

# The Open Source CFD Toolbox

# Programmer's Guide

Version 2.2.1 17th June 2013 Copyright © 2011-2013 OpenFOAM Foundation.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivs 3.0 Unported License.

Typeset in LATEX.

#### License

THE WORK (AS DEFINED BELOW) IS PROVIDED UNDER THE TERMS OF THIS CREATIVE COMMONS PUBLIC LICENSE ("CCPL" OR "LICENSE"). THE WORK IS PROTECTED BY COPYRIGHT AND/OR OTHER APPLICABLE LAW. ANY USE OF THE WORK OTHER THAN AS AUTHORIZED UNDER THIS LICENSE OR COPYRIGHT LAW IS PROHIBITED.

BY EXERCISING ANY RIGHTS TO THE WORK PROVIDED HERE, YOU ACCEPT AND AGREE TO BE BOUND BY THE TERMS OF THIS LICENSE. TO THE EXTENT THIS LICENSE MAY BE CONSIDERED TO BE A CONTRACT, THE LICENSOR GRANTS YOU THE RIGHTS CONTAINED HERE IN CONSIDERATION OF YOUR ACCEPTANCE OF SUCH TERMS AND CONDITIONS.

#### 1. Definitions

- a. "Adaptation" means a work based upon the Work, or upon the Work and other preexisting works, such as a translation, adaptation, derivative work, arrangement of music or other alterations of a literary or artistic work, or phonogram or performance and includes cinematographic adaptations or any other form in which the Work may be recast, transformed, or adapted including in any form recognizably derived from the original, except that a work that constitutes a Collection will not be considered an Adaptation for the purpose of this License. For the avoidance of doubt, where the Work is a musical work, performance or phonogram, the synchronization of the Work in timed-relation with a moving image ("synching") will be considered an Adaptation for the purpose of this License.
- b. "Collection" means a collection of literary or artistic works, such as encyclopedias and anthologies, or performances, phonograms or broadcasts, or other works or subject matter other than works listed in Section 1(f) below, which, by reason of the selection and arrangement of their contents, constitute intellectual creations, in which the Work is included in its entirety in unmodified form along with one or more other contributions, each constituting separate and independent works in themselves, which together are assembled into a collective whole. A work that constitutes a Collection will not be considered an Adaptation (as defined above) for the purposes of this License.
- c. "Distribute" means to make available to the public the original and copies of the Work through sale or other transfer of ownership.
- d. "Licensor" means the individual, individuals, entity or entities that offer(s) the Work under the terms of this License.
- e. "Original Author" means, in the case of a literary or artistic work, the individual, individuals, entity or entities who created the Work or if no individual or entity can be identified, the publisher; and in addition (i) in the case of a performance the actors, singers, musicians, dancers, and other persons who act, sing, deliver, declaim, play in, interpret or otherwise perform literary or artistic works or expressions of folklore; (ii) in the case of a phonogram the producer being the person or legal entity who first fixes the sounds of

- a performance or other sounds; and, (iii) in the case of broadcasts, the organization that transmits the broadcast.
- f. "Work" means the literary and/or artistic work offered under the terms of this License including without limitation any production in the literary, scientific and artistic domain, whatever may be the mode or form of its expression including digital form, such as a book, pamphlet and other writing; a lecture, address, sermon or other work of the same nature; a dramatic or dramatico-musical work; a choreographic work or entertainment in dumb show; a musical composition with or without words; a cinematographic work to which are assimilated works expressed by a process analogous to cinematography; a work of drawing, painting, architecture, sculpture, engraving or lithography; a photographic work to which are assimilated works expressed by a process analogous to photography; a work of applied art; an illustration, map, plan, sketch or three-dimensional work relative to geography, topography, architecture or science; a performance; a broadcast; a phonogram; a compilation of data to the extent it is protected as a copyrightable work; or a work performed by a variety or circus performer to the extent it is not otherwise considered a literary or artistic work.
- g. "You" means an individual or entity exercising rights under this License who has not previously violated the terms of this License with respect to the Work, or who has received express permission from the Licensor to exercise rights under this License despite a previous violation.
- h. "Publicly Perform" means to perform public recitations of the Work and to communicate to the public those public recitations, by any means or process, including by wire or wireless means or public digital performances; to make available to the public Works in such a way that members of the public may access these Works from a place and at a place individually chosen by them; to perform the Work to the public by any means or process and the communication to the public of the performances of the Work, including by public digital performance; to broadcast and rebroadcast the Work by any means including signs, sounds or images.
- i. "Reproduce" means to make copies of the Work by any means including without limitation by sound or visual recordings and the right of fixation and reproducing fixations of the Work, including storage of a protected performance or phonogram in digital form or other electronic medium.

### 2. Fair Dealing Rights.

Nothing in this License is intended to reduce, limit, or restrict any uses free from copyright or rights arising from limitations or exceptions that are provided for in connection with the copyright protection under copyright law or other applicable laws.

#### 3. License Grant.

Subject to the terms and conditions of this License, Licensor hereby grants You a worldwide, royalty-free, non-exclusive, perpetual (for the duration of the applicable copyright) license to exercise the rights in the Work as stated below:

- a. to Reproduce the Work, to incorporate the Work into one or more Collections, and to Reproduce the Work as incorporated in the Collections;
- b. and, to Distribute and Publicly Perform the Work including as incorporated in Collections.

The above rights may be exercised in all media and formats whether now known or hereafter devised. The above rights include the right to make such modifications as are technically necessary to exercise the rights in other media and formats, but otherwise you have no rights

to make Adaptations. Subject to 8(f), all rights not expressly granted by Licensor are hereby reserved, including but not limited to the rights set forth in Section 4(d).

#### 4. Restrictions.

The license granted in Section 3 above is expressly made subject to and limited by the following restrictions:

- a. You may Distribute or Publicly Perform the Work only under the terms of this License. You must include a copy of, or the Uniform Resource Identifier (URI) for, this License with every copy of the Work You Distribute or Publicly Perform. You may not offer or impose any terms on the Work that restrict the terms of this License or the ability of the recipient of the Work to exercise the rights granted to that recipient under the terms of the License. You may not sublicense the Work. You must keep intact all notices that refer to this License and to the disclaimer of warranties with every copy of the Work You Distribute or Publicly Perform. When You Distribute or Publicly Perform the Work, You may not impose any effective technological measures on the Work that restrict the ability of a recipient of the Work from You to exercise the rights granted to that recipient under the terms of the License. This Section 4(a) applies to the Work as incorporated in a Collection, but this does not require the Collection apart from the Work itself to be made subject to the terms of this License. If You create a Collection, upon notice from any Licensor You must, to the extent practicable, remove from the Collection any credit as required by Section 4(c), as requested.
- b. You may not exercise any of the rights granted to You in Section 3 above in any manner that is primarily intended for or directed toward commercial advantage or private monetary compensation. The exchange of the Work for other copyrighted works by means of digital file-sharing or otherwise shall not be considered to be intended for or directed toward commercial advantage or private monetary compensation, provided there is no payment of any monetary compensation in connection with the exchange of copyrighted works.
- c. If You Distribute, or Publicly Perform the Work or Collections, You must, unless a request has been made pursuant to Section 4(a), keep intact all copyright notices for the Work and provide, reasonable to the medium or means You are utilizing: (i) the name of the Original Author (or pseudonym, if applicable) if supplied, and/or if the Original Author and/or Licensor designate another party or parties (e.g., a sponsor institute, publishing entity, journal) for attribution ("Attribution Parties") in Licensor's copyright notice, terms of service or by other reasonable means, the name of such party or parties; (ii) the title of the Work if supplied; (iii) to the extent reasonably practicable, the URI, if any, that Licensor specifies to be associated with the Work, unless such URI does not refer to the copyright notice or licensing information for the Work. The credit required by this Section 4(c) may be implemented in any reasonable manner; provided, however, that in the case of a Collection, at a minimum such credit will appear, if a credit for all contributing authors of Collection appears, then as part of these credits and in a manner at least as prominent as the credits for the other contributing authors. For the avoidance of doubt, You may only use the credit required by this Section for the purpose of attribution in the manner set out above and, by exercising Your rights under this License, You may not implicitly or explicitly assert or imply any connection with, sponsorship or endorsement by the Original Author, Licensor and/or Attribution Parties, as appropriate, of You or Your use of the Work, without the separate, express prior written permission of the Original Author, Licensor and/or Attribution Parties.
- d. For the avoidance of doubt:

- i. Non-waivable Compulsory License Schemes. In those jurisdictions in which the right to collect royalties through any statutory or compulsory licensing scheme cannot be waived, the Licensor reserves the exclusive right to collect such royalties for any exercise by You of the rights granted under this License;
- ii. Waivable Compulsory License Schemes. In those jurisdictions in which the right to collect royalties through any statutory or compulsory licensing scheme can be waived, the Licensor reserves the exclusive right to collect such royalties for any exercise by You of the rights granted under this License if Your exercise of such rights is for a purpose or use which is otherwise than noncommercial as permitted under Section 4(b) and otherwise waives the right to collect royalties through any statutory or compulsory licensing scheme; and,
- iii. Voluntary License Schemes. The Licensor reserves the right to collect royalties, whether individually or, in the event that the Licensor is a member of a collecting society that administers voluntary licensing schemes, via that society, from any exercise by You of the rights granted under this License that is for a purpose or use which is otherwise than noncommercial as permitted under Section 4(b).
- e. Except as otherwise agreed in writing by the Licensor or as may be otherwise permitted by applicable law, if You Reproduce, Distribute or Publicly Perform the Work either by itself or as part of any Collections, You must not distort, mutilate, modify or take other derogatory action in relation to the Work which would be prejudicial to the Original Author's honor or reputation.

#### 5. Representations, Warranties and Disclaimer

UNLESS OTHERWISE MUTUALLY AGREED BY THE PARTIES IN WRITING, LICENSOR OFFERS THE WORK AS-IS AND MAKES NO REPRESENTATIONS OR WARRANTIES OF ANY KIND CONCERNING THE WORK, EXPRESS, IMPLIED, STATUTORY OR OTHERWISE, INCLUDING, WITHOUT LIMITATION, WARRANTIES OF TITLE, MERCHANTIBILITY, FITNESS FOR A PARTICULAR PURPOSE, NONINFRINGEMENT, OR THE ABSENCE OF LATENT OR OTHER DEFECTS, ACCURACY, OR THE PRESENCE OF ABSENCE OF ERRORS, WHETHER OR NOT DISCOVERABLE. SOME JURISDICTIONS DO NOT ALLOW THE EXCLUSION OF IMPLIED WARRANTIES, SO SUCH EXCLUSION MAY NOT APPLY TO YOU.

#### 6. Limitation on Liability.

EXCEPT TO THE EXTENT REQUIRED BY APPLICABLE LAW, IN NO EVENT WILL LICENSOR BE LIABLE TO YOU ON ANY LEGAL THEORY FOR ANY SPECIAL, INCIDENTAL, CONSEQUENTIAL, PUNITIVE OR EXEMPLARY DAMAGES ARISING OUT OF THIS LICENSE OR THE USE OF THE WORK, EVEN IF LICENSOR HAS BEEN ADVISED OF THE POSSIBILITY OF SUCH DAMAGES.

#### 7. Termination

- a. This License and the rights granted hereunder will terminate automatically upon any breach by You of the terms of this License. Individuals or entities who have received Collections from You under this License, however, will not have their licenses terminated provided such individuals or entities remain in full compliance with those licenses. Sections 1, 2, 5, 6, 7, and 8 will survive any termination of this License.
- b. Subject to the above terms and conditions, the license granted here is perpetual (for the duration of the applicable copyright in the Work). Notwithstanding the above, Licensor reserves the right to release the Work under different license terms or to stop distributing the Work at any time; provided, however that any such election will not serve to withdraw

this License (or any other license that has been, or is required to be, granted under the terms of this License), and this License will continue in full force and effect unless terminated as stated above.

#### 8. Miscellaneous

- a. Each time You Distribute or Publicly Perform the Work or a Collection, the Licensor offers to the recipient a license to the Work on the same terms and conditions as the license granted to You under this License.
- b. If any provision of this License is invalid or unenforceable under applicable law, it shall not affect the validity or enforceability of the remainder of the terms of this License, and without further action by the parties to this agreement, such provision shall be reformed to the minimum extent necessary to make such provision valid and enforceable.
- c. No term or provision of this License shall be deemed waived and no breach consented to unless such waiver or consent shall be in writing and signed by the party to be charged with such waiver or consent.
- d. This License constitutes the entire agreement between the parties with respect to the Work licensed here. There are no understandings, agreements or representations with respect to the Work not specified here. Licensor shall not be bound by any additional provisions that may appear in any communication from You.
- e. This License may not be modified without the mutual written agreement of the Licensor and You. The rights granted under, and the subject matter referenced, in this License were drafted utilizing the terminology of the Berne Convention for the Protection of Literary and Artistic Works (as amended on September 28, 1979), the Rome Convention of 1961, the WIPO Copyright Treaty of 1996, the WIPO Performances and Phonograms Treaty of 1996 and the Universal Copyright Convention (as revised on July 24, 1971). These rights and subject matter take effect in the relevant jurisdiction in which the License terms are sought to be enforced according to the corresponding provisions of the implementation of those treaty provisions in the applicable national law. If the standard suite of rights granted under applicable copyright law includes additional rights not granted under this License, such additional rights are deemed to be included in the License; this License is not intended to restrict the license of any rights under applicable law.

#### **Trademarks**

ANSYS is a registered trademark of ANSYS Inc.

CFX is a registered trademark of Ansys Inc.

CHEMKIN is a registered trademark of Reaction Design Corporation

EnSight is a registered trademark of Computational Engineering International Ltd.

Fieldview is a registered trademark of Intelligent Light

Fluent is a registered trademark of Ansys Inc.

GAMBIT is a registered trademark of Ansys Inc.

Icem-CFD is a registered trademark of Ansys Inc.

I-DEAS is a registered trademark of Structural Dynamics Research Corporation

JAVA is a registered trademark of Sun Microsystems Inc.

Linux is a registered trademark of Linus Torvalds

OpenFOAM is a registered trademark of SGI Corp.

ParaView is a registered trademark of Kitware

STAR-CD is a registered trademark of Computational Dynamics Ltd.

UNIX is a registered trademark of The Open Group

# Contents

$\mathbf{C}$	opyri	ight No	otice	P-2
	1. D	efinition	ns	P-2
	2. F	air Deal	ling Rights	P-3
	3. L	icense (	Grant	P-3
	4. R	Restriction	ons	P-4
	5. R	depresen	tations, Warranties and Disclaimer	P-5
	6. L	imitatio	on on Liability.	P-5
	7. T	erminat	<del>ion</del>	P-5
	8. N	Iiscellan	leous	P-6
Ti	rader	narks		P-7
$\mathbf{C}$	onter	nts		P-9
1	Ten	sor ma	athematics	P-11
	1.1	Coordi	inate system	P-11
	1.2	Tensor	S	P-11
		1.2.1	Tensor notation	P-13
	1.3	Algebr	raic tensor operations	P-13
		1.3.1	The inner product	P-14
		1.3.2	The double inner product of two tensors	P-15
		1.3.3	The triple inner product of two third rank tensors	P-15
		1.3.4	The outer product	P-15
		1.3.5	The cross product of two vectors	P-15
		1.3.6	Other general tensor operations	P-16
		1.3.7	Geometric transformation and the identity tensor	P-16
		1.3.8	Useful tensor identities	P-17
		1.3.9	Operations exclusive to tensors of rank 2	P-17
			Operations exclusive to scalars	P-18
	1.4		FOAM tensor classes	P-19
		1.4.1	Algebraic tensor operations in OpenFOAM	P-19
	1.5	Dimen	sional units	P-21
2			tion procedures	P-23
	2.1		ential operators	P-23
		2.1.1	Gradient	P-23
		2.1.2	Divergence	P-24
		2.1.3	Curl	P-24
		2.1.4	Laplacian	P-24
		2.1.5	Temporal derivative	P-24
	2.2	()vervi	iew of discretisation	P-25

P-10 Contents

		2.2.1	OpenFOAM lists and fields
	2.3	Discre	tisation of the solution domain
		2.3.1	Defining a mesh in OpenFOAM
		2.3.2	Defining a geometricField in OpenFOAM
	2.4	Equati	ion discretisation
		2.4.1	The Laplacian term
		2.4.2	The convection term
		2.4.3	First time derivative
		2.4.4	Second time derivative
		2.4.5	Divergence
		2.4.6	Gradient
		2.4.7	Grad-grad squared
		2.4.8	Curl
		2.4.9	Source terms
			Other explicit discretisation schemes
	2.5		oral discretisation
	2.0	2.5.1	Treatment of temporal discretisation in OpenFOAM
	2.6	_	lary Conditions
	2.0	2.6.1	Physical boundary conditions
		2.0.1	1 hysical boundary conditions
3	Exa	mples	of the use of OpenFOAM
	3.1	_	around a cylinder
		3.1.1	Problem specification
		3.1.2	Note on potentialFoam
		3.1.3	Mesh generation
		3.1.4	Boundary conditions and initial fields
		3.1.5	Running the case
	3.2		turbulent flow over a backward-facing step
	J.2	3.2.1	Problem specification
		3.2.2	Mesh generation
		3.2.2	Boundary conditions and initial fields
		3.2.4	Case control
		3.2.4 $3.2.5$	Running the case and post-processing
	3.3		sonic flow over a forward-facing step
	5.5	3.3.1	
		3.3.2	Problem specification
			Mesh generation
		3.3.3	Running the case
	9 1	3.3.4	Exercise
	3.4		appression of a tank internally pressurised with water
		3.4.1	Problem specification
		3.4.2	Mesh Generation
		3.4.3	Preparing the Run
		3.4.4	Running the case
		3.4.5	Improving the solution by refining the mesh
	3.5	_	etohydrodynamic flow of a liquid
		3.5.1	Problem specification
		3.5.2	Mesh generation
		3.5.3	Running the case
_	_		
n	dex		

# Chapter 1

# Tensor mathematics

This Chapter describes tensors and their algebraic operations and how they are represented in mathematical text in this book. It then explains how tensors and tensor algebra are programmed in OpenFOAM.

# 1.1 Coordinate system

OpenFOAM is primarily designed to solve problems in continuum mechanics, *i.e.* the branch of mechanics concerned with the stresses in solids, liquids and gases and the deformation or flow of these materials. OpenFOAM is therefore based in 3 dimensional space and time and deals with physical entities described by tensors. The coordinate system used by OpenFOAM is the right-handed rectangular Cartesian axes as shown in Figure 1.1. This system of axes is constructed by defining an origin O from which three lines are drawn at right angles to each other, termed the Ox, Oy, Oz axes. A right-handed set of axes is defined such that to an observer looking down the Oz axis (with O nearest them), the arc from a point on the Ox axis to a point on the Oy axis is in a clockwise sense.

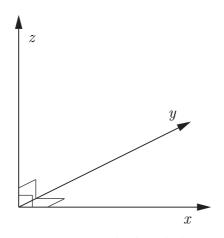


Figure 1.1: Right handed axes

## 1.2 Tensors

The term tensor describes an entity that belongs to a particular space and obeys certain mathematical rules. Briefly, tensors are represented by a set of *component values* relating to a set of unit base vectors; in OpenFOAM the unit base vectors  $\mathbf{i}_x$ ,  $\mathbf{i}_y$  and  $\mathbf{i}_z$  are

P-12 Tensor mathematics

aligned with the right-handed rectangular Cartesian axes x, y and z respectively. The base vectors are therefore orthogonal, i.e. at right-angles to one another. Every tensor has the following attributes:

**Dimension** d of the particular space to which they belong, *i.e.* d = 3 in OpenFOAM;

**Rank** An integer  $r \geq 0$ , such that the number of component values  $= d^r$ .

While OpenFOAM 1.x is set to 3 dimensions, it offers tensors of ranks 0 to 3 as standard while being written in such a way to allow this basic set of ranks to be extended indefinitely. Tensors of rank 0 and 1, better known as scalars and vectors, should be familiar to readers; tensors of rank 2 and 3 may not be so familiar. For completeness all ranks of tensor offered as standard in OpenFOAM 1.x are reviewed below.

- Rank 0 'scalar' Any property which can be represented by a single real number, denoted by characters in italics, e.g. mass m, volume V, pressure p and viscosity  $\mu$ .
- **Rank 1 'vector'** An entity which can be represented physically by both magnitude and direction. In component form, the vector  $\mathbf{a} = (a_1, a_2, a_3)$  relates to a set of Cartesian axes x, y, z respectively. The *index notation* presents the same vector as  $a_i$ , i = 1, 2, 3, although the list of indices i = 1, 2, 3 will be omitted in this book, as it is intuitive since we are always dealing with 3 dimensions.
- Rank 2 'tensor' or second rank tensor, T has 9 components which can be expressed in array notation as:

$$\mathbf{T} = T_{ij} = \begin{pmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{pmatrix}$$
(1.1)

The components  $T_{ij}$  are now represented using 2 indices since r=2 and the list of indices i, j=1,2,3 is omitted as before. The components for which i=j are referred to as the diagonal components, and those for which  $i \neq j$  are referred to as the off-diagonal components. The *transpose* of **T** is produced by exchanging components across the diagonal such that

$$\mathbf{T}^{\mathrm{T}} = T_{ji} = \begin{pmatrix} T_{11} & T_{21} & T_{31} \\ T_{12} & T_{22} & T_{32} \\ T_{13} & T_{23} & T_{33} \end{pmatrix}$$
(1.2)

Note: a rank 2 tensor is often colloquially termed 'tensor' since the occurrence of higher order tensors is fairly rare.

- **Symmetric rank 2** The term 'symmetric' refers to components being symmetric about the diagonal, *i.e.*  $T_{ij} = T_{ji}$ . In this case, there are only 6 independent components since  $T_{12} = T_{21}$ ,  $T_{13} = T_{31}$  and  $T_{23} = T_{32}$ . OpenFOAM distinguishes between symmetric and non-symmetric tensors to save memory by storing 6 components rather than 9 if the tensor is symmetric. Most tensors encountered in continuum mechanics are symmetric.
- **Rank 3** has 27 components and is represented in index notation as  $P_{ijk}$  which is too long to represent in array notation as in Equation 1.1.
- **Symmetric rank 3** Symmetry of a rank 3 tensor is defined in OpenFOAM to mean that  $P_{ijk} = P_{ikj} = P_{jik} = P_{jki} = P_{kij} = P_{kji}$  and therefore has 10 independent components. More specifically, it is formed by the outer product of 3 identical vectors, where the outer product operation is described in Section 1.3.4.

#### 1.2.1 Tensor notation

This is a book on computational continuum mechanics that deals with problems involving complex PDEs in 3 spatial dimensions and in time. It is vital from the beginning to adopt a notation for the equations which is compact yet unambiguous. To make the equations easy to follow, we must use a notation that encapsulates the idea of a tensor as an entity in the own right, rather than a list of scalar components. Additionally, any tensor operation should be perceived as an operation on the entire tensor entity rather than a series of operations on its components.

Consequently, in this book the *tensor notation* is preferred in which any tensor of rank 1 and above, *i.e.* all tensors other than scalars, are represented by letters in bold face, *e.g.* a. This actively promotes the concept of a tensor as a entity in its own right since it is denoted by a single symbol, and it is also extremely compact. The potential drawback is that the rank of a bold face symbol is not immediately apparent, although it is clearly not zero. However, in practice this presents no real problem since we are aware of the property each symbol represents and therefore intuitively know its rank, *e.g.* we know velocity **U** is a tensor of rank 1.

A further, more fundamental idea regarding the choice of notation is that the mathematical representation of a tensor should not change depending on our coordinate system, *i.e.* the vector **a**is the same vector irrespective of where we view it from. The tensor notation supports this concept as it implies nothing about the coordinate system. However, other notations, *e.g.*  $a_i$ , expose the individual components of the tensor which naturally implies the choice of coordinate system. The unsatisfactory consequence of this is that the tensor is then represented by a set of values which are not unique — they depend on the coordinate system.

That said, the index notation, introduced in Section 1.2, is adopted from time to time in this book mainly to expand tensor operations into the constituent components. When using the index notation, we adopt the *summation convention* which states that whenever the same letter subscript occurs twice in a term, the that subscript is to be given all values, *i.e.* 1, 2, 3, and the results added together, *e.g.* 

$$a_i b_i = \sum_{i=1}^3 a_i b_i = a_1 b_1 + a_2 b_2 + a_3 b_3 \tag{1.3}$$

In the remainder of the book the symbol  $\sum$  is omitted since the repeated subscript indicates the summation.

# 1.3 Algebraic tensor operations

This section describes all the algebraic operations for tensors that are available in Open-FOAM. Let us first review the most simple tensor operations: addition, subtraction, and scalar multiplication and division. Addition and subtraction are both commutative and associative and are only valid between tensors of the same rank. The operations are performed by addition/subtraction of respective components of the tensors, e.g. the subtraction of two vectors  $\mathbf{a}$  and  $\mathbf{b}$  is

$$\mathbf{a} - \mathbf{b} = a_i - b_i = (a_1 - b_1, a_2 - b_2, a_3 - b_3) \tag{1.4}$$

Multiplication of any tensor  $\mathbf{a}$  by a scalar s is also commutative and associative and is performed by multiplying all the tensor components by the scalar. For example,

$$s\mathbf{a} = sa_i = (sa_1, sa_2, sa_3)$$
 (1.5)

P-14 Tensor mathematics

Division between a tensor  $\mathbf{a}$  and a scalar is only relevant when the scalar is the second argument of the operation, *i.e.* 

$$\mathbf{a}/s = a_i/s = (a_1/s, a_2/s, a_3/s) \tag{1.6}$$

Following these operations are a set of more complex products between tensors of rank 1 and above, described in the following Sections.

## 1.3.1 The inner product

The inner product operates on any two tensors of rank  $r_1$  and  $r_2$  such that the rank of the result  $r = r_1 + r_2 - 2$ . Inner product operations with tensors up to rank 3 are described below:

• The inner product of two vectors **a** and **b** is commutative and produces a scalar  $s = \mathbf{a} \cdot \mathbf{b}$  where

$$s = a_i b_i = a_1 b_1 + a_2 b_2 + a_3 b_3 \tag{1.7}$$

• The inner product of a tensor  $\mathbf{T}$  and vector  $\mathbf{a}$  produces a vector  $\mathbf{b} = \mathbf{T} \cdot \mathbf{a}$ , represented below as a column array for convenience

$$b_{i} = T_{ij}a_{j} = \begin{pmatrix} T_{11}a_{1} + T_{12}a_{2} + T_{13}a_{3} \\ T_{21}a_{1} + T_{22}a_{2} + T_{23}a_{3} \\ T_{31}a_{1} + T_{32}a_{2} + T_{33}a_{3} \end{pmatrix}$$

$$(1.8)$$

It is non-commutative if **T** is non-symmetric such that  $\mathbf{b} = \mathbf{a} \cdot \mathbf{T} = \mathbf{T}^{\mathrm{T}} \cdot \mathbf{a}$  is

$$b_{i} = a_{j}T_{ji} = \begin{pmatrix} a_{1}T_{11} + a_{2}T_{21} + a_{3}T_{31} \\ a_{1}T_{12} + a_{2}T_{22} + a_{3}T_{32} \\ a_{1}T_{13} + a_{2}T_{23} + a_{3}T_{33} \end{pmatrix}$$

$$(1.9)$$

ullet The inner product of two tensors  ${f T}$  and  ${f S}$  produces a tensor  ${f P}={f T}ullet {f S}$  whose components are evaluated as:

$$P_{ij} = T_{ik} S_{kj} (1.10)$$

It is non-commutative such that  $\mathbf{T} \cdot \mathbf{S} = (\mathbf{S}^T \cdot \mathbf{T}^T)^T$ 

• The inner product of a vector  $\mathbf{a}$  and third rank tensor  $\mathbf{P}$  produces a second rank tensor  $\mathbf{T} = \mathbf{a} \cdot \mathbf{P}$  whose components are

$$T_{ij} = a_k P_{kij} (1.11)$$

Again this is non-commutative so that  $\mathbf{T} = \mathbf{P} \cdot \mathbf{a}$  is

$$T_{ij} = P_{ijk}a_k \tag{1.12}$$

• The inner product of a second rank tensor T and third rank tensor P produces a third rank tensor  $Q = T \cdot P$  whose components are

$$Q_{ijk} = T_{il}P_{ljk} \tag{1.13}$$

Again this is non-commutative so that  $\mathbf{Q} = \mathbf{P} \cdot \mathbf{T}$  is

$$Q_{ijk} = P_{ijl}T_{lk} \tag{1.14}$$

## 1.3.2 The double inner product of two tensors

The double inner product of two second-rank tensors T and S produces a scalar s = T : S which can be evaluated as the sum of the 9 products of the tensor components

$$s = T_{ij}S_{ij} = T_{11}S_{11} + T_{12}S_{12} + T_{13}S_{13} + T_{21}S_{21} + T_{22}S_{22} + T_{23}S_{23} + T_{31}S_{31} + T_{32}S_{32} + T_{33}S_{33}$$

$$(1.15)$$

The double inner product between a second rank tensor T and third rank tensor P produces a vector  $\mathbf{a} = T \ P$  with components

$$a_i = T_{jk} P_{jki} \tag{1.16}$$

This is non-commutative so that  $\mathbf{a} = \mathbf{P} \cdot \mathbf{T}$  is

$$a_i = P_{ijk}T_{jk} \tag{1.17}$$

## 1.3.3 The triple inner product of two third rank tensors

The triple inner product of two third rank tensors  $\mathbf{P}$  and  $\mathbf{Q}$  produces a scalar  $s = \mathbf{P} \ \mathbf{Q}$  which can be evaluated as the sum of the 27 products of the tensor components

$$s = P_{ijk}Q_{ijk} \tag{1.18}$$

## 1.3.4 The outer product

The outer product operates between vectors and tensors as follows:

• The outer product of two vectors  $\mathbf{a}$  and  $\mathbf{b}$  is non-commutative and produces a tensor  $\mathbf{T} = \mathbf{a}\mathbf{b} = (\mathbf{b}\mathbf{a})^{\mathrm{T}}$  whose components are evaluated as:

$$T_{ij} = a_i b_j = \begin{pmatrix} a_1 b_1 & a_1 b_2 & a_1 b_3 \\ a_2 b_1 & a_2 b_2 & a_2 b_3 \\ a_3 b_1 & a_3 b_2 & a_3 b_3 \end{pmatrix}$$

$$(1.19)$$

• An outer product of a vector  $\mathbf{a}$  and second rank tensor  $\mathbf{T}$  produces a third rank tensor  $\mathbf{P} = \mathbf{a}\mathbf{T}$  whose components are

$$P_{ijk} = a_i T_{jk} (1.20)$$

This is non-commutative so that P = Ta produces

$$P_{ijk} = T_{ij}a_k (1.21)$$

## 1.3.5 The cross product of two vectors

The cross product operation is exclusive to vectors only. For two vectors  $\mathbf{a}$  with  $\mathbf{b}$ , it produces a vector  $\mathbf{c} = \mathbf{a} \times \mathbf{b}$  whose components are

$$c_i = e_{ijk}a_ib_k = (a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1)$$

$$(1.22)$$

where the *permutation symbol* is defined by

$$e_{ijk} = \begin{cases} 0 & \text{when any two indices are equal} \\ +1 & \text{when } i,j,k \text{ are an even permutation of } 1,2,3 \\ -1 & \text{when } i,j,k \text{ are an odd permutation of } 1,2,3 \end{cases}$$

$$(1.23)$$

in which the even permutations are 123, 231 and 312 and the odd permutations are 132, 213 and 321.

