STUDY ON APPLICATIONS OF GRAPH THEORY

Dissertation Submitted in partial fulfillment of the requirements for the award of

BACHELORS DEGREE IN MATHEMATICS

Mahatma Gandhi University

Kottayam, Kerala

Under the guidance of

Mr. Amal Shaji

Department of Mathematics

Newman College, Thodupuzha

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CERTIFICATE

This is to certify that the work entitled "**A STUDY ON APPLICATIONS OF GRAPH THEORY** " is a Bonafede record of the work done by **ANITTA THOMAS (Reg.No:170021031369), EBIN SIBY(Reg.No:170021031380), NEFY SAJU (Reg.No:170021031391),** Newman College, Thodupuzha under the supervision and guidance of **Mr.AMAL SHAJI**, Assistant Professor on contract, Department of Mathematics, Newman College, Thodupuzha on partial fulfillment for the award of Bachelor's Degree in Mathematics of Mahatma Gandhi University, Kottayam.

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LEAD KINDLY LIGHT

CERTIFICATE

This is to certify that the work entitled "**A STUDY ON APPLICATIONS OF GRAPH THEORY** " is a Bonafede record of the work done by **ANITTA THOMAS (Reg.No: 170021031369), EBIN SIBY(Reg.No:170021031380), NEFY SAJU (Reg.No:170021031391),** Newman College, Thodupuzha under the supervision and guidance of **Mr.AMAL SHAJI**, Assistant Professor on contract, Department of Mathematics, Newman College, Thodupuzha during the year 2017-2020

Mr. AMAL SHAJI

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DECLARATION

We **ANITTA THOMAS (Reg.No: 170021031369), EBIN SIBY (Reg.No:170021031380), NEFY SAJU (Reg.No:170021031391),** hereby declare that the project report entitled **"A STUDY ON APPLICATIONS OF GRAPH THEORY"** submitted in partial fulfillment of the requirement for the award of Bachelor's Degree in Mathematics of Mahatma Gandhi University, Kottayam is our dissertation work. The contents of the study, in full or parts have not been submitted to any other institution or university for the award of any degree or diploma.

ANITTA THOMAS

EBIN SIBY

NEFY SAJU

Place: Thodupuzha
Date:

ACKNOWLEDGEMENT

On the occasion of presenting this project report, we wish to express my deep and profound feelings of gratitude to a number of persons who have contributed to the completion of this project.

First of all, we express our gratitude to **GOD ALMIGHTY,** the supreme guide for bestowing his blessings upon our entire endeavor. We are grateful to our respected Principal **Dr. THOMSON JOSEPH** for providing all facilities for the development of this project.

Sincere obeisance is paid to **Smt. NANCY JACOB,** Head of the Department of Mathematics, Newman College, who has always been guiding and motivating force behind all our aspirations and accomplishments.

With great pleasure we express our sincere gratitude to our guide **Mr.AMAL SHAJI,** Assistant Professor on contract, Department of Mathematics, Newman College, Thodupuzha for the completion of this project work successfully.

We express our thanks with pleasure to all other teachers of the Department for the encouragement and assistance in taking up this project. Last but not least thanks are dedicated to the Library staff, to my classmates and to my parents for the help extended in finishing this work.

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CONTENTS

INTRODUCTION

Graph theory has established itself as an important mathematical tool in a wide variety of subjects ranging from operational research and chemistry to genetics and linguistics and from electric engineering to sociology, architecture etc. At the same time, it has also emerged as a worthwhile mathematical discipline in its own right, because of its inherent simplicity graph theory has a wide range of applications.

A graph can be used to represent almost any physical situation involving discrete objects and a relationship among them. Graphs are one of the prime objects of study in discrete Mathematics. In general, a graph is represented as a set of vertices (nodes or points) connected by edges (arcs or line). We all know the saying,” Mathematics is not needed in real life.”, but graph theory is actually applicable in real life. In fact, this is how it was discovered. The paper written by Leonhard Euler on the seven Konigsberg and published in 1736 is regarded as the first paper in the history of graph theory.

Graph theory also used in real life situations such as Chinese postman problem travelling salesman problem and personal assignment problems. It can be used to minimize total cost, time and distance in travelling salesman problem, Chinese postman problem and in personal assignment problem. Graph theory is increasingly significant as it is applied to other areas of mathematics, science and technology. The powerful combinational method found in graph theory has also being used to prove fundamental results in the other areas of the mathematics. New applications to computer networking security using minimum vertex covers in graphs are discussed

Map coloring and GSM mobile phone networks can also be associated through graph theory. One of the beauties of graph theory is that it depends very little on other branches of mathematics. Graph theory has proven useful in the design of integrated circuits for computers and other electronic devices. These components more often called chips that contain complex layered micro circuits that can be represented as set of points interconnected by lines or arcs.

Using graph theory, engineers develop chips with maximum component density and minimum total interconnecting conductor length. This is important for optimizing processing speed and electrical deficiency. Use of graphs is a visualization technique. It is incredibly useful and helps businesses make better data-driven decisions. But to understand the concepts of graphs in detail, we must first understand its base.

Graph theory also widely used in sociology as a way, for example to measure actors' prestige or to explore rumor spreading notably through the use of social network analysis software. Under the umbrella of social networks are many different types of graphs.

CHAPTER-1

BASICS OF GRAPH THEORY

A graph is a drawing or a diagram consisting of a collection of vertices (dots and points) together with edges (line) joining certain pairs of these vertices. A graph G(V(G),E(G))or G(V,E) consist of two finite sets V(G) which is the vertex set of graph G usually denoted by V which is a non-empty set of elements called vertices and E(G) which is the edge set of graph often denoted by E which may be empty and whose elements are called edges.

- **Simple graph: -**

  A graph is called simple if it has no loops and no parallel edges.

- **Null graph: -**

  A graph G (V, E) is a null graph if the edge set E is empty. That is the graph without any edge is called null graph.

- **Subgraph: -**

  Let H be the graph with vertex se V(H) and edge set E(H) and G be graph with vertex set V(G) and edge set E(G). Then H is a subgraph of G if V(H) ≤ V(G) and E(H) ≤ E(G)

- **Loop: -**

  If an edge has identical end vertices, that is if the edge joins a vertex to itself. Then it is called a loop.

- **Edge loop: -**

  An edge with the same end vertices is called a loop.

- **Isolated vertex: -**

  A vertex of G which is not the end of any edge is called an isolated vertex.

- **Parallel edges: -**

  If two or more edges of a graph G have the same end vertices then these edges are called parallel edges or multiple edges.

- **Neighborhood: -**

  Two vertices which are joined by an edge are said to be adjacent / neighbors.

- **Pendant vertex: -**

  A vertex with only one edge incident with it is called a pendant vertex.

- **Trail: -**

  If the edges of walk are distant then it is a trail.

- **Path: -**

  If the vertices of a walk are distinct then it is a path.

- **Tree: -**

  A graph G is called a tree if it is a connected cyclic graph.

- **Bridges: -**

  An edge e of a graph G is called a bridge or a cut edge if the subgraph G-e has more connected components than G has.

