

Binary, Octal and Hexadecimal Numbers

October 30, 2023

Administrative Notes

Project 1 is due next Monday - November 6

Next week is exam week, which means:

- No discussion NEXT week
- Sample exam 2 will be out later this week

Number bases

As humans, we tend to count things by powers of 10: 10, 100, 1000, ..

We have only ten symbols, or digits, used to count

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Why? Because most humans have ten digits - fingers, or toes - and that made it easy to keep track of things

While that's the best way for humans to count, it's not the best way for machines to count

Why not?

Most computers (and similar machines) are made up of electrical switches.

Electrical switches have two states: **ON** and **OFF**

We can say that if something is ON it has the value 1 and if it is OFF it has the value 0.

This is called “binary” numbering. We only need two digits, 0 and 1, to represent all numbers, using powers of 2. So, there are only two binary digits

“**B**inary digit” was a pain to say all the time, so it got shortened to ... “bit”

There are 10 types of people in the world - those who understand binary and those who don't.

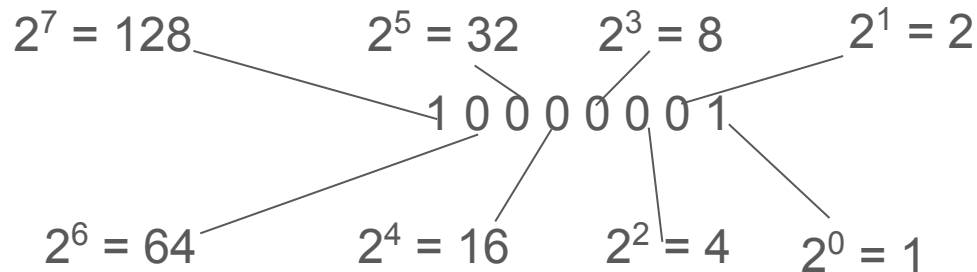
$$147355 = 5 * 10^0 + 5 * 10^1 + 3 * 10^2 + 7 * 10^3 + 4 * 10^4 + 1 * 10^5$$

A binary number

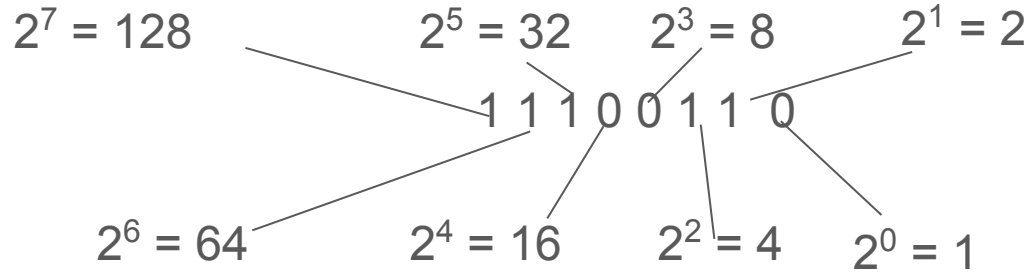
1 0 0 0 0 0 0 1 in base 10 = 129

What's its value in human-speak i.e. decimal?

Places mean the same as in decimal - the “least significant” is on the right and each place to the left is one power of two greater



Another example



$$2 + 4 + 32 + 64 + 128 = 230$$

Binary Math - just like decimal math (mostly)

Addition: start on the right; add the numbers in the column and carry when needed

$$\begin{array}{r} 1010 \\ + \quad 110 \\ \hline 10000 \end{array}$$

Subtraction: start on the right; borrow from the next place over when needed

$$\begin{array}{r} 1010 \\ - 101 \\ \hline 101 \end{array}$$

Multiplication: go column by column; add products

So how do you convert from decimal to binary?

Suppose I gave you the decimal number 10,000,001 and asked you to tell me what it is in binary?

1001

You have to subtract powers of two from it until you get zero.