P-16 Tensor mathematics

## 1.3.6 Other general tensor operations

Some less common tensor operations and terminology used by OpenFOAM are described below.

**Square** of a tensor is defined as the outer product of the tensor with itself, e.g. for a vector  $\mathbf{a}$ , the square  $\mathbf{a}^2 = \mathbf{a}\mathbf{a}$ .

**nth power** of a tensor is evaluated by n outer products of the tensor, e.g. for a vector  $\mathbf{a}$ , the 3rd power  $\mathbf{a}^3 = \mathbf{a}\mathbf{a}\mathbf{a}$ .

**Magnitude squared** of a tensor is the rth inner product of the tensor of rank r with itself, to produce a scalar. For example, for a second rank tensor  $\mathbf{T}$ ,  $|\mathbf{T}|^2 = \mathbf{T} \cdot \mathbf{T}$ .

**Magnitude** is the square root of the magnitude squared, e.g. for a tensor  $\mathbf{T}$ ,  $|\mathbf{T}| = \sqrt{\mathbf{T} \cdot \mathbf{T}}$ . Vectors of unit magnitude are referred to as unit vectors.

**Component maximum** is the component of the tensor with greatest value, inclusive of sign, *i.e.* not the largest magnitude.

Component minimum is the component of the tensor with smallest value.

Component average is the mean of all components of a tensor.

**Scale** As the name suggests, the scale function is a tool for scaling the components of one tensor by the components of another tensor of the same rank. It is evaluated as the product of corresponding components of 2 tensors, *e.g.*, scaling vector **a** by vector **b** would produce vector **c** whose components are

$$c_i = \text{scale}(\mathbf{a}, \mathbf{b}) = (a_1 b_1, a_2 b_2, a_3 b_3)$$
 (1.24)

# 1.3.7 Geometric transformation and the identity tensor

A second rank tensor  $\mathbf{T}$  is strictly defined as a linear vector function, i.e. it is a function which associates an argument vector  $\mathbf{a}$  to another vector  $\mathbf{b}$  by the inner product  $\mathbf{b} = \mathbf{T} \cdot \mathbf{a}$ . The components of  $\mathbf{T}$  can be chosen to perform a specific geometric transformation of a tensor from the x, y, z coordinate system to a new coordinate system  $x^*, y^*, z^*$ ;  $\mathbf{T}$  is then referred to as the transformation tensor. While a scalar remains unchanged under a transformation, the vector  $\mathbf{a}$  is transformed to  $\mathbf{a}^*$  by

$$\mathbf{a}^* = \mathbf{T} \cdot \mathbf{a} \tag{1.25}$$

A second rank tensor S is transformed to  $S^*$  according to

$$\mathbf{S}^* = \mathbf{T} \cdot \mathbf{S} \cdot \mathbf{T}^{\mathrm{T}} \tag{1.26}$$

The *identity tensor*  $\mathbf{I}$  is defined by the requirement that it transforms another tensor onto itself. For all vectors  $\mathbf{a}$ 

$$\mathbf{a} = \mathbf{I} \cdot \mathbf{a} \tag{1.27}$$

and therefore

$$\mathbf{I} = \delta_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{1.28}$$

where  $\delta_{ij}$  is known as the *Kronecker delta* symbol.

### 1.3.8 Useful tensor identities

Several identities are listed below which can be verified by under the assumption that all the relevant derivatives exist and are continuous. The identities are expressed for scalar s and vector  $\mathbf{a}$ .

$$\nabla \cdot (\nabla \times \mathbf{a}) \equiv 0$$

$$\nabla \times (\nabla s) \equiv \mathbf{0}$$

$$\nabla \cdot (s\mathbf{a}) \equiv s \nabla \cdot \mathbf{a} + \mathbf{a} \cdot \nabla s$$

$$\nabla \times (s\mathbf{a}) \equiv s \nabla \times \mathbf{a} + \nabla s \times \mathbf{a}$$

$$\nabla (\mathbf{a} \cdot \mathbf{b}) \equiv \mathbf{a} \times (\nabla \times \mathbf{b}) + \mathbf{b} \times (\nabla \times \mathbf{a}) + (\mathbf{a} \cdot \nabla) \mathbf{b} + (\mathbf{b} \cdot \nabla) \mathbf{a}$$

$$\nabla \cdot (\mathbf{a} \times \mathbf{b}) \equiv \mathbf{b} \cdot (\nabla \times \mathbf{a}) - \mathbf{a} \cdot (\nabla \times \mathbf{b})$$

$$\nabla \times (\mathbf{a} \times \mathbf{b}) \equiv \mathbf{a} (\nabla \cdot \mathbf{b}) - \mathbf{b} (\nabla \cdot \mathbf{a}) + (\mathbf{b} \cdot \nabla) \mathbf{a} - (\mathbf{a} \cdot \nabla) \mathbf{b}$$

$$\nabla \times (\nabla \times \mathbf{a}) \equiv \nabla (\nabla \cdot \mathbf{a}) - \nabla^2 \mathbf{a}$$

$$(\nabla \times \mathbf{a}) \times \mathbf{a} \equiv \mathbf{a} \cdot (\nabla \mathbf{a}) - \nabla (\mathbf{a} \cdot \mathbf{a})$$

It is sometimes useful to know the  $e-\delta$  identity to help to manipulate equations in index notation:

$$e_{ijk}e_{irs} = \delta_{jr}\delta_{ks} - \delta_{js}\delta_{kr} \tag{1.30}$$

## 1.3.9 Operations exclusive to tensors of rank 2

There are several operations that manipulate the components of tensors of rank 2 that are listed below:

**Transpose** of a tensor  $T = T_{ij}$  is  $T^T = T_{ji}$  as described in Equation 1.2.

Symmetric and skew (antisymmetric) tensors As discussed in section 1.2, a tensor is said to be symmetric if its components are symmetric about the diagonal, i.e.  $\mathbf{T} = \mathbf{T}^{\mathrm{T}}$ . A skew or antisymmetric tensor has  $\mathbf{T} = -\mathbf{T}^{\mathrm{T}}$  which intuitively implies that  $T_{11} = T_{22} = T_{33} = 0$ . Every second order tensor can be decomposed into symmetric and skew parts by

$$\mathbf{T} = \underbrace{\frac{1}{2}(\mathbf{T} + \mathbf{T}^{\mathrm{T}})}_{symmetric} + \underbrace{\frac{1}{2}(\mathbf{T} - \mathbf{T}^{\mathrm{T}})}_{skew} = \operatorname{symm} \mathbf{T} + \operatorname{skew} \mathbf{T}$$
(1.31)

**Trace** The trace of a tensor **T** is a scalar, evaluated by summing the diagonal components

$$\operatorname{tr} \mathbf{T} = T_{11} + T_{22} + T_{33} \tag{1.32}$$

 ${f Diagonal}$  returns a vector whose components are the diagonal components of the second rank tensor  ${f T}$ 

$$\operatorname{diag} \mathbf{T} = (T_{11}, T_{22}, T_{33}) \tag{1.33}$$

**Deviatoric and hydrostatic tensors** Every second rank tensor  $\mathbf{T}$  can be decomposed into a deviatoric component, for which  $\operatorname{tr} \mathbf{T} = 0$  and a hydrostatic component of the form  $\mathbf{T} = s\mathbf{I}$  where s is a scalar. Every second rank tensor can be decomposed into deviatoric and hydrostatic parts as follows:

$$\mathbf{T} = \underbrace{\mathbf{T} - \frac{1}{3} (\operatorname{tr} \mathbf{T}) \mathbf{I}}_{deviatoric} + \underbrace{\frac{1}{3} (\operatorname{tr} \mathbf{T}) \mathbf{I}}_{hydrostatic} = \operatorname{dev} \mathbf{T} + \operatorname{hyd} \mathbf{T}$$
(1.34)

P-18 Tensor mathematics

**Determinant** The determinant of a second rank tensor is evaluated by

$$\det \mathbf{T} = \begin{vmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{vmatrix} = T_{11}(T_{22}T_{33} - T_{23}T_{32}) - T_{12}(T_{21}T_{33} - T_{23}T_{31}) + T_{13}(T_{21}T_{32} - T_{22}T_{31})$$

$$= \frac{1}{6}e_{ijk}e_{pqr}T_{ip}T_{jq}T_{kr}$$
(1.35)

**Cofactors** The *minors* of a tensor are evaluated for each component by deleting the row and column in which the component is situated and evaluating the resulting entries as a  $2 \times 2$  determinant. For example, the minor of  $T_{12}$  is

$$\begin{vmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{vmatrix} = \begin{vmatrix} T_{21} & T_{23} \\ T_{31} & T_{33} \end{vmatrix} = T_{21}T_{33} - T_{23}T_{31}$$

$$(1.36)$$

The cofactors are *signed minors* where each minor is component is given a sign based on the rule

+ve if 
$$i + j$$
 is even  
-ve if  $i + j$  is odd (1.37)

The cofactors of T can be evaluated as

$$\operatorname{cof} \mathbf{T} = \frac{1}{2} e_{jkr} e_{ist} T_{sk} T_{tr} \tag{1.38}$$

**Inverse** The inverse of a tensor can be evaluated as

$$\operatorname{inv} \mathbf{T} = \frac{\operatorname{cof} \mathbf{T}^{\mathrm{T}}}{\det \mathbf{T}} \tag{1.39}$$

Hodge dual of a tensor is a vector whose components are

$$*\mathbf{T} = (T_{23}, -T_{13}, T_{12}) \tag{1.40}$$

# 1.3.10 Operations exclusive to scalars

OpenFOAM supports most of the well known functions that operate on scalars, e.g. square root, exponential, logarithm, sine, cosine etc.., a list of which can be found in Table 1.2. There are 3 additional functions defined within OpenFOAM that are described below:

**Sign** of a scalar s is

$$\operatorname{sgn}(s) = \begin{cases} 1 & \text{if } s \ge 0, \\ -1 & \text{if } s < 0. \end{cases}$$
 (1.41)

**Positive** of a scalar s is

$$pos(s) = \begin{cases} 1 & \text{if } s \ge 0, \\ 0 & \text{if } s < 0. \end{cases}$$
 (1.42)

**Limit** of a scalar s by the scalar n

$$\lim_{n \to \infty} \lim_{n \to \infty} f(s, n) = \begin{cases} s & \text{if } s < n, \\ 0 & \text{if } s \ge n. \end{cases} \tag{1.43}$$

# 1.4 OpenFOAM tensor classes

OpenFOAM contains a C++ class library primitive that contains the classes for the tensor mathematics described so far. The basic tensor classes that are available as standard in OpenFOAM are listed in Table 1.1. The Table also lists the functions that allow the user to access individual components of a tensor, known as access functions.

Rank	Common name	Basic class	Access functions
0	Scalar	scalar	
1	Vector	vector	x(), y(), z()
2	Tensor	tensor	xx(), xy(), xz()

Table 1.1: Basic tensor classes in OpenFOAM

We can declare the tensor

$$\mathbf{T} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \tag{1.44}$$

in OpenFOAM by the line:

We can then access the component  $T_{13}$ , or  $T_{xz}$  using the xz() access function. For instance the code

Info << 
$$``Txz = '' << T.xz() << endl;$$

outputs to the screen:

Txz = 3

# 1.4.1 Algebraic tensor operations in OpenFOAM

The algebraic operations described in Section 1.3 are all available to the OpenFOAM tensor classes using syntax which closely mimics the notation used in written mathematics. Some functions are represented solely by descriptive functions, e.g.symm(), but others can also be executed using symbolic operators, e.g.\*. All functions are listed in Table 1.2.

Operation	Comment	Mathematical	Description
		Description	in OpenFOAM
Addition		a + b	a + b
Subtraction		a - b	a - b
Scalar multiplication		$s\mathbf{a}$	s * a
Scalar division		$\mathbf{a}/s$	a / s
Outer product	$rank  \mathbf{a}, \mathbf{b}  = 1$	ab	a * b
Inner product	$rank  \mathbf{a}, \mathbf{b}  = 1$	a•b	a & b
Double inner product	$rank \mathbf{a}, \mathbf{b} >= 2$	a:b	a && b
Cross product	$rank \ \mathbf{a}, \mathbf{b} = 1$	$\mathbf{a} \times \mathbf{b}$	a ^ b
Square		$\mathbf{a}^2$	sqr(a)
		Cox	ntinued on next page

P-20 Tensor mathematics

Continued from previous page			
Operation	Comment	Mathematical	Description
		Description	in OpenFOAM
Magnitude squared		$ \mathbf{a} ^2$	magSqr(a)
Magnitude		$ \mathbf{a} $	mag(a)
Power	n = 0, 1,, 4	$\mathbf{a}^n$	pow(a,n)
Component average	i = 1,, N	$\overline{a_i}$	cmptAv(a)
Component maximum	i = 1,, N	$\max(a_i)$	max(a)
Component minimum	i = 1,, N	$\min(a_i)$	min(a)
Scale		$scale(\mathbf{a}, \mathbf{b})$	scale(a,b)
Geometric transformation	transforms $\mathbf{a}$ u	$\sin g \tan \sigma T$	<pre>transform(T,a)</pre>
Operations exclusive to tens	sors of rank 2		
Transpose		$\mathbf{T}^{\mathrm{T}}$	T.T()
Diagonal		$\operatorname{diag}\mathbf{T}$	diag(T)
Trace		$\operatorname{tr} \mathbf{T}$	tr(T)
Deviatoric component		$\operatorname{dev} \mathbf{T}$	dev(T)
Symmetric component		$\operatorname{symm} \mathbf{T}$	symm(T)
Skew-symmetric component		$\operatorname{skew} \mathbf{T}$	skew(T)
Determinant		$\det \mathbf{T}$	det(T)
Cofactors		$\cot \mathbf{T}$	cof(T)
Inverse		$\operatorname{inv} \mathbf{T}$	inv(T)
Hodge dual		$*\mathbf{T}$	*T
Troage datar		-	
Operations exclusive to scal	ars		
Sign (boolean)		$\operatorname{sgn}(s)$	sign(s)
Positive (boolean)		s >= 0	pos(s)
Negative (boolean)		s < 0	neg(s)
Limit	n scalar	$\lim_{\longrightarrow} \operatorname{t}(s,n)$	<pre>limit(s,n)</pre>
Square root		$\sqrt{S}$	sqrt(s)
Exponential		$\exp s$	exp(s)
Natural logarithm		$\ln s$	log(s)
Base 10 logarithm		$\log_{10} s$	log10(s)
Sine		$\sin s$	sin(s)
Cosine		$\cos s$	cos(s)
Tangent		$\tan s$	tan(s)
Arc sine		asin s	asin(s)
Arc cosine		$a\cos s$	acos(s)
Arc tangent		a tan s	atan(s)
Hyperbolic sine		$\sinh s$	sinh(s)
Hyperbolic cosine		$\cosh s$	cosh(s)
Hyperbolic tangent		$\tanh s$	tanh(s)
Hyperbolic arc sine		a s inh s	asinh(s)
Hyperbolic arc cosine		$a \cosh s$	acosh(s)
Hyperbolic arc tangent		$\operatorname{atanh} s$	atanh(s)
Error function		$\operatorname{erf} s$	erf(s)
Complement error function		$\operatorname{erfc} s$	erfc(s)
Logarithm gamma function		$\ln \Gamma s$	lgamma(s)
Type 1 Bessel function of order		$J_0 s$	j0(s)
Type 1 Bessel function of order	1	$J_1 s$	j1(s)
		Con	tinued on next page

1.5 Dimensional units P-21

Continued from previous page			
Operation	Comment	Mathematical	Description
		Description	in OpenFOAM
Type 2 Bessel function of order	$Y_0 s$	y0(s)	
Type 2 Bessel function of order	1	$Y_1 s$	y1(s)

a, b are tensors of arbitrary rank unless otherwise stated

Table 1.2: Algebraic tensor operations in OpenFOAM

## 1.5 Dimensional units

In continuum mechanics, properties are represented in some chosen units, e.g. mass in kilograms (kg), volume in cubic metres (m³), pressure in Pascals (kg m s⁻²). Algebraic operations must be performed on these properties using consistent units of measurement; in particular, addition, subtraction and equality are only physically meaningful for properties of the same dimensional units. As a safeguard against implementing a meaningless operation, OpenFOAM encourages the user to attach dimensional units to any tensor and will then perform dimension checking of any tensor operation.

Units are defined using the dimensionSet class, e.g.

dimensionSet pressureDims(1, -1, -2, 0, 0, 0, 0);

No.	Property	Unit	Symbol
1	Mass	kilogram	k
2	Length	metre	m
3	Time	second	S
4	Temperature	Kelvin	K
5	Quantity	moles	mol
6	Current	ampere	A
7	Luminous intensity	candela	$\operatorname{cd}$

Table 1.3: S.I. base units of measurement

where each of the values corresponds to the power of each of the S.I. base units of measurement listed in Table 1.3. The line of code declares pressureDims to be the dimensionSet for pressure  $kg\,m\,s^{-2}$  since the first entry in the pressureDims array, 1, corresponds to  $k^1$ , the second entry, -1, corresponds to  $m^{-1}$  etc.. A tensor with units is defined using the dimensioned<Type> template class, the <Type> being scalar, vector, tensor, etc.. The dimensioned<Type> stores a variable name of class word, the value <Type> and a dimensionSet

```
dimensionedTensor sigma
   (
        "sigma",
        dimensionSet(1, -1, -2, 0, 0, 0, 0),
        tensor(1e6,0,0,0,1e6,0,0,0,1e6),
   );
```

s is a scalar, N is the number of tensor components

P-22 Tensor mathematics

creates a tensor with correct dimensions of pressure, or stress

$$\sigma = \begin{pmatrix} 10^6 & 0 & 0 \\ 0 & 10^6 & 0 \\ 0 & 0 & 10^6 \end{pmatrix} \tag{1.45}$$

# Chapter 2

# Discretisation procedures

So far we have dealt with algebra of tensors at a point. The PDEs we wish to solve involve derivatives of tensors with respect to time and space. We therefore need to extend our description to a tensor field, i.e. a tensor that varies across time and spatial domains. In this Chapter we will first present a mathematical description of all the differential operators we may encounter. We will then show how a tensor field is constructed in OpenFOAM and how the derivatives of these fields are discretised into a set of algebraic equations.

# 2.1 Differential operators

Before defining the spatial derivatives we first introduce the nabla vector operator  $\nabla$ , represented in index notation as  $\partial_i$ :

$$\nabla \equiv \partial_i \equiv \frac{\partial}{\partial x_i} \equiv \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\right) \tag{2.1}$$

The nabla operator is a useful notation that obeys the following rules:

- it operates on the tensors to its right and the conventional rules of a derivative of a product, e.g.  $\partial_i ab = (\partial_i a) b + a (\partial_i b)$ ;
- otherwise the nabla operator behaves like any other vector in an algebraic operation.

### 2.1.1 Gradient

If a scalar field s is defined and continuously differentiable then the gradient of s,  $\nabla s$  is a vector field

$$\nabla s = \partial_i s = \left(\frac{\partial s}{\partial x_1}, \frac{\partial s}{\partial x_2}, \frac{\partial s}{\partial x_3}\right) \tag{2.2}$$

The gradient can operate on any tensor field to produce a tensor field that is one rank higher. For example, the gradient of a vector field **a** is a second rank tensor field

$$\nabla \mathbf{a} = \partial_i a_j = \begin{pmatrix} \partial a_1 / \partial x_1 & \partial a_2 / \partial x_1 & \partial a_3 / \partial x_1 \\ \partial a_1 / \partial x_2 & \partial a_2 / \partial x_2 & \partial a_3 / \partial x_2 \\ \partial a_1 / \partial x_3 & \partial a_2 / \partial x_3 & \partial a_3 / \partial x_3 \end{pmatrix}$$
(2.3)

## 2.1.2 Divergence

If a vector field  $\mathbf{a}$  is defined and continuously differentiable then the divergence of  $\mathbf{a}$  is a scalar field

$$\nabla \cdot \mathbf{a} = \partial_i a_i = \frac{\partial a_1}{\partial x_1} + \frac{\partial a_2}{\partial x_2} + \frac{\partial a_3}{\partial x_3}$$
 (2.4)

The divergence can operate on any tensor field of rank 1 and above to produce a tensor that is one rank lower. For example the divergence of a second rank tensor field **T** is a vector field (expanding the vector as a column array for convenience)

$$\nabla \cdot \mathbf{T} = \partial_i T_{ij} = \begin{pmatrix} \partial T_{11}/\partial x_1 + \partial T_{21}/\partial x_2 + \partial T_{31}/\partial x_3 \\ \partial T_{12}/\partial x_1 + \partial T_{22}/\partial x_2 + \partial T_{32}/\partial x_3 \\ \partial T_{13}/\partial x_1 + \partial T_{23}/\partial x_2 + \partial T_{33}/\partial x_3 \end{pmatrix}$$
(2.5)

### 2.1.3 Curl

If a vector field **a** is defined and continuously differentiable then the curl of **a**,  $\nabla \times \mathbf{a}$  is a vector field

$$\nabla \times \mathbf{a} = e_{ijk} \partial_j a_k = \left( \frac{\partial a_3}{\partial x_2} - \frac{\partial a_2}{\partial x_3}, \frac{\partial a_1}{\partial x_3} - \frac{\partial a_3}{\partial x_1}, \frac{\partial a_2}{\partial x_1} - \frac{\partial a_1}{\partial x_2} \right)$$
(2.6)

The curl is related to the gradient by

$$\nabla \times \mathbf{a} = 2 \,(* \,\mathrm{skew} \,\nabla \mathbf{a}) \tag{2.7}$$

## 2.1.4 Laplacian

The Laplacian is an operation that can be defined mathematically by a combination of the divergence and gradient operators by  $\nabla^2 \equiv \nabla \cdot \nabla$ . However, the Laplacian should be considered as a single operation that transforms a tensor field into another tensor field of the same rank, rather than a combination of two operations, one which raises the rank by 1 and one which reduces the rank by 1.

In fact, the Laplacian is best defined as a *scalar operator*, just as we defined nabla as a vector operator, by

$$\nabla^2 \equiv \partial^2 \equiv \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}$$
 (2.8)

For example, the Laplacian of a scalar field s is the scalar field

$$\nabla^2 s = \partial^2 s = \frac{\partial^2 s}{\partial x_1^2} + \frac{\partial^2 s}{\partial x_2^2} + \frac{\partial^2 s}{\partial x_3^2}$$
 (2.9)

# 2.1.5 Temporal derivative

There is more than one definition of temporal, or time, derivative of a tensor. To describe the temporal derivatives we must first recall that the tensor relates to a property of a volume of material that may be moving. If we track an infinitesimally small volume of material, or particle, as it moves and observe the change in the tensorial property  $\phi$  in time, we have the *total*, or *material* time derivative denoted by

$$\frac{D\phi}{Dt} = \lim_{\Delta t \to 0} \frac{\Delta\phi}{\Delta t} \tag{2.10}$$

However in continuum mechanics, particularly fluid mechanics, we often observe the change of a  $\phi$  in time at a fixed point in space as different particles move across that point. This change at a point in space is termed the *spatial* time derivative which is denoted by  $\partial/\partial t$  and is related to the material derivative by:

$$\frac{D\phi}{Dt} = \frac{\partial\phi}{\partial t} + \mathbf{U} \cdot \nabla\phi \tag{2.11}$$

where **U** is the velocity field of property  $\phi$ . The second term on the right is known as the convective rate of change of  $\phi$ .

## 2.2 Overview of discretisation

The term discretisation means approximation of a problem into discrete quantities. The FV method and others, such as the finite element and finite difference methods, all discretise the problem as follows:

**Spatial discretisation** Defining the solution domain by a set of points that fill and bound a region of space when connected;

**Temporal discretisation** (For transient problems) dividing the time domain into into a finite number of time intervals, or steps;

**Equation discretisation** Generating a system of algebraic equations in terms of discrete quantities defined at specific locations in the domain, from the PDEs that characterise the problem.

## 2.2.1 OpenFOAM lists and fields

OpenFOAM frequently needs to store sets of data and perform functions, such as mathematical operations, on the data. OpenFOAM therefore provides an array template class List<Type>, making it possible to create a list of any object of class Type that inherits the functions of the Type. For example a List of vector is List<vector>.

Lists of the tensor classes are defined as standard in OpenFOAM by the template class Field<Type>. For better code legibility, all instances of Field<Type>, e.g.Field<vector>, are renamed using typedef declarations as scalarField, vectorField, tensorField, symmTensor-Field, tensorThirdField and symmTensorThirdField. Algebraic operations can be performed between Fields subject to obvious restrictions such as the fields having the same number of elements. OpenFOAM also supports operations between a field and single tensor, e.g. all values of a Field U can be multiplied by the scalar 2 with the operation U = 2.0 \* U.

## 2.3 Discretisation of the solution domain

Discretisation of the solution domain is shown in Figure 2.1. The space domain is discretised into computational mesh on which the PDEs are subsequently discretised. Discretisation of time, if required, is simple: it is broken into a set of time steps  $\Delta t$  that may change during a numerical simulation, perhaps depending on some condition calculated during the simulation.

On a more detailed level, discretisation of space requires the subdivision of the domain into a number of cells, or control volumes. The cells are contiguous, i.e. they do not overlap one another and completely fill the domain. A typical cell is shown in Figure 2.2.

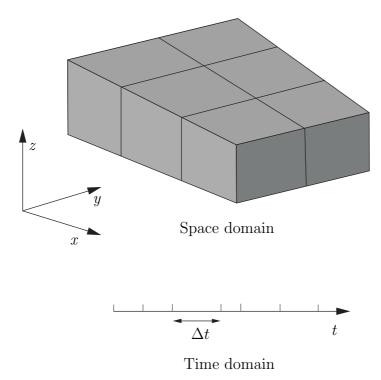


Figure 2.1: Discretisation of the solution domain

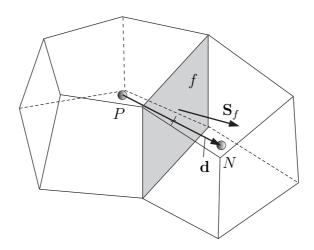


Figure 2.2: Parameters in finite volume discretisation

Dependent variables and other properties are principally stored at the cell centroid P although they may be stored on faces or vertices. The cell is bounded by a set of flat faces, given the generic label f. In OpenFOAM there is no limitation on the number of faces bounding each cell, nor any restriction on the alignment of each face. This kind of mesh is often referred to as "arbitrarily unstructured" to differentiate it from meshes in which the cell faces have a prescribed alignment, typically with the coordinate axes. Codes with arbitrarily unstructured meshes offer greater freedom in mesh generation and manipulation in particular when the geometry of the domain is complex or changes over time.