- **Walk: -**

  A walk in a graph G is a finite sequence $W=v_o c_1 v_1 c_2 v_2 \ldots\ldots\ldots\ldots c_k v_k$ whose terms are alternatively vertices and edges with end vertices v_{i-1}, v_i for all i such that $1 \leq i \leq k$.

- **Euler's Tour: -**

  Euler's tour is a closed walk which include every edge exactly once.

- **Hamiltonian path: -**

  A Hamiltonian path in a graph G is a path which contain every vertex of G exactly once.

- **Hamiltonian Cycle: -**

  A Hamiltonian cycle in a graph G is cycle in which contains every vertex of G exactly once.

- **Vertex coloring: -**

  Let G be a graph a vertex coloring of G assigns colors denoted by 1,2,3……. to the vertices of G one color per vertex so that adjacent vertices are assigned different colors.

CHAPTER-2

SOME REAL-LIFE APPLICATIONS

Graph theoretical concepts are widely used to study and model various applications, in different areas. They include, study of molecules, construction of bonds in chemistry and the study of atoms. Similarly, graph theory is used in sociology for example to measure actor’s prestige or to explore diffusion mechanisms. Graph theory is used in biology and conservation efforts where a vertex represents regions where certain species exist and the edges represent migration path or movement between the regions. This information is important when looking at breeding patterns or tracking the spread of disease, parasites and to study the impact of migration that affect other species.

2.1 GRAPHS IN CHEMISTRY

’Arthur Cayley’ used trees to represent the structure of organic molecules 100 years ago; it is only recently that graph theoretic of chemical compound. This is due to;

1→The advent of the electronic computer with its ability to handle graphs

2→ The ever-intersecting need of the chemist to have a mechanized information retrieval system capable of dealing with the millions of organic compounds known today

The structural graph of a chemical compound in general contains much more information than the molecular formula for example the structural graph of amino acetone c_3h_7 no with its H atoms stripped off its shown

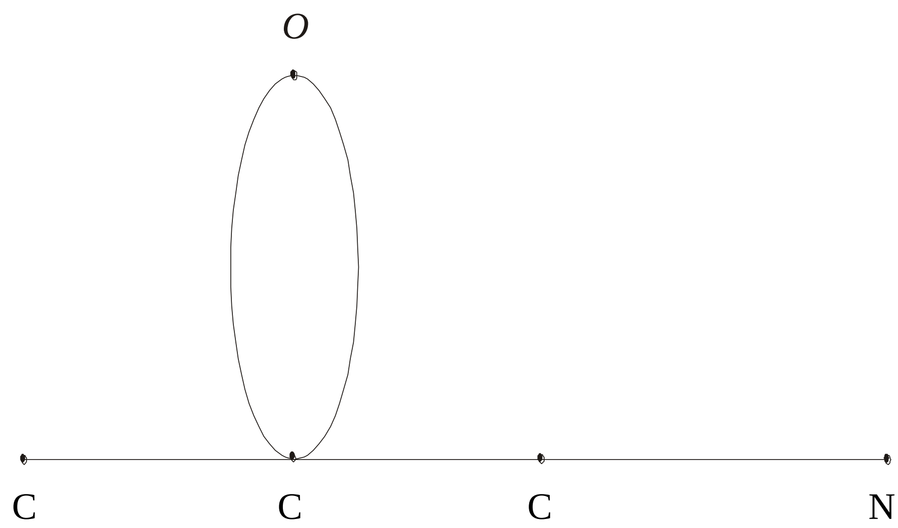

Figure 2.1: Structural graph gives a reasonably adequate description of a chemical compound.

2.2 APPLICATIONS IN BIOLOGY

Within the field of biology and medicine potential applications of network analysis by using graph theory include identifying drug tangents, determining the role of proteins and genes of unknown functions. There are several biological domains where graph theory techniques are applied for knowledge extraction from data.

As an effective modeling analysis and computational tool graph theory is widely used in biological mathematics to deal with various biology problems. In the field of micro-biology, graphs can express the molecular structure, where cell, gene or protein can be denoted as vertex and the connect element can be regarded as an edge. In this way, the biological activity characteristics can be measured via topological index computing in the corresponding graphs.

The aim of scientific researcher in biology and medicine is to describe and -perhaps understand structural, functional relation between elements of given systems.

Calculus of weighted directed graphs can be used to model functional relations directly. Consequently, the theoretical model of a graph can give both a first insight into the structure as well as a description of the functional relations of the elements of biological systems.

2.3 MAP COLOURING

Geographical maps of countries or states are colored using graph coloring where no two adjacent cites cant assign same color by four color theorem given any separation of a plane into continues regions producing a figure called map no more than four color are required to color the map so that no two adjacent regions have the same color .two regions are said to be adjacent if they shares a common boundary that is not corners where a corner are the points shared by s3 or more regions world map can be colored using just 4 colors having no 2 adjacent countries have the same color.

2.4 GRAPH COLORING

Graph coloring is one of the most important concepts in graph theory and is used in many real time applications in computer science. Various coloring methods are available and can be used on requirement basis. The proper coloring of a graph is the coloring of the vertices and edges with minimal number of colors such that no two vertices should have the same color. The minimum number of colors is called as the chromatic number and the graph is called properly colored graph

When we color the map of a country, we see to it that is adjacent states are colored differently for easy identification of the states. One might ask: what is the minimum no of colors required to color any map so that adjacent states receive different colors? The famous four-color conjecture states that four colors are sufficient. The question was first raised by Francis G Morgan in 1852. For a long time it was a fascinating open problem defying a solution despite innovative attempts by several mathematics ultimately Kenneth Appel and Wolfgang harken proved the conjecture in 1976.The graph coloring problem has a huge number of application.

2.5 VERTEX COLOURING

A vertex is an assignment of labels or each vertex of a graph such that no edge connects two identically colored vertices the most common type of vertex

coloring seeks to minimize the no of coloring of a given graph such a coloring is known as minimum vertex coloring

Let be G a graph a (vertex) coloring of g assigns colors usually denoted by 1,2,3 …. to the vertices of G one color per vertex so that the adjacent vertices are assigned different colors.

A k- coloring of G is a to be k coloring which consists a k different colors add. In this case, G is said to be k-colorable

Chromic number : painting all the vertices of a graph with colors such that no to adjacent vertices have the same color is called the proper coloring (or sometimes simply coloring) of a graph a graph in which every vertex has been assigned a color according to a proper coloring is called a properly colored graph .usually a given graph can be properly colored in many different ways.

