## ciphart —

# memory-hard key derivation with easier measurable security

caveman<sup>2</sup> 2021-02-13 14:03:13+00:00

argon2<sup>2</sup> is mostly nice, but trying to interpret its contribution to the protection against password brute-forcing attacks remains more difficult than it should be. this vagueness is a problem that is not limited to *argon2*, but also shared with every other key derivation function that i've known so far.

when one uses argon2, his derived key will surely have superior protection against password brute-forcing attacks, but by how much? to answer this, one would need to survey the industry that manufactures application-specific integrated circuits (asics) to obtain a map between time and money, in order to get an estimation on how much would it cost the adversary to discover the password in a given time window.

while the approach of surveying the asics industry is not wrong, it is largely subjective, with expensive housekeeping, and practically leads the user to rely on vague foundations to build his security on. this vagueness is not nice, and it would be better if we had an objective measure to quantify the security of our memory-hard key derivation functions.

resolving this vagueness is not a mere luxury to have, but a necessity for maximising survival, because it hinders the process of studying the cost-value of memory-hard key derivation functions, which, effectively, increases the risk of having a false sense of security.

so i propose *ciphart* — a memory-hard key derivation function with a security contribution that is measured in a unit that i call *caveman's entropy bits*. this unit is measured objectively and is guaranteed to be true irrespective of whatever alien technology that the adversary might have.

libciphart<sup>3</sup> is a library that implements *ciphart* very closely to this paper, without much fluff. this should make integrating *ciphart* into other systems more convenient.

ciphart<sup>4</sup> is an application for encrypting and decrypting files that makes use of libciphart. this application is intended for use by end-users or scripts, henceforth it has some fluff to treat mankind with dignity.

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## 1 shannon's entropy

we've got password p with H(p) many shannon's entropy bits worth of information in it. so what does this mean?

fundamentally, it means that, on average, we'd need to ask H(p) many perfect binary questions<sup>5</sup> in order to fully resolve all ambiguities about p; i.e. to fully get every bit of p.

but people use it to do less orthodox things, such as quantifying the amount of security p has against, say, brute-forcing attacks.

#### 1.1 an unorthodox use

say that we've got a 8V bit key  $k \leftarrow \mathtt{hash}(p||s,8V)$ , derived from password p, where s is a salt, and attacker has a cipher-text block  $m_0 \leftarrow \mathtt{enc}(\text{"goodbye}, \text{world"}, 0, k)^6$ . attacker's goal is to eventually decrypt the sequence  $m_0, m_1, \ldots$  back to original plain-text by using  $\mathtt{enc}$ 's corresponding  $\mathtt{dec}$  function. the attacker has these options:

1. try to decrypt  $m_0$ , each time with a new key, until it results in "goodbye, world". i.e. test whether "goodbye, world" =  $dec(m_0, 0, k)$  is true. this way the attacker will find k but not p. this will require him to perform  $2^{8V}$  dec calls in order to guarantee

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<sup>&</sup>lt;sup>2</sup>https://github.com/P-H-C/phc-winner-argon2

<sup>3</sup>https://github.com/Al-Caveman/libciphart

<sup>4</sup>https://github.com/Al-Caveman/ciphart

 $<sup>^5\</sup>mathrm{one}$  which, if answered, and on average, gets the search space reduced in half.

<sup>&</sup>lt;sup>6</sup> "goodbye, world" is to be used ever since dennis ritchie passed away. the colourful "Hello, world!", including the capitalised first letter, was only valid when he was alive. despite his achievements, he died unnoticed by almost everyone, as a friend found him dead in his house a week after he was dead already. steve jobs, who did almost nothing really useful, got loads of attention, some of whom even called him the messiah of computing. this is wrong, and must change. henceforth i propose: all "Hello, world!" s to become "goodbye, world"s, and all caps, except constants, to be lowered and remain so to eternity. using caps for other than constants is useless anyway.

finding k. or,  $2^{8V}/2$  on average<sup>7</sup>. both of which are insanely slow and would take many centuries to complete. e.g. my laptop would take  $4.28 \times 10^{58}$  many centuries in order to increment a mere counter for  $2^{8V}$  times. so this is not practical.