Whilst most properties are defined at the cell centroids, some are defined at cell faces. There are two types of cell face.

Internal faces Those faces that connect two cells (and it can never be more than two). For each internal face, OpenFOAM designates one adjoining cell to be the face owner and the other to be the neighbour;

**Boundary faces** Those belonging to one cell since they coincide with the boundary of the domain. These faces simply have an owner cell.

## 2.3.1 Defining a mesh in OpenFOAM

There are different levels of mesh description in OpenFOAM, beginning with the most basic mesh class, named polyMesh since it is based on polyhedra. A polyMesh is constructed using the minimum information required to define the mesh geometry described below and presented in Figure 2.3:

**Points** A list of cell vertex point coordinate vectors, *i.e.* a vectorField, that is renamed pointField using a typedef declaration;

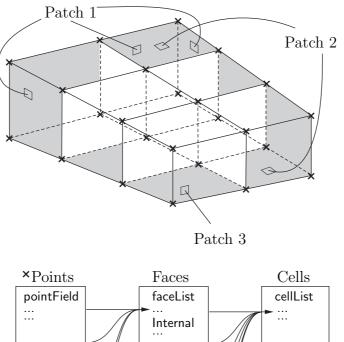
**Faces** A list of cell faces List<face>, or faceList, where the face class is defined by a list of vertex numbers, corresponding to the pointField;

Cells a list of cells List<cell>, or cellList, where the cell class is defined by a list of face numbers, corresponding to the faceList described previously.

Boundary a polyBoundaryMesh decomposed into a list of patches, polyPatchList representing different regions of the boundary. The boundary is subdivided in this manner to allow different boundary conditions to be specified on different patches during a solution. All the faces of any polyPatch are stored as a single block of the faceList, so that its faces can be easily accessed using the slice class which stores references to the first and last face of the block. Each polyPatch is then constructed from

- a slice;
- a word to assign it a name.

FV discretisation uses specific data that is derived from the mesh geometry stored in polyMesh. OpenFOAM therefore extends the polyMesh class to fvMesh which stores the additional data needed for FV discretisation. fvMesh is constructed from polyMesh and stores the data in Table 2.1 which can be updated during runtime in cases where the mesh moves, is refined *etc.*.



pointField faceList cellList cellList iiii linternal iiii Boundary polyPatchList Patch 1 Patch 2 Patch 3

Figure 2.3: Schematic of the basic mesh description used in OpenFOAM

# 2.3.2 Defining a geometricField in OpenFOAM

So far we can define a field, *i.e.* a list of tensors, and a mesh. These can be combined to define a tensor field relating to discrete points in our domain, specified in OpenFOAM by the template class geometricField<Type>. The Field values are separated into those defined within the internal region of the domain, *e.g.* at the cell centres, and those defined on the domain boundary, *e.g.* on the boundary faces. The geometricField<Type> stores the following information:

Internal field This is simply a Field<Type>, described in Section 2.2.1;

BoundaryField This is a GeometricBoundaryField, in which a Field is defined for the faces of each patch and a Field is defined for the patches of the boundary. This is then a field of fields, stored within an object of the FieldField<Type> class. A reference to the fvBoundaryMesh is also stored [\*\*].

**Mesh** A reference to an fvMesh, with some additional detail as to the whether the field is defined at cell centres, faces, etc..

**Dimensions** A dimensionSet, described in Section 4.2.6.

Old values Discretisation of time derivatives requires field data from previous time steps. The geometricField<Type> will store references to stored fields from the previous, or old, time step and its previous, or old-old, time step where necessary.

Class	Description	Symbol	Access function
volScalarField	Cell volumes	V	V()
surfaceVectorField	Face area vectors	$\mathbf{S}_f$	Sf()
surfaceScalarField	Face area magnitudes	$ \mathbf{S}_f $	magSf()
volVectorField	Cell centres	$\mathbf{C}$	C()
surfaceVectorField	Face centres	$\mathbf{C}_f$	Cf()
surfaceScalarField	Face motion fluxes **	$\phi_g$	phi()

Table 2.1: fvMesh stored data.

Previous iteration values The iterative solution procedures can use under-relaxation which requires access to data from the previous iteration. Again, if required, geometricField<Type> stores a reference to the data from the previous iteration.

As discussed in Section 2.3, we principally define a property at the cell centres but quite often it is stored at the cell faces and on occasion it is defined on cell vertices. The geometricField<Type> is renamed using typedef declarations to indicate where the field variable is defined as follows:

volField<Type> A field defined at cell centres;

surfaceField<Type> A field defined on cell faces;

pointField<Type> A field defined on cell vertices.

These typedef field classes of geometricField<Type>are illustrated in Figure 2.4. A geometricField<Type> inherits all the tensor algebra of Field<Type> and has all operations subjected to dimension checking using the dimensionSet. It can also be subjected to the FV discretisation procedures described in the following Section. The class structure used to build geometricField<Type> is shown in Figure 2.5<sup>1</sup>.

# 2.4 Equation discretisation

Equation discretisation converts the PDEs into a set of algebraic equations that are commonly expressed in matrix form as:

$$[A][x] = [b] \tag{2.12}$$

where [A] is a square matrix, [x] is the column vector of dependent variable and [b] is the source vector. The description of [x] and [b] as 'vectors' comes from matrix terminology rather than being a precise description of what they truly are: a list of values defined at locations in the geometry, i.e. a geometricField<Type>, or more specifically a volField<Type> when using FV discretisation.

[A] is a list of coefficients of a set of algebraic equations, and cannot be described as a geometricField<Type>. It is therefore given a class of its own: fvMatrix. fvMatrix<Type> is created through discretisation of a geometric<Type>Field and therefore inherits the <Type>. It supports many of the standard algebraic matrix operations of addition +, subtraction - and multiplication \*.

Each term in a PDE is represented individually in OpenFOAM code using the classes of static functions finiteVolumeMethod and finiteVolumeCalculus, abbreviated by a typedef

<sup>&</sup>lt;sup>1</sup>The diagram is not an exact description of the class hierarchy, rather a representation of the general structure leading from some primitive classes to geometric<Type>Field.

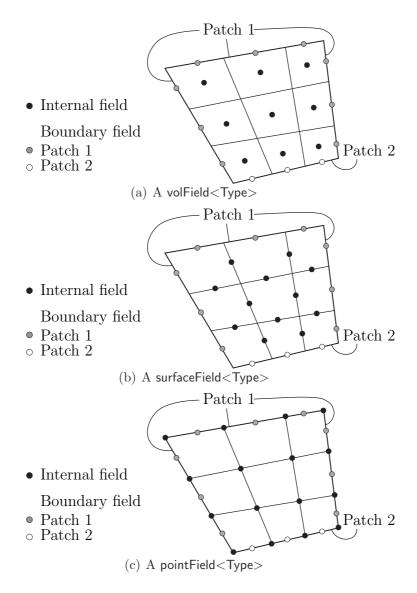


Figure 2.4: Types of geometricField<Type> defined on a mesh with 2 boundary patches (in 2 dimensions for simplicity)

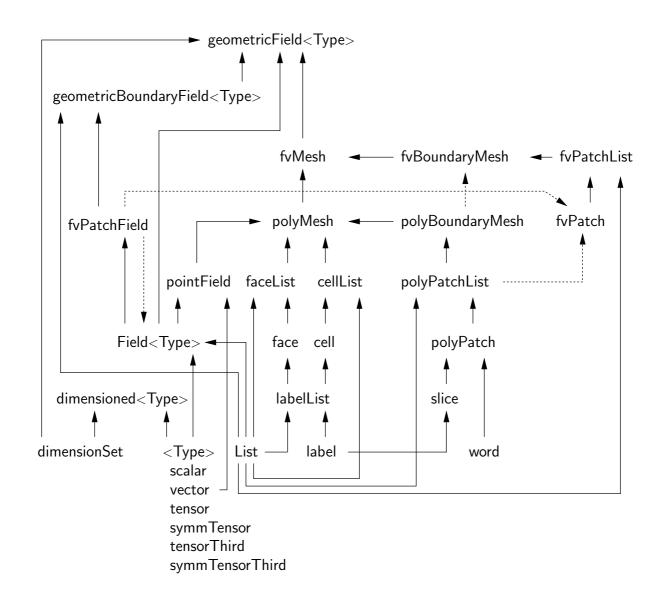


Figure 2.5: Basic class structure leading to geometricField<Type>

to fvm and fvc respectively. fvm and fvc contain static functions, representing differential operators, e.g.  $\nabla^2$ ,  $\nabla \cdot$  and  $\partial/\partial t$ , that discretise geometricField<Type>s. The purpose of defining these functions within two classes, fvm and fvc, rather than one, is to distinguish:

- functions of fvm that calculate implicit derivatives of and return an fvMatrix<Type>
- some functions of fvc that calculate explicit derivatives and other explicit calculations, returning a geometricField<Type>.

Figure 2.6 shows a geometricField<Type> defined on a mesh with 2 boundary patches and illustrates the explicit operations merely transform one field to another and drawn in 2D for simplicity.

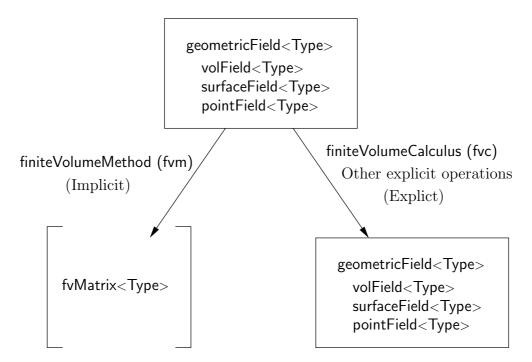


Figure 2.6: A geometricField<Type> and its operators

Table 2.2 lists the main functions that are available in  $\mathsf{fvm}$  and  $\mathsf{fvc}$  to discretise terms that may be found in a PDE. FV discretisation of each term is formulated by first integrating the term over a cell volume V. Most spatial derivative terms are then converted to integrals over the cell surface S bounding the volume using Gauss's theorem

$$\int_{V} \nabla \star \phi \ dV = \int_{S} d\mathbf{S} \star \phi \tag{2.13}$$

where **S** is the surface area vector,  $\phi$  can represent any tensor field and the star notation  $\star$  is used to represent any tensor product, *i.e.* inner, outer and cross and the respective derivatives: divergence  $\nabla \cdot \phi$ , gradient  $\nabla \phi$  and  $\nabla \times \phi$ . Volume and surface integrals are then linearised using appropriate schemes which are described for each term in the following Sections. Some terms are always discretised using one scheme, a selection of schemes is offered in OpenFOAM for the discretisation of other terms. The choice of scheme is either made by a direct specification within the code or it can be read from an input file at job run-time and stored within an fvSchemes class object.

Term description	Implicit /	Text	fvm::/fvc:: functions
	Explicit	expression	
Laplacian	Imp/Exp	$ abla^2 \phi$	laplacian(phi)
		$ abla \cdot \Gamma  abla \phi$	laplacian(Gamma, phi)
Time derivative	Imp/Exp	$\frac{\partial \phi}{\partial t}$	ddt(phi)
		$\frac{\partial \rho \phi}{\partial t}$	ddt(rho,phi)
Second time derivative	Imp/Exp	$\frac{\partial}{\partial t} \left( \rho \frac{\partial \phi}{\partial t} \right)$	d2dt2(rho, phi)
Convection	Imp/Exp	$\nabla \cdot (\psi)$	div(psi,scheme)*
		$\nabla \cdot (\psi \phi)$	div(psi, phi, word)*
			div(psi, phi)
Divergence	Exp	$\nabla \cdot \chi$	div(chi)
Gradient	Exp	$\nabla \chi$	grad(chi)
		$ abla \phi$	gGrad(phi)
			lsGrad(phi)
			<pre>snGrad(phi)</pre>
			<pre>snGradCorrection(phi)</pre>
Grad-grad squared	Exp	$ \nabla\nabla\phi ^2$	sqrGradGrad(phi)
Curl	Exp	$\nabla \times \phi$	curl(phi)
Source	Imp	$\rho\phi$	Sp(rho,phi)
+£	Imp/Exp†		SuSp(rho,phi)

†fvm::SuSp source is discretised implicit or explicit depending on the sign of rho. †An explicit source can be introduced simply as a vol<Type>Field, e.g.rho\*phi. Function arguments can be of the following classes:

phi: vol<Type>Field

Gamma: scalar volScalarField, surfaceScalarField, volTensorField, surfaceTensorField.

rho: scalar, volScalarField psi: surfaceScalarField.

chi: surface<Type>Field, vol<Type>Field.

Table 2.2: Discretisation of PDE terms in OpenFOAM

## 2.4.1 The Laplacian term

The Laplacian term is integrated over a control volume and linearised as follows:

$$\int_{V} \nabla \cdot (\Gamma \nabla \phi) \ dV = \int_{S} d\mathbf{S} \cdot (\Gamma \nabla \phi) = \sum_{f} \Gamma_{f} \mathbf{S}_{f} \cdot (\nabla \phi)_{f}$$
(2.14)

The face gradient discretisation is implicit when the length vector  $\mathbf{d}$  between the centre of the cell of interest P and the centre of a neighbouring cell N is orthogonal to the face plane, *i.e.* parallel to  $\mathbf{S}_f$ :

$$\mathbf{S}_f \bullet (\nabla \phi)_f = |S_f| \frac{\phi_N - \phi_P}{|\mathbf{d}|} \tag{2.15}$$

In the case of non-orthogonal meshes, an additional explicit term is introduced which is evaluated by interpolating cell centre gradients, themselves calculated by central differencing cell centre values.

### 2.4.2 The convection term

The convection term is integrated over a control volume and linearised as follows:

$$\int_{V} \nabla \cdot (\rho \mathbf{U}\phi) \ dV = \int_{S} d\mathbf{S} \cdot (\rho \mathbf{U}\phi) = \sum_{f} \mathbf{S}_{f} \cdot (\rho \mathbf{U})_{f} \phi_{f} = \sum_{f} F \phi_{f}$$
 (2.16)

The face field  $\phi_f$  can be evaluated using a variety of schemes:

Central differencing (CD) is second-order accurate but unbounded

$$\phi_f = f_x \phi_P + (1 - f_x) \phi_N \tag{2.17}$$

where  $f_x \equiv \overline{fN}/\overline{PN}$  where  $\overline{fN}$  is the distance between f and cell centre N and  $\overline{PN}$  is the distance between cell centres P and N.

Upwind differencing (UD) determines  $\phi_f$  from the direction of flow and is bounded at the expense of accuracy

$$\phi_f = \begin{cases} \phi_P & \text{for } F \ge 0\\ \phi_N & \text{for } F < 0 \end{cases} \tag{2.18}$$

Blended differencing (BD) schemes combine UD and CD in an attempt to preserve boundedness with reasonable accuracy,

$$\phi_f = (1 - \gamma) \left(\phi_f\right)_{UD} + \gamma \left(\phi_f\right)_{CD} \tag{2.19}$$

OpenFOAM has several implementations of the Gamma differencing scheme to select the blending coefficient  $\gamma$  but it offers other well-known schemes such as van Leer, SUPERBEE, MINMOD etc..

#### 2.4.3 First time derivative

The first time derivative  $\partial/\partial t$  is integrated over a control volume as follows:

$$\frac{\partial}{\partial t} \int_{V} \rho \phi \ dV \tag{2.20}$$

The term is discretised by simple differencing in time using:

**new values**  $\phi^n \equiv \phi(t + \Delta t)$  at the time step we are solving for;

old values  $\phi^o \equiv \phi(t)$  that were stored from the previous time step;

old-old values  $\phi^{oo} \equiv \phi(t - \Delta t)$  stored from a time step previous to the last.

One of two discretisation schemes can be declared using the timeScheme keyword in the appropriate input file, described in detail in section 4.4 of the User Guide.

Euler implicit scheme, timeScheme EulerImplicit, that is first order accurate in time:

$$\frac{\partial}{\partial t} \int_{V} \rho \phi \ dV = \frac{(\rho_P \phi_P V)^n - (\rho_P \phi_P V)^o}{\Delta t}$$
 (2.21)

Backward differencing scheme, timeScheme BackwardDifferencing, that is second order accurate in time by storing the old-old values and therefore with a larger overhead in data storage than EulerImplicit:

$$\frac{\partial}{\partial t} \int_{V} \rho \phi \ dV = \frac{3 \left(\rho_{P} \phi_{P} V\right)^{n} - 4 \left(\rho_{P} \phi_{P} V\right)^{o} + \left(\rho_{P} \phi_{P} V\right)^{oo}}{2\Delta t} \tag{2.22}$$

### 2.4.4 Second time derivative

The second time derivative is integrated over a control volume and linearised as follows:

$$\frac{\partial}{\partial t} \int_{V} \rho \frac{\partial \phi}{\partial t} dV = \frac{(\rho_P \phi_P V)^n - 2(\rho_P \phi_P V)^o + (\rho_P \phi_P V)^{oo}}{\Delta t^2}$$
(2.23)

It is first order accurate in time.

## 2.4.5 Divergence

The divergence term described in this Section is strictly an explicit term that is distinguished from the convection term of Section 2.4.2, *i.e.* in that it is not the divergence of the product of a velocity and dependent variable. The term is integrated over a control volume and linearised as follows:

$$\int_{V} \nabla \cdot \phi \ dV = \int_{S} d\mathbf{S} \cdot \phi = \sum_{f} \mathbf{S}_{f} \cdot \phi_{f}$$
(2.24)

The fvc::div function can take as its argument either a surface<Type>Field, in which case  $\phi_f$  is specified directly, or a vol<Type>Field which is interpolated to the face by central differencing as described in Section 2.4.10:

### 2.4.6 Gradient

The gradient term is an explicit term that can be evaluated in a variety of ways. The scheme can be evaluated either by selecting the particular grad function relevant to the discretisation scheme, e.g.fvc::gGrad, fvc::lsGrad etc., or by using the fvc::grad function combined with the appropriate timeScheme keyword in an input file

Gauss integration is invoked using the fvc::grad function with timeScheme Gauss or directly using the fvc::gGrad function. The discretisation is performed using the standard method of applying Gauss's theorem to the volume integral:

$$\int_{V} \nabla \phi \ dV = \int_{S} d\mathbf{S} \, \phi = \sum_{f} \mathbf{S}_{f} \phi_{f} \tag{2.25}$$

As with the fvc::div function, the Gaussian integration fvc::grad function can take either a surfaceField<Type> or a volField<Type> as an argument.

Least squares method is based on the following idea:

- 1. a value at point P can be extrapolated to neighbouring point N using the gradient at P;
- 2. the extrapolated value at N can be compared to the actual value at N, the difference being the error;
- 3. if we now minimise the sum of the square of weighted errors at all neighbours of P with the respect to the gradient, then the gradient should be a good approximation.

Least squares is invoked using the fvc::grad function with timeScheme leastSquares or directly using the fvc::lsGrad function. The discretisation is performed as by first calculating the tensor G at every point P by summing over neighbours N:

$$\mathbf{G} = \sum_{N} w_{N}^{2} \mathbf{dd} \tag{2.26}$$

where **d** is the vector from P to N and the weighting function  $w_N = 1/|\mathbf{d}|$ . The gradient is then evaluated as:

$$(\nabla \phi)_P = \sum_N w_N^2 \mathbf{G}^{-1} \cdot \mathbf{d} (\phi_N - \phi_P)$$
(2.27)

Surface normal gradient The gradient normal to a surface  $\mathbf{n}_f \cdot (\nabla \phi)_f$  can be evaluated at cell faces using the scheme

$$(\nabla \phi)_f = \frac{\phi_N - \phi_P}{|\mathbf{d}|} \tag{2.28}$$

This gradient is called by the function fvc::snGrad and returns a surfaceField<Type>. The scheme is directly analogous to that evaluated for the Laplacian discretisation scheme in Section 2.4.1, and in the same manner, a correction can be introduced to improve the accuracy of this face gradient in the case of non-orthogonal meshes. This correction is called using the function fvc::snGradCorrection [Check\*\*].

## 2.4.7 Grad-grad squared

The grad-grad squared term is evaluated by: taking the gradient of the field; taking the gradient of the resulting gradient field; and then calculating the magnitude squared of the result. The mathematical expression for grad-grad squared of  $\phi$  is  $|\nabla (\nabla \phi)|^2$ .

#### 2.4.8 Curl

The curl is evaluated from the gradient term described in Section 2.4.6. First, the gradient is discretised and then the curl is evaluated using the relationship from Equation 2.7, repeated here for convenience

$$\nabla \times \phi = 2 * (\text{skew } \nabla \phi)$$

### 2.4.9 Source terms

Source terms can be specified in 3 ways

**Explicit** Every explicit term is a volField<Type>. Hence, an explicit source term can be incorporated into an equation simply as a field of values. For example if we wished to solve Poisson's equation  $\nabla^2 \phi = f$ , we would define phi and f as volScalarField and then do

Implicit An implicit source term is integrated over a control volume and linearised by

$$\int_{V} \rho \phi \ dV = \rho_P V_P \phi_P \tag{2.29}$$

Implicit/Explicit The implicit source term changes the coefficient of the diagonal of the matrix. Depending on the sign of the coefficient and matrix terms, this will either increase or decrease diagonal dominance of the matrix. Decreasing the diagonal dominance could cause instability during iterative solution of the matrix equation. Therefore OpenFOAM provides a mixed source discretisation procedure that is implicit when the coefficients that are greater than zero, and explicit for the coefficients less than zero. In mathematical terms the matrix coefficient for node P is  $V_P \max(\rho_P, 0)$  and the source term is  $V_P \phi_P \min(\rho_P, 0)$ .

## 2.4.10 Other explicit discretisation schemes

There are some other discretisation procedures that convert volField<Type>s into surface<Type>Fields and visa versa.

Surface integral fvc::surfaceIntegrate performs a summation of surface<Type>Field face values bounding each cell and dividing by the cell volume, i.e.  $(\sum_f \phi_f)/V_P$ . It returns a volField<Type>.

Surface sum fvc::surfaceSum performs a summation of surface<br/>Type>Field face values bounding each cell, i.e.  $\sum_f \phi_f$  returning a volField<br/>Type>.

Average fvc::average produces an area weighted average of surface<Type>Field face values, i.e.  $(\sum_f S_f \phi_f) / \sum_f S_f$ , and returns a volField<Type>.

#### Reconstruct

Face interpolate The geometric<Type>Field function faceInterpolate() interpolates volField<Type> cell centre values to cell faces using central differencing, returning a surface<Type>Field.

## 2.5 Temporal discretisation

Although we have described the discretisation of temporal derivatives in Sections 2.4.3 and 2.4.4, we need to consider how to treat the spatial derivatives in a transient problem. If we denote all the spatial terms as  $\mathcal{A}\phi$  where  $\mathcal{A}$  is any spatial operator, e.g. Laplacian, then we can express a transient PDE in integral form as

$$\int_{t}^{t+\Delta t} \left[ \frac{\partial}{\partial t} \int_{V} \rho \phi \ dV + \int_{V} \mathcal{A}\phi \ dV \right] \ dt = 0$$
 (2.30)

Using the Euler implicit method of Equation 2.21, the first term can be expressed as

$$\int_{t}^{t+\Delta t} \left[ \frac{\partial}{\partial t} \int_{V} \rho \phi \ dV \right] dt = \int_{t}^{t+\Delta t} \frac{(\rho_{P} \phi_{P} V)^{n} - (\rho_{P} \phi_{P} V)^{o}}{\Delta t} dt$$

$$= \frac{(\rho_{P} \phi_{P} V)^{n} - (\rho_{P} \phi_{P} V)^{o}}{\Delta t} \Delta t \tag{2.31}$$

The second term can be expressed as

$$\int_{t}^{t+\Delta t} \left[ \int_{V} \mathcal{A}\phi \ dV \right] \ dt = \int_{t}^{t+\Delta t} \mathcal{A}^{*}\phi \ dt \tag{2.32}$$

where  $\mathcal{A}^*$  represents the spatial discretisation of  $\mathcal{A}$ . The time integral can be discretised in three ways:

Euler implicit uses implicit discretisation of the spatial terms, thereby taking current values  $\phi^n$ .

$$\int_{t}^{t+\Delta t} \mathcal{A}^* \phi \ dt = \mathcal{A}^* \phi^n \Delta t \tag{2.33}$$

It is first order accurate in time, guarantees boundedness and is unconditionally stable.

**Explicit** uses explicit discretisation of the spatial terms, thereby taking old values  $\phi^o$ .

$$\int_{t}^{t+\Delta t} \mathcal{A}^* \phi \ dt = \mathcal{A}^* \phi^o \Delta t \tag{2.34}$$

It is first order accurate in time and is unstable if the Courant number Co is greater than 1. The Courant number is defined as

$$Co = \frac{\mathbf{U}_f \cdot \mathbf{d}}{|\mathbf{d}|^2 \Delta t} \tag{2.35}$$

where  $\mathbf{U}_f$  is a characteristic velocity, e.g. velocity of a wave front, velocity of flow.

**Crank Nicholson** uses the trapezoid rule to discretise the spatial terms, thereby taking a mean of current values  $\phi^n$  and old values  $\phi^o$ .

$$\int_{t}^{t+\Delta t} \mathcal{A}^* \phi \ dt = \mathcal{A}^* \left( \frac{\phi^n + \phi^o}{2} \right) \Delta t \tag{2.36}$$

It is second order accurate in time, is unconditionally stable but does not guarantee boundedness.

## 2.5.1 Treatment of temporal discretisation in OpenFOAM

At present the treatment of the temporal discretisation is controlled by the implementation of the spatial derivatives in the PDE we wish to solve. For example, let us say we wish to solve a transient diffusion equation

$$\frac{\partial \phi}{\partial t} = \kappa \nabla^2 \phi \tag{2.37}$$

An Euler implicit implementation of this would read

```
solve(fvm::ddt(phi) == kappa*fvm::laplacian(phi))
```

where we use the fvm class to discretise the Laplacian term implicitly. An explicit implementation would read

```
solve(fvm::ddt(phi) == kappa*fvc::laplacian(phi))
```

where we now use the fvc class to discretise the Laplacian term explicitly. The Crank Nicholson scheme can be implemented by the mean of implicit and explicit terms:

```
solve
   (
   fvm::ddt(phi)
   ==
   kappa*0.5*(fvm::laplacian(phi) + fvc::laplacian(phi))
)
```

## 2.6 Boundary Conditions

Boundary conditions are required to complete the problem we wish to solve. We therefore need to specify boundary conditions on all our boundary faces. Boundary conditions can be divided into 2 types:

**Dirichlet** prescribes the value of the dependent variable on the boundary and is therefore termed 'fixed value' in this guide;

**Neumann** prescribes the gradient of the variable normal to the boundary and is therefore termed 'fixed gradient' in this guide.

When we perform discretisation of terms that include the sum over faces  $\sum_f$ , we need to consider what happens when one of the faces is a boundary face.

**Fixed value** We specify a fixed value at the boundary  $\phi_b$ 

- We can simply substitute  $\phi_b$  in cases where the discretisation requires the value on a boundary face  $\phi_f$ , e.g. in the convection term in Equation 2.16.
- In terms where the face gradient  $(\nabla \phi)_f$  is required, e.g. Laplacian, it is calculated using the boundary face value and cell centre value,

$$\mathbf{S}_f \bullet (\nabla \phi)_f = |S_f| \frac{\phi_b - \phi_P}{|\mathbf{d}|} \tag{2.38}$$

**Fixed gradient** The fixed gradient boundary condition  $g_b$  is a specification on inner product of the gradient and unit normal to the boundary, or

$$g_b = \left(\frac{\mathbf{S}}{|\mathbf{S}|} \bullet \nabla \phi\right)_f \tag{2.39}$$

• When discretisation requires the value on a boundary face  $\phi_f$  we must interpolate the cell centre value to the boundary by

$$\phi_f = \phi_P + \mathbf{d} \cdot (\nabla \phi)_f$$

$$= \phi_P + |\mathbf{d}| q_h$$
(2.40)

•  $\phi_b$  can be directly substituted in cases where the discretisation requires the face gradient to be evaluated,

$$\mathbf{S}_f \bullet (\nabla \phi)_f = |S_f| \, g_b \tag{2.41}$$

## 2.6.1 Physical boundary conditions

The specification of boundary conditions is usually an engineer's interpretation of the true behaviour. Real boundary conditions are generally defined by some physical attributes rather than the numerical description as described of the previous Section. In incompressible fluid flow there are the following physical boundaries

**Inlet** The velocity field at the inlet is supplied and, for consistency, the boundary condition on pressure is zero gradient.