The proper coloring which is of interest to us is one that requires the minimum no. of colors a graph G that requires k different colors of its proper coloring and no less is called a k –chromatic graph and the number k is called the chromatic number of G

The minimum number n for which there is an n-coloring of a graph G is called the chromatic index (for chromatic number) of a G and is denoted by X(G). If X(G)=k, we say that G is k –chromatic

Some observations that follows directly from the definitions just introduced are

- Vertex colorable graph is loop less.
- A graph consisting of only isolated vertices is i-chromatic
- A graph with one or more edges (not a self - loop of course) is at least 2 chromatic (also called bi-chromatic)

2.6 EDGE COLOURING

In graph theory, an edge coloring of a graph is an assignment of colors to the edge of the graph so that no two adjacent edges have the same colors. recall that the two edges of a graph are called adjacent edges have the same vertex in common .A graph G with no laps , is said to be k- edge colorable if it as possible to assign to each edge one coloring from a set of k colors such that no two edge with a vertex in common get the same color . A k- edge colorable graph is a k-edge chromatic graph if it is not (k-1) edge colorable and if it’s chromatic index X ‘(G) is k

Furthermore, the edge in an edge coloring of a graph that get the same color constitute a matching in that graph. therefore, the chromatic index of the graph is also equal to the minimum number of matching into which the edge set of the graph can be partitioned in particular ‘both the chromatic index is the same for any cyclic path.

CHAPTER-3

APPLICATION IN OPERATONS RESEARCH AND ALGORITHMS

Graph theoretical concepts are widely used in Operations Research. For example, the traveling salesman problem, the shortest spanning tree in a weighted graph, obtaining an optimal match of job sandmen and locating the shortest path between two vertices in a graph

3.1 LIST COLORING

In list coloring problem, each vertex v has a list of available colors and we have to find a coloring where the color of each vertex is taken from the list of available colors. This list coloring can be used to model situations where a job can be processed only in certain time slots or can be processed only by certain machines

3.2 MINIMUM SUM COLORING

In minimum sum coloring, the sum of the colors assigned to the vertices is minimal in the graph. The minimum sum coloring technique can be applied to the scheduling theory of minimizing the sum of completion times of the jobs. The multicolor version of the problem can be used to model jobs with arbitrary lengths. Here, the finish time of a vertex is the largest color assigned to it and the sum of coloring is the sum of the finish time of the vertices. That is the sum of the finish times in a multi-coloring is equal to the sum of completion times in the corresponding schedule.

3.3 PRECOLORING EXTENSION

In certain scheduling problems, the assignments of jobs are already decided. In such cases precoloring technique can be adopted. Here some vertices of the graph will have preassigned color and the precoloring problem has to be solved by extending the coloring of the vertices for the whole graph using minimum number of colors.

3.4 BI-PROCESSOR TASKS

Assume that there is a set of processors and set of tasks. Each task has to be executed on two processors simultaneously and these two processors must be pre assigned to the task. A processor cannot work on two jobs simultaneously. This type of tasks will arise when scheduling of file transfers between processors or in case of mutual diagnostic besting of processors. This can be modeled by considering a graph whose vertices correspond to the processes and if there is any task that has to be executed on processors i and j, then and edge to be added between the two vertices. Now the scheduling problem is to assign colors to edges in such a way that every color appears at most once at a vertex. If there are no multiple edges in the graph (i.e) no two tasks require the same two processors then the edge coloring technique can be adopted. The authors have developed an algorithm for multiple edges which gives an 1-1 approximate solution.

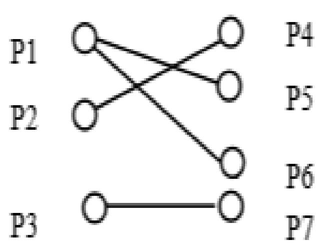

Figure 3.1: Tasks allotted to processors

The diagram shows the tasks namely task1, task2, task3 and task4 are allocated to the processors (P1, P5); (P1, P6); (P2, P4) and (P3, P7) respectively

3.5 AIRCRAFT SCHEDULING

Assuming that there are k aircrafts and they have to be assigned n flights. The i^{th} flight should be during the time interval (ai, bi). If two flights overlap, then the same aircraft cannot be assigned to both the flights. This problem is modeled as a graph as follows. The vertices of the graph correspond to the flights. Two vertices will be connected, if the corresponding time intervals overlap. Therefore, the graph is an interval graph that can be colored optimally in polynomial time.

3.6 TIME TABLE SCHEDULING

Allocation of classes and subjects to the professors is one of the major issues if the constraints are complex. Graph theory plays an important role in this problem. For m professors with n subjects the available number of p periods timetable has to be prepared. This is done as follows. A bipartite graph (or bi-graph is a graph whose vertices can be divided into two disjoint sets U and V such that every edge connects a vertex in U to one in V; that is, U and V are independent sets G where the vertices are the number of professors say m1, m2, m3, m4, ……. m_k and n number of subjects say n1, n2, n3, n4, ……. nm such that the vertices are connected by pi edges. It is presumed that at any one period each professor can teach at most one subject and that each subject can be taught by maximum one professor. Consider the first period. The timetable for this single period corresponds to a matching in the graph and conversely, each matching corresponds to a possible assignment of professors to subjects taught during that period. So, the solution for the timetabling problem will be obtained by partitioning the edges of graph G into minimum number of matching. Also, the edges have to be colored with minimum number of colors. This problem can also be solved by vertex coloring algorithm. "The line graph L(G) of G has equal number of vertices and edges of G and two vertices in L(G) are connected by an edge if the corresponding edges of G have a vertex in common. The line graph L(G) is a simple graph and a proper vertex coloring of L(G) gives a proper edge coloring of G by the same number of colors. So, the problem can be solved by finding minimum proper vertex coloring of L(G).” For example, consider there are 4 professors namely m1, m2, m3, m4, and 5 subjects say n1, n2, n3, n4, n5 to be taught. The teaching requirement matrix p = [p_{ij}] is given below.

p	n_1	n_2	n_3	n_4	n_5
m_1	2	0	1	1	0
m_2	0	1	0	1	0
m_3	0	1	1	1	0
m_4	0	0	0	1	1

Figure 3.2: The teaching requirement matrix for four professors and five subjects.

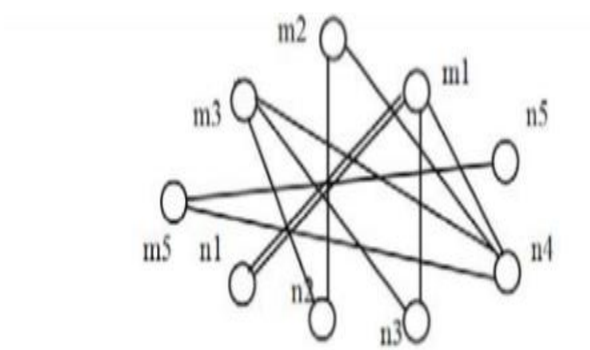

Figure 3.3: Bipartite graph with 4 professors and 5 subjects

Finally, the authors found that proper coloring of the above-mentioned graph can be done by 4 colors using the vertex coloring algorithm which leads to the edge coloring of the bipartite multigraph G.   Four colors are interpreted to four periods.

