Principia Mathematica

Al Caveman

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Contents

Chapter 1

Integration

The goal here is to integrate the area under $f(x)$ from when $x = 0$ until we reach $x = x_{end}$. What follows here is bunch of steps that shows my thinking process because shadodaemon asked for it (and also because I am happy now).

1.1 Areas of Lots of Extremely Tiny Rectangular Columns

So to integrate $f(x) = x^2$ we have to keep summing extremely skinny columns of, each of width d.

$$\begin{aligned}
&(x + d - x)(x)^2 + \\
&(x + 2d - (x + d))(x + d)^2 + \\
&(x + 3d - (x + 2d))(x + 2d)^2 + \\
&(x + 4d - (x + 3d))(x + 3d)^2 + \\
&(x + 5d - (x + 4d))(x + 4d)^2 + \\
&(x + 6d - (x + 5d))(x + 5d)^2 + \\
&(x + 7d - (x + 6d))(x + 6d)^2 + \\
&(x + 8d - (x + 7d))(x + 7d)^2 + \\
&(x + 9d - (x + 8d))(x + 8d)^2 + \\
&(x + 10d - (x + 9d))(x + 9d)^2 + \\
&(x + 11d - (x + 10d))(x + 10d)^2 + \\
&\ldots
\end{aligned}$$

You see, if d is extremely tiny (near zero), then we will have to sum an infinite number of those tiny skinny areas. But for simplicity I put ... instead.

We can simplify things:

$$
\begin{aligned}
d(x)^2&+\\
d(x+d)^2&+\\
d(x+2d)^2&+\\
d(x+3d)^2&+\\
d(x+4d)^2&+\\
d(x+5d)^2&+\\
d(x+6d)^2&+\\
d(x+7d)^2&+\\
d(x+8d)^2&+\\
d(x+9d)^2&+\\
d(x+10d)^2&+\\
&\dots
\end{aligned}
$$

Let's expand those squares:

$$
\begin{aligned}
d(x)(x)&+\\
d(x+d)(x+d)&+\\
d(x+2d)(x+2d)&+\\
d(x+3d)(x+3d)&+\\
d(x+4d)(x+4d)&+\\
d(x+5d)(x+5d)&+\\
d(x+6d)(x+6d)&+\\
d(x+7d)(x+7d)&+\\
d(x+8d)(x+8d)&+\\
d(x+9d)(x+9d)&+\\
d(x+10d)(x+10d)&+\\
&\dots
\end{aligned}
$$

Let's multiply them square bitches:

$$\begin{aligned}
dx^2 + \\
d(x^2 + 2dx + d^2) + \\
d(x^2 + 4dx + 4d^2) + \\
d(x^2 + 6dx + 9d^2) + \\
d(x^2 + 8dx + 16d^2) + \\
d(x^2 + 10dx + 25x^2) + \\
d(x^2 + 12dx + 36d^2) + \\
d(x^2 + 14dx + 49d^2) + \\
d(x^2 + 16dx + 64d^2) + \\
d(x^2 + 18dx + 81d^2) + \\
d(x^2 + 20dx + 100d^2) + \\
\ldots
\end{aligned}$$

Let's now multiply those bitches with d so that the shit gets spread even more:

$$\begin{aligned}
dx^2 + \\
dx^2 + 2d^2x + d^3 + \\
dx^2 + 4d^2x + 4d^3 + \\
dx^2 + 6d^2x + 9d^3 + \\
dx^2 + 8d^2x + 16d^3 + \\
dx^2 + 10d^2x + 25x^3 + \\
dx^2 + 12d^2x + 36d^3 + \\
dx^2 + 14d^2x + 49d^3 + \\
dx^2 + 16d^2x + 64d^3 + \\
dx^2 + 18d^2x + 81d^3 + \\
dx^2 + 20d^2x + 100d^3 + \\
\ldots
\end{aligned}$$

1.2 Approximating the Area

Now this is a critical point. Below is basically saying that each row is an approximation for the area under $f(x)$ from $x = 0$ till $x = x_{end}$. So the 1^{st} one is a shit approximation where were approximate the area under that curve by only one big fat column; so d is so huge here, in fact $d = x_{end}$.

Then, the 2nd line show a slightly less shit approximation where we approximate the area under the curve by two fat ass rectangular columns. So here $d = \frac{x_{end}}{2}$.