**Outlet** The pressure field at the outlet is supplied and a zero gradient boundary condition on velocity is specified.

No-slip impermeable wall The velocity of the fluid is equal to that of the wall itself, *i.e.* a fixed value condition can be specified. The pressure is specified zero gradient since the flux through the wall is zero.

In a problem whose solution domain and boundary conditions are symmetric about a plane, we only need to model half the domain to one side of the symmetry plane. The boundary condition on the plane must be specified according to

Symmetry plane The symmetry plane condition specifies the component of the gradient normal to the plane should be zero. [Check\*\*]

# Chapter 3

# Examples of the use of OpenFOAM

In this section we shall describe several test cases supplied with the OpenFOAM distribution. The intention is to provide example cases, including those in the tutorials in chapter 2 of the User Guide, for every standard solver. The examples are designed to introduce certain tools and features of OpenFOAM, e.g. within pre-/post-processing, numerical schemes, algorithms. They also provide a means for validation of solvers although that is not their principal function.

Each example contains a description of the problem: the geometry, initial and boundary conditions, a brief description of the equations being solved, models used, and physical properties required. The solution domain is selected which may be a portion of the original geometry, e.g. if we introduce symmetry planes. The method of meshing, usually blockMesh, is specified; of course the user can simply view the mesh since every example is distributed with the polyMesh directory containing the data files that describe the mesh.

The examples coexist with the tutorials in the *tutorials* subdirectory of the OpenFOAM installation. They are organised into a set of subdirectories by solver, *e.g.* all the icoFoam cases are stored within a subdirectory *icoFoam*. Before running a particular example, the user is urged to copy it into their user account. We recommend that the user stores all OpenFOAM cases in a directory we recommend that the tutorials are copied into a directory *\$FOAM\_RUN*. If this directory structure has not yet been created in the user's account, it can be created with

```
mkdir -p $FOAM_RUN
```

The tutorials can then be copied into this directory with

```
cp -r $FOAM_TUTORIALS/* $FOAM_RUN
```

## 3.1 Flow around a cylinder

In this example we shall investigate potential flow around a cylinder using potentialFoam. This example introduces the following OpenFOAM features:

- non-orthogonal meshes;
- generating an analytical solution to a problem in OpenFOAM.

## 3.1.1 Problem specification

The problem is defined as follows:

**Solution domain** The domain is 2 dimensional and consists of a square domain with a cylinder collocated with the centre of the square as shown in Figure 3.1.

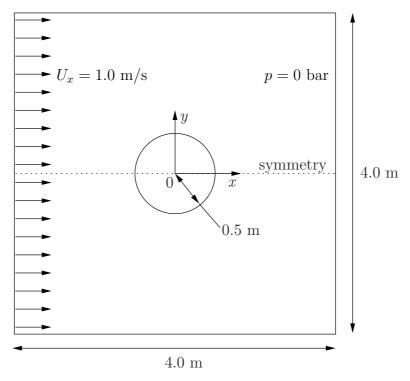


Figure 3.1: Geometry of flow round a cylinder

#### Governing equations

• Mass continuity for an incompressible fluid

$$\nabla \cdot \mathbf{U} = 0 \tag{3.1}$$

• Pressure equation for an incompressible, irrotational fluid assuming steadystate conditions

$$\nabla^2 p = 0 \tag{3.2}$$

#### Boundary conditions

- Inlet (left) with fixed velocity U = (1, 0, 0) m/s.
- Outlet (right) with a fixed pressure p = 0 Pa.
- No-slip wall (bottom);
- Symmetry plane (top).

**Initial conditions** U = 0 m/s, p = 0 Pa — required in OpenFOAM input files but not necessary for the solution since the problem is steady-state.

**Solver name** potentialFoam: a potential flow code, *i.e.* assumes the flow is incompressible, steady, irrotational, inviscid and it ignores gravity.

Case name cylinder case located in the \$FOAM\_TUTORIALS/potentialFoam directory.

## 3.1.2 Note on potentialFoam

potentialFoam is a useful solver to validate OpenFOAM since the assumptions of potential flow are such that an analytical solution exists for cases whose geometries are relatively simple. In this example of flow around a cylinder an analytical solution exists with which we can compare our numerical solution. potentialFoam can also be run more like a utility to provide a (reasonably) conservative initial U field for a problem. When running certain cases, this can useful for avoiding instabilities due to the initial field being unstable. In short, potentialFoam creates a conservative field from a non-conservative initial field supplied by the user.

## 3.1.3 Mesh generation

Mesh generation using blockMesh has been described in tutorials in the User Guide. In this case, the mesh consists of 10 blocks as shown in Figure 3.2. Remember that all

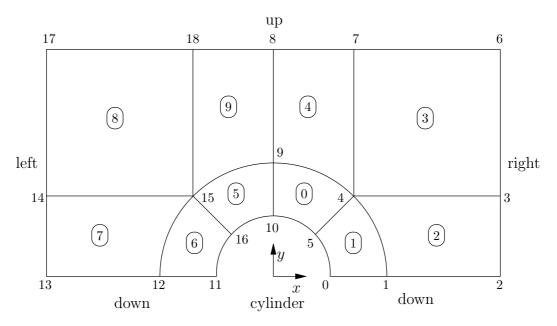


Figure 3.2: Blocks in cylinder geometry

meshes are treated as 3 dimensional in OpenFOAM. If we wish to solve a 2 dimensional problem, we must describe a 3 dimensional mesh that is only one cell thick in the third direction that is not solved. In Figure 3.2 we show only the back plane of the geometry, along z = -0.5, in which the vertex numbers are numbered 0-18. The other 19 vertices in the front plane, z = +0.5, are numbered in the same order as the back plane, as shown in the mesh description file below:

```
2
                                      OpenFOAM: The Open Source CFD Toolbox
                    ield
3
                   O peration
                                      Version:
                                                 2.2.1
                                                 www.OpenFOAM.org
5
                   A nd
                                      Web:
                   M anipulation
6
    FoamFile
                      2.0;
         version
         format
                      ascii;
11
                      dictionary
         class
                      blockMeshDict
13
         object
14
15
16
    convertToMeters 1;
```

```
18
    vertices #codeStream
19
20
         codeInclude
21
         #{
22
              #include "pointField.H"
23
24
         #};
25
         code
26
27
              pointField points(19);
28
              points[0]
                          = point(0.5, 0, -0.5);
29
                           = point(1, 0, -0.5);
              points[1]
30
              points[2]
                          = point(2, 0, -0.5);
                          = point(2, 0.707107, -0.5);
              points[3]
32
                           = point(0.707107, 0.707107, -0.5);
              points[4]
33
                           = point(0.353553, 0.353553, -0.5);
34
              points[5]
              points[6]
                           = point(2, 2, -0.5);
35
              points[7]
                           = point(0.707107, 2, -0.5);
                          = point(0, 2, -0.5);
= point(0, 1, -0.5);
              points[8]
37
              points[9]
38
              points[10]
                          = point(0, 0.5, -0.5)
              points[11] = point(-0.5, 0, -0.5);
40
              points[12] = point(-1, 0, -0.5);
41
              points[13] = point(-2, 0, -0.5);
42
              points[14] = point(-2, 0.707107,
                                                    -0.5);
43
                          = point(-0.707107, 0.707107, -0.5);
              points[15]
              points[16] = point(-0.353553, 0.353553, -0.5);
45
              points[17] = point(-2, 2, -0.5);
46
              points[18] = point(-0.707107, 2, -0.5);
47
              // Duplicate z points
49
              label sz = points.size();
50
              points.setSize(2*sz);
              for (label i = 0; i < sz; i++)
52
53
                   const point& pt = points[i];
54
                   points[i+sz] = point(pt.x(), pt.y(), -pt.z());
55
              }
57
              os
                  << points;
         #};
59
    };
60
61
62
    blocks
63
64
         hex (5 4 9 10 24 23 28 29) (10 10 1) simpleGrading (1 1 1)
65
         hex (0 1 4 5 19 20 23 24) (10 10 1) simpleGrading (1 1 1)
66
         hex (1 2 3 4 20 21 22 23) (20 10 1) simpleGrading (1 1 1)
67
         hex (4 3 6 7 23 22 25 26) (20 20 1) simpleGrading (1 1 1)
68
         hex (9 4 7 8 28 23 26 27) (10 20 1) simpleGrading (1 1 1)
69
         hex (15 16 10 9 34 35 29 28) (10 10 1) simpleGrading (1 1 1)
70
         hex (12 11 16 15 31 30 35 34) (10 10 1) simpleGrading (1 1 1)
71
         hex (13 12 15 14 32 31 34 33) (20 10 1) simpleGrading (1 1 1)
72
         hex (14 15 18 17 33 34 37 36) (20 20 1) simpleGrading (1 1 1)
         hex (15 9 8 18 34 28 27 37) (10 20 1) simpleGrading (1 1 1)
74
    );
75
76
    edges
77
78
         arc 0 5 (0.469846 0.17101 -0.5)
         arc 5 10 (0.17101 0.469846 -0.5)
arc 1 4 (0.939693 0.34202 -0.5)
arc 4 9 (0.34202 0.939693 -0.5)
         arc 19 24 (0.469846 0.17101 0.5)
arc 24 29 (0.17101 0.469846 0.5)
                     (0.469846 0.17101 0.5)
84
         arc 20 23
arc 23 28
                     (0.939693 0.34202 0.5)
(0.34202 0.939693 0.5)
(-0.469846 0.17101 -0.5)
(-0.17101 0.469846 -0.5)
         arc 11 16
88
         arc 16 10
                    (-0.939693 0.34202 -0.5)
(-0.34202 0.939693 -0.5)
         arc 12
89
                 15
         arc 15 9
90
         arc 30 35
arc 35 29
                     (-0.469846 \ 0.17101 \ 0.5)
                     (-0.17101 0.469846 0.5)
92
                     (-0.939693 0.34202
              31
                 34
93
         arc
         arc 34 28
                    (-0.34202 0.939693 0.5)
94
    );
95
96
```

```
boundary
98
           down
99
100
                 type symmetryPlane;
101
                 faces
102
103
                      (0 1 20 19)
(1 2 21 20)
(12 11 30 31)
(13 12 31 32)
104
105
106
107
                 );
108
109
           right
110
111
112
                 type patch;
                 faces
113
114
115
116
                 );
117
118
119
           up
120
                 type symmetryPlane;
121
122
123
                      (7 8 27 26)
(6 7 26 25)
(8 18 37 27)
(18 17 36 37)
124
125
126
127
                 );
           }
left
129
130
132
                 type patch;
133
134
                      (14 13 32 33)
(17 14 33 36)
136
137
           cylinder
139
140
                 type symmetryPlane;
141
                 faces
142
143
                       (10 5 24 29)
144
                      (5 0 19 24)
(16 10 29 35)
(11 16 35 30)
145
146
147
                 );
148
           }
149
150
151
      mergePatchPairs
152
153
154
155
```

## 3.1.4 Boundary conditions and initial fields

Using FoamX or editing case files by hand, set the boundary conditions in accordance with the problem description in Figure 3.1, i.e. the left boundary should be an Inlet, the right boundary should be an Outlet and the down and cylinder boundaries should be symmetryPlane. The top boundary conditions is chosen so that we can make the most genuine comparison with our analytical solution which uses the assumption that the domain is infinite in the y direction. The result is that the normal gradient of U is small along a plane coinciding with our boundary. We therefore impose the condition that the normal component is zero, i.e. specify the boundary as a symmetryPlane, thereby ensuring that the comparison with the analytical is reasonable.

## 3.1.5 Running the case

No fluid properties need be specified in this problem since the flow is assumed to be incompressible and inviscid. In the *system* subdirectory, the *controlDict* specifies the control parameters for the run. Note that since we assume steady flow, we only run for 1 time step:

```
---*- C++ -*---
2
                   F ield
                                       OpenFOAM: The Open Source CFD Toolbox
3
4
                   O peration
                                       Version:
                                                  2.2.1
5
                   A nd
                                       Web:
                                                   www.OpenFOAM.org
                   {\tt M} anipulation
6
    FoamFile
10
         version
                       2.0;
11
         format
                       ascii;
                       dictionary;
"system";
         class
         location
         object
14
                       controlDict;
15
                      * * * * * * *
16
17
    application
                       potentialFoam;
18
19
    startFrom
                       startTime;
20
21
    startTime
                       0;
22
23
    stopAt
                       endTime;
24
25
    endTime
                       1;
26
27
    deltaT
                       1;
28
29
    writeControl
                       timeStep;
30
31
    writeInterval
                       1;
32
33
    purgeWrite
34
35
    writeFormat
                       ascii;
36
37
38
    writePrecision 6;
39
    writeCompression off;
40
41
42
    timeFormat
                       general;
43
    timePrecision
44
46
    runTimeModifiable true;
47
    functions
48
         difference
              // Load the library containing the 'coded' functionObject
52
             functionObjectLibs ("libutilityFunctionObjects.so");
53
              type coded;
54
             // Name of on-the-fly generated functionObject
redirectType error;
55
56
              code
                  // Lookup U
                  Info<< "Looking up field U\n" << endl;
60
                  const volVectorField& U = mesh().lookupObject<volVectorField>("U");
61
62
                  Info<< "Reading inlet velocity uInfX\n" << endl;</pre>
63
64
                  scalar ULeft = 0.0;
label leftI = mesh().boundaryMesh().findPatchID("left");
65
66
                  const fvPatchVectorField& fvp = U.boundaryField()[leftI];
67
                  if (fvp.size())
68
                  {
                       ULeft = fvp[0].x();
70
71
                  reduce(ULeft, maxOp<scalar>());
72
73
                  dimensionedScalar uInfX
74
```

```
(
                      "uInfx"
76
                      dimensionSet(0, 1, -1, 0, 0),
77
78
                      ULeft.
                  );
79
80
                  Info << "U at inlet = " << uInfX.value() << " m/s" << endl;</pre>
81
82
83
                  scalar magCylinder = 0.0;
84
                  label cylI = mesh().boundaryMesh().findPatchID("cylinder");
85
                  const fvPatchVectorField& cylFvp = mesh().C().boundaryField()[cylI];
86
                  if (cylFvp.size())
87
89
                      magCylinder = mag(cylFvp[0]);
90
                  reduce(magCylinder, maxOp<scalar>());
91
92
                  dimensionedScalar radius
93
94
                      "radius"
95
                      dimensionSet(0, 1, 0, 0, 0),
96
                      magCylinder
97
98
                  );
99
                  Info << "Cylinder radius = " << radius.value() << " m" << endl;</pre>
100
101
                  volVectorField UA
103
                      IOobject
104
105
                          "UA"
106
                          mesh().time().timeName(),
U.mesh(),
IOobject::NO_READ,
107
108
109
                          IOobject::AUTO_WRÍTE
110
111
                      )
U
112
                  );
113
114
                  Info<< "\nEvaluating analytical solution" << endl;</pre>
115
                  const volVectorField& centres = UA.mesh().C();
117
                  volScalarField magCentres(mag(centres));
118
                  volScalarField theta(acos((centres & vector(1,0,0))/magCentres));
119
120
                  volVectorField cs2theta
121
122
                      cos(2*theta)*vector(1,0,0)
123
                    + sin(2*theta)*vector(0,1,0)
124
125
126
                  UA = uInfX*(dimensionedVector(vector(1,0,0))
127
                    - pow((radius/magCentres),2)*cs2theta);
128
129
                  // Force writing of UA (since time has not changed)
130
132
                  volScalarField error("error", mag(U-UA)/mag(UA));
133
134
                  Info<<"Writing relative error in U to " << error.objectPath()</pre>
                      << endl;
136
137
                  error.write();
138
             #};
139
         }
140
141
142
143
```

potentialFoam executes an iterative loop around the pressure equation which it solves in order that explicit terms relating to non-orthogonal correction in the Laplacian term may be updated in successive iterations. The number of iterations around the pressure equation is controlled by the nNonOrthogonalCorrectors keyword in *controlDict*. In the first instance we can set nNonOrthogonalCorrectors to 0 so that no loops are performed, *i.e.* the pressure equation is solved once, and there is no non-orthogonal correction. The solution is shown in Figure 3.3(a) (at t = 1, when the steady-state simulation is complete). We expect the solution to show smooth streamlines passing across the domain

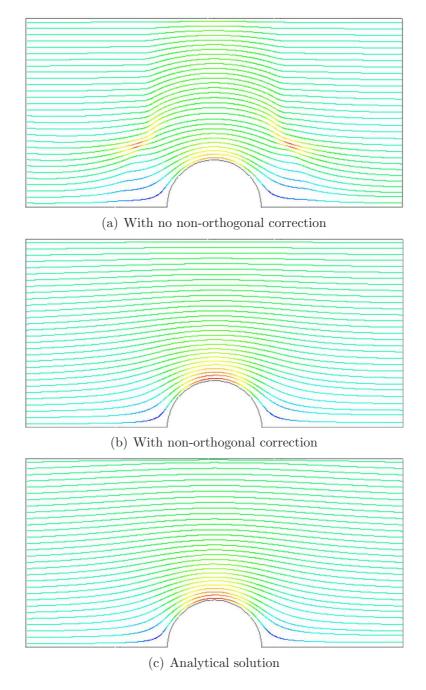


Figure 3.3: Streamlines of potential flow

as in the analytical solution in Figure 3.3(c), yet there is clearly some error in the regions where there is high non-orthogonality in the mesh, e.g. at the join of blocks 0, 1 and 3. The case can be run a second time with some non-orthogonal correction by setting nNonOrthogonalCorrectors to 3. The solution shows smooth streamlines with no significant error due to non-orthogonality as shown in Figure 3.3(b).

## 3.2 Steady turbulent flow over a backward-facing step

In this example we shall investigate steady turbulent flow over a backward-facing step. The problem description is taken from one used by Pitz and Daily in an experimental investigation [\*\*] against which the computed solution can be compared. This example introduces the following OpenFOAM features for the first time:

- generation of a mesh using blockMesh using full mesh grading capability;
- steady turbulent flow.

## 3.2.1 Problem specification

The problem is defined as follows:

**Solution domain** The domain is 2 dimensional, consisting of a short inlet, a backward-facing step and converging nozzle at outlet as shown in Figure 3.4.

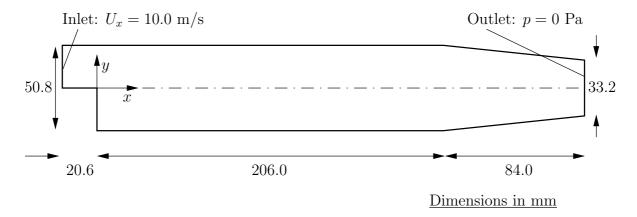


Figure 3.4: Geometry of backward-facing step

#### Governing equations

• Mass continuity for incompressible flow

$$\nabla \cdot \mathbf{U} = 0 \tag{3.3}$$

• Steady flow momentum equation

$$\nabla \cdot (\mathbf{U}\mathbf{U}) + \nabla \cdot \mathbf{R} = -\nabla p \tag{3.4}$$

where p is kinematic pressure and (in slightly over-simplistic terms)  $\mathbf{R} = \nu_{eff} \nabla \mathbf{U}$  is the viscous stress term with an effective kinematic viscosity  $\nu_{eff}$ , calculated from selected transport and turbulence models.

**Initial conditions** U = 0 m/s, p = 0 Pa — required in OpenFOAM input files but not necessary for the solution since the problem is steady-state.

### Boundary conditions

- Inlet (left) with fixed velocity U = (10, 0, 0) m/s;
- Outlet (right) with fixed pressure p = 0 Pa;
- No-slip walls on other boundaries.

### Transport properties

• Kinematic viscosity of air  $\nu = \mu/\rho = 18.1 \times 10^{-6}/1.293 = 14.0 \ \mu m^2/s$ 

#### Turbulence model

- Standard  $k \epsilon$ ;
- Coefficients:  $C_{\mu} = 0.09$ ;  $C_1 = 1.44$ ;  $C_2 = 1.92$ ;  $\alpha_k = 1$ ;  $\alpha_{\epsilon} = 0.76923$ .

**Solver name** simpleFoam: an implementation for steady incompressible flow.

Case name pitzDaily, located in the \$FOAM\_TUTORIALS/simpleFoam directory.

The problem is solved using simpleFoam, so-called as it is an implementation for steady flow using the SIMPLE algorithm [\*\*]. The solver has full access to all the turbulence models in the incompressibleTurbulenceModels library and the non-Newtonian models incompressibleTransportModels library of the standard OpenFOAM release.

## 3.2.2 Mesh generation

We expect that the flow in this problem is reasonably complex and an optimum solution will require grading of the mesh. In general, the regions of highest shear are particularly critical, requiring a finer mesh than in the regions of low shear. We can anticipate where high shear will occur by considering what the solution might be in advance of any calculation. At the inlet we have strong uniform flow in the x direction and, as it passes over the step, it generates shear on the fluid below, generating a vortex in the bottom half of the domain. The regions of high shear will therefore be close to the centreline of the domain and close to the walls.

The domain is subdivided into 12 blocks as shown in Figure 3.5.

The mesh is 3 dimensional, as always in OpenFOAM, so in Figure 3.5 we are viewing the back plane along z = -0.5. The full set of vertices and blocks are given in the mesh description file below:

```
2
                   F ield
                                       OpenFOAM: The Open Source CFD Toolbox
3
4
                   O peration
                                       Version: 2.2.1
                                                  www.OpenFOAM.org
5
                   A nd
                                       Web:
                   M anipulation
6
    FoamFile
         version
                       2.0;
         format
                       ascii;
                       dictionary;
blockMeshDict;
12
14
15
16
    convertToMeters 0.001:
17
18
```

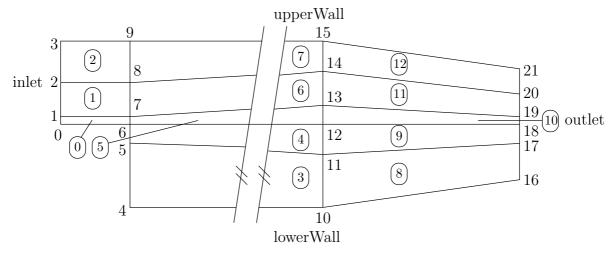


Figure 3.5: Blocks in backward-facing step

```
vertices
19
20
                (-20.6 0 -0.5)
(-20.6 3 -0.5)
(-20.6 12.7 -0.5)
(-20.6 25.4 -0.5)
(0 -25.4 -0.5)
(0 -5 -0.5)
2.1
22
23
24
25
26
                (0 -5 -0.5)

(0 0 -0.5)

(0 3 -0.5)

(0 12.7 -0.5)

(0 25.4 -0.5)

(206 -25.4 -0.5)

(206 -8.5 -0.5)

(206 6 5 -0.5)
27
28
29
30
31
32
33
                (206 6.5 -0.5)
(206 17 -0.5)
(206 25.4 -0.5)
34
35
36
                (290 -16.6 -0.5)
(290 -6.3 -0.5)
(290 0 -0.5)
(290 4.5 -0.5)
37
38
39
40
                (290 11 -0.5)
(290 16.6 -0.5)
41
               (290 11 -0.5)

(290 16.6 -0.5)

(-20.6 0 0.5)

(-20.6 3 0.5)

(-20.6 12.7 0.5)

(-20.6 25.4 0.5)

(0 -25.4 0.5)

(0 0 0.5)

(0 3 0.5)

(0 12.7 0.5)

(0 25.4 0.5)

(0 25.4 0.5)

(206 -25.4 0.5)

(206 -25.4 0.5)

(206 6.5 0.5)

(206 0 0.5)

(206 17 0.5)

(206 25.4 0.5)

(206 25.4 0.5)

(206 17 0.5)

(206 25.4 0.5)

(206 17 0.5)

(206 25.4 0.5)

(209 -16.6 0.5)

(290 -6.3 0.5)

(290 4.5 0.5)

(290 11 0.5)

(290 16.6 0.5)
43
44
^{45}
46
47
49
51
53
55
57
59
60
61
62
63
64
        );
65
66
        blocks
67
68
               hex (0 6 7 1 22 28 29 23) (18 7 1) simpleGrading (0.5 1.8 1)
69
70
               hex (1 7 8 2 23 29 30 24) (18 10 1) simpleGrading (0.5 4 1)
               hex (2 8 9 3 24 30 31 25) (18 13 1) simpleGrading (0.5 0.25 1)
71
               hex (4 10 11 5 26 32 33 27) (180 18 1) simpleGrading (4 1 1)
72
               hex (5 11 12 6 27 33 34 28)
                                                                     (180 9 1) edgeGrading (4 4 4 4 0.5 1 1 0.5 1 1 1 1)
73
               hex (6 12 13 7 28 34 35 29) (180 7 1) edgeGrading (4 4 4 4 1.8 1 1 1.8 1 1 1)
74
               hex (7 13 14 8 29 35 36 30) (180 10 1) edgeGrading (4 4 4 4 4 1 1 4 1 1 1 1)
75
               hex (8 14 15 9 30 36 37 31) (180 13 1) simpleGrading (4 0.25 1)
76
               hex (10 16 17 11 32 38 39 33) (25 18 1) simpleGrading (2.5 1 1)
77
```

```
hex (11 17 18 12 33 39 40 34) (25 9 1) simpleGrading (2.5 1 1)
 78
                hex (12 18 19 13 34 40 41 35) (25 7 1) simpleGrading (2.5 1 1)
 79
                 hex (13 19 20 14 35 41 42 36) (25 10 1) simpleGrading (2.5 1 1)
 80
                 hex (14 20 21 15 36 42 43 37) (25 13 1) simpleGrading (2.5 0.25 1)
 81
 82
 83
         edges
 84
 85
 86
 87
         boundary
 88
 89
                inlet {
 90
 91
                         type patch;
faces
 92
 93
 94
                                 (0 22 23 1)
(1 23 24 2)
(2 24 25 3)
 95
 96
 97
                         );
 98
 99
                outlet {
100
101
                         type patch;
102
                         faces
103
104
                                        17 39 38)
105
                                 (16
                                 (17 18 40 39)
(18 19 41 40)
(19 20 42 41)
(20 21 43 42)
106
107
109
110
                        );
111
                 upperWall
112
113
                         type wall;
114
                         faces
115
116
                                 (3 25 31 9)
(9 31 37 15)
(15 37 43 21)
117
118
119
                        );
120
121
                 lowerWall
122
123
                         type wall;
faces
124
125
126
                                 (0 6 28 22)
(6 5 27 28)
(5 4 26 27)
(4 10 32 26)
127
128
129
130
                                 (10 16 38 32)
131
                         );
132
133
                 frontAndBack
134
135
                         type empty;
136
137
                         faces
                                (22 28 29 23)

(23 29 30 24)

(24 30 31 25)

(26 32 33 27)

(27 33 34 28)

(28 34 35 29)

(29 35 36 30)

(30 36 37 31)

(32 38 39 33)

(33 39 40 34)

(34 40 41 35)

(35 41 42 36)

(36 42 43 37)

(0 1 7 6)

(1 2 8 7)

(2 3 9 8)

(4 5 11 10)

(5 6 12 11)

(6 7 13 12)

(7 8 14 13)

(8 9 15 14)

(10 11 17 16)

(11 12 18 17)

(12 13 19 18)
138
139
141
142
145
149
150
151
152
153
154
155
156
157
158
159
160
161
162
```

A major feature of this problem is the use of the full mesh grading capability of blockMesh that is described in section 5.3.1 of the User Guide. The user can see that blocks 4,5 and 6 use the full list of 12 expansion ratios. The expansion ratios correspond to each edge of the block, the first 4 to the edges aligned in the local  $x_1$  direction, the second 4 to the edges in the local  $x_2$  direction and the last 4 to the edges in the local  $x_3$  direction. In blocks 4, 5, and 6, the ratios are equal for all edges in the local  $x_1$  and  $x_3$  directions but not for the edges in the  $x_2$  direction that corresponds in all blocks to the global y. If we consider the ratios used in relation to the block definition in section 5.3.1 of the User Guide, we realize that different gradings have been prescribed along the left and right edges in blocks 4,5 and 6 in Figure 3.5. The purpose of this differential grading is to generate a fine mesh close to the most critical region of flow, the corner of the step, and allow it to expand into the rest of the domain.

The mesh can be generated using blockMesh from the command line or from within FoamX and viewed as described in previous examples.