----	1	2	3	4
m_1	n_1	n_1	n_3	n_4

Figure 3.4: The schedule for the four subjects

3.7 <u>ALGORITHMS AND GRAPH THEORY</u>

The major role of graph theory in computer applications is the development of graph algorithms. Numerous algorithms are used to solve problems that are modeled in the form of graphs. These algorithms are used to solve the graph theoretical concepts which intern used to solve the corresponding computer science application problems. Some algorithms are as follows:

1. Shortest path algorithm in a network

2. Finding a minimum spanning tree

3. Finding graph planarity

4. Algorithms to find adjacency matrices.

5. Algorithms to find the connectedness

6. Algorithms to find the cycles in a graph

7. Algorithms for searching an element in a data structure (DFS, BFS) and so on.

Various computer languages are used to support the graph theory concepts. The main goal of such languages is to enable the user to formulate operations on graphs in a compact and natural manner Some graph theoretic languages are

1. SPANTREE – To find a spanning tree in the given graph. 2.
2. GTPL – Graph Theoretic Language
3. GASP – Graph Algorithm Software Package
4. HINT – Extension of LISP
5. GRASPE – Extension of LISP
6. IGTS – Extension of FORTRAN
7. GEA – Graphic Extended ALGOL (Extension of ALGOL)
8. AMBIT – To manipulate digraphs
9. GIRL – Graph Information Retrieval Language
10. FGRAAL – FORTRAN Extended Graph Algorithmic Language

3.7.1 Graph algorithm in computer network security

The vertex cover algorithm (Given as input a simple graph G with n vertices labeled 1, 2, …, n, search for a vertex cover of size at most k. At each stage, if the vertex cover obtained has size at most k, then stop.) is used to simulate the propagation of stealth worms on large computer networks and design optimal strategies to protect the network against virus attacks in real time. Simulation was carried out in large internet like virtual network and showed that the topology routing has big impact on worm propagation. The importance of finding the worm propagation is to hinder them in real time. The main idea here is to find a minimum vertex cover in the graph whose vertices are the routing servers and the edges are the connections between the routing servers. Then an optimal solution is found for worm propagation and a network defense strategy is defined. In a graph G, a set of edges g is said to cover G if every vertex in G is incident on at least one edge in g. The set of edges that covers a graph G is said to be an edge covering or a covering subgraph or simply a covering of G. Example: A spanning tree in a connected graph is a covering. A Hamiltonian circuit is also a covering. The sample computer network with corresponding minimum vertex cover is shown below

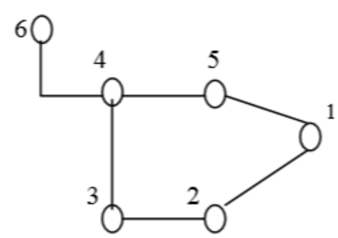

Figure 3.5: The vertex Set g= {2,4,5} covers all the vertices in G

3.7.2 <u>Automatic Channel Allocation For Small Wireless Local Area Networks(using graph coloring algorithm approach)</u>

It deals the channel allocation issue in wireless LAN by means of modeling the network in the form of a graph and solving it using graph coloring methodology. The graph model is constructed and called as interference graph since the access points are interfering with some other access points in the same region. The graph is called as interference graph, which is constructed by the access points as nodes. An undirected edge is connecting these nodes if the nodes interfere with each other when using the same channel. Now, the channel allocation problem is converted into graph coloring problem. i.e. vertex coloring problem. A vertex coloring function

f: v(G)→C where C is the set of colors corresponds to the channels on the access points. These channels are preferably non overlapping edges. A coloring algorithm is developed by the authors called DSATUR (Degree of Saturation) for coloring purpose. The algorithm is a heuristic search. i.e. It finds vertices with largest number of differently colored neighbors. If this subset contains only one vertex it is chosen for coloring. If the subset contains more than one vertex then the coloring is done based on the order of decreasing number of uncolored neighbors. If more than one candidate vertex is available then the final selection is replaced by a deterministic selection function to select the vertex. The protocol operation is done by identifying the neighbors by means of listening the messages generated by the access points. The protocol operation finishes when a message is rebroadcasted by the access points. After finishing this, the interference graph is constructed and the coloring algorithm is applied. The correspondence between the channels and the graph is that as the channels listen the messages in regular intervals as the same way the coloring algorithm should be kept running at regular intervals. A floorplan and the corresponding interference graph is given below.

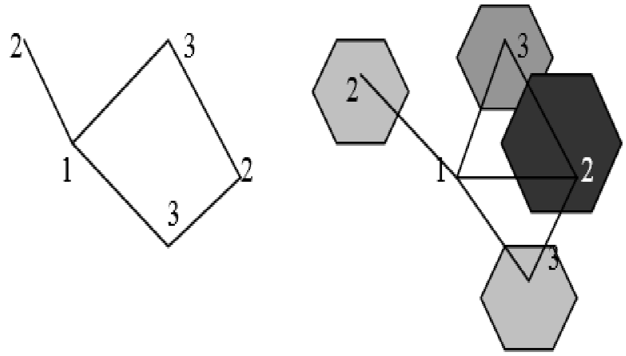

Figure 3.6: Interference graph

3.7.3 Connector Problems

Suppose we are to connect n stations V_1, V_2 V_3 .... V_N through a network of roads. Also let the cost of building a direct road between Vi and V_j is known wherever such direct roads can be built . The connecter problem is to find a network of roads that connects all the n stations together such that the cost of construction is the cheapest.

This network is obviously a **spanning tree**. Thus, the problem of connecting n station with a least expensive network is the problem of finding a minimum weight spanning tree in a connected weight graph of n vertices

A weighted graph is a graph G in which each edge 'e' has been assigned a real number called the weight or length of 'e'. if it is a subgraph of a weighted graph the weight w(H) of H is the sum of the weights of edges in H.

The graph model of the above problem as follows. Let G be a graph whose Vertex set is the set of all stations and xy is an edge in G if it is possible to build a pipe line joining the village x and y. We can then make G into a weighted graph by assigning to each, the cost of constructing the corresponding pipe line.

For example: Let there are six village A, B, C, D, E and F and we get the weighted graph G

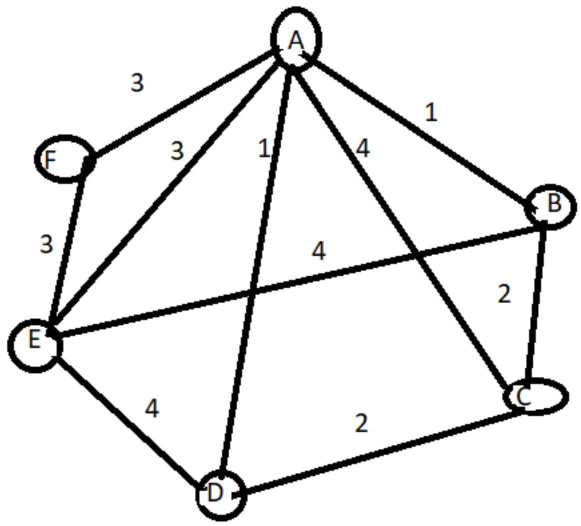

Figure 3.7: A weighted Graph

The lack of an edge from B to D indicates that it is not possible to build a pipe line from B to D. The number 4 assigned to the edge AC indicates that the cost of building pipe line from A to C is 4. We must get a cheapest spanning tree of G, that is a spanning tree with minimum weight. Such a spanning tree is called a minimal tree or an option tree of G.

