# **PHILOSOPHIÆ** NATURALIS **PRINCIPIA** MATHEMATICA

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# Contents

# Chapter 1

# Integration

The goal here is to integrate the area under $f(x)$ from when $x = 0$ until we reach $x = x_{end}$. What follows here is bunch of steps that shows my thinking process because *shadowdaemon* asked for it (and also because I am happy now).

## 1.1 Areas of Lots of Extremely Tiny Rectangular Columns

So to integrate $f(x) = x^2$ we have to keep summing extremely skinny columns[1] of, each of width $d$.

$$\begin{aligned}
(x + d - x)(x)^2 &+ \\
(x + 2d - (x + d))(x + d)^2 &+ \\
(x + 3d - (x + 2d))(x + 2d)^2 &+ \\
(x + 4d - (x + 3d))(x + 3d)^2 &+ \\
(x + 5d - (x + 4d))(x + 4d)^2 &+ \\
(x + 6d - (x + 5d))(x + 5d)^2 &+ \\
(x + 7d - (x + 6d))(x + 6d)^2 &+ \\
(x + 8d - (x + 7d))(x + 7d)^2 &+ \\
(x + 9d - (x + 8d))(x + 8d)^2 &+ \\
(x + 10d - (x + 9d))(x + 9d)^2 &+ \\
(x + 11d - (x + 10d))(x + 10d)^2 &+ \\
\ldots &
\end{aligned}$$

You see, if $d$ is extremely tiny (near zero), then we will have to sum an infinite number of those tiny skinny areas. But for simplicity I put $\ldots$ instead.

[1]**Note:** *fefelix* of *freenode/#gentoo-chat-exile* tried to look smart by attacking my rigor by saying that the term *skinny columns* is wrong and that it must be replaced by the term *infinitesimal* (facepalm moment here). He also tried to look even smarter by using the phrase *In the realm of* $\mathbb{R}$. Obviously, any half-assed mathematician knows that $\mathbb{R}$ has only a single infinitesimal which is zero. Yep, zero bitch. So the term *infinitesimal* is absolutely wrong in this context, thus even worse than *skinny columns*. The only exception is if *fefelix* wishes to live in the 1600s.

We can simplify things:

$$
\begin{aligned}
d(x)^2 &+ \\
d(x+d)^2 &+ \\
d(x+2d)^2 &+ \\
d(x+3d)^2 &+ \\
d(x+4d)^2 &+ \\
d(x+5d)^2 &+ \\
d(x+6d)^2 &+ \\
d(x+7d)^2 &+ \\
d(x+8d)^2 &+ \\
d(x+9d)^2 &+ \\
d(x+10d)^2 &+ \\
\ldots
\end{aligned}
$$

Let's expand those squares:

$$
\begin{aligned}
d(x)(x) &+ \\
d(x+d)(x+d) &+ \\
d(x+2d)(x+2d) &+ \\
d(x+3d)(x+3d) &+ \\
d(x+4d)(x+4d) &+ \\
d(x+5d)(x+5d) &+ \\
d(x+6d)(x+6d) &+ \\
d(x+7d)(x+7d) &+ \\
d(x+8d)(x+8d) &+ \\
d(x+9d)(x+9d) &+ \\
d(x+10d)(x+10d) &+ \\
\ldots
\end{aligned}
$$

Let's multiply them square bitches:

$$
\begin{aligned}
dx^2+ \\
d(x^2 + 2dx + d^2)+ \\
d(x^2 + 4dx + 4d^2)+ \\
d(x^2 + 6dx + 9d^2)+ \\
d(x^2 + 8dx + 16d^2)+ \\
d(x^2 + 10dx + 25x^2)+ \\
d(x^2 + 12dx + 36d^2)+ \\
d(x^2 + 14dx + 49d^2)+ \\
d(x^2 + 16dx + 64d^2)+ \\
d(x^2 + 18dx + 81d^2)+ \\
d(x^2 + 20dx + 100d^2)+ \\
\ldots
\end{aligned}
$$

Let's now multiply those bitches with $d$ so that the shit gets spread even more:

$$
\begin{aligned}
dx^2+ \\
dx^2 + 2d^2x + d^3+ \\
dx^2 + 4d^2x + 4d^3+ \\
dx^2 + 6d^2x + 9d^3+ \\
dx^2 + 8d^2x + 16d^3+ \\
dx^2 + 10d^2x + 25x^3+ \\
dx^2 + 12d^2x + 36d^3+ \\
dx^2 + 14d^2x + 49d^3+ \\
dx^2 + 16d^2x + 64d^3+ \\
dx^2 + 18d^2x + 81d^3+ \\
dx^2 + 20d^2x + 100d^3+ \\
\ldots
\end{aligned}
$$

## 1.2 Approximating the Area

Now this is a critical point. Below is basically saying that each row is an approximation for the area under $f(x)$ from $x = 0$ till $x = x_{end}$. So the $1^{st}$ one is a shit approximation where were approximate the area under that curve by only one big fat column; so $d$ is so huge here, in fact $d = x_{end}$.

Then, the 2nd line show a slightly less shit approximation where we approximate the area under the curve by two fat ass rectangular columns. So here $d = \frac{x_{end}}{2}$.