## 3.2.3 Boundary conditions and initial fields

The case files can be viewed, or edited from within FoamX or by hand. In this case, we are required to set the initial and boundary fields for velocity  $\mathbf{U}$ , pressure p, turbulent kinetic energy k and dissipation rate  $\varepsilon$ . The boundary conditions can be specified by setting the physical patch types in FoamX: the upper and lower walls are set to Wall, the left patch to Inlet and the right patch to Outlet. These physical boundary conditions require us to specify a fixedValue at the inlet on  $\mathbf{U}$ , k and  $\varepsilon$ .  $\mathbf{U}$  is given in the problem specification, but the values of k and  $\epsilon$  must be chosen by the user in a similar manner to that described in section 2.1.8.1 of the User Guide. We assume that the inlet turbulence is isotropic and estimate the fluctuations to be 5% of  $\mathbf{U}$  at the inlet. We have

$$U'_x = U'_y = U'_z = \frac{5}{100} 10 = 0.5 \text{ m/s}$$
 (3.5)

and

$$k = \frac{3}{2}(0.5)^2 = 0.375 \text{ m}^2/\text{s}^2$$
 (3.6)

If we estimate the turbulent length scale l to be 10% of the width of the inlet then

$$\varepsilon = \frac{C_{\mu}^{0.75} k^{1.5}}{l} = \frac{0.09^{0.75} 0.375^{1.5}}{0.1 \times 25.4 \times 10^{-3}} = 14.855 \,\mathrm{m}^2/\mathrm{s}^3$$
(3.7)

At the outlet we need only specify the pressure p = 0Pa.

#### 3.2.4 Case control

The choices of *fvSchemes* are as follows: the timeScheme should be SteadyState; the gradScheme and laplacianScheme should be set as default to Gauss; and, the divScheme should be set to UD to ensure boundedness.

Special attention should be paid to the settings of *fvTolerances*. Although the top level simpleFoam code contains only equations for p and  $\mathbf{U}$ , the turbulent model solves equations for k,  $\varepsilon$  and  $\mathbf{R}$ , and tolerance settings are required for all 5 equations. A solverTolerance of  $10^{-5}$  and solverRelativeTolerance of 0.1 are acceptable for all variables with the exception of p when  $10^{-6}$  and 0.01 are recommended. Under-relaxation of the solution is required since the problem is steady. A relaxationFactor of 0.7 is acceptable for  $\mathbf{U}$ , k,  $\varepsilon$  and  $\mathbf{R}$  but 0.3 is required for p to avoid numerical instability.

Finally, in *controlDict*, the time step deltaT should be set to 1 since in steady state cases such as this is effectively an iteration counter. With benefit of hindsight we know that the solution requires 1000 iterations reach reasonable convergence, hence endTime is set to 1000. Ensure that the writeFrequency is sufficiently high, e.g. 50, that you will not fill the hard disk with data during run time.

## 3.2.5 Running the case and post-processing

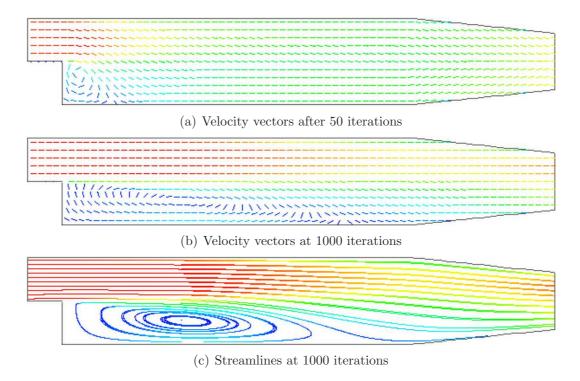


Figure 3.6: Development of a vortex in the backward-facing step.

Run the case and post-process the results. After a few iterations, e.g. 50, a vortex develops beneath the corner of the step that is the height of the step but narrow in the x-direction as shown by the vector plot of velocities is shown Figure 3.6(a). Over several iterations the vortex stretches in the x-direction from the step to the outlet until at 1000 iterations the system reaches a steady-state in which the vortex is fully developed as shown in Figure 3.6(b-c).

## 3.3 Supersonic flow over a forward-facing step

In this example we shall investigate supersonic flow over a forward-facing step. The problem description involves a flow of Mach 3 at an inlet to a rectangular geometry with a step near the inlet region that generates shock waves.

This example introduces the following OpenFOAM features for the first time:

• supersonic flow;

## 3.3.1 Problem specification

The problem is defined as follows:

**Solution domain** The domain is 2 dimensional and consists of a short inlet section followed by a forward-facing step of 20% the height of the section as shown in Figure 3.7

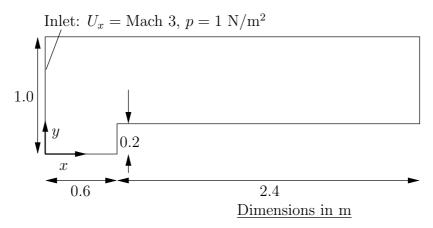


Figure 3.7: Geometry of the forward step geometry

## Governing equations

• Mass continuity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0 \tag{3.8}$$

• Ideal gas

$$p = \rho RT \tag{3.9}$$

• Momentum equation for Newtonian fluid

$$\frac{\partial \rho \mathbf{U}}{\partial t} + \nabla \cdot (\rho \mathbf{U} \mathbf{U}) - \nabla \cdot \mu \nabla \mathbf{U} = -\nabla p \tag{3.10}$$

• Energy equation for fluid (ignoring some viscous terms),  $e = C_v T$ , with Fourier's Law  $\mathbf{q} = -k \nabla T$ 

$$\frac{\partial \rho e}{\partial t} + \nabla \cdot (\rho \mathbf{U} e) - \nabla \cdot \left(\frac{k}{C_v}\right) \nabla e = p \nabla \cdot \mathbf{U}$$
(3.11)

**Initial conditions** U = 0 m/s, p = 1 Pa, T = 1 K.

## Boundary conditions

- Inlet (left) with fixedValue for velocity U = 3 m/s = Mach 3, pressure p = 1 Pa and temperature T = 1 K;
- Outlet (right) with zeroGradient on U, p and T;
- No-slip adiabatic wall (bottom);

• Symmetry plane (top).

## Transport properties

• Dynamic viscosity of air  $\mu = 18.1 \mu Pa s$ 

### Thermodynamic properties

- Specific heat at constant volume  $C_v = 1.78571 \text{ J/kg K}$
- Gas constant R = 0.714286 J/kg K
- Conductivity  $k = 32.3 \, \mu \text{W/m K}$

Case name forwardStep case located in the \$FOAM\_TUTORIALS/sonicFoam directory.

**Solver name** sonicFoam: an implementation for compressible trans-sonic/supersonic laminar gas flow.

The case is designed such that the speed of sound of the gas  $c = \sqrt{\gamma RT} = 1$  m/s, the consequence being that the velocities are directly equivalent to the Mach number, e.g. the inlet velocity of 3 m/s is equivalent to Mach 3. This speed of sound calculation can be verified using the relationship for a perfect gas,  $C_p - Cv = R$ , i.e. the ratio of specific heats

$$\gamma = C_p/C_v = \frac{R}{C_v} + 1 \tag{3.12}$$

## 3.3.2 Mesh generation

The mesh used in this case is relatively simple, specified with uniform rectangular cells of length 0.06 m in the x direction and 0.05 m in the y direction. The geometry can simply be divided into 3 blocks, one below the top of the step, and two above the step, one either side of the step front. The full set of vertices and blocks are given in the mesh description file below:

```
---*- C++ -*---
2
                                              OpenFOAM: The Open Source CFD Toolbox
3
                         ield
                                              Version: 2.2.1
                       O peration
4
                       A nd
                                                            www.OpenFOAM.org
                       M anipulation
6
7
     FoamFile
8
9
                           2.0;
           version
10
           format
11
           class
                           dictionary
12
13
14
15
16
     convertToMeters 1;
17
18
19
     vertices
20
           (0 0 -0.05)
(0.6 0 -0.05)
(0 0.2 -0.05)
(0.6 0.2 -0.05)
21
22
23
24
                      -0.05)
25
26
27
28
29
30
              0.2 0.05)
.6 0.2 0.0
31
                       0.05)
32
33
```

```
(0 1 0.05)
(0.6 1 0.05)
(3 1 0.05)
35
36
     );
37
38
     blocks
39
40
          hex (0 1 3 2 8 9 11 10) (25 10 1) simpleGrading (1 1 1)
41
          hex (2 3 6 5 10 11 14 13) (25 40 1) simpleGrading (1 1 1)
42
43
          hex (3 4 7 6 11 12 15 14) (100 40 1) simpleGrading (1 1 1)
     );
44
45
     edges
46
     );
48
49
50
     boundary
51
          inlet {
52
53
               type patch;
54
               faces
55
56
                     (0 \ 8 \ 10 \ 2)
57
                     (2 10 13 5)
58
59
60
          outlet {
61
62
               type patch;
63
               faces
64
65
                     (471512)
66
67
68
          bottom
69
70
71
               type symmetryPlane;
72
73
                     (0 1 9 8)
74
               );
75
          }
76
          top
77
78
               type symmetryPlane;
79
80
81
                       13 14 6)
14 15 7)
82
83
84
85
          obstacle
86
87
               type patch;
88
               faces
89
90
                     (1 \ 3 \ 11 \ 9)
91
                     (3 \ 4 \ 12 \ 11)
92
93
94
     );
95
96
    mergePatchPairs
97
```

## 3.3.3 Running the case

The case approaches a steady-state at some time after 5 s. The results for pressure at 10 s are shown in Figure 3.8. The results clearly show discontinuities in pressure, *i.e.* shock waves, emanating from ahead of the base of the step.

#### 3.3.4 Exercise

The user can examine the effect on the solution of increasing the inlet velocity.

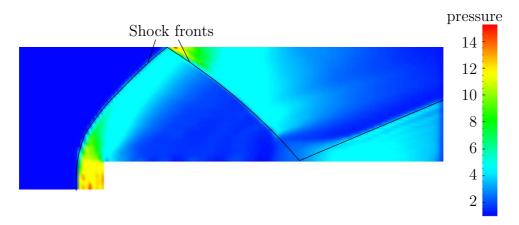


Figure 3.8: Shock fronts in the forward step problem

# 3.4 Decompression of a tank internally pressurised with water

In this example we shall investigate a problem of rapid opening of a pipe valve close to a pressurised liquid-filled tank. The prominent feature of the result in such cases is the propagation of pressure waves which must therefore be modelled as a compressible liquid.

This tutorial introduces the following OpenFOAM features for the first time:

- Mesh refinement
- Pressure waves in liquids

## 3.4.1 Problem specification

**Solution domain** The domain is 2 dimensional and consists of a tank with a small outflow pipe as shown in Figure 3.9

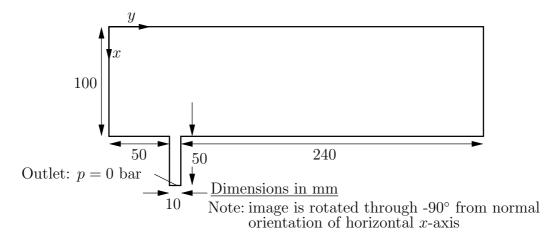


Figure 3.9: Geometry of a tank with outflow pipe

Governing equations This problem requires a model for compressibility  $\psi$  in the fluid in order to be able to resolve waves propagating at a finite speed. A barotropic relationship is used to relate density  $\rho$  and pressure p are related to  $\psi$ .

• Mass continuity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0 \tag{3.13}$$

• The barotropic relationship

$$\frac{\partial \rho}{\partial p} = \frac{\rho}{K} = \psi \tag{3.14}$$

where K is the bulk modulus

• Equation 3.14 is linearised as

$$\rho \approx \rho_0 + \psi \left( p - p_0 \right) \tag{3.15}$$

where  $\rho_0$  and  $p_0$  are the reference density and pressure respectively such that  $\rho(p_0) = \rho_0$ .

• Momentum equation for Newtonian fluid

$$\frac{\partial \rho \mathbf{U}}{\partial t} + \nabla \cdot (\rho \mathbf{U} \mathbf{U}) - \nabla \cdot \mu \nabla \mathbf{U} = -\nabla p \tag{3.16}$$

**Boundary conditions** Using FoamX the following physical boundary conditions can be set:

- outerWall is specified the wall condition;
- axis is specified as the symmetryPlane;
- nozzle is specified as a pressureOutlet where p = 0 bar.
- front and back boundaries are specified as empty.

**Initial conditions** U = 0 m/s, p = 100 bar.

Transport properties

• Dynamic viscosity of water  $\mu = 1.0 \text{ mPa s}$ 

Thermodynamic properties

- Density of water  $\rho = 1000 \text{ kg/m}^3$
- Reference pressure  $p_0 = 1$  bar
- Compressibility of water  $\psi = 4.54 \times 10^{-7} \text{ s}^2/\text{m}^2$

Solver name sonicLiquidFoam: a compressible sonic laminar liquid flow code.

Case name decompressionTank case located in the \$FOAM\_TUTORIALS/sonicLiquidFoam directory.

#### 3.4.2 Mesh Generation

The full geometry is modelled in this case; the set of vertices and blocks are given in the mesh description file below:

```
11
                format
                                       ascii;
                                      dictionary;
blockMeshDict;
12
                class
13
                object
14
15
16
        convertToMeters 0.1;
17
18
        vertices
19
                (0 0 -0.1)

(1 0 -0.1)

(0 0.5 -0.1)

(1 0.5 -0.1)

(1 0.5 -0.1)

(1 0.6 -0.1)

(1 0.6 -0.1)

(1 0.6 -0.1)

(1 0.0 0.1)

(1 0 0.1)

(1 0.5 0.1)

(1 0.5 0.1)

(1 0.5 0.1)

(1 0.6 0.1)

(1 0.6 0.1)

(1 0.6 0.1)

(1 0.6 0.1)

(1 0.5 0.1)

(1 0.6 0.1)

(1 0.6 0.1)

(1 0.5 0.1)

(1 0.6 0.1)

(1 0.6 0.1)

(1 0.5 0.1)

(1 0.6 0.1)

(1 0.5 0.1)

(1 0.6 0.1)

(1 0.5 0.1)
20
21
22
23
24
25
26
27
28
31
32
37
39
40
        );
41
42
        blocks
43
44
                hex (0 1 3 2 10 11 13 12) (30 20 1) simpleGrading (1 1 1)
45
               hex (2 3 6 5 12 13 16 15) (30 5 1) simpleGrading (1 1 1)
46
               hex (3 4 7 6 13 14 17 16) (25 5 1) simpleGrading (1 1 1)
47
               hex (5 6 9 8 15 16 19 18) (30 95 1) simpleGrading (1 1 1)
48
49
50
        edges
51
        ();
52
53
54
55
        boundary
56
        (
               outerWall {
57
58
                        type wall;
59
                        faces
60
61
                               (0 1 11 10)
(1 3 13 11)
(3 4 14 13)
(7 6 16 17)
(6 9 19 16)
(9 8 18 19)
62
63
64
65
66
67
                       );
68
                }
69
                axis
70
71
                        type symmetryPlane;
                       faces
74
                                    10 12 2)
12 15 5)
15 18 8)
                               (0
(2
(5
75
                       );
79
                }
80
                nozzle
82
                        type patch;
                       faces
83
84
                                (471714)
85
                       );
86
                }
back
87
88
89
90
                       type empty;
                       faces
91
92
                               (0 2 3
(2 5 6
(3 6 7
                                            1)
3)
4)
93
94
95
```

```
(5896)
97
98
99
100
101
             type empty;
102
103
                 (10 11 13 12)
(12 13 16 15)
(13 14 17 16)
104
105
106
107
108
109
110
111
    mergePatchPairs
112
113
114
```

In order to improve the numerical accuracy, we shall use the reference level of 1 bar for the pressure field. Note that both the internal field level and the boundary conditions are offset by the reference level.

## 3.4.3 Preparing the Run

Before we commence the setup of the calculation, we need to consider the characteristic velocity of the phenomenon we are trying to capture. In the case under consideration, the fluid velocity will be very small, but the pressure wave will propagate with the speed of sound in water. The speed of sound is calculated as:

$$c = \sqrt{\frac{1}{\psi}} = \sqrt{\frac{1}{4.54 \times 10^{-7}}} = 1483.2 \text{m/s}.$$
 (3.17)

For the mesh described above, the characteristic mesh size is approximately 2 mm (note the scaling factor of 0.1 in the *blockMeshDict* file). Using

$$Co = \frac{U\,\Delta t}{\Delta x}\tag{3.18}$$

a reasonable time step is around  $\Delta t = 5 \times 10^{-7} \text{s}$ , giving the Co number of 0.35, based on the speed of sound. Also, note that the reported Co number by the code (associated with the convective velocity) will be two orders of magnitude smaller. As we are interested in the pressure wave propagation, we shall set the simulation time to 0.25 ms. For reference, the *controlDict* file is quoted below.

```
---*- C++ -*----
2
                    ield
                                     OpenFOAM: The Open Source CFD Toolbox
3
                                                2.2.1
                  O peration
                                     Version:
                  A nd
                                     Web:
                                                www.OpenFOAM.org
                  M anipulation
6
    FoamFile
10
         version
                      2.0;
                      ascii;
11
         format
                      dictionary;
         location
                      "system"
13
                      controlDict;
         object
15
16
17
                      sonicLiquidFoam;
    application
18
19
    startFrom
                      startTime:
20
```

```
startTime
                       0;
22
23
    stopAt
                       endTime;
24
25
    endTime
                       0.0001;
26
27
    deltaT
                       5e-07;
28
29
    writeControl
                       timeStep;
30
31
    writeInterval
                       20;
32
33
    purgeWrite
                       0;
34
35
                       ascii;
    writeFormat
36
37
    writePrecision
38
39
    writeCompression off;
40
41
    timeFormat
                       general;
42
43
    timePrecision
44
45
    runTimeModifiable true;
46
47
48
```

## 3.4.4 Running the case

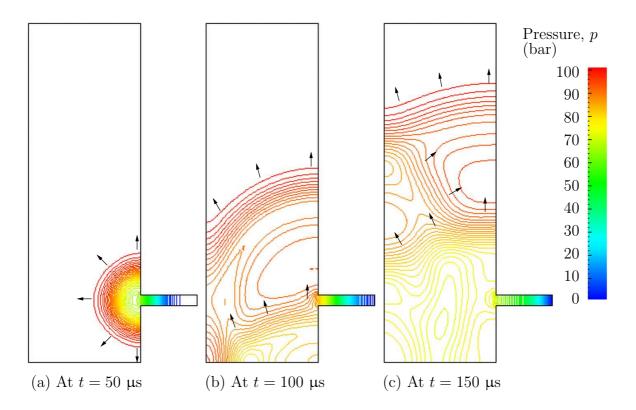


Figure 3.10: Propagation of pressure waves

The user can run the case and view results in dxFoam. The liquid flows out through the nozzle causing a wave to move along the nozzle. As it reaches the inlet to the tank, some of the wave is transmitted into the tank and some of it is reflected. While a wave is reflected up and down the inlet pipe, the waves transmitted into the tank expand and propagate through the tank. In Figure 3.10, the pressures are shown as contours so that the wave fronts are more clearly defined than if plotted as a normal isoline plot.

If the simulation is run for a long enough time for the reflected wave to return to the pipe, we can see that negative absolute pressure is detected. The modelling permits this and has some physical basis since liquids can support tension, *i.e.* negative pressures. In reality, however, impurities or dissolved gases in liquids act as sites for cavitation, or vapourisation/boiling, of the liquid due to the low pressure. Therefore in practical situations, we generally do not observe pressures falling below the vapourisation pressure of the liquid; not at least for longer than it takes for the cavitation process to occur.

## 3.4.5 Improving the solution by refining the mesh

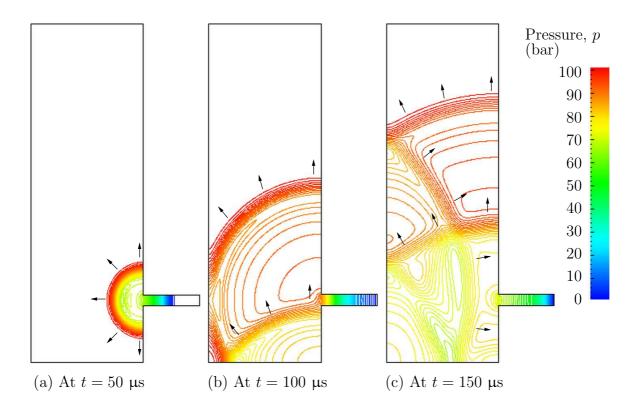


Figure 3.11: Propagation of pressure waves with refined mesh

Looking at the evolution of the resulting pressure field in time, we can clearly see the propagation of the pressure wave into the tank and numerous reflections from the inside walls. It is also obvious that the pressure wave is smeared over a number of cells. We shall now refine the mesh and reduce the time step to obtain a sharper front resolution. Simply edit the *blockMeshDict* and increase the number of cells by a factor of 4 in the x and y directions, *i.e.* block 0 becomes (120 80 1) from (30 20 1) and so on. Run blockMesh on this file. In addition, in order to maintain a Courant number below 1, the time step must be reduced accordingly to  $\Delta t = 10^{-7}$  s. The second simulation gives considerably better resolution of the pressure waves as shown in Figure 3.11.

## 3.5 Magnetohydrodynamic flow of a liquid

In this example we shall investigate an flow of an electrically-conducting liquid through a magnetic field. The problem is one belonging to the branch of fluid dynamics known as magnetohydrodynamics (MHD) that uses mhdFoam.

## 3.5.1 Problem specification

The problem is known as the Hartmann problem, chosen as it contains an analytical solution with which mhdFoam can be validated. It is defined as follows:

**Solution domain** The domain is 2 dimensional and consists of flow along two parallel plates as shown in Fig. 3.12.

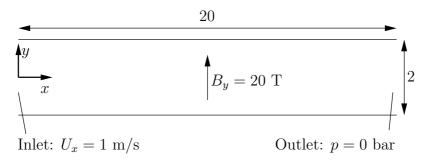


Figure 3.12: Geometry of the Hartmann problem

### Governing equations

• Mass continuity for incompressible fluid

$$\nabla \cdot \mathbf{U} = 0 \tag{3.19}$$

• Momentum equation for incompressible fluid

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot (\mathbf{U}\mathbf{U}) + \nabla \cdot (2\mathbf{B}\Gamma_{\mathbf{B}\mathbf{U}}\mathbf{B}) + \nabla \cdot (\nu \mathbf{U}) + \nabla (\Gamma_{\mathbf{B}\mathbf{U}}\mathbf{B} \cdot \mathbf{B}) = -\nabla p \ (3.20)$$

where **B** is the magnetic flux density,  $\Gamma_{\mathbf{B}\mathbf{U}} = (2\mu\rho)^{-1}$ .

• Maxwell's equations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{3.21}$$

where **E** is the electric field strength.

$$\nabla \cdot \mathbf{B} = 0 \tag{3.22}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J} \tag{3.23}$$

assuming  $\partial \mathbf{D}/\partial t \ll \mathbf{J}$ . Here, **H** is the magnetic field strength, **J** is the current density and **D** is the electric flux density.

• Charge continuity

$$\nabla \cdot \mathbf{J} = 0 \tag{3.24}$$

• Constitutive law

$$\mathbf{B} = \mu \mathbf{H} \tag{3.25}$$

• Ohm's law

$$\mathbf{J} = \sigma \left( \mathbf{E} + \mathbf{U} \times \mathbf{B} \right) \tag{3.26}$$

• Combining Equation 3.21, Equation 3.23, Equation 3.26, and taking the curl

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\mathbf{U}\mathbf{B}) - \nabla \cdot (\phi_{\mathbf{B}}\mathbf{U}) - \nabla \cdot (\Gamma_{\mathbf{B}}\mathbf{B}) = 0$$
(3.27)

## Boundary conditions

- inlet is specified the inlet condition with fixed velocity U = (1, 0, 0) m/s;
- outlet is specified as the outlet with with fixed pressure p = 0 Pa;
- upperWall is specified as a wall where  $\mathbf{B} = (0, 20, 0) \mathrm{T}$ .
- lowerWall is specified as a wall where  $\mathbf{B} = (0, 20, 0)$  T.
- front and back boundaries are specified as empty.

Initial conditions U = 0 m/s, p = 100 Pa, B = (0, 20, 0) T.

## Transport properties

- Kinematic viscosity  $\nu = 1 \text{ Pas}$
- Density  $\rho = 1 \text{ kg m/s}$
- Electrical conductivity  $\sigma = 1 \ (\Omega \, \text{m})^{-1}$
- Permeability  $\mu = 1 \text{ H/m}$

Solver name mhdFoam: an incompressible laminar magneto-hydrodynamics code.

Case name hartmann case located in the \$FOAM\_TUTORIALS/mhdFoam directory.

## 3.5.2 Mesh generation

The geometry is simply modelled with 100 cells in the x-direction and 40 cells in the y-direction; the set of vertices and blocks are given in the mesh description file below:

```
----*- C++ -*-----
1
2
                    ield
                                      OpenFOAM: The Open Source CFD Toolbox
3
                                                 2.2.1
                                      Version:
4
                  O peration
                   A nd
                                                 www.OpenFOAM.org
                                      Web:
5
                  {\tt M} anipulation
    FoamFile
9
                      2.0;
ascii;
         version
10
11
         format
                      dictionary;
blockMeshDict;
12
         class
13
14
15
16
    convertToMeters 1;
17
18
    vertices
19
20
         (0 -1 0)
         (20 -1 0)
22
          20 1 0)
24
            -1 0.1)
             -1 0.1)
         (20\ 1\ 0.1)
         (0\ 1\ 0.1)
28
    );
30
    blocks
        hex (0 1 2 3 4 5 6 7) (100 40 1) simpleGrading (1 1 1)
    );
34
```

```
edges
36
37
     ();
38
39
     boundary
40
           inlet
43
44
                 type patch;
45
46
                      (0 4 7 3)
48
49
           outlet
50
51
                type patch;
52
54
                      (2651)
55
56
57
           ĺowerWall
58
59
                type patch;
faces
60
61
62
                      (1540)
63
64
65
           upperWall
66
67
                type patch;
                      (3762)
           frontAndBack
                type empty;
                      (0 \ 3 \ 2 \ 1)
(4 \ 5 \ 6 \ 7)
80
81
           }
82
     );
84
     mergePatchPairs
86
87
```

## 3.5.3 Running the case

The user can run the case and view results in dxFoam. It is also useful at this stage to run the Ucomponents utility to convert the U vector field into individual scalar components. MHD flow is governed by, amongst other things, the Hartmann number which is a measure of the ratio of electromagnetic body force to viscous force

$$M = BL\sqrt{\frac{\sigma}{\rho\nu}} \tag{3.28}$$

where L is the characteristic length scale. In this case with  $B_y=20$  T, M=20 and the electromagnetic body forces dominate the viscous forces. Consequently with the flow fairly steady at t=2 s the velocity profile is almost planar, viewed at a cross section midway along the domain x=10 m. The user can plot a graph of the profile of  $U_x$  in dxFoam. Now the user should reduce the magnetic flux density  $\mathbf{B}$  to 1 Tand re-run the code and Ucomponents. In this case, M=1 and the electromagnetic body forces no longer dominate. The velocity profile consequently takes on the parabolic form, characteristic

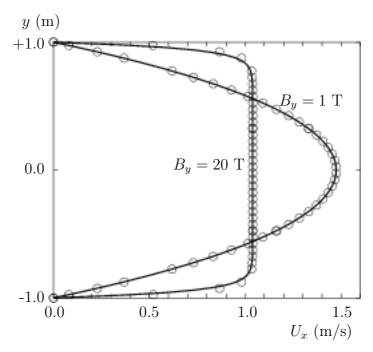


Figure 3.13: Velocity profile in the Hartmann problem for  $B_y=1~\mathrm{T}$  and  $B_y=20~\mathrm{T}$ .

of Poiseuille flow as shown in Figure 3.13. To validate the code the analytical solution for the velocity profile  $U_x$  is superimposed in Figure 3.13, given by:

$$\frac{U_x(y)}{U_x(0)} = \frac{\cosh M - \cosh M(y/L)}{\cosh M - 1} \tag{3.29}$$

where the characteristic length L is half the width of the domain, *i.e.* 1 m.