3.7.3. A. Kruskal's Algorithm

Let a be a simple connected weighted graph. The three steps of the Algorithm are as follows

Step 1: Choose an edge e, such that $w[e_1]$ is as small as possible.

Step 2: If edges $e_{1,}e_2\ldots..e_i$, $i\geq 1$ have been chosen, then choose an edge e_{i+1} from

E= {$e_{1,}$ e_2…. e_i} such that

1)The subgraph induced by {e_1,….. e_{i+1}}is acyclic

2)w(e_{i+1}) as small as possible subject to (i)

Step 3: Stop when step 2 can be implemented further

When the graph is not weighted, we can assign weight of 1 to each of its edges and then apply the algorithms.

Illustration

The distance in miles between some of the Indian cities connected by Indian airlines are given in table A below

	Mumbai	Hyderabad	Nagpur	Calcutta	New Delhi	Chennai
Mumbai						
Hyderabad	385					
Nagpur	425	255				
Calcutta	1035	740	679			
New Delhi	708	773	531	816		
Chennai	644	329		860	1095	

Table 3.1

Determine a minimum- cost operational system so that every city is connected to every other city. Assume that the cost of operation is directly proportional to the distance.

Let G be a graph with the set of cities as its vertex set. An edge corresponds to a pair of cities for which ticked mileage is indicated. The ticked mileage is the weight of the corresponding edge (see table 3.1).

The required operation system demands a minimum cost spanning tree of G. We shall apply Kruskal's algorithm and determine such a system. The following is a sequence of edges selected according to the algorithm-HN, HCh, HM, ND, NCa.

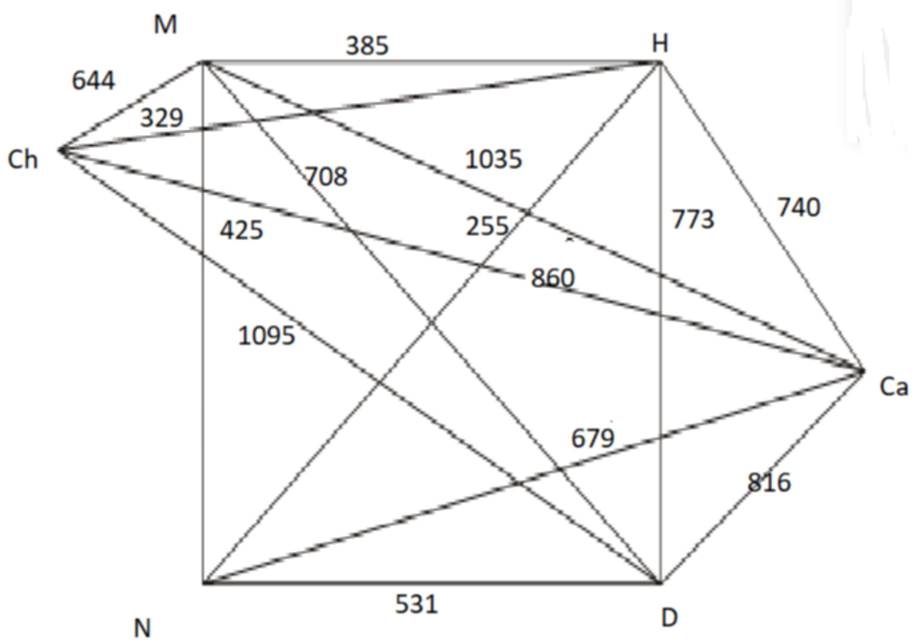

Figure 3.8: shows graph of mileage b/w cities

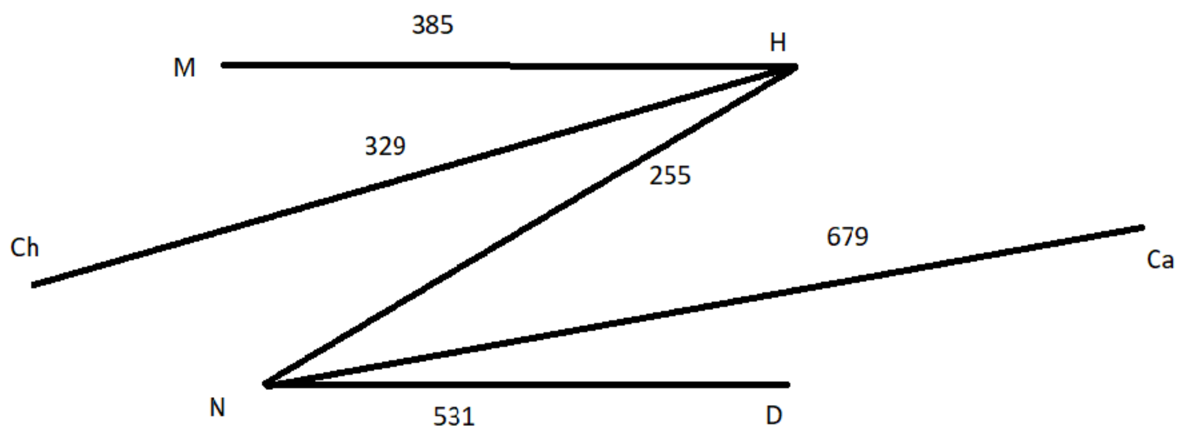

Figure 3.9: a minimum spanning tree graph of above figure

3.7.3. B. Prim's Algorithm

Let G be a simple connected weighted graph having n vertices. Let the vertices of G be labeled as $V_{1,}V_{2,\ldots\ldots,}$ Let W= W(G)=[W_{ij}] be the weight matrix of G . That is ,W is the n×n matrix with

Step1:$W_{ii}=\infty$ for $1\leq i\leq n$

Step 2: $W_{ij}=W_{ji}$=the weight of the edge V_i,V_j if Vi and V_j are adjacent

Step 3 $W_{ij}=W_{ij}=\infty$, if V_i and are not adjacent.

The algorithm gives a minimum weight spanning tree

Step 1: Start with V_1 and connect to V_K where V_k is a nearest vertex to V_1 (V_k is nearest to V_1 if $V_{1,}$ V_k is an edge with minimum possible weight).The vertex V_k can be easily determined by observing the matrix W. actually ,V_k is a vertex corresponding to which the entry in row 1 of W is minimum

Step 2: Having chosen V_k, let $V_i \neq V_1$ or V_k be a vertex corresponding to the smallest entry in rows 1 and K put together. Then V_i is the vertex 'nearest' to the edge subgraph defined by the edge V_1, V_k. Connect V_i to V_1 or V_k recording as whether the entry is in the first row or k^{th} row . suppose it is , say in the k^{th} row then it is the $(k,i)^{th}$ entry of W

Step 3: consider the edge subgraph defined by the edge set$\{V_1 V_k, V_k V_j\}$. determine the nearest neighbor to the set of vertices $\{V_1, V_k, V_j\}$

Step 4: continue the process until all the n vertices have been connected by (n-1) edges . this results in a minimum – cost spanning tree

Illustration

Consider the weighted graph G shown in fig 2.10 the weight matrix W of G is shown below

