

**PHILOSOPHIÆ  
NATURALIS  
PRINCIPIA  
MATHEMATICA**

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# Chapter 1

## Integration

The goal here is to integrate the area under  $f(x)$  from when  $x = 0$  until we reach  $x = x_{end}$ . What follows here is bunch of steps that shows my thinking process because *shadowdaemon* asked for it (and also because I am happy now).

### 1.1 Areas of Lots of Extremely Tiny Rectangular Columns

So to integrate  $f(x) = x^2$  we have to keep summing extremely skinny columns<sup>1</sup> of, each of width  $d$ .

$$\begin{aligned} & (x + d - x)(x)^2 + \\ & (x + 2d - (x + d))(x + d)^2 + \\ & (x + 3d - (x + 2d))(x + 2d)^2 + \\ & (x + 4d - (x + 3d))(x + 3d)^2 + \\ & (x + 5d - (x + 4d))(x + 4d)^2 + \\ & (x + 6d - (x + 5d))(x + 5d)^2 + \\ & (x + 7d - (x + 6d))(x + 6d)^2 + \\ & (x + 8d - (x + 7d))(x + 7d)^2 + \\ & (x + 9d - (x + 8d))(x + 8d)^2 + \\ & (x + 10d - (x + 9d))(x + 9d)^2 + \\ & (x + 11d - (x + 10d))(x + 10d)^2 + \\ & \dots \end{aligned}$$

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<sup>1</sup>**Note:** *fefelix* of *freenode/#gentoo-chat-exile* tried to look smart by attacking my rigor by saying that the term *skinny columns* is wrong and that it must be replaced by the term *infinitesimal* (facepalm moment here). He also tried to look even smarter by using the phrase *In the realm of  $\mathbb{R}$* . Obviously, any half-assed mathematician knows that  $\mathbb{R}$  has only a single infinitesimal which is zero. Yep, zero bitch.

You see, if  $d$  is extremely tiny (near zero), then we will have to sum an infinite number of those tiny skinny areas. But for simplicity I put  $\dots$  instead.

We can simplify things:

$$\begin{aligned}
 & d(x)^2 + \\
 & d(x+d)^2 + \\
 & d(x+2d)^2 + \\
 & d(x+3d)^2 + \\
 & d(x+4d)^2 + \\
 & d(x+5d)^2 + \\
 & d(x+6d)^2 + \\
 & d(x+7d)^2 + \\
 & d(x+8d)^2 + \\
 & d(x+9d)^2 + \\
 & d(x+10d)^2 + \\
 & \dots
 \end{aligned}$$

Let's expand those squares:

$$\begin{aligned}
 & d(x)(x) + \\
 & d(x+d)(x+d) + \\
 & d(x+2d)(x+2d) + \\
 & d(x+3d)(x+3d) + \\
 & d(x+4d)(x+4d) + \\
 & d(x+5d)(x+5d) + \\
 & d(x+6d)(x+6d) + \\
 & d(x+7d)(x+7d) + \\
 & d(x+8d)(x+8d) + \\
 & d(x+9d)(x+9d) + \\
 & d(x+10d)(x+10d) + \\
 & \dots
 \end{aligned}$$

Let's multiply them square bitches:

$$\begin{aligned}
& dx^2 + \\
& d(x^2 + 2dx + d^2) + \\
& d(x^2 + 4dx + 4d^2) + \\
& d(x^2 + 6dx + 9d^2) + \\
& d(x^2 + 8dx + 16d^2) + \\
& d(x^2 + 10dx + 25d^2) + \\
& d(x^2 + 12dx + 36d^2) + \\
& d(x^2 + 14dx + 49d^2) + \\
& d(x^2 + 16dx + 64d^2) + \\
& d(x^2 + 18dx + 81d^2) + \\
& d(x^2 + 20dx + 100d^2) + \\
& \dots
\end{aligned}$$

Let's now multiply those bitches with  $d$  so that the shit gets spread even more:

$$\begin{aligned}
& dx^2 + \\
& dx^2 + 2d^2x + d^3 + \\
& dx^2 + 4d^2x + 4d^3 + \\
& dx^2 + 6d^2x + 9d^3 + \\
& dx^2 + 8d^2x + 16d^3 + \\
& dx^2 + 10d^2x + 25d^3 + \\
& dx^2 + 12d^2x + 36d^3 + \\
& dx^2 + 14d^2x + 49d^3 + \\
& dx^2 + 16d^2x + 64d^3 + \\
& dx^2 + 18d^2x + 81d^3 + \\
& dx^2 + 20d^2x + 100d^3 + \\
& \dots
\end{aligned}$$

## 1.2 Approximating the Area

Now this is a critical point. Below is basically saying that each row is an approximation for the area under  $f(x)$  from  $x = 0$  till  $x = x_{end}$ . So the 1<sup>st</sup> one is a shit approximation where we approximate the area under that curve by only one big fat column; so  $d$  is so huge here, in fact  $d = x_{end}$ .

Then, the 2nd line show a slightly less shit approximation where we approximate the area under the curve by two fat ass rectangular columns. So here  $d = \frac{x_{end}}{2}$ .

