filter (HPF):** Enhances edges and fine details while removing low-frequency information, resulting in sharper images.
- **Band-Pass Filter (BPF):** Retains mid-range frequency components, providing a balance between smooth and fine details.

Comparison Between Butterworth and Gaussian Filters with Ideal Filters

Butterworth and Gaussian filters offer different characteristics compared to ideal filters:

- **Butterworth Filters:** Provide a smooth transition between passed and blocked frequencies, avoiding the sharp cutoff of ideal filters. This results in less ringing.

- **Gaussian Filters:** Similar to Butterworth filters, but with an even smoother transition and less sharpness at the cutoff.
- **Ideal Filters:** Have an abrupt transition that can introduce ringing artifacts in the reconstructed image.

Effect of Varying ‘n‘ in Butterworth Filtering

The parameter ‘n‘ in Butterworth filtering determines the sharpness of the filter. A higher ‘n‘ results in a sharper cutoff, which means more high-frequency details are preserved in high-pass filtering and more low-frequency details are preserved in low-pass filtering. A lower ‘n‘ produces a more gradual transition between passed and blocked frequencies.

Conclusion

The experiment demonstrates the effectiveness of different frequency-domain filters, including Butterworth and Gaussian filters, in modifying image features. The Butterworth filters provide a sharper transition compared to the Gaussian filters, while the ideal filters show sharp cutoffs that can introduce artifacts. The contrast level of the image plays a crucial role in determining how the frequency components are distributed, with high-contrast images emphasizing high-frequency details.