	M	H	N	Ca	D	Ch
M	∞	385	425	1035	708	644
H	385	∞	455	740	773	329
N	425	255	∞	679	531	∞
Ca	1035	740	679	∞	816	860
D	708	773	531	816	∞	1095
Ch	644	329	∞	860	1095	∞

Table 2

In row M (ie; in the row corresponding to the city m , namely ,Mumbai) , the smallest weight is 385 which occurs in column H. hence , join M and H now , after omitting columns M and H 255 is the minimum weight in the rows M and H put together .it occurs in row H and column N . Hence, join H and N. now omitting column M, H and N. the smallest number in the rows M, H and N put together is 329 and it occurs in row H and column Ch, so join H and Ch. Again, the smallest entry in rows M, H, N and Ch not belonging to the corresponding columns is 531 and it occurs in the N and column D, so join N and H. Now, the least entry in rows M, H,N,D and Ch not belonging to the corresponding columns is 679 , and it occurs in row N and column Ca .so join N and Ca . This construction gives some minimum weight spanning tree of figure below

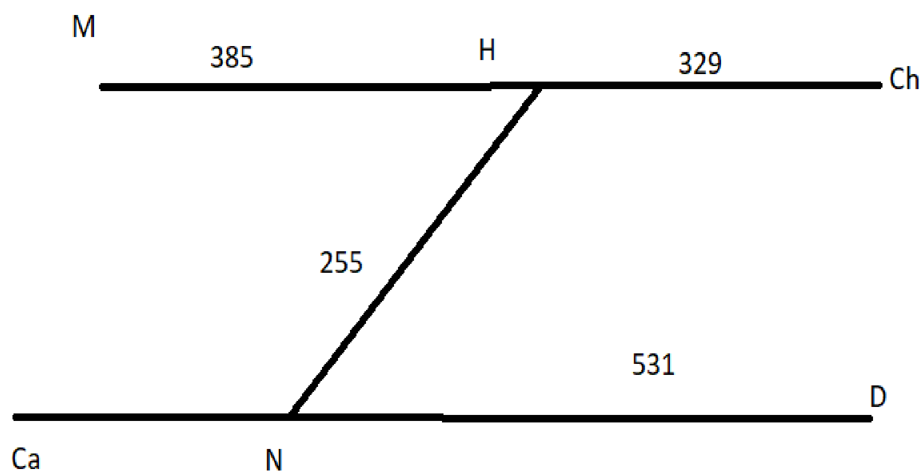

Figure 3.10: minimumweight spanning tree

Remark: In each iteration of prim's algorithm, a subtree of a minimum weight spanning tree is obtained, whereas the sub graph constructed in any step of Kruskal's algorithm is just a sub graph of a minimum – weight spanning tree

3.7.4 <u>Shortest- Path Algorithm</u>

A manufacturing concern has a ware house at location X and the market for the product at another location Y. Given the various routes of transporting the product from x to Y and the cost of operating them, what is the economical way of transporting the materials?

This problem can be tackled by graph theory all such optimization problems come under a type of graph theoretic problem known as "shortest path problems" three types of shortest – path problems are well-known. Let G be a connected weighted graph .

1: Determine a shortest path , that is , a minimum- weight path b/w two specified vertices of G .

2: Determine a set of shortest paths b/w all pairs of vertices of G .

3: Determine a set of shortest paths from a specified vertex to all other vertices of G

We consider only the first problem. The other two problem are similar, we describe Dijkstra's algorithm for determining the shortest path between two specified vertices. Once again, it is clear that in shortest –path problems, we could restrict ourselves to simple connected weighted graphs.

3.7.4. A. Dijkstra's Algorithm

Let G be a simple connected weighted graph having vertices $V_{1,}V_{2......}$,V. let 'S' and 'T' be the two specified vertices of G . we want to determine a shortest path from S to T. let W be the weight matrix of G . Dijkstra's algorithm allots weights to the vertices of G . at each stage of a algorithm. Some vertices have permanent weights and others have temporary weights.

To start with, the vertex S is allotted the permanent weight O and all other vertices the temporary weight ∞. In each iteration of the algorithm, one new vertex is allotted a permanent weight by the following rules;

Rule 1: If V_j is a vertex that has not yet been allotted a permanent weight , determine , for each vertex V_I that had already been allotted a permanent weight

A_{ij} =min{old weight of V_i(old weight of V_i)+W_{ij}

Let W_{ij}= mini A_{ij} . then wij is a new temporary weight of V_j not exceeding the previous temporary weight.

RULE 2 : Determine the smallest among the W_j . if this smallest weight is at V_k . W_k becomes the permanent weight of $V_{k.}$ in case there is a tie , any vertex is taken for allotting a permanent weight .

The algorithm stops when the vertex t gets a permanent weight.

It is a clear from that the permanent weight of vertex is the shortest weighted distance from S to that vertex. The shortest path from S and T is constructed by working backward from the vertex T

Illustration

We present below an illustration example. Let us find the shortest path from A to B in the graph below.

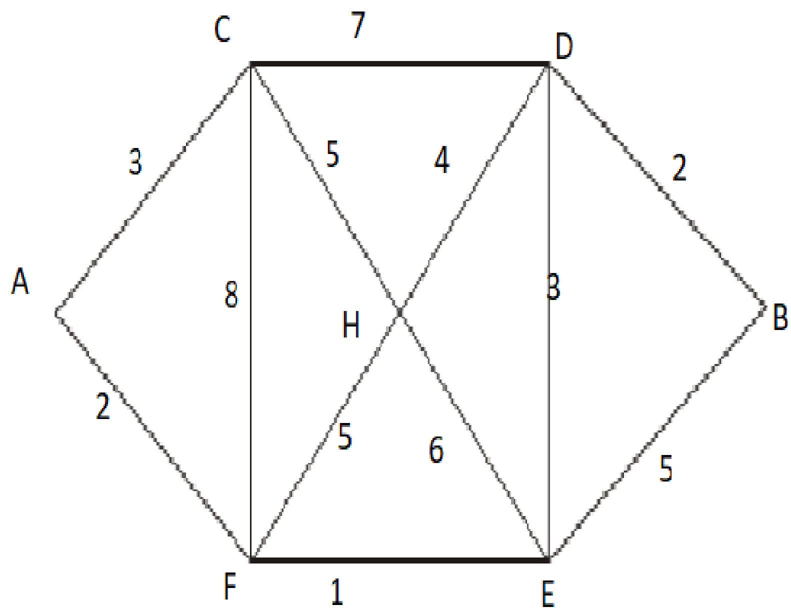

Figure 3.10: Graph G for shortest path problem

We present the various iterations of the algorithm by arrays of weights of the vertices one consisting of weights before iteration and another after it temporary weights will be enclosed in double squares. the steps of the algorithm for determining the shortest path from vertex A to vertex B in graph G of figure above are given table below

In our example, B is the last vertex to get a permanent weight. hence, the algorithm stops after iteration 6, in which is allotted the permanent weight. however, the algorithm may be stopped as soon as vertex B gets the permanent weight.