adversary's cost<sup>8</sup> in this option is:

$$2^{8V} \begin{pmatrix} \operatorname{cost}(\operatorname{dec}) \\ + \operatorname{cost}(\operatorname{if} c_0 = \operatorname{"goodbye, world"}) \end{pmatrix}$$
 (1)

- 2. if enc is broken enough by cryptanalysis, then using approach in item 1 could become feasible by finding a much cheaper decryption function dec than dec. but only an idiot would use a broken enc that has dec, and such people need a different type of help than what i try to offer here. so this option is off-topic.
- 3. explore key candidates by trying different passwords. while each step is more expensive than those in item 1 as it also requires calling **hash** before **dec**, the total number of iterations is  $2^{H(p)}$ , which is usually small enough to make this strategy practically viable for the adversary to bet on. in this paper i want to enhance our security against this option. there are subscenarios:
  - (a) attacker doesn't know k. here, every time the attacker calls hash to generate a key from a candidate password, he also needs to call dec in order to see if it leads to output "goodbye, world" so that he knows that the key that he found is actually k, so that he goes on to decrypt  $m_0, m_1, \ldots$  his cost<sup>9</sup> in this sub-scenario is:

$$2^{H(p)} \begin{pmatrix} \operatorname{cost}(\operatorname{hash}) + \operatorname{cost}(\operatorname{dec}) \\ + \operatorname{cost}(\operatorname{if} c_0 = \operatorname{"goodbye..."}) \end{pmatrix}$$
(2)

(b) attacker knows k. here, he doesn't need to call dec, so his  $cost^{10}$  is:

$$2^{H(p)} \left( \texttt{cost}(\texttt{hash}) + \texttt{cost}(\text{if } \hat{k} = k) \right) \tag{3}$$

#### 1.1.1 calculating the security

**definition 1.** the security of a system is the cost of the cheapest method that can break it.

i think ways to go forward, to make total costs in (1), (2) and (3) calculable so that we can get an understanding of the quantity of security that we have, are:

• estimate the cost function. e.g. by surveying the asics industry to get an idea how much money it costs to get a given key, or password, space brute-forced within a target time frame<sup>11</sup>.

this has expensive housekeeping as one needs to keep a constant eye on the industry of asic manufacturing. plus it is too heuristic-y to build security on. until i learn more about the theoretical cost guarantees about the involved function calls, i'm going to ignore this option.

not saying that it's a bad idea. in fact i think it's the most accurate approach to measure security, as i think security against breakage *is* the cost of breaking it. it is just that this approach seems to be too expensive for most people to adopt.

• ignore anything that has no cryptographic guarantee. so, in (1), cryptography guarantees<sup>12</sup> that  $2^{8V}$  many tests are needed to be performed in order to find the key k with a probability of 1.

but there is no cryptographic guarantee on how slow calling dec is across hardware implementations, other than that it will cost a unit of time greater than zero. this way (1) becomes just:

$$2^{8V} \times 1$$

$$= 2^{8V} \tag{4}$$

likewise, (2) and (3) become just:

$$2^{H(p)} \times 1$$

$$= 2^{H(p)} \tag{5}$$

until i learn more about the guarantees of the involved function calls, i'm adopting this conservative thinking in this paper.

request 1. drop me a mail if you know other ways to go forward, even if not necessarily related to this paper's conclusions or its style of thinking.

i think eqs. (4) and (5) is the reason it made sense to people to use entropy as a measure of password security. they just look at it in  $log_2$  scale. i.e. eqs. (4) and (5) become:

$$2^{8V} \times 1 \\
= 2^{8V} \tag{6}$$

and

$$2^{H(p)} \times 1$$

$$= 2^{H(p)} \tag{7}$$

<sup>&</sup>lt;sup>7</sup>in the limit as our repetition of this experiment, except for using different passwords, approaches infinity.