Index P-69

# Index

## Symbols Numbers A B C D E F G H I J K L M N O P Q R S T U V W X Z

Symbols	algorithms tools, $U-95$
*	alphaContactAngle
tensor member function, P-21	boundary condition, U-57
+	analytical solution, P-41
tensor member function, P-21	Animations window panel, U-168
-	anisotropicFilter model, U-99
tensor member function, P-21	Annotation window panel, U-24, U-167
/	ansysToFoam utility, U-89
tensor member function, P-21	APIfunctions model, U-98
/**/	applications, U-67
C++ syntax, U-76	Apply button, U-164, U-168
//	applyBoundaryLayer utility, U-88
C++ syntax, U-76	applyWallFunctionBoundaryConditions utility,
OpenFOAM file syntax, U-104	U-88
# include	arbitrarily unstructured, P-27
C++ syntax, U-70, U-76	arc
&	keyword entry, U-139
tensor member function, P-21	arc keyword, U-138
&&	As keyword, U-182
tensor member function, P-21	ascii
	keyword entry, U-112
tensor member function, P-21	attachMesh utility, U-89
<pre><lesmodel>Coeffs keyword, U-184</lesmodel></pre>	Auto Accept button, U-168
<rasmodel>Coeffs keyword, U-184</rasmodel>	autoMesh
<pre><delta>Coeffs keyword, U-184</delta></pre>	library, U-96
0.000000e+00 directory, U-104	autoPatch utility, U-89
1-dimensional mesh, U-130	autoRefineMesh utility, U-90
1D mesh, U-130	axes
2-dimensional mesh, U-130	right-handed, U-136
2D mesh, U-130	right-handed rectangular Cartesian, P-11,
Numbers	U-18
0 directory, U-104	axi-symmetric cases, U-135, U-144
	axi-symmetric mesh, U-130
$\mathbf{A}$	D
access functions, P-19	В
addLayersControls keyword, U-146	background
adiabaticFlameT utility, U-94	process, U-24, U-79
adjointShapeOptimizationFoam solver, U-83	backward
adjustableRunTime	keyword entry, U-119
keyword entry, U-60, U-111	Backward differencing, P-35
adjustTimeStep keyword, U-60, U-112	barotropicCompressibilityModels
agglomerator keyword II-123	library II_08

P-70 Index

$basic Multi Component Mixture \mod el, \qquad U-97,$	slip, $U$ - $137$
U-180	supersonicFreeStream, U-137
basicSolidThermo	surfaceNormalFixedValue, U-137
library, U-99	symmetryPlane, P-59, U-134
basic Thermophysical Models	totalPressure, U-137
library, U-97	turbulentInlet, $U$ -137
binary	wall, U-40
keyword entry, U-112	wall, $P-59$ , $P-65$ , $U-57$ , $U-134$
BirdCarreau model, U-100	wedge, U-130, U-135, U-144
blended differencing, P-34	zeroGradient, U-136
block	boundary conditions, P-39
expansion ratio, U-140	Dirichlet, P-39
block keyword, U-138	inlet, P-40
blocking	Neumann, P-39
keyword entry, U-78	no-slip impermeable wall, P-40
blockMesh	outlet, P-40
library, U-96	physical, P-40
blockMesh solver, P-43	symmetry plane, P-40
blockMesh utility, U-36, U-88, U-136	boundaryField keyword, U-21, U-108
blockMesh executable	boundaryFoam solver, U-84
vertex numbering, U-140	bounded
blockMeshDict	keyword entry, U-117, U-118
dictionary, U-18, U-20, U-35, U-48, U-136,	- · · · · · · · · · · · · · · · · · · ·
U-144	boxTurb utility, U-88
blocks keyword, U-20, U-30, U-140	breaking of a dam, U-55
boundaries, U-132	buoyantBoussinesqPimpleFoam solver, U-86
boundary, U-132	buoyantBoussinesqSimpleFoam solver, U-86
boundary	buoyantPimpleFoam solver, U-86
dictionary, U-129, U-136	buoyantPressure
boundary keyword, U-141	boundary condition, U-137
boundary condition	buoyantSimpleFoam solver, U-86
alphaContactAngle, U-57	button
buoyantPressure, U-137	Apply, U-164, U-168
calculated, U-136	Auto Accept, U-168
cyclic, U-135, U-142	Choose Preset, U-166
directionMixed, U-136	Delete, U-164
empty, P-59, P-65, U-18, U-130, U-135	Edit Color Map, U-165
fixedGradient, U-136	Enable Line Series, U-34
fixedValue, U-136	Orientation Axes, U-24, U-167
fluxCorrectedVelocity, U-137	Refresh Times, U-25
inlet, P-65 inletOutlet, U-137	Rescale to Data Range, U-25
mixed, U-136	Reset, U-164
movingWallVelocity, U-137	Set Ambient Color, U-166
outlet, P-65	Update GUI, U-165
outlet, U-137	Use Parallel Projection, U-24
partialSlip, U-137	Use parallel projection, U-167
patch, U-134	$\mathbf{C}$
pressureDirectedInletVelocity, U-137	C++ syntax
pressureInletVelocity, U-137	/**/, U-76
pressureOutlet, P-59	//, U-76
pressure Transmissive, U-137	# include, U-70, U-76
processor, U-135	cacheAgglomeration keyword, U-123
setup, U-20	calculated

Index P-71

boundary condition, U-136	pointField, P-27
cAlpha keyword, U-61	polyBoundaryMesh, P-27
cases, U-103	polyMesh, P-27, U-127, U-129
castellatedMesh keyword, U-146	polyPatchList, P-27
castellatedMeshControls	polyPatch, P-27
dictionary, U-147–U-149	scalarField, P-25
castellatedMeshControls keyword, U-146	scalar, P-19
cavitatingDyMFoam solver, U-85	slice, P-27
cavitatingFoam solver, U-85	symmTensorField, P-25
cavity flow, U-17	symmTensorThirdField, P-25
CELARCH	tensorField, P-25
environment variable, U-173	tensorThirdField, P-25
CEI_HOME	tensor, P-19
environment variable, U-173	vectorField, P-25
cell	vector, P-19, U-107
expansion ratio, U-140	word, P-21, P-27
cell class, P-27	class keyword, U-105
cell	clockTime
keyword entry, U-174	keyword entry, U-111
cellLimited	cloud keyword, U-175
keyword entry, U-117	cmptAv
cellPoint	-
	tensor member function, P-21
keyword entry, U-174 cellPointFace	Co utility, U-91
	coalChemistryFoam solver, U-86 coalCombustion
keyword entry, U-174	
cells	library, U-96
dictionary, U-136	cofactors
central differencing, P-34	tensor member function, P-21
cfdTools tools, U-95	coldEngineFoam solver, U-86
cfx4ToFoam utility, U-89, U-154	collapseEdges utility, U-90
changeDictionary utility, U-88	Color By menu, U-166
Charts window panel, U-168	Color Legend window, U-27
checkMesh utility, U-89, U-155	Color Legend window panel, U-166
chemFoam solver, U-86	Color Scale window panel, U-166
chemistryModel	Colors window panel, U-168
library, U-98	combinePatchFaces utility, U-90
chemistryModel model, U-98	comments, U-76
chemistrySolver model, U-98	commsType keyword, U-78
chemkinToFoam utility, U-94	compressed
Choose Preset button, U-166	keyword entry, U-112
chtMultiRegionFoam solver, U-86	compressibleInterDyMFoam solver, U-85
chtMultiRegionSimpleFoam solver, U-86	compressibleInterFoam solver, U-85
Chung	compressibleLESModels
library, U-98	library, U-100
class	compressibleRASModels
cell, P-27	library, U-99
dimensionSet, P-21, P-28, P-29	constant directory, U-104, U-179
face, P-27	constLaminarFlameSpeed model, U-98
finiteVolumeCalculus, P-29	constTransport model, U-98, U-180
finiteVolumeMethod, P-29	containers tools, U-95
fvMesh, P-27	continuum
fvSchemes, P-32	mechanics, P-11
fvc, P-32	control
fvm, P-32	of time, U-111

P-72 Index

controlDict	db tools, U-95
dictionary, P-61, U-21, U-30, U-41, U-50,	ddt
U-60, U-104, U-160	fvc member function, P-33
controlDict file, P-46	fvm member function, P-33
convection, see divergence, P-34	DeardorffDiffStress model, U-100
convergence, U-38	debug keyword, U-146
conversion	decomposePar utility, U-79, U-80, U-94
library, U-96	decomposeParDict
convertToMeters keyword, U-138	dictionary, U-79
coordinate	decomposition
system, P-11	of field, U-79
coordinate system, U-18	of mesh, U-79
corrected	decompositionMethods
keyword entry, U-117, U-118	library, U-96
Courant number, P-38, U-22	decompression of a tank, P-58
Cp keyword, U-181	defaultFieldValues keyword, U-58
cpuTime	deformedGeom utility, U-89
keyword entry, U-111	Delete button, U-164
Crank Nicholson	delta keyword, U-81, U-184
temporal discretisation, P-38	deltaT keyword, U-111
CrankNicholson	dependencies, U-70
keyword entry, U-119	dependency lists, U-70
createBaffles utility, U-89	det
createPatch utility, U-89	tensor member function, P-21
createTurbulenceFields utility, U-92	determinant, see tensor, determinant
cross product, see tensor, vector cross product	dev
CrossPowerLaw	tensor member function, P-21
keyword entry, U-59	diag
CrossPowerLaw model, U-100	tensor member function, P-21
cubeRootVolDelta model, U-99	diagonal
cubicCorrected	keyword entry, U-121, U-122
keyword entry, U-119	DIC
cubicCorrection	keyword entry, U-122
keyword entry, U-116	DICGaussSeidel
curl, P-33	keyword entry, U-122
curl	dictionary
fvc member function, P-33	LESProperties, U-184
Current Time Controls menu, U-25, U-165	<i>PISO</i> , U-23
curve keyword, U-175	blockMeshDict, U-18, U-20, U-35, U-48,
Cv keyword, U-182	U-136, U-144
cyclic	boundary, U-129, U-136
boundary condition, U-135, U-142	castellated Mesh Controls, U-147-U-149
cyclic	cells, U-136
keyword entry, U-135	controlDict, P-61, U-21, U-30, U-41, U-50,
cylinder	U-60, U-104, U-160
flow around a, P-41	decomposeParDict, U-79
	faces, U-129, U-136
D	fvSchemes, U-60, U-61, U-104, U-113
d2dt2	fvSolution, U-104, U-120
fvc member function, P-33	mechanicalProperties, U-49
fvm member function, P-33	neighbour, U-129
dam	owner, U-129
breaking of a, U-55	points, U-129, U-136
datToFoam utility, U-89	thermalProperties, U-50

thermophysicalProperties, U-179	dnsFoam solver, U-86
transportProperties, U-21, U-38, U-41	doLayers keyword, U-146
turbulenceProperties, U-40, U-59, U-184	double inner product, see tensor, double inner
dieselMixture model, U-97, U-180	product
dieselSpray	dsmc
library, U-96	library, U-96
differencing	dsmcFieldsCalc utility, U-92
Backward, P-35	dsmcFoam solver, U-87
blended, P-34	dsmcInitialise utility, U-88
central, P-34	dx
Euler implicit, P-35	keyword entry, U-174
Gamma, P-34	dynamicFvMesh
MINMOD, P-34	library, U-96
SUPERBEE, P-34	dynamicMesh
upwind, P-34	library, U-96
van Leer, P-34	dynLagrangian model, U-100
DILU	dynMixedSmagorinsky model, U-100
keyword entry, U-122	dynOneEqEddy model, U-100
dimension	
checking in OpenFOAM, P-21, U-107	dynSmagorinsky model, U-100
	${f E}$
dimensional units, U-107	eConstThermo model, U-98, U-179
dimensioned <type> template class, P-21</type>	edgeGrading keyword, U-140
dimensionedTypes tools, U-95	edgeMesh
dimensions keyword, U-20, U-108	•
dimensionSet class, P-21, P-28, P-29	library, U-96
dimensionSet tools, U-95	edges keyword, U-138
directionMixed	Edit menu, U-167, U-168
boundary condition, U-136	Edit Color Map button, U-165
directory	egrMixture model, U-97, U-180
0.000000e+00,  U-104	electrostaticFoam solver, U-87
0, U-104	empty
Make, U-71	boundary condition, P-59, P-65, U-18,
constant, $U-104$ , $U-179$	U-130, U-135
fluentInterface, U-170	empty
polyMesh, U-104, U-129	keyword entry, U-135
processorN, U-80	Enable Line Series button, U-34
run, U-103	endTime keyword, U-22, U-111
system, P-46, U-104	energy keyword, U-181
tutorials, P-41, U-17	engine
discretisation	library, U-96
equation, P-29	engineCompRatio utility, U-92
Display window panel, U-23, U-25, U-164,	engineFoam solver, U-86
U-165	engineSwirl utility, U-88
distance	ensight74FoamExec utility, U-172
keyword entry, U-149, U-175	ENSIGHT7_INPUT
distributed model, U-96	environment variable, U-173
distributed keyword, U-81, U-82	ENSIGHT7_READER
distribution Models	environment variable, U-173
library, U-96	ensightFoamReader utility, U-91
div	enstrophy utility, U-91
fvc member function, P-33	environment variable
fvm member function, P-33	CELARCH, U-173
divergence, P-33, P-35	CEL-HOME, U-173
divSchemes keyword, U-114	ENSIGHT7_INPUT, U-173
GIVE TO THE MEDIAN WOLL, WILLIAM	

P-74 Index

ENSIGHT7_READER, U-173	FDIC
FOAM_RUN, U-103	keyword entry, U-122
WM_ARCH_OPTION, U-74	featureAngle keyword, U-152
WM_ARCH, U-74	features keyword, U-147, U-148
WM_COMPILER_BIN, U-74	field
WM_COMPILER_DIR, U-74	U, U-22
WM_COMPILER_LIB, U-74	p, U-22
WM_COMPILER, U-74	decomposition, U-79
WM_COMPILE_OPTION, U-74	FieldField <type> template class, P-28</type>
WM_DIR, U-74	fieldFunctionObjects
WM_MPLIB, U-74	library, U-95
WM_OPTIONS, U-74	fields, P-25
WM_PRECISION_OPTION, U-74	mapping, U-160
WM_PROJECT_DIR, U-74	fields tools, U-95
WM_PROJECT_INST_DIR, U-74	fields keyword, U-174
•	Field <type> template class, P-25</type>
WM_PROJECT_USER_DIR, U-74	fieldValues keyword, U-58
WM_PROJECT_VERSION, U-74	file
WM_PROJECT, U-74	
wmake, U-73	Make/files, U-72
ePsiThermo model, U-97, U-180	controlDict, P-46
equationOfState keyword, U-181	files, U-71
equilibriumCO utility, U-94	g, U-59
equilibriumFlameT utility, U-94	options, U-71
errorReduction keyword, U-153	snappyHexMeshDict, U-145
Euler	transportProperties, U-59
keyword entry, U-119	file format, U-104
Euler implicit	fileFormats
differencing, P-35	library, U-96
temporal discretisation, P-38	fileModificationChecking keyword, U-78
examples	fileModificationSkew keyword, U-78
decompression of a tank, P-58	files file, U-71
flow around a cylinder, P-41	filteredLinear2
flow over backward step, P-49	keyword entry, U-116
Hartmann problem, P-63	finalLayerThickness keyword, U-152
supersonic flow over forward step, P-54	financialFoam solver, U-88
execFlowFunctionObjects utility, U-92	finite volume
expandDictionary utility, U-94	discretisation, P-23
expansionRatio keyword, U-152	$\operatorname{mesh}, P-27$
explicit	finiteVolume
temporal discretisation, P-38	library, U-95
extrude2DMesh utility, U-88	finiteVolume tools, U-95
extrudeMesh utility, U-88	finiteVolumeCalculus class, P-29
extrudeToRegionMesh utility, U-89	finiteVolumeMethod class, P-29
<b>3</b>	fireFoam solver, U-86
${f F}$	firstTime keyword, U-111
face class, P-27	fixed
face keyword, U-175	keyword entry, U-112
faceAgglomerate utility, U-88	fixedGradient
faceAreaPair	boundary condition, U-136
keyword entry, U-123	fixedValue
faceLimited	boundary condition, U-136
keyword entry, U-117	flattenMesh utility, U-89
faces	floatTransfer keyword, U-78
dictionary, U-129, U-136	flow

free surface, U-55	ddt, P-33
laminar, U-17	div, P-33
steady, turbulent, P-49	gGrad, P-33
supersonic, P-55	grad, P-33
turbulent, U-17	laplacian, P-33
flow around a cylinder, P-41	lsGrad, P-33
flow over backward step, P-49	snGrad, P-33
flowType utility, U-91	snGradCorrection, P-33
fluent3DMeshToFoam utility, U-89	sgrGradGrad, P-33
fluentInterface directory, U-170	fvDOM
fluentMeshToFoam utility, U-89, U-154	library, U-98
fluxCorrectedVelocity	fvm class, P-32
boundary condition, U-137	fvm member function
fluxRequired keyword, U-114	d2dt2, P-33
OpenFOAM	ddt, P-33
cases, U-103	div, P-33
FOAM_RUN	laplacian, P-33
environment variable, U-103	Su, P-33
foamCalc utility, U-32, U-92	SuSp, P-33
foamCalcFunctions	fvMatrices tools, U-95
library, U-95	fvMatrix template class, P-29
foamCorrectVrt script/alias, U-158	fvMesh class, P-27
foamDataToFluent utility, U-91, U-170	fvMesh tools, U-95
foamDebugSwitches utility, U-94	fvMotionSolvers
FoamFile keyword, U-105	library, U-96
foamFile	fvSchemes
keyword entry, U-174	dictionary, U-60, U-61, U-104, U-113
foamFormatConvert utility, U-94	fvSchemes class, P-32
foamInfoExec utility, U-94	fvSchemes
foamJob script/alias, U-177	menu entry, U-51
foamListTimes utility, U-92	fvSolution
foamLog script/alias, U-177	dictionary, U-104, U-120
foamMeshToFluent utility, U-89, U-170	
foamToEnsight utility, U-91	G
foamToEnsightParts utility, U-91	g file, U-59
foamToGMV utility, U-91	gambitToFoam utility, U-89, U-154
foamToStarMesh utility, U-89	GAMG
foamToSurface utility, U-89	keyword entry, U-52, U-121, U-122
foamToTecplot360 utility, U-91	Gamma
foamToVTK utility, U-91	keyword entry, U-116
foamUpgradeCyclics utility, U-88	Gamma differencing, P-34
foamUpgradeFvSolution utility, U-88	Gauss
forces	keyword entry, U-117
library, U-95	Gauss's theorem, P-32
foreground	GaussSeidel
process, U-24	keyword entry, U-122
format keyword, U-105	General window panel, U-167, U-168
fourth	general
keyword entry, U-117, U-118	keyword entry, U-112
functions keyword, U-113	genericFvPatchField
fvc class, P-32	library, U-96
fvc member function	geometric-algebraic multi-grid, U-122
curl, P-33	GeometricBoundaryField template class, P-28
d2dt2, P-33	geometricField <type> template class, P-28</type>

P-76 Index

geometry keyword, U-146	identity, see tensor, identity
gGrad	incompressibleLESModels
fvc member function, P-33	library, U-100
global tools, U-95	incompressibleRASModels
gmshToFoam utility, U-89	library, U-99
gnuplot	incompressibleTransportModels
keyword entry, U-112, U-174	library, P-50, U-100
grad	incompressibleTurbulenceModels
fvc member function, P-33	library, P-50
(Grad Grad) squared, P-33	index
gradient, P-33, P-36	notation, P-12, P-13
Gauss scheme, P-36	Information window panel, U-164
Gauss's theorem, U-51	inhomogeneousMixture model, U-97, U-180
least square fit, U-51	inlet
least squares method, P-36, U-51	boundary condition, P-65
surface normal, P-36	inletOutlet
gradSchemes keyword, U-114	boundary condition, U-137
graph tools, U-95	inner product, see tensor, inner product
graphFormat keyword, U-112	inotify
GuldersEGRLaminarFlameSpeed model, U-98	keyword entry, U-78
GuldersLaminarFlameSpeed model, U-98	inotifyMaster
	keyword entry, U-78
Н	inside
hConstThermo model, U-98, U-179	keyword entry, U-149
Help menu, U-167	insideCells utility, U-89
HerschelBulkley model, U-100	interDyMFoam solver, U-85
Hf keyword, U-181, U-182	interfaceProperties
hhuMixtureThermo model, U-97, U-180	library, U-100
hierarchical	interfaceProperties model, U-100
keyword entry, U-80, U-81	interFoam solver, U-85
highCpCoeffs keyword, U-182	interMixingFoam solver, U-85
homogenousDynSmagorinsky model, U-100	internalField keyword, U-21, U-108
homogeneousMixture model, U-97, U-180	interPhaseChangeFoam $solver, U-85$
hPolynomialThermo model, U-98, U-179	interpolation tools, U-95
hPsiMixtureThermo model, U-97, U-180	interpolationScheme keyword, U-174
hPsiThermo model, U-97, U-180	interpolations tools, U-95
hRhoMixtureThermo model, U-97, U-180	interpolationSchemes keyword, U-114
hRhoThermo model, U-97, U-180	inv
hsPsiMixtureThermo model, U-97, U-180	tensor member function, P-21
hsPsiThermo model, U-97, U-180	iterations
hsRhoMixtureThermo model, U-97, U-180	maximum, U-121
hsRhoThermo model, U-97, U-180	т.
I	J
	janafThermo model, U-98, U-179
I tensor member function, P-21	jobControl
•	library, U-95
icoFoam solver, U-17, U-21, U-22, U-24, U-84	jplot
icoPolynomial model, U-98, U-179	keyword entry, U-112, U-174
$\begin{tabular}{ll} icoUncoupledKinematicParcelDyMFoam & solver, \\ U-86 & \\ \end{tabular}$	K
icoUncoupledKinematicParcelFoam solver, U-87	kEpsilon model, U-99
ideasToFoam utility, U-154	keyword
ideasUnvToFoam utility, U-89	As, U-182
identities, see tensor, identities	Cp, U-181

a II 100	6 L II 105
Cv, U-182	format, U-105
FoamFile, U-105	functions, U-113
Hf, U-181, U-182	geometry, U-146
LESModel, U-184	gradSchemes, U-114
Pr, U-182	graphFormat, U-112
RASModel, U-184	highCpCoeffs, U-182
Tcommon, U-182	internalField, U-21, U-108
Thigh, U-182	interpolationSchemes, U-114
Tlow, U-182	interpolationScheme, U-174
Ts, U-182	laplacianSchemes, U-114
addLayersControls, U-146	latestTime, U-38
adjustTimeStep, U-60, U-112	layers, U-152
agglomerator, U-123	leastSquares, U-51
arc, U-138	levels, U-150
blocks, U-20, U-30, U-140	libs, U-78, U-112
block, U-138	locationInMesh, U-148, U-149
boundaryField, U-21, U-108	location, U-105
boundary, U-141	lowCpCoeffs, U-182
boxToCell, U-58	manualCoeffs, U-81
cAlpha, U-61	maxAlphaCo, U-60
cacheAgglomeration, U-123	maxBoundarySkewness, U-153
castellatedMeshControls, U-146	maxConcave, U-153
castellatedMesh, U-146	maxCo, U-60, U-112
class, U-105	maxDeltaT, U-60
cloud, U-175	maxFaceThicknessRatio, U-152
commsType, U-78	maxGlobalCells, U-148
convertToMeters, U-138	maxInternalSkewness, U-153
curve, U-175	maxIter, U-121
debug, U-146	maxLocalCells, U-148
defaultFieldValues, U-58	maxNonOrtho, U-153
deltaT, U-111	maxThicknessToMedialRatio, U-152
delta, U-81, U-184	mergeLevels, U-123
dimensions, U-20, U-108	mergePatchPairs, U-138
distributed, U-81, U-82	mergeTolerance, U-146
divSchemes, U-114	meshQualityControls, U-146
doLayers, U-146	method, U-81
edgeGrading, U-140	midPointAndFace, U-175
edges, U-138	midPoint, U-175
endTime, U-22, U-111	minArea, U-153
energy, U-181	minDeterminant, U-153
equationOfState, U-181	minFaceWeight, U-153
errorReduction, U-153	minFlatness, U-153
expansionRatio, U-152	minMedianAxisAngle, U-152
face, U-175	minRefinementCells, U-148
featureAngle, U-152	minThickness, U-152
features, U-147, U-148	minTriangleTwist, U-153
fieldValues, U-58	minTwist, U-153
fields, U-174	minVolRatio, U-153
fileModificationChecking, U-78	minVol, U-153
fileModificationSkew, U-78	mode, U-149
finalLayerThickness, U-152	molWeight, U-181
firstTime, U-111	mu, U-182
floatTransfer, U-78	nAlphaSubCycles, U-61
fluxRequired, U-114	nBufferCellsNoExtrude, U-152

P-78 Index

G 13 D	1 II FO II 101
nCellsBetweenLevels, U-148	solver, U-52, U-121
nFaces, U-130	specie, U-181
nFinestSweeps, U-123	spline, U-138
nGrow, U-152	startFace, U-130
nLayerIter, U-152	startFrom, U-22, U-111
nMoles, U-181	startTime, U-22, U-111
nPostSweeps, U-123	stopAt, U-111
nPreSweeps, U-123	strategy, U-80, U-81
nRelaxIter, U-150, U-152	surfaceFormat, U-174
nRelaxedIter, U-152	surfaces, U-174
nSmoothNormals, U-152	thermoType, U-179
nSmoothPatch, U-150	thermodynamics, U-181
nSmoothScale, U-153	timeFormat, U-112
nSmoothSurfaceNormals, U-152	timePrecision, U-112
nSmoothThickness, U-152 nSolveIter, U-150	timeScheme, U-114 tolerance, U-52, U-121, U-150
neighbourPatch, U-142	topoSetSource, U-58
numberOfSubdomains, U-81	traction, U-49
,	transport, U-181
n, U-81 object, U-105	turbulence, U-184
order, U-81	type, U-132, U-133
pRefCell, U-23, U-125	uniform, U-175
pRefValue, U-23, U-125	valueFraction, U-136
p_rhgRefCell, U-125	value, U-21, U-136
p_rhgRefValue, U-125	version, U-105
patchMap, U-160	vertices, U-20, U-138, U-139
patches, U-138	writeCompression, U-112
preconditioner, U-121, U-122	writeControl, U-22, U-60, U-111
pressure, U-49	writeFormat, U-54, U-112
printCoeffs, U-41, U-184	writeInterval, U-22, U-31, U-111
processorWeights, U-80	writePrecision, U-112
processorWeights, U-81	<pre><lesmodel>Coeffs, U-184</lesmodel></pre>
purgeWrite, U-112	<pre><rasmodel>Coeffs, U-184</rasmodel></pre>
refGradient, U-136	<delta>Coeffs, U-184</delta>
refinementRegions, U-148, U-150	keyword entry
refinementSurfaces, U-148	CrankNicholson, U-119
refinementRegions, U-149	CrossPowerLaw, U-59
regions, U-58	DICGaussSeidel, U-122
relTol, U-52, U-121	DIC, U-122
${ t relative Sizes, U-152}$	DILU, $U-122$
$\mathtt{relaxed}, \mathrm{U}\text{-}153$	Euler, U- $119$
${ t resolve Feature Angle,\ U-148}$	FDIC, U-122
$\mathtt{roots},\ \mathrm{U}\text{-}81,\ \mathrm{U}\text{-}82$	GAMG, $U-52$ , $U-121$ , $U-122$
$\verb"runTimeModifiable", U-112"$	${\tt Gamma},\ U\text{-}116$
$\mathtt{scotchCoeffs}, \mathrm{U}\text{-}81$	${\tt GaussSeidel}, \hbox{U-}122$
$\mathtt{setFormat},\ U\text{-}174$	${\tt Gauss}, {\tt U\text{-}}117$
sets, U-174	LESModel, $U-40$ , $U-184$
$ ext{simpleGrading},   ext{U-}140$	$ t MGridGen, \ U-123$
$\mathtt{simulationType},\ U\text{-}40,\ U\text{-}59,\ U\text{-}184$	MUSCL, U-116
smoother, U-123	Newtonian, U-59
snGradSchemes, U-114	PBiCG, U-121
snapControls, U-146	PCG, U-121
snap, U-146	QUICK, U-119
solvers, U-120	RASModel, U-40, U-184