The shortest distances from A to B is **8**. a shortest part is A F E D B with weight 8

	A	B	C	D	E	F	H
Integration 0	0	∞	∞	∞	∞	∞	∞
Integration 1	0	∞	3	∞	∞	2	∞
Integration 2	0	∞	3	∞	3	2	7
Integration 3	0	∞	3	10	3	2	7
Integration 4	0	8	3	6	3	2	7
Integration 5	0	8	3	6	3	2	7
Integration 6	0	8	3	6	3	2	7

Table 3. Steps of algorithm path from A to B

3.7.5 Clustering of Web Documents Using Graph Model

Here discussed the enhanced representation of web documents by means of clustering them. They used graphs instead of vectors. Here they have used the classical **k-means clustering algorithm** which uses the maximum common subgraph distance measure instead of usual distance measure and the concept of median graphs instead of centroid calculations. (k-means clustering is a method of cluster analysis which aims to partition n observations into k clusters in which each observation belongs to the cluster with the nearest mean.[19]) (A median graph is an undirected graph in which any three vertices a, b, and c have a unique median: a vertex m(a,b,c) that belongs to shortest paths between any two of a, b, and c[16]

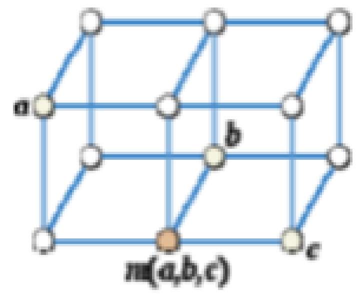

Figure 3.11: The median of three vertices in a median graph

Since, traditional clustering methods are working purely on numeric feature vectors, the origi9nal data needs to be converted to a vector of numeric values by discarding possibly useful structural information. Otherwise, new customized algorithms have to be developed for specific representation. Modeling the web documents as graphs has two significant benefits.

1. It keeps the inherent structure of the original documents, rather to arrive numeric feature vectors that contain term frequencies.

2. There is no necessity to build new clustering algorithm completely from scratch. But the extension of classical algorithms can be developed to deal with graphs that use numerical vectors.

The graph model of the web documents has constructed by the following method.
1. Each word that appears in the web document except stop words is a vertex in the graph representing that document. This is executed by a node labeling function which gives labels to each node. Even though a word is repeated more than once, it is represented as only one vertex. Therefore each vertex in the graph represents a unique word and is labeled with a unique term which is not used to label any other node.

2. If any word say b follows another word say a then there is a directed edge between these two words a and b. If these two words are in a section say s, then the edge between a and b will be labeled as s.

3. Some punctuation marks are not taken into account for edge creation.

4. Three sections are defined here. They are sections for title tag, Section link and Section text.

5. The nodes and their corresponding incident edges to the words such as the, end, of, a, for, to etc., are removed because these words don't play much role.

6. A stemming check is performed for plural forms.

7. The most infrequent words are removed from each page by leaving maximum nodes say m for each and every graph where m is the user defined parameter. The example of the graph representation of the web document is given below. The edges are labeled based on the titles, links and texts.

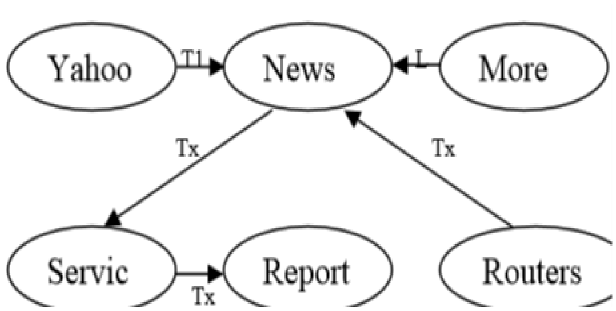

Figure 3.12: Web document – Graph representation

3.7.6 Graph-based and structural methods for fingerprint classification

Fingerprint classification is mainly used in criminal investigation. This classification of fingerprints uses databases for storage of fingerprints. But the database will become large and the storage capacity will be more if it is stored as it is. Various approaches are available for the classification namely structural approaches, statistical approaches and graph-based approaches. Here it is discussed only the graph-based approaches. Previously, the work was done on the basis of segmenting the fingerprint images into regions containing ridges having homogeneous orientations. But this type of structural information is not useful in identifying the fingerprints based on other classes, since they have also the same structural arrangements. The graph-based approach uses simple relational graphs. The following diagram shows relational graph for fingerprint orientation field segmentation.

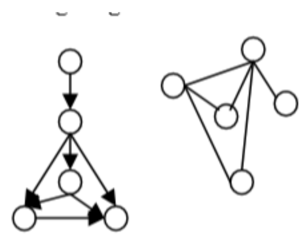

Figure 3.13: Relational graph

This method serves as a guide for segmentation. Relational graphs appear to be more appropriate since the nodes naturally correspond to the regions extracted by the **segmentation algorithm**. Each graph node can be associated to a segmentation region and the edges join two nodes according to the adjacency relationship of the respective regions. To compute relational graphs, the error correcting graph matching is used. This method computes a measure of the dissimilarity between the graph representing the input pattern to be classified and a certain graph prototype. This similarity measure is called edit distance. This is done by deformation model which can be done by substituting, inserting or deleting nodes or edges. The main idea stressed here is the graph based representation of the fingerprints is much useful for classification purpose than the other structural methods.

CHAPTER-4

APPLICATIONS IN NETWORKS AND GAMES

Graph theory is also used in modeling transport networks, activity networks and theory of games. The network activity is used to solve large number of combinatorial problems. The most popular and successful applications of networks in OR is the planning and scheduling of large complicated projects. The best well known problems are PERT (Project Evaluation Review Technique) and CPM (Critical Path Method). Next, Game theory is applied to the problems in engineering, economics and war science to find optimal way to perform certain tasks in competitive environments. To represent the method of finite game a digraph is used.

4.1 GRAPH THEORY IN OR

Graph theory is a very natural and powerful tool in combinatorial operations research. Some important OR problems that can be solved using graphs are given here. A network called transport network where a graph is used to model the transportation of commodity from one place to another. The objective is to maximize the flow or minimize the cost within the prescribed flow. The graph theoretic approach is found more efficient for these types of problems though they have more constraint.

4.2 COMPUTER NETWORK SECURITY

A team of computer scientists led by Eric Filiol at the Virology and Cryptology Lab, ESAT, and the French Navy, ESCANSIC, have recently used the vertex cover algorithm to simulate the propagation of stealth worms on large computer networks and design optimal strategies for protecting the network against virus attacks.