So the approximation of the area under the curve gets more and more accurate in each line.

$$\begin{aligned}
& dx^2 \\
& 2dx^2 + 2d^2x + d^3 \\
& 3dx^2 + 6d^2x + 5d^3 \\
& 4dx^2 + 12d^2x + 14d^3 \\
& 5dx^2 + 20d^2x + 30d^3 \\
& 6dx^2 + 30d^2x + 55d^3 \\
& 7dx^2 + 42d^2x + 91d^3 \\
& 8dx^2 + 56d^2x + 140d^3 \\
& 9dx^2 + 72d^2x + 204d^3 \\
& 10dx^2 + 90d^2x + 285d^3 \\
& 11dx^2 + 110d^2x + 385d^3
\end{aligned}$$

So basically, you see there is a pattern. The coefficient of the  $1^{st}$  term is easy peasy (just incrementing from 1 to  $\infty$ ). The coefficient from the  $2^{nd}$  term is kinda interesting, it follows the equation  $(i^2 + i)$  where  $i$  is the line number. Note that we start counting lines from 0. So the  $1^{st}$  has  $i = 0$  and the  $2^{nd}$  line has  $i = 1$ , etc. Finally, the last term is kinda cool, it follows the pattern  $\frac{(i^2+i)(2i+1)}{6}$ .

Now you may ask, how did I find these patterns? Well these are well known number series. You can look them up in the On-Line Encyclopedia of Integer Sequences<sup>2</sup>.

So, the area under the curve of  $f(x)$  from  $x = 0$  up to  $x_{end}$ , by any  $d$  (and its corresponding  $i$ ) is:

$$dx^2 + (i^2 + i)d^2x + \frac{(i^2 + i)(2i + 1)}{6}d^3$$

Now we are almost done. We know that  $x = 0$ , so we can cancel a few terms:

$$\begin{aligned}
d0^2 + (i^2 + i)d^20 + \frac{(i^2 + i)(2i + 1)}{6}d^3 \\
\frac{(i^2 + i)(2i + 1)}{6}d^3
\end{aligned} \tag{1.1}$$

Of course, we could've canceled those terms that multiply against zero earlier, but I didn't for random reasons. I just didn't. That's the randomness of life. But it's all mathematically correct as my caveman balls tell.

You can code a simple script that you give it  $x_{end}$  and  $d$ , by which it automatically finds  $i = x_{end}/d$ . You will notice that as  $d$  gets smaller, you end up approaching some limit after which reduction in  $d$  does not cause any change in the estimated area under the curve.

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<sup>2</sup><http://oeis.org/>

### 1.3 The Precise Area Under The Bitch

Now let's find the ultimate precision in the limit as  $i \rightarrow \infty$  which also means that  $d \rightarrow 0$ . But how about not? Cause it's too hard to solve the limit when two variables are approaching different limits.

To simplify the limits in an easier way, let's represent  $i$  in terms of  $d$  and  $x_{end}$  as follows  $i = x_{end}/d$ . Then the same equation would become as follows:

$$\begin{aligned}
& \frac{((x_{end}/d)^2 + (x_{end}/d))(2(x_{end}/d) + 1)}{6} d^3 \\
& \frac{(\frac{x_{end}^2}{d^2} + \frac{x_{end}}{d})(\frac{2x_{end}}{d} + 1)}{6} d^3 \\
& \frac{(\frac{x_{end}^2}{d^2} + \frac{x_{end}}{d})(\frac{2x_{end}}{d} + 1)}{6} d^3 \\
& \frac{\frac{1}{d}(\frac{x_{end}^2}{d} + x_{end})(\frac{2x_{end}}{d} + 1)}{6} d^3 \\
& \frac{(\frac{x_{end}^2}{d} + x_{end})(\frac{2x_{end}}{d} + 1)}{6} d^2 \\
& \frac{\frac{2x_{end}^3}{d^2} + \frac{x_{end}^2}{d} + \frac{2x_{end}^2}{d} + x_{end}}{6} d^2 \\
& \frac{\frac{2x_{end}^3 d^2}{d^2} + \frac{x_{end}^2 d^2}{d} + \frac{2x_{end}^2 d^2}{d} + x_{end} d^2}{6} \\
& \frac{2x_{end}^3 + x_{end}^2 d + 2x_{end}^2 d + x_{end} d^2}{6}
\end{aligned}$$

Now, it's super easy. We have to find the limit of that equation as a single variable approaches 0 (we got rid of  $i$ ). The equation becomes:

$$\begin{aligned}
& \frac{2x_{end}^3}{6} \\
& \frac{x_{end}^3}{3}
\end{aligned}$$

That's it. Integration re-invented bitch :) —  $\frac{x_{end}^2}{3}$ .  
Q.E. freaking DEE.

## Chapter 2

# Differentiation

Here we want to find the slope of  $f(x)$  at point  $x$ . This is easy so I won't say much here.

Differentiate  $x^2$ .

$$\frac{f(x+d)-f(x)}{x+d-x}$$

$$\frac{(x+d)^2-x^2}{x+d-x}$$

$$\frac{(x+d)^2-x^2}{d}$$

$$\frac{(x+d)(x+d)-x^2}{d}$$

$$\frac{x^2+xd+xd+d^2-x^2}{d}$$

$$\frac{x^2}{d} + \frac{2xd}{d} + \frac{d^2}{d} - \frac{x^2}{d}$$

$$\frac{x^2}{d} + 2x + d - \frac{x^2}{d}$$

$$2x + d$$

Now, as  $d \rightarrow \infty$ , it becomes  $2x$ . Done.