SFCD, U-116, U-119	null, U-174
UMIST, U-115	outside, $ ext{U-}149$
adjustableRunTime, $U$ - $60$ , $U$ - $111$	$\mathtt{patch},\ \mathrm{U\text{-}}135,\ \mathrm{U\text{-}}176$
arc, U-139	polyLine, $U-139$
ascii, $U$ -112	polySpline, $U-139$
backward, U-119	${\tt processor},  {\tt U\text{-}}135$
binary, $U-112$	${ t raw},\ { t U-112},\ { t U-174}$
blocking, U-78	$\mathtt{runTime},\ U\text{-}31,\ U\text{-}111$
bounded, U-117, U-118	scheduled, U-78
$\mathtt{cellLimited}, \textcolor{red}{\text{U-}117}$	scientific, $U-112$
$\mathtt{cellPointFace},\ \mathrm{U}\text{-}174$	$\mathtt{scotch},\ U\text{-}80,\ U\text{-}81$
cellPoint, U-174	$\mathtt{simpleSpline}, U\text{-}139$
cell, U-174	$\mathtt{simple},\ U\text{-}80,\ U\text{-}81$
clockTime, U-111	$\mathtt{skewLinear},  U\text{-}116,  U\text{-}119$
${\tt compressed},  \hbox{U-}112$	${ t smoothSolver},\ U ext{-}121$
corrected, U-117, U-118	$\mathtt{startTime},\ \mathtt{U-22},\ \mathtt{U-111}$
$\mathtt{cpuTime},\ U\text{-}111$	${ t steadyState}, { t U-119}$
cubicCorrected, U-119	$stl, \mathrm{U}\text{-}174$
cubicCorrection, $U$ -116	$\mathtt{symmetryPlane},  \text{U-}135$
cyclic, $U-135$	timeStampMaster, U-78
$\mathtt{diagonal},\ U\text{-}121,\ U\text{-}122$	timeStamp, U-78
distance, U-149, U-175	timeStep, U-22, U-31, U-111
dx, U-174	uncompressed, U-112
empty, $U-135$	uncorrected, U-117, U-118
${ t faceAreaPair,U-123}$	upwind, U-116, U-119
${\tt faceLimited}, U\text{-}117$	vanLeer, U-116
filteredLinear2, U-116	vtk, U-174
$\mathtt{fixed}, U\text{-}112$	wall, U-135
foamFile, U-174	wedge, U-135
fourth, U-117, U-118	writeControl, U-111
general, U-112	writeNow, U-111
gnuplot, U-112, U-174	xmgr, U-112, U-174
hierarchical, U-80, U-81	xyz, U-175
inotifyMaster, U-78	x, U-175
${\tt inotify}, {\tt U-78}$	y, U-175
inside, $U$ -149	z, U-175
jplot, U-112, U-174	kivaToFoam utility, U-89
laminar, U-40, U-184	kOmega model, U-99
$\mathtt{latestTime},\ U\text{-}111$	kOmegaSST model, U-99
leastSquares, U-117	kOmegaSSTSAS model, U-100
${\tt limitedCubic}, {\tt U-116}$	Kronecker delta, P-16
limitedLinear, U-116	,
limited, U-117, U-118	${f L}$
linearUpwind, U-116, U-119	lagrangian
linear, U-116, U-119	library, U-96
line, U-139	lagrangianIntermediate
${ t localEuler, U-119}$	library, U-96
manual, U-80, U-81	Lambda2 utility, U-91
metis, U-81	LamBremhorstKE $model, U-99$
midPoint, U-116	laminar model, U-99
nextWrite, U-111	laminar
noWriteNow, U-111	keyword entry, U-40, U-184
nonBlocking, U-78	laminarFlameSpeedModels
none, U-115, U-122	library, U-98
	- /

P-80 Index

lenle collitor model II 00	domo II 06
laplaceFilter model, U-99	dsmc, U-96
Laplacian, P-34	dynamicFvMesh, U-96
laplacian, P-33	dynamicMesh, U-96
laplacian	edgeMesh, U-96
fvc member function, P-33	engine, U-96
fvm member function, P-33	fieldFunctionObjects, U-95
laplacianFoam solver, U-83	fileFormats, U-96
laplacianSchemes keyword, U-114	finiteVolume, U-95
latestTime	foamCalcFunctions, U-95
keyword entry, U-111	forces, U-95
latestTime keyword, U-38	fvDOM, U-98
LaunderGibsonRSTM model, U-99	fvMotionSolvers, U-96
LaunderSharmaKE model, U-99	genericFvPatchField, U-96
layers keyword, U-152	incompressibleLESModels, U-100
leastSquares	incompressibleRASModels, U-99
keyword entry, U-117	incompressibleTransportModels, P-50, U-100
leastSquares keyword, U-51	incompressibleTurbulenceModels, P-50
LESdeltas	interfaceProperties, U-100
library, U-99	jobControl, U-95
LESfilters	lagrangianIntermediate, U-96
library, U-99	lagrangian, U-96
LESModel	laminarFlameSpeedModels, U-98
keyword entry, U-40, U-184	linear, U-98
LESModel keyword, U-184	liquidMixtureProperties, U-99
LESProperties	liquidProperties, U-98
dictionary, U-184	meshTools, U-96
levels keyword, U-150	molecularMeasurements, U-96
libraries, U-67	molecule, U-96
library	pairPatchAgglomeration, U-96
Chung, U-98	postCalc, U-95
LESdeltas, U-99	potential, U-96
LESfilters, U-99	primitive, P-19
${\sf MGridGenGAMGAgglomeration},\ U\text{-}96$	radiationModels, U-98
ODE, U-96	randomProcesses, U-96
OSspecific, U-96	reaction Thermophysical Models, $U-97$
OpenFOAM, $U$ - $95$	sampling, $U$ - $95$
P1, U-98	solidMixtureProperties, U-99
PV3FoamReader, U-163	solidParticle, U-96
PVFoamReader, U-163	solidProperties, U-99
SLGThermo, U-99	solid, U-99
Wallis, U-98	specie, U-98
autoMesh, U-96	surfMesh, U-96
barotropic Compressibility Models, $U-98$	surfaceFilmModels, U-101
basicSolidThermo, $U$ -99	systemCall, U-96
basicThermophysicalModels, U-97	thermalPorousZone, U-99
blockMesh, U-96	thermophysicalFunctions, U-98
chemistryModel, U-98	thermophysical, U-179
coalCombustion, U-96	topoChangerFvMesh, U-96
compressibleLESModels, U-100	triSurface, U-96
compressible RAS Models, U-99	twoPhaseInterfaceProperties, U-100
conversion, U-96	utilityFunctionObjects, U-96
decompositionMethods, U-96	viewFactor, U-98
dieselSpray, U-96	vtkFoam, U-163
distributionModels, U-96	vtkPV3Foam, U-163

libs keyword, U-78, U-112	manual
lid-driven cavity flow, U-17	keyword entry, U-80, U-81
LienCubicKE model, U-99	manualCoeffs keyword, U-81
LienCubicKELowRe model, U-99	mapFields utility, U-30, U-37, U-41, U-54, U-88,
LienLeschzinerLowRe model, U-99	U-160
Lights window panel, U-167	mapping
limited	fields, U-160
keyword entry, U-117, U-118	Marker Style menu, U-34
limitedCubic	matrices tools, U-95
keyword entry, U-116	max
limitedLinear	tensor member function, P-21
keyword entry, U-116	maxAlphaCo keyword, U-60
line	maxBoundarySkewness keyword, U-153
keyword entry, U-139	maxCo keyword, U-60, U-112
Line Style menu, U-34	maxConcave keyword, U-153
linear	maxDeltaT keyword, U-60
library, U-98	maxDeltaxyz model, U-100
linear	maxFaceThicknessRatio keyword, U-152
keyword entry, U-116, U-119	maxGlobalCells keyword, U-148
linearUpwind	maximum iterations, U-121
keyword entry, U-116, U-119	maxInternalSkewness keyword, U-153
liquid	maxIter keyword, U-121
electrically-conducting, P-63	maxLocalCells keyword, U-148
liquidMixtureProperties	maxNonOrtho keyword, U-153
library, U-99	maxThicknessToMedialRatio keyword, U-152
liquidProperties	mdEquilibrationFoam solver, U-87
library, U-98	mdFoam solver, U-87
lists, P-25	mdInitialise utility, U-88
List <type> template class, P-25</type>	mechanicalProperties
localEuler	dictionary, U-49
keyword entry, U-119	memory tools, U-95
location keyword, U-105	menu
locationInMesh keyword, U-148, U-149	Color By, U-166
locDynOneEqEddy model, U-100	Current Time Controls, U-25, U-165
lowCpCoeffs keyword, U-182	Edit, U-167, U-168
lowReOneEqEddy model, U-100	Help, $U-167$
LRDDiffStress model, U-100	Line Style, U-34
LRR model, U-99	Marker Style, U-34
lsGrad	VCR Controls, U-25, U-165
fvc member function, P-33	View, $U-167$
LTSInterFoam solver, U-85	menu entry
LTSReactingParcelFoam solver, U-87	Plot Over Line, $U-33$
3	Save Animation, U-169
${f M}$	Save Screenshot, U-169
Mach utility, U-91	Settings, U-168
mag	Show Color Legend, U-25
tensor member function, P-21	Solid Color, U-166
magneticFoam solver, U-87	Toolbars, U-167
magnetohydrodynamics, P-63	View Settings, $U$ -24
magSqr	View Settings, U-24, U-167
tensor member function, P-21	Wireframe, U-166
Make directory, U-71	fvSchemes, U-51
make script/alias, U-69	mergeLevels keyword, U-123
Make/files file, U-72	mergeMeshes utility, U-90

P-82

${\sf mergeOrSplitBaffles\ utility,\ U-90}$	minTriangleTwist keyword, U-153
mergePatchPairs keyword, U-138	minTwist keyword, U-153
mergeTolerance keyword, U-146	minVol keyword, U-153
mesh	minVolRatio keyword, U-153
1-dimensional, U-130	mirrorMesh utility, U-90
1D, U-130	mixed
2-dimensional, U-130	boundary condition, U-136
2D, U-130	mixedSmagorinsky model, U-100
axi-symmetric, U-130	mixtureAdiabaticFlameT utility, U-94
basic, P-27	mode keyword, U-149
block structured, U-136	model
decomposition, U-79	APIfunctions, U-98
description, U-127	BirdCarreau, U-100
finite volume, P-27	CrossPowerLaw, $U-100$
generation, U-136, U-145	DeardorffDiffStress, U-100
grading, U-136, U-140	Gulders EGR Laminar Flame Speed, $U-98$
grading, example of, P-49	GuldersLaminarFlameSpeed, U-98
non-orthogonal, P-41	HerschelBulkley, U-100
refinement, P-58	LRDDiffStress, U-100
resolution, U-30	LRR, U-99
specification, U-127	LamBremhorstKE, U-99
split-hex, U-145	LaunderGibsonRSTM, U-99
Stereolithography (STL), U-145	LaunderSharmaKE, U-99
surface, U-145	LienCubicKELowRe, U-99
validity constraints, U-127	LienCubicKE, U-99
Mesh Parts window panel, U-23	LienLeschzinerLowRe, U-99
meshes tools, U-95	NSRDSfunctions, U-98
meshQualityControls keyword, U-146	Newtonian, U-100
meshTools	NonlinearKEShih, U-99
library, U-96	PrandtlDelta, U-99
message passing interface	RNGkEpsilon, U-99
openMPI, U-81	Smagorinsky2, U-100
method keyword, U-81	Smagorinsky, U-100
metis	SpalartAllmarasDDES, U-100
keyword entry, U-81	SpalartAllmarasIDDES, U-100
MGridGenGAMGAgglomeration	SpalartAllmaras, U-99, U-100
library, U-96	anisotropicFilter, U-99
MGridGen	basicMultiComponentMixture, U-97, U-180
keyword entry, U-123	chemistryModel, U-98
mhdFoam solver, P-65, U-87	chemistrySolver, U-98
midPoint	constLaminarFlameSpeed, U-98
keyword entry, U-116	constTransport, U-98, U-180
midPoint keyword, U-175	cubeRootVolDelta, U-99
midPointAndFace keyword, U-175	dieselMixture, U-97, U-180
min	distributed, U-96
tensor member function, P-21	dynLagrangian, U-100
minArea keyword, U-153	dyn $MixedSmagorinsky, U-100$
minDeterminant keyword, U-153	dynOneEqEddy, $U$ - $100$
minFaceWeight keyword, U-153	dynSmagorinsky, U-100
minFlatness keyword, U-153	eConstThermo, U-98, U-179
minMedianAxisAngle keyword, U-152	ePsiThermo, U-97, U-180
MINMOD differencing, P-34	egrMixture, U-97, U-180
minRefinementCells keyword, U-148	hConstThermo, U-98, U-179
minThickness keyword, U-152	hPolynomialThermo, U-98, U-179

hPsiMixtureThermo, U-97, U-180	boundary condition, U-137
hPsiThermo, U-97, U-180	MPI
hRhoMixtureThermo, U-97, U-180	openMPI, U-81
hRhoThermo, U-97, U-180	MRFInterFoam solver, U-85
hhuMixtureThermo, U-97, U-180	MRFMultiphaseInterFoam solver, U-85
homogenousDynSmagorinsky, U-100	mshToFoam utility, U-89
homogeneousMixture, U-97, U-180	mu keyword, U-182
hsPsiMixtureThermo, U-97, U-180	multiComponentMixture model, U-97, U-180
hsPsiThermo, U-97, U-180	multigrid
hsRhoMixtureThermo, U-97, U-180	geometric-algebraic, U-122
hsRhoThermo, U-97, U-180	multiphaseEulerFoam solver, U-85
icoPolynomial, U-98, U-179	multiphaseInterFoam solver, U-85
inhomogeneousMixture, U-97, U-180	MUSCL
interfaceProperties, U-100	keyword entry, U-116
janafThermo, U-98, U-179	key word energy, C-110
kEpsilon, U-99	${f N}$
kOmegaSSTSAS, U-100	n keyword, U-81
kOmegaSST, U-99	nabla
kOmega, U-99	operator, P-23
laminar, U-99	nAlphaSubCycles keyword, U-61
laplaceFilter, U-99	nBufferCellsNoExtrude keyword, U-152
locDynOneEqEddy, U-100	nCellsBetweenLevels keyword, U-148
lowReOneEqEddy, U-100	neighbour
maxDeltaxyz, U-100	dictionary, U-129
mixedSmagorinsky, U-100	neighbourPatch keyword, U-142
<b>G</b> • • • • • • • • • • • • • • • • • • •	netgenNeutralToFoam utility, U-89
multiComponentMixture, U-97, U-180 oneEqEddy, U-100	Newtonian
. • ,	keyword entry, U-59
perfectGas, U-98, U-179	Newtonian model, U-100
polynomialTransport, U-98, U-180	nextWrite
powerLaw, U-100	keyword entry, U-111
ptsotchDecomp, U-97	
pureMixture, U-97, U-180	nFaces keyword, U-130 nFinestSweeps keyword, U-123
qZeta, U-99	. ,
reactingMixture, U-97, U-180	nGrow keyword, U-152
realizableKE, U-99	nLayerIter keyword, U-152
reconstruct, U-96	nMoles keyword, U-181
scaleSimilarity, U-100	non-orthogonal mesh, P-41
scotchDecomp, U-97	nonBlocking
simpleFilter, U-99	keyword entry, U-78
smoothDelta, U-100	none
specieThermo, U-98, U-180	keyword entry, U-115, U-122
spectEddyVisc, U-100	NonlinearKEShih model, U-99
sutherlandTransport, U-98, U-180	nonNewtonianIcoFoam solver, U-84
veryInhomogeneousMixture, U-97, U-180	noWriteNow
modifyMesh utility, U-90	keyword entry, U-111
molecularMeasurements	nPostSweeps keyword, U-123
library, U-96	nPreSweeps keyword, U-123
molecule	nRelaxedIter keyword, U-152
library, U-96	nRelaxIter keyword, U-150, U-152
molWeight keyword, U-181	nSmoothNormals keyword, U-152
moveDynamicMesh utility, U-90	nSmoothPatch keyword, U-150
moveEngineMesh utility, U-90	nSmoothScale keyword, U-153
moveMesh utility, U-90	nSmoothSurfaceNormals keyword, U-152
movingWallVelocity	nSmoothThickness keyword, U-152

P-84 Index

nSolveIter keyword, U-150	partialSlip
NSRDSfunctions model, U-98	boundary condition, U-137
null	particleTracks utility, U-92
keyword entry, U-174	patch
numberOfSubdomains keyword, U-81	boundary condition, U-134
	patch
O	keyword entry, U-135, U-176
object keyword, U-105	patchAverage utility, U-92
objToVTK utility, U-90	patches keyword, U-138
ODE	patchIntegrate utility, U-92
library, U-96	patchMap keyword, U-160
oneEqEddy model, U-100	patchSummary utility, U-94
Opacity text box, U-167	PBiCG
OpenFOAM	keyword entry, U-121
applications, U-67	PCG
file format, U-104	keyword entry, U-121
libraries, U-67	pdfPlot utility, U-92
OpenFOAM	PDRFoam solver, U-86
library, U-95	PDRMesh utility, U-90
OpenFOAM file syntax	Pe utility, U-91
//, U-104	perfectGas model, U-98, U-179
openMPI	permutation symbol, P-15
message passing interface, U-81	pimpleDyMFoam solver, U-84
MPI, U-81	pimpleFoam solver, U-84
operator	Pipeline Browser window, U-23, U-164
scalar, P-24	PISO
vector, P-23	dictionary, U-23
Options window, U-168	pisoFoam solver, U-17, U-84
options file, U-71	Plot Over Line
order keyword, U-81	menu entry, U-33
Orientation Axes button, U-24, U-167	plot3dToFoam utility, U-89
OSspecific	pointField class, P-27
library, U-96	pointField <type> template class, P-29</type>
outer product, see tensor, outer product	points H 100 H 100
outlet	dictionary, U-129, U-136
boundary condition, P-65	polyBoundaryMesh class, P-27
outletInlet	polyDualMesh utility, U-90
boundary condition, U-137	polyLine
outside	keyword entry, U-139
keyword entry, U-149	polyMesh directory, U-104, U-129
owner II 100	polyMesh class, P-27, U-127, U-129
dictionary, U-129	polynomialTransport model, U-98, U-180
P	polyPatch class, P-27
p field, U-22	polyPatchList class, P-27
P1	polySpline keyword entry, U-139
library, U-98	porousInterFoam solver, U-85
p_rhgRefCell keyword, U-125	porousSimpleFoam solver, U-84
p_rhgRefValue keyword, U-125	post-processing, U-163
pairPatchAgglomeration	post-processing post-processing
library, U-96	paraFoam, U-163
paraFoam, U-23, U-163	postCalc
parallel	library, U-95
running, U-79	postChannel utility, U-92
· · · · · · · · · · · · · · · · · · ·	posteriumor autituy, V V2

potential	${f R}$
library, U-96	R utility, U-92
potentialFoam solver, P-42, U-83	radiationModels
potentialFreeSurfaceFoam solver, U-84	library, U-98
pow	randomProcesses
tensor member function, P-21	library, U-96
powerLaw model, U-100	RASModel
pPrime2 utility, U-91	keyword entry, U-40, U-184
Pr keyword, U-182	RASModel keyword, U-184
PrandtlDelta model, U-99	raw
preconditioner keyword, U-121, U-122	keyword entry, U-112, U-174
pRefCell keyword, U-23, U-125	reactingFoam solver, U-86
pRefValue keyword, U-23, U-125	reactingMixture model, U-97, U-180
pressure keyword, U-49	reactingParcelFilmFoam solver, U-87
pressure waves	reactingParcelFoam solver, U-87
in liquids, P-58	reaction Thermophysical Models
pressureDirectedInletVelocity	library, U-97
boundary condition, U-137	realizableKE model, U-99
pressureInletVelocity	reconstruct model, U-96
boundary condition, U-137	reconstructPar utility, U-83, U-94
pressureOutlet	reconstructParMesh utility, U-94
boundary condition, P-59	refGradient keyword, U-136
pressureTransmissive	refineHexMesh utility, U-90
boundary condition, U-137	refinementRegions keyword, U-149
primitive	refinementLevel utility, U-90
library, P-19	refinementRegions keyword, U-148, U-150
primitives tools, U-95	refinementSurfaces keyword, U-148
printCoeffs keyword, U-41, U-184	refineMesh utility, U-90
processorWeights keyword, U-80	refineWallLayer utility, U-90
probeLocations utility, U-92	Refresh Times button, U-25
process	regions keyword, U-58
background, U-24, U-79	relative tolerance, U-121
foreground, U-24	relativeSizes keyword, U-152
processor	relaxed keyword, U-153
boundary condition, U-135	relTol keyword, U-52, U-121
processor	removeFaces utility, U-90
keyword entry, U-135	Render View window, U-168
processorN directory, U-80	Render View window panel, U-168
processorWeights keyword, U-81	renumberMesh utility, U-90
Properties window panel, U-25, U-164	Rescale to Data Range button, U-25
ptot utility, U-92	Reset button, U-164
ptsotchDecomp model, U-97	resolveFeatureAngle keyword, U-148
pureMixture model, U-97, U-180	restart, U-38
purgeWrite keyword, U-112	Reynolds number, U-17, U-21
PV3FoamReader	rhoPorousSimpleFoam solver, U-84
library, U-163	rhoCentralDyMFoam solver, U-84
PVFoamReader	rhoCentralFoam solver, U-84
library, U-163	rhoLTSPimpleFoam solver, U-84
$\cap$	rhoPimpleFoam solver, U-84
Outility II 01	rhoPimplecFoam solver, U-84
Q utility, U-91	rhoReactingFoam solver, U-86
QUICK	rhoSimpleFoam solver, U-84
keyword entry, U-119 qZeta model, U-99	rhoSimplecFoam solver, U-84
qzeta model, 0-33	rmdepall script/alias, U-74

P-86 Index

RNGkEpsilon model, U-99	menu entry, U-168
roots keyword, U-81, U-82	settlingFoam solver, U-86
rotateMesh utility, U-90	SFCD
run	keyword entry, U-116, U-119
parallel, U-79	shallowWaterFoam solver, U-84
run directory, U-103	shape, U-140
runTime	Show Color Legend
keyword entry, U-31, U-111	menu entry, U-25
runTimeModifiable keyword, U-112	SI units, U-107
Tunitimonatification neg word, 6 112	simple
${f S}$	keyword entry, U-80, U-81
sammToFoam utility, U-89	simpleFilter model, U-99
sample utility, U-92, U-173	simpleFoam solver, P-50, U-84
sampling	simpleGrading keyword, U-140
library, U-95	simpleReactingParcelFoam solver, U-87
Save Animation	simpleSpline
menu entry, U-169	keyword entry, U-139
Save Screenshot	simulationType keyword, U-40, U-59, U-184
menu entry, U-169	singleCellMesh utility, U-90
scalar, P-12	skew
operator, P-24	tensor member function, P-21
scalar class, P-19	skewLinear
scalarField class, P-25	keyword entry, U-116, U-119
scalarTransportFoam solver, U-83	SLGThermo
scale	library, U-99
tensor member function, P-21	slice class, P-27
scalePoints utility, U-157	slip
scaleSimilarity model, U-100	•
scheduled	boundary condition, U-137
keyword entry, U-78	Smagorinsky model, U-100
scientific	Smagorinsky2 model, U-100
keyword entry, U-112	smapToFoam utility, U-91
scotch	smoothDelta model, U-100
keyword entry, U-80, U-81	smoother keyword, U-123
scotchCoeffs keyword, U-81	smoothSolver
scotchDecomp model, U-97	keyword entry, U-121
•	snap keyword, U-146
script/alias	snapControls keyword, U-146
foamCorrectVrt, U-158 foamJob, U-177	snappyHexMesh utility
foamLog, U-177	background mesh, U-146
<u>.</u>	cell removal, U-149
make, U-69	cell splitting, U-147
rmdepall, U-74 wclean, U-73	mesh layers, U-150
,	meshing process, U-145
wmake, U-69	snapping to surfaces, U-150
second time derivative, P-33	snappyHexMesh utility, U-89, U-145
Seed window, U-169	snappyHexMeshDict file, U-145
selectCells utility, U-90	snGrad
Set Ambient Color button, U-166	fvc member function, P-33
setFields utility, U-58, U-88	snGradCorrection
setFormat keyword, U-174	fvc member function, P-33
sets keyword, U-174	snGradSchemes keyword, U-114
setSet utility, U-90	solid
setsToZones utility, U-90	library, U-99
Settings	Solid Color

menu entry, U-166	multiphaseEulerFoam, U-85
solidDisplacementFoam solver, U-87	multiphaseInterFoam, U-85
solidDisplacementFoam solver, U-50	nonNewtonianIcoFoam, U-84
solidEquilibriumDisplacementFoam solver, U-87	pimpleDyMFoam, U-84
solidMixtureProperties	pimpleFoam, U-84
library, U-99	pisoFoam, U-17, U-84
solidParticle	porousInterFoam, U-85
library, U-96	porousSimpleFoam, U-84
solidProperties	potentialFoam, P-42, U-83
library, U-99	potentialFreeSurfaceFoam, U-84
solver	reactingFoam, U-86
LTSInterFoam, U-85	reactingParcelFilmFoam, U-87
LTSReactingParcelFoam, U-87	reactingParcelFoam, U-87
MRFInterFoam, U-85	rhoCentralDyMFoam, U-84
MRFMultiphaseInterFoam, U-85	rhoCentralFoam, U-84
PDRFoam, U-86	rhoLTSPimpleFoam, U-84
SRFPimpleFoam, U-84	rhoPimpleFoam, U-84
SRFSimpleFoam, U-84	rhoPimplecFoam, U-84
XiFoam, U-86	rhoReactingFoam, U-86
adjointShapeOptimizationFoam, U-83	rhoSimpleFoam, U-84
blockMesh, P-43	rhoSimpler Gam, U-84
boundaryFoam, U-84	rhoPorousSimpleFoam, U-84
buoyantBoussinesqPimpleFoam, U-86	scalarTransportFoam, U-83
	settlingFoam, U-86
buoyantBoussinesqSimpleFoam, U-86	<u> </u>
buoyantPimpleFoam, U-86	shallowWaterFoam, U-84
buoyantSimpleFoam, U-86	simpleFoam, P-50, U-84
cavitatingDyMFoam, U-85	simpleReactingParcelFoam, U-87
cavitatingFoam, U-85	solidDisplacementFoam, U-87
chemFoam, U-86	solidDisplacementFoam, U-50
chtMultiRegionFoam, U-86	solidEquilibriumDisplacementFoam, U-87
chtMultiRegionSimpleFoam, U-86	sonicDyMFoam, U-84
coalChemistryFoam, U-86	sonicFoam, P-56, U-85
coldEngineFoam, U-86	sonicLiquidFoam, P-59, U-85
compressibleInterDyMFoam, U-85	sprayEngineFoam, U-87
compressibleInterFoam, U-85	sprayFoam, U-87
dnsFoam, U-86	twoLiquidMixingFoam, U-86
dsmcFoam, U-87	twoPhaseEulerFoam, U-86
electrostatic $Foam, U-87$	uncoupled Kinematic Parcel Foam, $U-87$
engineFoam, U-86	solver keyword, U-52, U-121
financialFoam, U-88	solver relative tolerance, U-121
fireFoam, U-86	solver tolerance, U-121
icoFoam, U-17, U-21, U-22, U-24, U-84	solvers keyword, U-120
icoUncoupled Kinematic Parcel DyMFoam,	sonicDyMFoam solver, U-84
U-86	sonicFoam solver, P-56, U-85
icoUncoupledKinematicParcelFoam, $U-87$	sonicLiquidFoam solver, P-59, U-85
interDyMFoam, $U$ - $85$	source, P-33
interFoam, $U-85$	SpalartAllmaras model, U-99, U-100
interMixingFoam, $ ext{U-}85$	SpalartAllmarasDDES model, U-100
interPhaseChangeFoam, $U$ -85	SpalartAllmarasIDDES $model$ , $U-100$
laplacian $Foam, U-83$	specie
magneticFoam, U-87	library, U-98
mdEquilibrationFoam, U-87	specie keyword, U-181
$mdFoam,\ \mathrm{U}\text{-}87$	specieThermo model, U-98, U-180
mhdFoam, $P-65$ , $U-87$	spectEddyVisc model, U-100