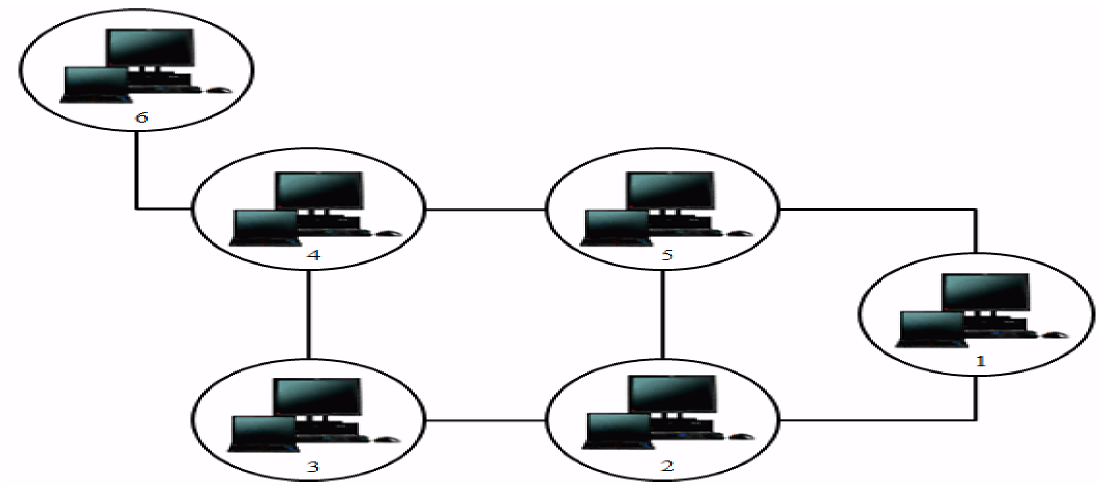

Figure 4.1: The set {2, 4, 5} is a minimum vertex cover in this computer network

The simulation was carried out on a large internet-like virtual network and showed that the combinatorial topology of routing may have a huge impact on the worm propagation and thus some servers play a more essential and significant role than others. The real-time capability to identify them is essential to greatly hinder worm propagation. The idea is to find a minimum vertex cover in the graph whose vertices are the routing servers and whose edges are the (possibly dynamic) connections between routing servers. This is an optimal solution for worm propagation and an optimal solution for designing the network defense strategy. Figure 4.1 above shows a simple computer network and a corresponding minimum vertex cover {2, 4, 5}.

4.3 KNIGHT’S TOURS

In 840 A.D., al-Adli , a renowned shatranj (chess) player of Baghdad is said to have discovered the first re-entrant knight's tour, a sequence of moves that takes the knight to each square on an 8×8 chessboard exactly once, returning to the original square. Many other re-entrant knight's tours were subsequently discovered but Euler was the first mathematician to do a systematic analysis in 1766, not only for the 8×8 chessboard, but for re-entrant knight's tours on the general $n \times n$ chessboard. Given an $n \times n$ chessboard, define a knight's graph with a vertex corresponding to each square of the chessboard and an edge connecting vertex i with vertex j if and only if there is a legal knight's move from the square corresponding to vertex i to the square corresponding to vertex j. Thus, a re-entrant knight's tour on the chessboard corresponds to a Hamiltonian circuit in the knight's graph. The Hamiltonian circuit algorithm has been used to find re-entrant knights tours on chessboards of various dimensions.

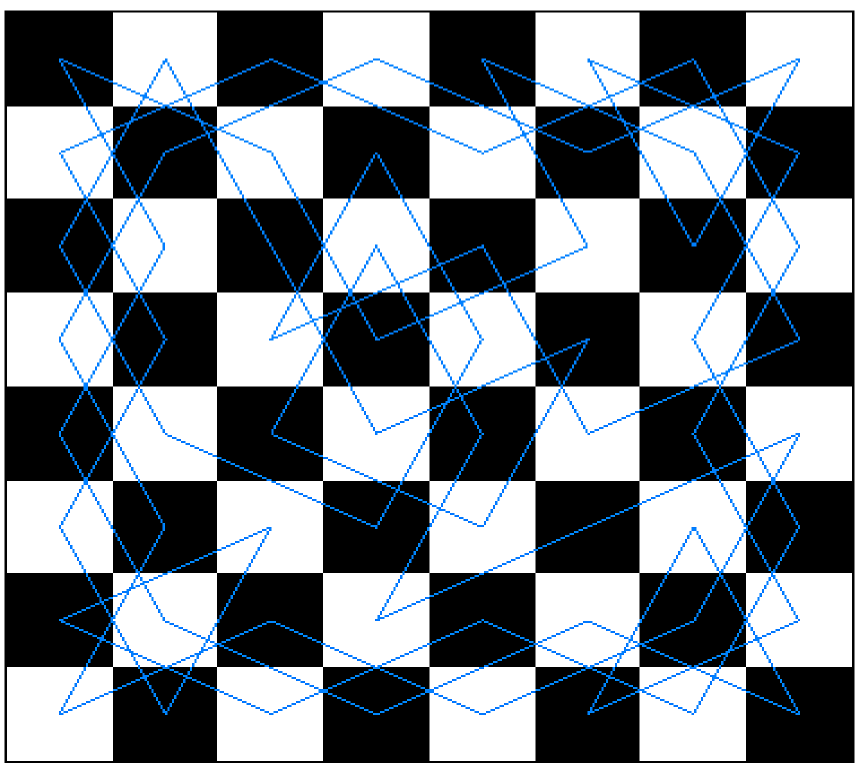

Figure 4.2: A re-entrant knight's tour on the 8×8 chessboard

4. 4 SUDOKU

Enumerate the 81 cells of the Sudoku board with numbers from 1 to 81 then the graph associated with a Sudoku puzzle consists of 81 vertices (one for each cell of the board), together with edges as follows ,two vertices are connected by an edges if the cells that they correspond to are in the same column ,row or 3×3 box we have thus represented the Sudoku grid as a graph . With 81 vertices and several hundred edges it would be a big graph if one wanted to draw it so let us just thick about it without attempting to produce a graphical representation. what about the numbers in the cells of the Sudoku puzzle how do we represent those? Again each number from 1 to 9 a color now color the vertices corresponding to cell that contains a given number in the color of the number it is now easy to see that completing a Sudoku puzzle without violating the Sudoku condition is equivalent to coloring the vertices of the corresponding graph while ensuring that no two adjacent vertices have the same color.

CONCLUSION

Graph theory is rapidly moving into the mainstreams of mathematics mainly because of its applications in diverse fields which includes computer and mobile networks operation research and so on. The powerful combinatorial methods found in graph theory have also been used to prove real life situations also.

The various applications that we discussed are map coloring, computer network security, applications of different kinds of algorithms, job scheduling, processor tasks so on. These all shows the simplicity of graph theory which is differ from another mathematical fields. Graphs are considered as an excellent modeling tool which can be used to model many types of relations amongst any physical situations. Many problems of real world can be represented by the graphs

There is wide use of graphs in providing problem solving technique, because it gives an intuitive manner prior to present formal definition. To analyze the graph theory application two problem areas are considered, which are classical problems and problems from applications. Classical problems are defined with the help of graph theory as connectivity, cuts, paths and flows, coloring problems and theoretical aspects of graph drawing. Whereas, problems from application particularly emphasis on experimental research and the implementation of the graph theory algorithms. Depending on the particular situation, restrictions are imposed on the type edges we allow. For some problems directed edges are applied and for other problems undirected edges are applied from one vertex to other. So graphs give as many techniques and flexibility while defining and solving a real life problem.

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