<sup>&</sup>lt;sup>8</sup>cost(enc) is the cost of calling enc, and cost(if  $c_0$  = "goodbye, world") is that of testing whether  $c_0 \leftarrow \text{dec}(m_0, 0, \hat{k})$  equals the expected clear-text "goodbye, world".

<sup>&</sup>lt;sup>9</sup>where cost(hash) is the cost of a single call to hash.

<sup>&</sup>lt;sup>10</sup>where  $\hat{k} \leftarrow \text{hash}(p||s, 8V)$ .

 $<sup>^{11}\</sup>mathrm{see}$  the scrypt paper for an example.

<sup>&</sup>lt;sup>12</sup>statistically by confidence earned through peer review and attempts to break encryption algorithms.

## 2 caveman's entropy

but this simple 1 to 1 mapping between cost and entropy is broken when expensive key derivation functions is used. say that the key k was obtained by recursively calling the hash function N many times. i.e. hash(hash(...hash(p||s,8V),...,8V), 8V). this way

## 3 ciphart

### 3.1 parameters

enc encryption function.

p password.

s salt.

M total memory in bytes.

L number of memory lanes for concurrency.

T number of tasks per lane segment.

B minimum quantity of increased protection against password brute-forcing attacks in the unit of caveman's entropy bits.

K output key size in bytes.

#### 3.2 internal variables

hash hashing function.

$$C \qquad \leftarrow \begin{cases} 64 \text{ bytes} & \text{if enc is } xchacha20\\ 16 \text{ bytes} & \text{if enc is } aes\\ \dots \end{cases}$$

this to reflect the block size of the encryption algorithm that implements enc.

$$V \qquad \leftarrow \begin{cases} 32 \text{ bytes} & \text{if enc is } xchacha20\\ 16 \text{ bytes} & \text{if enc is } aes\text{-}128\\ 32 \text{ bytes} & \text{if enc is } aes\text{-}256\\ \dots \end{cases}$$

this is the size of the encryption key that's used to solve ciphart's tasks. this is different than the enc-independent K which is possibly used by other encryption algorithms in later stages<sup>13</sup>.

 $\hat{T}$   $\leftarrow \max(\lceil VC^{-1} \rceil, T)$ . this is to ensure that we have enough encrypted bytes for new keys.

 $\hat{T}$   $\leftarrow \hat{T} - (\hat{T} \mod 2) + 2$ . this is to ensure that there is an even number of tasks in a segment. why? because we need a buffer for storing the clear-text and another for storing the output ciphertext.

 $G \leftarrow \hat{M}C^{-1}\hat{T}^{-1}L^{-1}$ . total number of segments per lane.

 $N \leftarrow 0$ . actual number of times enc is called, where  $\hat{N} \geq 2^B$ .

 $m_i$  C-bytes memory for  $i^{th}$  task in the  $\hat{M}$ -bytes pad.

 $n_l \leftarrow lG\hat{T}$ . nonce variable for  $l^{th}$  lane with at least 64 bits.

 $f \leftarrow 0$ . a flag indicating whether the  $\hat{M}$ -bytes pad

 $v \leftarrow * \text{hash}(p \parallel s, V)$ . a pointer to the first byte where V-bytes key is stored.

#### 3.3 output

k K-bytes key.

 $\hat{B}$  actual quantity of increased security against password brute-forcing attacks in the unit of *caveman's* entropy bits, where  $\hat{B} \geq B$ .

#### 3.4 steps

steps of *ciphart* is shown in algorithm 1. this corresponds to *argon2d*. adding a *ciphart-i* variant is a trivial matter, i just didn't do it yet because my threat model currently doesn't benefit from a password independent variant.

<sup>&</sup>lt;sup>13</sup>at the expense of losing the meaning of caveman's entropy bits.