So the approximation of the area under the curve gets more and more accurate in each line.

$$dx^2$$
$$2dx^2 + 2d^2x + d^3$$
$$3dx^2 + 6d^2x + 5d^3$$
$$4dx^2 + 12d^2x + 14d^3$$
$$5dx^2 + 20d^2x + 30d^3$$
$$6dx^2 + 30d^2x + 55d^3$$
$$7dx^2 + 42d^2x + 91d^3$$
$$8dx^2 + 56d^2x + 140d^3$$
$$9dx^2 + 72d^2x + 204d^3$$
$$10dx^2 + 90d^2x + 285d^3$$
$$11dx^2 + 110d^2x + 385d^3$$

So basically, you see there is a pattern. The coefficient of the 1^{st} term is easy peasy (just incrementing from 1 to ∞). The coefficient from the 2^{nd} term is kinda interesting, it follows the equation $(i^2 + i)$ where i is the line number. Note that we start counting lines from 0. So the 1^{st} has $i = 0$ and the 2^{nd} line has $i = 1$, etc. Finally, the last term is kinda cool, it follows the pattern $\frac{(i^2+i)(2i+1)}{6}$.

Now you may ask, how did I find these patterns? Well these are well known number series. You can look them up in the On-Line Encyclopedia of Integer Sequences[1].

So, the area under the curve of $f(x)$ from $x = 0$ up to x_{end}, by any d (and its corresponding i) is:

$$dx^2 + (i^2 + i)d^2x + \frac{(i^2 + i)(2i + 1)}{6}d^3$$

Now we are almost done. We know that $x = 0$, so we can cancel a few terms:

$$\begin{aligned} d0^2 + (i^2 + i)d^20 + &\frac{(i^2 + i)(2i + 1)}{6}d^3 \\ &\frac{(i^2 + i)(2i + 1)}{6}d^3 \end{aligned} \tag{1.1}$$

Of course, we could've canceled those terms that multiply against zero earlier, but I didn't for random reasons. I just didn't. That's the randomness of life. But it's all mathematically correct as my caveman balls tell.

You can code a simple script that you give it x_{end} and d, by which it automatically finds $i = x_{end}/d$. You will notice that as d gets smaller, you end up approaching some limit after which reduction in d does not cause any change in the estimated area under the curve.

[1] http://oeis.org/

1.3 The Precise Area Under The Bitch

Now let's find the ultimate precision in the limit as $i \to \infty$ which also means that $d \to 0$. But how about not? Cause it's too hard to solve the limit when two variables are approaching different limits.

To simplify the limits in an easier way, let's represent i in terms of d and x_{end} as follows $i = x_{end}/d$. Then the same equation would become as follows:

$$\frac{((x_{end}/d)^2 + (x_{end}/d))(2(x_{end}/d)+1)}{6}d^3$$
$$\frac{(\frac{x_{end}^2}{d^2} + \frac{x_{end}}{d})(\frac{2x_{end}}{d}+1)}{6}d^3$$
$$\frac{(\frac{x_{end}^2}{d^2} + \frac{x_{end}}{d})(\frac{2x_{end}}{d}+1)}{6}d^3$$
$$\frac{\frac{1}{d}(\frac{x_{end}^2}{d} + x_{end})(\frac{2x_{end}}{d}+1)}{6}d^3$$
$$\frac{(\frac{x_{end}^2}{d} + x_{end})(\frac{2x_{end}}{d}+1)}{6}d^2$$
$$\frac{\frac{2x_{end}^3}{d^2} + \frac{x_{end}^2}{d} + \frac{2x_{end}^2}{d} + x_{end}}{6}d^2$$
$$\frac{\frac{2x_{end}^3d^2}{d^2} + \frac{x_{end}^2d^2}{d} + \frac{2x_{end}^2d^2}{d} + x_{end}d^2}{6}$$
$$\frac{2x_{end}^3 + x_{end}^2d + 2x_{end}^2d + x_{end}d^2}{6}$$

Now, it's super easy. We have to find the limit of that equation as a single variable approaches 0 (we got rid of i). The equation becomes:

$$\frac{2x_{end}^3}{6}$$
$$\frac{x_{end}^3}{3}$$

That's it. Integration re-invented bitch :) — $\frac{x_{end}^2}{3}$.
Q.E. freaking DEE.

Chapter 2

Differentiation

Here we want to find the slope of $f(x)$ at point x. This is easy so I won't say much here.

Differentiate x^2.

$\frac{f(x+d)-f(x)}{x+d-x}$

$\frac{(x+d)^2-x^2}{x+d-x}$

$\frac{(x+d)^2-x^2}{d}$

$\frac{(x+d)(x+d)-x^2}{d}$

$\frac{x^2+xd+xd+d^2-x^2}{d}$

$\frac{x^2}{d} + \frac{2xd}{d} + \frac{d^2}{d} - \frac{x^2}{d}$

$\frac{x^2}{d} + 2x + d - \frac{x^2}{d}$

$2x + d$

Now, as $d \to \infty$, it becomes $2x$. Done.