Huh? $10,000,001 - 8,388,608 = 1,611,393 - 1,048,576 = 562,817$

What is the largest power of two that is less than 10,000,001?

n	2 ⁿ	n	2 ⁿ	n	2 ⁿ
1	2	6	64	22	4,194,304
2	4	10	1024	23	8,388,608
3	8	15	32768	24	16,777,216
4	16	20	1,048,576		
5	32	21	2,097,152		

Converting decimal to binary:

10,000,001 decimal: write down 1 in the 2^{23} column, subtract 8,388,608, from 10,000,001 = 1,611,393. The largest power of 2 that's less than that is 2^{20} . Write down zeroes in the next two columns, and then put a 1 in the 2^{20} column. Subtract $1,611,393 - 1,048,576 = 562,817$.

Keep doing this, and you eventually wind up with:

100110001001011010000001

Fortunately, Python makes this somewhat easier

To indicate that a number represents a binary number, preface it with “0b”

```
x = 0b110111011
```

Tells python you mean this to be the binary number, NOT the decimal number 110,111,011

To convert decimal numbers to Binary, use the built-in Python function “bin”.

```
i = 54
```

```
bin_i = bin(i)
```

As before, we can convert (almost) anything to an integer - base 10 - by using `int()`

Awkward! That's big number and hard to deal with

Luckily, there's another numbering system: base 16, or "hexadecimal"

- "hex" for short

We need 16 different symbols to represent hexadecimal "digits"

We have 0,1,2,3,4,5,6,7,8,9 - that's 10. We need six more. So we use the first six letters of the alphabet.

A in hex = 10 in decimal; B = 11; C = 12; D = 13; E = 14 and F = 15. That's all we need, because 16 in decimal is 10 in hex. We go by powers of 16

Converting Decimal to Hex

The algorithm is similar to binary

Fortunately for us, there's a built-in function in Python3 that does the conversion for us

`hex(int)` takes a decimal integer value and returns its hexadecimal value

`0x` means a hexadecimal number

$$\begin{aligned} 0x1234 \text{ to decimal} &= 4 * 1 + 3 * 16 + 2 * 16^{**2} + \\ &1 * 16^{**3} \\ &4 * 1 + 3 * 16 + 2 * 256 + 1 * 4096 \end{aligned}$$

Ox ab123

3 * 16 to the 0 power ($3 * 1$)

2 * 16 to the 1 power ($3 * 16 = 48$)

1 * 16 to the 2nd power ($2 * 256 = 512$)

B or 11 * 16 to the 3rd power ($11 * 4096 = 45056$)

A or 10 * 16 to the 4th power ($10 * 65,536 = 655,360$)

$3 + 48 + 512 + 45,056 + 655,360 =$

Converting binary to hexadecimal

This is actually easier - and more important

Four binary digits - four bits - together represent one hexadecimal digit, because four bits can represent exactly 16 different values

So you simply replace four bits with their hex equivalent

1001 1000 1001 0110 1000 0001

Start from the left - 1001 = 9 hex; 1000 = 8 hex; 1001 = 9 hex; 0110 = 6 hex; 1000 = 8 hex; 0001 = 1 hex. So the binary string above, converted into hex, is:

0x989681

A Handy Conversion Table

Binary	Hex	Decimal	Binary	Hex	Decimal
0000	0	0	1000	8	8
0001	1	1	1001	9	9
0010	2	2	1010	A	10
0011	3	3	1011	B	11
0100	4	4	1100	C	12
0101	5	5	1101	D	13
0110	6	6	1110	E	14
0111	7	7	1111	F	15

Octal is base 8

Octal digits are 0,1,2, 3, 4, 5, 6, and 7

Octal numbers are represented in Python as 0o(numbers)

That's a 0, then a lower-case o

Representation and examples

In python - and most other languages - hex numbers are prefaced with “0x” to indicate that they are NOT decimal numbers

Let's use the built-in `hex()` function to do some conversions