#### algorithm 1: ciphart 1 while 1 do for q = 0, 1, ..., G - 1 do 2 for l = 0, 1, ..., L - 1 do 3 for t = 0, 1, ..., T - 1 do 4 $i \leftarrow qLT + lT + t;$ 5 if t < T - 1 then 6 $i \leftarrow i + 1$ ; 7 else if t = T - 1 then 8 $j \leftarrow i - T + 1;$ 9 $m_i \leftarrow \texttt{enc}(m_i, n_l, v);$ 10 $n_l \leftarrow n_l + 1;$ 11 if f = 0 then 12 $v \leftarrow m_i \mod (qLTC + tC - V);$ 13 if $v \geq gLTC - V$ then 14 $v \leftarrow v + lTC;$ 15 else 16 $v \leftarrow m_j \mod (\hat{M} - LTC + tC - V);$ 17 if $v \ge qLTC + tC - V$ then 18 $v \leftarrow v + LTC;$ 19 else if $v \geq gLTC - V$ then 20 $v \leftarrow v + lTC;$ 21 $N \leftarrow N + LT;$ 22 if $N \geq 2^B$ then 23 $g_{\text{last}} \leftarrow g;$ 24 go to line 27; 25 $f \leftarrow 1$ ; 2627 $i \leftarrow g_{\text{last}} LT$ ; 28 $k \leftarrow \text{hash}(m_{i+0T} || m_{i+1T} || \dots || m_{i+(L-1)T}, K);$ **29** $\hat{B} \leftarrow log_2N$ ; 30 return k, $\hat{B}$

## 4 parallelism

since iterations of the loop in line 3 in algorithm 1 are fully independent of one other, they can quite happily utilise L cpu cores, specially when segment sizes, T, are larger.

## 5 memory-hardness

*Proof.* algorithm 1 is just a variation of argon2d, except that it uses an encryption function, enc, instead of a hashing functionn. so if argon2d is memory-hard, then so is ciphart.

## 6 security interpretation

**note 1.** i assume that the decryption part of the encryption algorithm enc costs the same as the encryption. this is true for algorithms such as xchacha20. and in cases where it is not true, such as with aes, ciphart can simply

encrypt using the decryption function. this way we guarantee that the cost are identical between ciphart's encryption, and the cipher-text decryption that the adversary does to test a given key.

let's say that we used block encryption function enc and a key  $v \leftarrow \mathtt{hash}(p||s,V)$  to encrypt some clear-text into a sequence of C-byte cipher-text blocks  $m_0, m_1, \ldots$  let's say that the adversary got those  $m_0, m_1, \ldots$ 

adversary's goal is to decrypt those cipher-text blocks back into the original clear-text. so what options does he have?

#### 6.1 key brute-forcing

brute-force the V-bytes key space. in order to get a probability of 1 of finding the key v, the adversary would need to evaluate  $2^{8V}$  many keys<sup>14</sup>.

this works by having the adversary repeatedly decrypting  $m_0$  with enc, each time using a new key among

- $\hat{v}_0 \leftarrow \texttt{0x00...0},$
- $\hat{v}_1 \leftarrow \texttt{0x00...1}$ ,
- •
- $\hat{v}_{2^{8V-1}} \leftarrow \texttt{Oxff...f},$

until the adversary finds a key that manages to decrypt  $m_0$ .

the adversary could be extremely lucky and have  $v_0$  manage to decrypt  $m_0$ , hence needing to call enc only once

or he might be extremely unlucky and need to keep trying until  $v_{2^{8V}-1}$  manages to do it, hence needing to call enc for  $2^{8V}$  many times.

usually it's sometime in between. asymptotically n on average, the adversary would need to call enc for  $2^{8V}/2$  many times.

but in order to guarantee finding v, the brute-forcing process would need to run  $2^{8V}$  many evaluations, hence calling enc for  $2^{8V}$  many times.

that said, the adversary would be fossilised long before his application completes. e.g. since 8V=256 is common for ciphers nowadays, on average while considering the lucky and the unlucky cases, it would take my laptop  $4.28\times 10^{58}$  centuries to just increment a counter for  $2^{256}$  many times. so if the adversary's best hope is to brute-force keys, our system has reached maximum security.

security interpretation 1. your protection against key brute-forcing attacks with key brute-forcing is  $2^{8V} \cos(\text{enc})$ , where  $\cos(\text{enc})$  is the cost of executing enc a single time.

this is usually called 8V entropy bits. but for the purpose of helping later sections, i think it's better to call it 8V entropy bits from the viewing angle of enc, or, for short:

<sup>&</sup>lt;sup>14</sup>assuming that each byte is 8 bits.