So the approximation of the area under the curve gets more and more accurate in each line.

$$\begin{aligned}
dx^2 \\
2dx^2 + 2d^2x + d^3 \\
3dx^2 + 6d^2x + 5d^3 \\
4dx^2 + 12d^2x + 14d^3 \\
5dx^2 + 20d^2x + 30d^3 \\
6dx^2 + 30d^2x + 55d^3 \\
7dx^2 + 42d^2x + 91d^3 \\
8dx^2 + 56d^2x + 140d^3 \\
9dx^2 + 72d^2x + 204d^3 \\
10dx^2 + 90d^2x + 285d^3 \\
11dx^2 + 110d^2x + 385d^3
\end{aligned}$$

So basically, you see there is a pattern. The coefficient of the $1^{st}$ term is easy peasy (just incrementing from 1 to $\infty$). The coefficient from the $2^{nd}$ term is kinda interesting, it follows the equation $(i^2 + i)$ where $i$ is the line number. Note that we start counting lines from 0. So the $1^{st}$ has $i = 0$ and the $2^{nd}$ line has $i = 1$, etc. Finally, the last term is kinda cool, it follows the pattern $\frac{(i^2+i)(2i+1)}{6}$.

Now you may ask, how did I find these patterns? Well these are well known number series. You can look them up in the On-Line Encyclopedia of Integer Sequences[2].

So, the area under the curve of $f(x)$ from $x = 0$ up to $x_{end}$, by any $d$ (and its corresponding $i$) is:

$$dx^2 + (i^2 + i)d^2x + \frac{(i^2 + i)(2i + 1)}{6}d^3$$

Now we are almost done. We know that $x = 0$, so we can cancel a few terms:

$$\begin{aligned}
d0^2 + (i^2 + i)d^2 0 + \frac{(i^2 + i)(2i + 1)}{6}d^3 \\
\frac{(i^2 + i)(2i + 1)}{6}d^3
\end{aligned} \tag{1.1}$$

Of course, we could've canceled those terms that multiply against zero earlier, but I didn't for random reasons. I just didn't. That's the randomness of life. But it's all mathematically correct as my caveman balls tell.

You can code a simple script that you give it $x_{end}$ and $d$, by which it automatically finds $i = x_{end}/d$. You will notice that as $d$ gets smaller, you end up approaching some limit after which reduction in $d$ does not cause any change in the estimated area under the curve.

[2] http://oeis.org/

## 1.3 The Precise Area Under The Bitch

Now let's find the ultimate precision in the limit as $i \to \infty$ which also means that $d \to 0$. But how about not? Cause it's too hard to solve the limit when two variables are approaching different limits.

To simplify the limits in an easier way, let's represent $i$ in terms of $d$ and $x_{end}$ as follows $i = x_{end}/d$. Then the same equation would become as follows:

$$\frac{((x_{end}/d)^2 + (x_{end}/d))(2(x_{end}/d) + 1)}{6} d^3$$
$$\frac{(\frac{x_{end}^2}{d^2} + \frac{x_{end}}{d})(\frac{2x_{end}}{d} + 1)}{6} d^3$$
$$\frac{(\frac{x_{end}^2}{d^2} + \frac{x_{end}}{d})(\frac{2x_{end}}{d} + 1)}{6} d^3$$
$$\frac{\frac{1}{d}(\frac{x_{end}^2}{d} + x_{end})(\frac{2x_{end}}{d} + 1)}{6} d^3$$
$$\frac{(\frac{x_{end}^2}{d} + x_{end})(\frac{2x_{end}}{d} + 1)}{6} d^2$$
$$\frac{\frac{2x_{end}^3}{d^2} + \frac{x_{end}^2}{d} + \frac{2x_{end}^2}{d} + x_{end}}{6} d^2$$
$$\frac{\frac{2x_{end}^3 d^2}{d^2} + \frac{x_{end}^2 d^2}{d} + \frac{2x_{end}^2 d^2}{d} + x_{end} d^2}{6}$$
$$\frac{2x_{end}^3 + x_{end}^2 d + 2x_{end}^2 d + x_{end} d^2}{6}$$

Now, it's super easy. We have to find the limit of that equation as a single variable approaches 0 (we got rid of $i$). The equation becomes:

$$\frac{2x_{end}^3}{6}$$
$$\frac{x_{end}^3}{3}$$

That's it. Integration re-invented bitch :) — $\frac{x_{end}^2}{3}$.
Q.E. freaking DEE.

# Chapter 2

# Differentiation

Here we want to find the slope of $f(x)$ at point $x$. This is easy so I won't say much here.

Differentiate $x^2$.

$\frac{f(x+d)-f(x)}{x+d-x}$

$\frac{(x+d)^2-x^2}{x+d-x}$

$\frac{(x+d)^2-x^2}{d}$

$\frac{(x+d)(x+d)-x^2}{d}$

$\frac{x^2+xd+xd+d^2-x^2}{d}$

$\frac{x^2}{d} + \frac{2xd}{d} + \frac{d^2}{d} - \frac{x^2}{d}$

$\frac{x^2}{d} + 2x + d - \frac{x^2}{d}$

$2x + d$

Now, as $d \to 0$, it becomes $2x$. Done.