**definition 2.** HENC is entropy bits from the viewing angle of enc.

HENC is specific only to enc's algorithm, so must hold with whatever alien technology that the adversary may have, for as long as enc, as an algorithm, has no cryptanalysis. if there is any cryptanalysis, we'll have to subtract bits from 8V, e.g. 8V-z HENC, where z is number of reduced bits due to cryptanalysis.

#### 6.2 normal password brute-forcing

brute-force the H(p) bits password space. where H(p) is the amount of entropy bits in p. in order to get a probability of 1 of finding the password p, the adversary would need to evaluate  $2^{H(p)}$  many keys<sup>15</sup>.

this works by having the adversary repeatedly decrypting  $m_0$  with enc, each time using a new key among:

- $\hat{v}_0 \leftarrow \text{hash}(\hat{p}_0 || s, V)$ ,
- $\hat{v}_1 \leftarrow \text{hash}(\hat{p}_1 || s, V),$
- :
- $\bullet \ \hat{v}_{2^{H(p)}-1} \leftarrow \mathtt{hash}(\hat{p}_{2^{H(p)}-1} \| s, V),$

until the adversary finds a key that manages to decrypt  $m_0$ .

security interpretation 2. your protection against password brute-forcing attacks with normal hashed passwords is  $2^{H(p)} \cos t(hash) + 2^{H(p)} \cos t(enc)$ . the latter enc calls are due to trying to decrypt  $m_0$  at every attempt.

this is usually called H(p) entropy bits. but for the purpose of this paper, i think it's better to be more specific. from the viewing angle of enc, we get the entropy:

$$H(p)$$
 HENC

and from the viewing angle of hash, we get the entropy:

$$H(p)$$
 HHASH

this may seem silly as it is too obvious, but i think it helps me to communicate my thoughts in later sections.

definition 3. HHASH is entropy bits from the viewing angle of hash.

#### 6.3 with argon2

adversary evaluates keys from:

- $\hat{v}_0 \leftarrow \operatorname{argon2}(\hat{p}_0, N, M, \ldots),$
- $\hat{v}_1 \leftarrow \operatorname{argon2}(\hat{p}_1, N, M, \ldots),$

•

•  $\hat{v}_{2^{H(p)}-1} \leftarrow \operatorname{argon2}(\hat{p}_{2^{H(p)}-1}, N, M, ...),$ 

for every argon2 call, hash is called for N many times if there is M bytes of memory. so, from the viewing angle of hash, we get:

- $\hat{v}_0 \leftarrow \text{hash}(\hat{p}_0 || s_0, V),$
- $\hat{v}_1 \leftarrow \text{hash}(\hat{p}_1 || s_1, V),$
- •
- $\bullet \ \hat{v}_{N2^{H(p)}-1} \leftarrow \text{hash}(\hat{p}_{N2^{H(p)}-1} \| s_{N2^{H(p)}-1}, V),$

if an adversary lacks M bytes, he can still compute argon2(p, N, M, ...), but at the expense of needing exponentially more cpu time as his memory is linearly reduced.

security interpretation 3. your protection against password brute-forcing attacks under the argon2 protection is:

$$\begin{split} N2^{H(p)} \cos t(\texttt{hash}) &+ 2^{H(p)} \cos t(\texttt{enc}) \\ &= 2^{H(p) + \log_2 N} \cos t(\texttt{hash}) + 2^{H(p)} \cos t(\texttt{enc}) \end{split}$$

from the viewing angle of enc we get the entropy:

$$H(p)$$
 HENC

while from the viewing angle of hash we get the entropy:

$$H(p) + \log_2 N \ \mathrm{HHASH}$$

so which one to pick? i think people so far just pick H(p) HENC to reflect password's entropy, and seem to not pick  $H(p) + \log_2 N$  HHASH as they don't seem to consider it entropy. but i have two disagreements with people:

• i think not accepting that  $H(p) + \log_2 N$  HHASH is entropy is needlessly limiting. because i think  $H(p) + \log_2 N$  HHASH is entropy as much as H(p) HENC is entropy; it's just that they are measured from different viewing angles: former is measured from the hash viewing angle, while the latter is measured from the enc viewing angle.

i don't see any reason why any of them is more true than the other. i think that both of them are entropies, but of different units.

why do people only pick either one of them? it's technically false my view. in my view truth is: we're just dealing with two entropies measured in different units. so i think truth is that we have the following number of entropy bits:

$$H(p)$$
 HENC  
+ $H(p) + \log_2 N$  HHASH

which obviously looks a bit ugly, since we cannot sum them due to the terms having different units, which also gives our brain a hard time to get a feeling of what that even means.

<sup>&</sup>lt;sup>15</sup>the adversary does not know p, obviously, but he knows the process that the user used to generate p, henceforth he knows H(p).

so what's the solution here to this ugliness? should we ignore H(p) HENC+H(p)+ $\log_2 N$  HHASH as an entropy measure that quantifies the security of our protection against password brute-forcing, as people currently do, and measure it only in terms of the computational cost by surveying the industry of asics in order to find a map between time and money?

my answer to the questions above is:

- no. the right approach is to just admit that argon2's approach is dragging us into the situation where we end up with two entropies measured in different units.
- argon2's security contribution is measurable as entropy, except that it is ugly since it is made of two entropies in distinct units. if we ignore this, we won't solve the problem, but end up stashing the dirts under the carpet.
- of course, we are always free to also survey the industry of asics to derive time-money maps, but this doesn't have to be our only approach to quantify our security gain.

#### 6.4 with ciphart

adversary evaluates keys from:

- $\hat{v}_0 \leftarrow \text{ciphart}(\hat{p}_0, B, M, \ldots),$
- $\hat{v}_1 \leftarrow \text{ciphart}(\hat{p}_1, B, M, \ldots),$
- \_
- $\bullet \ \hat{v}_{2^{H(p)}-1} \leftarrow \operatorname{ciphart}(\hat{p}_{2^{H(p)}-1}, B, M, \ldots),$

mostly similar to argon2. differences related to this section is that ciphart calls enc instead of hash, and specifies B instead of N, where  $B \approx \log_2 N$ . similar exponential time penalty applies with memory less than M.

**security interpretation 4.** your protection against password brute-forcing attacks under the ciphart protection approach is:

$$\begin{split} &\left(2^{H(p)}+2^{\hat{B}}\right) \operatorname{cost}(\operatorname{enc}) + 2^{H(p)} \operatorname{cost}(\operatorname{enc}) \\ &= \left(2^{H(p)}+2^{\hat{B}}+2^{H(p)}\right) \operatorname{cost}(\operatorname{enc}) \\ &= \left(2\times 2^{H(p)}+2^{\hat{B}}\right) \operatorname{cost}(\operatorname{enc}) \\ &= \left(2^{H(p)+\log_2 2}+2^{\hat{B}}\right) \operatorname{cost}(\operatorname{enc}) \\ &= \left(2^{H(p)+1}+2^{\hat{B}}\right) \operatorname{cost}(\operatorname{enc}) \end{split}$$

from the viewing angle of enc we get the entropy:

$$H(p) + 1 + \hat{B}$$
 HENC

and there is no other viewing angle than enc since only enc is used! as a result our brain can easily interpret it.

plus, if we wish to study the industry of asics to obtain time-money maps, our job will be much easier as we can simply look at the cost of asics that are already implemented for enc<sup>16</sup>.

## 7 summary

 $<sup>^{16}</sup>$ e.g. if enc is a popular algorithm, such as aes-256, then we can get more specific data from manufacturers, ultimately giving us a more accurate time-money maps. but when enc is not popular, we may need to do rougher calculations based on the expected asics area as done in the scrupt paper.