P-88 Index

and in a borrowood II 190	surfaceFilmModels
spline keyword, U-138	
splitCells utility, U-91	library, U-101
splitMesh utility, U-90	surfaceFind utility, U-93
splitMeshRegions utility, U-90	surfaceFormat keyword, U-174
sprayEngineFoam solver, U-87	surfaceInertia utility, U-93
sprayFoam solver, U-87	surfaceMesh tools, U-95
sqr	surfaceMeshConvert utility, U-93
tensor member function, P-21	surfaceMeshConvertTesting utility, U-93
sqrGradGrad	surfaceMeshExport utility, U-93
fvc member function, P-33	surfaceMeshImport utility, U-93
SRFPimpleFoam solver, U-84	surfaceMeshInfo utility, U-93
SRFSimpleFoam solver, U-84	surfaceMeshTriangulate utility, U-93
star3ToFoam utility, U-89	surfaceNormalFixedValue
star4ToFoam utility, U-89	boundary condition, U-137
startFace keyword, U-130	surfaceOrient utility, U-93
startFrom keyword, U-22, U-111	surfacePointMerge utility, U-93
starToFoam utility, U-154	surfaceRedistributePar utility, U-93
startTime	surfaceRefineRedGreen utility, U-93
keyword entry, U-22, U-111	surfaces keyword, U-174
startTime keyword, U-22, U-111	surfaceSmooth utility, U-93
steady flow	surfaceSplitByPatch utility, U-93
turbulent, P-49	surfaceSplitNonManifolds utility, U-94
steadyParticleTracks utility, U-92	surfaceSubset utility, U-94
steadyState	surfaceToPatch utility, U-94
keyword entry, U-119	surfaceTransformPoints utility, U-94
Stereolithography (STL), U-145	surfMesh
stitchMesh utility, U-90	library, U-96
stl	SuSp
keyword entry, U-174	fvm member function, P-33
stopAt keyword, U-111	,
strategy keyword, U-80, U-81	sutherlandTransport model, U-98, U-180
streamFunction utility, U-91	symm
stress analysis of plate with hole, U-45	tensor member function, P-21
stressComponents utility, U-91	symmetryPlane
Style window panel, U-23, U-166	boundary condition, P-59, U-134
	symmetryPlane
Su formation D 22	keyword entry, U-135
fvm member function, P-33	symmTensorField class, P-25
subsetMesh utility, U-90	symmTensorThirdField class, P-25
summation convention, P-13	system directory, P-46, U-104
SUPERBEE differencing, P-34	systemCall
supersonic flow, P-55	library, U-96
supersonic flow over forward step, P-54	<b>T</b> D
supersonicFreeStream	T
boundary condition, U-137	T()
surface mesh, U-145	tensor member function, P-21
surfaceAdd utility, U-93	Tcommon keyword, U-182
surfaceAutoPatch utility, U-93	template class
surfaceCheck utility, U-93	GeometricBoundaryField, P-28
surfaceClean utility, U-93	fvMatrix, P-29
surfaceCoarsen utility, U-93	dimensioned <type>, P-21</type>
surfaceConvert utility, U-93	FieldField <type>, P-28</type>
$surface Feature Convert\ utility,\ U\text{-}93$	Field <type>, P-25</type>
surfaceFeatureExtract utility, U-93, U-148	geometricField <type>, P-28</type>
surfaceField <tyne> template class P-20</tyne>	List <tyne> P-25</tyne>

pointField <type>, P-29</type>	tensor class, P-19
surfaceField <type>, P-29</type>	tensor member function
volField <type>, P-29</type>	*, P-21
temporal discretisation, P-38	+, P-21
Crank Nicholson, P-38	-, P-21
Euler implicit, P-38	/, P-21
explicit, P-38	&, P-21
in OpenFOAM, P-39	&&, P-21
tensor, P-11	^, P-21
addition, P-13	cmptAv, P-21
algebraic operations, P-13	cofactors, P-21
algebraic operations in OpenFOAM, P-19	det, P-21
antisymmetric, see tensor, skew	dev, P-21
calculus, P-23	diag, P-21
classes in OpenFOAM, P-19	I, P-21
cofactors, P-18	inv, P-21
component average, P-16	mag, P-21
component maximum, P-16	magSqr, P-21
component minimum, P-16	max, P-21
determinant, P-18	min, P-21
deviatoric, P-17	pow, P-21
diagonal, P-17	scale, P-21
dimension, P-12	skew, P-21
double inner product, P-15	sqr, P-21
geometric transformation, P-16	symm, P-21
Hodge dual, P-18	T(), P-21
hydrostatic, P-17	tr, P-21
identities, P-17	transform, P-21
identity, P-16	tensorField class, P-25
inner product, P-14	tensorThirdField class, P-25
inverse, P-18	tetgenToFoam utility, U-89
magnitude, P-16	text box
magnitude squared, P-16	Opacity, U-167
mathematics, P-11	thermalPorousZone
notation, P-13	library, U-99
nth power, P-16	thermalProperties
outer product, P-15	dictionary, U-50
rank, P-12	thermodynamics keyword, U-181
rank 3, P-12	thermophysical
scalar division, P-14	library, U-179
scalar multiplication, P-13	thermophysical Functions
scale function, P-16	library, U-98
second rank, P-12	thermophysicalProperties
skew, P-17	dictionary, U-179
square of, P-16	thermoType keyword, U-179
subtraction, P-13	Thigh keyword, U-182
symmetric, P-17	time
symmetric rank 2, P-12	control, U-111
symmetric rank 2, P-12	time derivative, P-33
trace, P-17	first, P-35
transformation, P-16	second, P-33, P-35
transpose, P-12, P-17	time step, U-22
triple inner product, P-15	timeFormat keyword, U-112
vector cross product, P-15	timePrecision keyword, U-112
. South Stood Products, 1 10	12 mor 2 00 20 20 11 moy word, 0 112

P-90 Index

	W TT 00
timeScheme keyword, U-114	library, U-96
timeStamp	Ts keyword, U-182
keyword entry, U-78	turbulence
timeStampMaster	dissipation, U-39
keyword entry, U-78	kinetic energy, U-39
timeStep	length scale, U-40
keyword entry, U-22, U-31, U-111	turbulence keyword, U-184
Tlow keyword, U-182	turbulence model
tolerance	RAS, U-39
solver, U-121	turbulenceProperties
solver relative, U-121	dictionary, U-40, U-59, U-184
tolerance keyword, U-52, U-121, U-150	turbulent flow
Toolbars	steady, P-49
menu entry, U-167	turbulentInlet
tools	
	boundary condition, U-137
algorithms, U-95	tutorials
cfdTools, U-95	breaking of a dam, U-55
containers, U-95	lid-driven cavity flow, U-17
db, U-95	stress analysis of plate with hole, U-45
dimensionSet, U-95	tutorials directory, P-41, U-17
dimensioned Types, $U-95$	twoLiquidMixingFoam solver, U-86
fields, $U$ -95	twoPhaseEulerFoam solver, U-86
finiteVolume, U-95	two Phase Interface Properties
fvMatrices, U-95	library, U-100
fvMesh, $ ext{U-}95$	type keyword, U-132, U-133
global, $U$ - $95$	
graph, $ ext{U-95}$	${f U}$
	H C 11 H 90
interpolations, U-95	U field, U-22
interpolations, $U-95$ interpolation, $U-95$	Ucomponents utility, P-66
interpolation, U-95	•
interpolation, U-95 matrices, U-95	Ucomponents utility, P-66 UMIST
$\begin{array}{c} \text{interpolation, } U\text{-}95 \\ \text{matrices, } U\text{-}95 \\ \text{memory, } U\text{-}95 \end{array}$	Ucomponents utility, P-66 UMIST keyword entry, U-115
interpolation, U-95 matrices, U-95 memory, U-95 meshes, U-95	Ucomponents utility, P-66 UMIST keyword entry, U-115 uncompressed
interpolation, U-95 matrices, U-95 memory, U-95 meshes, U-95 primitives, U-95	Ucomponents utility, P-66 UMIST     keyword entry, U-115 uncompressed     keyword entry, U-112
interpolation, U-95 matrices, U-95 memory, U-95 meshes, U-95 primitives, U-95 surfaceMesh, U-95	Ucomponents utility, P-66 UMIST     keyword entry, U-115 uncompressed     keyword entry, U-112 uncorrected
interpolation, U-95 matrices, U-95 memory, U-95 meshes, U-95 primitives, U-95 surfaceMesh, U-95 volMesh, U-95	Ucomponents utility, P-66 UMIST     keyword entry, U-115 uncompressed     keyword entry, U-112 uncorrected     keyword entry, U-117, U-118
interpolation, U-95 matrices, U-95 memory, U-95 meshes, U-95 primitives, U-95 surfaceMesh, U-95 volMesh, U-95 topoChangerFvMesh	Ucomponents utility, P-66 UMIST  keyword entry, U-115 uncompressed keyword entry, U-112 uncorrected keyword entry, U-117, U-118 uncoupledKinematicParcelFoam solver, U-87
interpolation, U-95 matrices, U-95 memory, U-95 meshes, U-95 primitives, U-95 surfaceMesh, U-95 volMesh, U-95 topoChangerFvMesh library, U-96	Ucomponents utility, P-66 UMIST     keyword entry, U-115 uncompressed     keyword entry, U-112 uncorrected     keyword entry, U-117, U-118 uncoupledKinematicParcelFoam solver, U-87 uniform keyword, U-175
interpolation, U-95 matrices, U-95 memory, U-95 meshes, U-95 primitives, U-95 surfaceMesh, U-95 volMesh, U-95 topoChangerFvMesh library, U-96 topoSet utility, U-90	Ucomponents utility, P-66 UMIST  keyword entry, U-115 uncompressed keyword entry, U-112 uncorrected keyword entry, U-117, U-118 uncoupledKinematicParcelFoam solver, U-87 uniform keyword, U-175 units
interpolation, U-95 matrices, U-95 memory, U-95 meshes, U-95 primitives, U-95 surfaceMesh, U-95 volMesh, U-95 topoChangerFvMesh library, U-96 topoSet utility, U-90 topoSetSource keyword, U-58	Ucomponents utility, P-66 UMIST  keyword entry, U-115 uncompressed keyword entry, U-112 uncorrected keyword entry, U-117, U-118 uncoupledKinematicParcelFoam solver, U-87 uniform keyword, U-175 units base, U-107
interpolation, U-95 matrices, U-95 memory, U-95 meshes, U-95 primitives, U-95 surfaceMesh, U-95 volMesh, U-95 topoChangerFvMesh library, U-96 topoSet utility, U-90 topoSetSource keyword, U-58 totalPressure	Ucomponents utility, P-66 UMIST  keyword entry, U-115 uncompressed keyword entry, U-112 uncorrected keyword entry, U-117, U-118 uncoupledKinematicParcelFoam solver, U-87 uniform keyword, U-175 units base, U-107 of measurement, P-21, U-107
interpolation, U-95 matrices, U-95 memory, U-95 meshes, U-95 primitives, U-95 surfaceMesh, U-95 volMesh, U-95 topoChangerFvMesh library, U-96 topoSet utility, U-90 topoSetSource keyword, U-58	Ucomponents utility, P-66 UMIST  keyword entry, U-115 uncompressed keyword entry, U-112 uncorrected keyword entry, U-117, U-118 uncoupledKinematicParcelFoam solver, U-87 uniform keyword, U-175 units base, U-107 of measurement, P-21, U-107 S.I. base, P-21
interpolation, U-95 matrices, U-95 memory, U-95 meshes, U-95 primitives, U-95 surfaceMesh, U-95 volMesh, U-95 topoChangerFvMesh library, U-96 topoSet utility, U-90 topoSetSource keyword, U-58 totalPressure boundary condition, U-137	Ucomponents utility, P-66 UMIST  keyword entry, U-115 uncompressed keyword entry, U-112 uncorrected keyword entry, U-117, U-118 uncoupledKinematicParcelFoam solver, U-87 uniform keyword, U-175 units base, U-107 of measurement, P-21, U-107 S.I. base, P-21 SI, U-107
interpolation, U-95 matrices, U-95 memory, U-95 meshes, U-95 primitives, U-95 surfaceMesh, U-95 volMesh, U-95 topoChangerFvMesh library, U-96 topoSet utility, U-90 topoSetSource keyword, U-58 totalPressure boundary condition, U-137 tr tensor member function, P-21	Ucomponents utility, P-66 UMIST  keyword entry, U-115 uncompressed keyword entry, U-112 uncorrected keyword entry, U-117, U-118 uncoupledKinematicParcelFoam solver, U-87 uniform keyword, U-175 units base, U-107 of measurement, P-21, U-107 S.I. base, P-21 SI, U-107 Système International, U-107
interpolation, U-95 matrices, U-95 memory, U-95 meshes, U-95 primitives, U-95 surfaceMesh, U-95 volMesh, U-95 topoChangerFvMesh library, U-96 topoSet utility, U-90 topoSetSource keyword, U-58 totalPressure boundary condition, U-137	Ucomponents utility, P-66 UMIST     keyword entry, U-115 uncompressed     keyword entry, U-112 uncorrected     keyword entry, U-117, U-118 uncoupledKinematicParcelFoam solver, U-87 uniform keyword, U-175 units     base, U-107     of measurement, P-21, U-107     S.I. base, P-21     SI, U-107     Système International, U-107 United States Customary System, U-107
interpolation, U-95 matrices, U-95 memory, U-95 meshes, U-95 primitives, U-95 surfaceMesh, U-95 volMesh, U-95 topoChangerFvMesh library, U-96 topoSet utility, U-90 topoSetSource keyword, U-58 totalPressure boundary condition, U-137 tr tensor member function, P-21	Ucomponents utility, P-66 UMIST  keyword entry, U-115 uncompressed keyword entry, U-112 uncorrected keyword entry, U-117, U-118 uncoupledKinematicParcelFoam solver, U-87 uniform keyword, U-175 units base, U-107 of measurement, P-21, U-107 S.I. base, P-21 SI, U-107 Système International, U-107
interpolation, U-95 matrices, U-95 memory, U-95 meshes, U-95 primitives, U-95 surfaceMesh, U-95 volMesh, U-95 topoChangerFvMesh library, U-96 topoSet utility, U-90 topoSetSource keyword, U-58 totalPressure boundary condition, U-137 tr tensor member function, P-21 trace, see tensor, trace	Ucomponents utility, P-66 UMIST     keyword entry, U-115 uncompressed     keyword entry, U-112 uncorrected     keyword entry, U-117, U-118 uncoupledKinematicParcelFoam solver, U-87 uniform keyword, U-175 units     base, U-107     of measurement, P-21, U-107     S.I. base, P-21     SI, U-107     Système International, U-107 United States Customary System, U-107
interpolation, U-95 matrices, U-95 memory, U-95 meshes, U-95 primitives, U-95 surfaceMesh, U-95 volMesh, U-95 topoChangerFvMesh library, U-96 topoSet utility, U-90 topoSetSource keyword, U-58 totalPressure boundary condition, U-137 tr tensor member function, P-21 trace, see tensor, trace traction keyword, U-49	Ucomponents utility, P-66 UMIST  keyword entry, U-115 uncompressed keyword entry, U-112 uncorrected keyword entry, U-117, U-118 uncoupledKinematicParcelFoam solver, U-87 uniform keyword, U-175 units base, U-107 of measurement, P-21, U-107 S.I. base, P-21 SI, U-107 Système International, U-107 United States Customary System, U-107 USCS, U-107
interpolation, U-95 matrices, U-95 memory, U-95 meshes, U-95 primitives, U-95 surfaceMesh, U-95 volMesh, U-95 topoChangerFvMesh library, U-96 topoSet utility, U-90 topoSetSource keyword, U-58 totalPressure boundary condition, U-137 tr tensor member function, P-21 trace, see tensor, trace traction keyword, U-49 transform	Ucomponents utility, P-66 UMIST  keyword entry, U-115 uncompressed keyword entry, U-112 uncorrected keyword entry, U-117, U-118 uncoupledKinematicParcelFoam solver, U-87 uniform keyword, U-175 units base, U-107 of measurement, P-21, U-107 S.I. base, P-21 SI, U-107 Système International, U-107 United States Customary System, U-107 USCS, U-107 Update GUI button, U-165
interpolation, U-95 matrices, U-95 memory, U-95 meshes, U-95 primitives, U-95 surfaceMesh, U-95 volMesh, U-95 topoChangerFvMesh library, U-96 topoSet utility, U-90 topoSetSource keyword, U-58 totalPressure boundary condition, U-137 tr tensor member function, P-21 trace, see tensor, trace traction keyword, U-49 transform tensor member function, P-21	Ucomponents utility, P-66 UMIST  keyword entry, U-115 uncompressed keyword entry, U-112 uncorrected keyword entry, U-117, U-118 uncoupledKinematicParcelFoam solver, U-87 uniform keyword, U-175 units base, U-107 of measurement, P-21, U-107 S.I. base, P-21 SI, U-107 Système International, U-107 United States Customary System, U-107 USCS, U-107 Update GUI button, U-165 uprime utility, U-91
interpolation, U-95 matrices, U-95 memory, U-95 meshes, U-95 primitives, U-95 surfaceMesh, U-95 volMesh, U-95 topoChangerFvMesh library, U-96 topoSet utility, U-90 topoSetSource keyword, U-58 totalPressure boundary condition, U-137 tr tensor member function, P-21 trace, see tensor, trace traction keyword, U-49 transform tensor member function, P-21 transformPoints utility, U-90	Ucomponents utility, P-66 UMIST  keyword entry, U-115 uncompressed keyword entry, U-112 uncorrected keyword entry, U-117, U-118 uncoupledKinematicParcelFoam solver, U-87 uniform keyword, U-175 units base, U-107 of measurement, P-21, U-107 S.I. base, P-21 SI, U-107 Système International, U-107 United States Customary System, U-107 USCS, U-107 Update GUI button, U-165 uprime utility, U-91 upwind
interpolation, U-95 matrices, U-95 memory, U-95 meshes, U-95 primitives, U-95 surfaceMesh, U-95 volMesh, U-95 topoChangerFvMesh library, U-96 topoSet utility, U-90 topoSetSource keyword, U-58 totalPressure boundary condition, U-137 tr tensor member function, P-21 trace, see tensor, trace traction keyword, U-49 transform tensor member function, P-21 transformPoints utility, U-90 transport keyword, U-181 transportProperties	Ucomponents utility, P-66 UMIST     keyword entry, U-115 uncompressed     keyword entry, U-112 uncorrected     keyword entry, U-117, U-118 uncoupledKinematicParcelFoam solver, U-87 uniform keyword, U-175 units     base, U-107     of measurement, P-21, U-107     S.I. base, P-21     SI, U-107     Système International, U-107     United States Customary System, U-107     USCS, U-107 Update GUI button, U-165 uprime utility, U-91 upwind     keyword entry, U-116, U-119 upwind differencing, P-34, U-60
interpolation, U-95 matrices, U-95 memory, U-95 meshes, U-95 primitives, U-95 surfaceMesh, U-95 volMesh, U-95 topoChangerFvMesh library, U-96 topoSet utility, U-90 topoSetSource keyword, U-58 totalPressure boundary condition, U-137 tr tensor member function, P-21 trace, see tensor, trace traction keyword, U-49 transform tensor member function, P-21 transformPoints utility, U-90 transport keyword, U-181 transportProperties dictionary, U-21, U-38, U-41	Ucomponents utility, P-66 UMIST     keyword entry, U-115 uncompressed     keyword entry, U-112 uncorrected     keyword entry, U-117, U-118 uncoupledKinematicParcelFoam solver, U-87 uniform keyword, U-175 units     base, U-107     of measurement, P-21, U-107     S.I. base, P-21     SI, U-107     Système International, U-107     United States Customary System, U-107     USCS, U-107 Update GUI button, U-165 uprime utility, U-91 upwind     keyword entry, U-116, U-119 upwind differencing, P-34, U-60 USCS units, U-107
interpolation, U-95 matrices, U-95 memory, U-95 meshes, U-95 primitives, U-95 surfaceMesh, U-95 volMesh, U-95 topoChangerFvMesh library, U-96 topoSet utility, U-90 topoSetSource keyword, U-58 totalPressure boundary condition, U-137 tr tensor member function, P-21 trace, see tensor, trace traction keyword, U-49 transform tensor member function, P-21 transformPoints utility, U-90 transport keyword, U-181 transportProperties dictionary, U-21, U-38, U-41 transportProperties file, U-59	Ucomponents utility, P-66 UMIST     keyword entry, U-115 uncompressed     keyword entry, U-112 uncorrected     keyword entry, U-117, U-118 uncoupledKinematicParcelFoam solver, U-87 uniform keyword, U-175 units     base, U-107     of measurement, P-21, U-107     S.I. base, P-21     SI, U-107     Système International, U-107     United States Customary System, U-107     UsCS, U-107 Update GUI button, U-165 uprime utility, U-91 upwind     keyword entry, U-116, U-119 upwind differencing, P-34, U-60 USCS units, U-107 Use Parallel Projection button, U-24
interpolation, U-95 matrices, U-95 memory, U-95 meshes, U-95 primitives, U-95 surfaceMesh, U-95 volMesh, U-95 topoChangerFvMesh library, U-96 topoSet utility, U-90 topoSetSource keyword, U-58 totalPressure boundary condition, U-137 tr tensor member function, P-21 trace, see tensor, trace traction keyword, U-49 transform tensor member function, P-21 transformPoints utility, U-90 transport keyword, U-181 transportProperties dictionary, U-21, U-38, U-41	Ucomponents utility, P-66 UMIST     keyword entry, U-115 uncompressed     keyword entry, U-112 uncorrected     keyword entry, U-117, U-118 uncoupledKinematicParcelFoam solver, U-87 uniform keyword, U-175 units     base, U-107     of measurement, P-21, U-107     S.I. base, P-21     SI, U-107     Système International, U-107     United States Customary System, U-107     USCS, U-107 Update GUI button, U-165 uprime utility, U-91 upwind     keyword entry, U-116, U-119 upwind differencing, P-34, U-60 USCS units, U-107

C. II 01	Constitutions II 00
Co, U-91	foamListTimes, U-92
Lambda2, U-91	foamMeshToFluent, U-89, U-170
Mach, U-91	foamToEnsightParts, U-91
PDRMesh, U-90	foamToEnsight, U-91
Pe, U-91	foamToGMV, U-91
Q, U-91	foamToStarMesh, U-89
R, U-92	foamToSurface, U-89
Ucomponents, P-66	foamToTecplot360, U-91
adiabaticFlameT, U-94	foamToVTK, U-91
ansysToFoam, U-89	foamUpgradeCyclics, U-88
applyBoundaryLayer, U-88	foamUpgradeFvSolution, U-88
applyWallFunctionBoundaryConditions,	gambitToFoam, U-89, U-154
U-88	gmshToFoam, U-89
attachMesh, U-89	ideasToFoam, U-154
autoPatch, U-89	ideasUnvToFoam, U-89
autoRefineMesh, U-90	insideCells, U-89
blockMesh, U-36, U-88, U-136	kivaToFoam, U-89
boxTurb, U-88	mapFields, U-30, U-37, U-41, U-54, U-88,
cfx4ToFoam, $U$ -89, $U$ -154	U-160
changeDictionary, $U$ -88	mdInitialise, U-88
checkMesh, U-89, U-155	mergeMeshes, $U$ - $90$
chemkinToFoam, U-94	mergeOrSplitBaffles, $U$ -90
collapseEdges, $U$ - $90$	mirrorMesh, U-90
combinePatchFaces, U-90	mixtureAdiabaticFlameT, $U-94$
createBaffles, U-89	modifyMesh, U-90
createPatch, U-89	moveDynamicMesh, U-90
createTurbulenceFields, U-92	moveEngineMesh, U-90
datToFoam, U-89	moveMesh, U-90
$decomposePar,\ U\text{-}79,\ U\text{-}80,\ U\text{-}94$	mshToFoam, U-89
deformedGeom, U-89	netgenNeutralToFoam, $U$ -89
dsmcFieldsCalc, $U$ -92	objToVTK, U-90
dsmcInitialise, U-88	pPrime2, U-91
engineCompRatio, $U-92$	particleTracks, $U$ -92
engineSwirl, U-88	patchAverage, $U-92$
ensight74FoamExec, $U-172$	patchIntegrate, $U-92$
ensightFoamReader, $U$ - $91$	patchSummary, $U$ -94
enstrophy, $U$ - $91$	pdfPlot, U-92
equilibriumCO, U-94	plot3dToFoam, U-89
equilibriumFlameT, U-94	polyDualMesh, $U$ -90
execFlowFunctionObjects, $U-92$	postChannel, $U$ - $92$
expandDictionary, $U$ -94	probeLocations, U-92
extrude2DMesh, U-88	ptot, $U$ -92
extrudeMesh, U-88	${\sf reconstructParMesh},  U\text{-}94$
extrudeToRegionMesh, $U$ -89	reconstructPar, U-83, U-94
faceAgglomerate, $U$ - $88$	refineHexMesh, U-90
flattenMesh, U-89	refineMesh, U-90
flowType, U-91	refineWallLayer, $U$ -90
fluent3DMeshToFoam, $U-89$	refinementLevel, U-90
fluentMeshToFoam, $U$ -89, $U$ -154	removeFaces, $U$ - $90$
foamCalc, $U$ -32, $U$ -92	renumberMesh, $U$ -90
foamDataToFluent, $U-91$ , $U-170$	$rotateMesh,\ U\text{-}90$
foamDebugSwitches, $U-94$	sammToFoam, $U$ - $89$
${\sf foamFormatConvert},\ U\text{-}94$	sample, $U-92$ , $U-173$
foamInfoExec, $U$ -94	scalePoints, U-157

P-92 Index

selectCells, U-90	wdot, U-92
setFields, U-58, U-88	writeCellCentres, U-92
setSet, U-90	writeMeshObj, U-89
setsToZones, U-90	yPlusLES, U-92
singleCellMesh, U-90	yPlusRAS, U-92
smapToFoam, U-91	zipUpMesh, U-90
snappyHexMesh, U-89, U-145	utilityFunctionObjects
splitCells, U-91	library, U-96
splitMeshRegions, U-90	
splitMesh, U-90	$\mathbf{V}$
star3ToFoam, U-89	value keyword, U-21, U-136
star4ToFoam, U-89	valueFraction keyword, U-136
starToFoam, U-154	van Leer differencing, P-34
steadyParticleTracks, U-92	vanLeer
stitchMesh, U-90	keyword entry, U-116
streamFunction, U-91	VCR Controls menu, U-25, U-165
stressComponents, U-91	vector, P-12
subsetMesh, U-90	operator, P-23
surfaceAdd, U-93	unit, P-16
surfaceAutoPatch, U-93	vector class, P-19, U-107
surfaceCheck, U-93	vector product, see tensor, vector cross product
surfaceClean, U-93	vectorField class, P-25
surfaceCoarsen, U-93	version keyword, U-105
surfaceConvert, U-93	vertices keyword, U-20, U-138, U-139
surfaceFeatureConvert, U-93	veryInhomogeneousMixture model, U-97, U-180
surfaceFeatureExtract, U-93, U-148	View menu, U-167
surfaceFind, U-93	View Settings
surfaceIniti, 0-93	menu entry, U-24, U-167
surfaceMeshConvertTesting, U-93	View Settings (Render View) window, U-167
surfaceMeshConvert, U-93	- ` '
surfaceMeshExport, U-93	View Settings
surfaceMeshImport, U-93	menu entry, U-24 viewFactor
surfaceMeshInfo, U-93	
surfaceMeshTriangulate, U-93	library, U-98
surfaceOrient, U-93	viewFactorsGen utility, U-88
surfaceOrient, 0-93	viscosity
surfaceRedistributePar, U-93	kinematic, U-21, U-41
surfaceRefineRedGreen, U-93	volField <type> template class, P-29</type>
surfaceSmooth, U-93	volMesh tools, U-95
surfaceShlooth, U-93	vorticity utility, U-91
surfaceSplitNonManifolds, U-94	vtk
surfaceSubset, U-94	keyword entry, U-174
surfaceToPatch, U-94	vtkFoam
surfaceTransformPoints, U-94	library, U-163
tetgenToFoam, U-89	vtkPV3Foam
topoSet, U-90	library, U-163
transformPoints, U-90	$\mathbf{W}$
uprime, U-91	wall
viewFactorsGen, U-88	boundary condition, P-59, P-65, U-57,
vorticity, U-91	U-134
wallFunctionTable, U-88	wall
wallGradU, U-91	keyword entry, U-135
wallHeatFlux, U-91	wallFunctionTable utility, U-88
wallShearStress, U-92	wallGradU utility, U-91
wandical delega, U-34	wan Grado domby, 0-31

wallHeatFlux utility, U-91	environment variable, U-74
Wallis	WM_OPTIONS
library, U-98	environment variable, U-74
wallShearStress utility, U-92	WM_PRECISION_OPTION
wclean script/alias, U-73	environment variable, U-74
wdot utility, U-92	WM_PROJECT
wedge	environment variable, U-74
boundary condition, U-130, U-135, U-144	WM_PROJECT_DIR
wedge	environment variable, U-74
keyword entry, U-135	WM_PROJECT_INST_DIR
window	environment variable, U-74
Color Legend, U-27	WM_PROJECT_USER_DIR
Options, U-168	environment variable, U-74
Pipeline Browser, U-23, U-164	WM_PROJECT_VERSION
Render View, U-168	environment variable, U-74
Seed, U-169	wmake
View Settings (Render View), U-167	platforms, U-70
window panel	wmake script/alias, U-69
Animations, U-168	word class, P-21, P-27
Annotation, U-24, U-167	writeCellCentres utility, U-92
Charts, U-168	writeCompression keyword, U-112
Color Legend, U-166	writeControl
Color Scale, U-166	keyword entry, U-111
Colors, U-168	writeControl keyword, U-22, U-60, U-111
Display, U-23, U-25, U-164, U-165	writeFormat keyword, U-54, U-112
General, U-167, U-168	writeInterval keyword, U-22, U-31, U-111
Information, U-164	writeMeshObj utility, U-89
Lights, U-167	writeNow
Mesh Parts, U-23	keyword entry, U-111
Properties, U-25, U-164	writePrecision keyword, U-112
Render View, U-168	77
Style, U-23, U-166	$\mathbf{X}$
Wireframe	X
menu entry, U-166	keyword entry, U-175
WM_ARCH	XiFoam solver, U-86
environment variable, U-74	xmgr
WM_ARCH_OPTION	keyword entry, U-112, U-174
environment variable, U-74	xyz
WM_COMPILE_OPTION (	keyword entry, U-175
environment variable, U-74	Y
WM_COMPILER	<del>-</del>
environment variable, U-74	y keyword entry, U-175
WM_COMPILER_BIN	yPlusLES utility, U-92
environment variable, U-74	yPlusRAS utility, U-92
WM_COMPILER_DIR	yriusivas utinity, 0-92
environment variable, U-74	${f Z}$
WM_COMPILER_LIB	z
environment variable, U-74	keyword entry, U-175
WM_DIR	zeroGradient
environment variable, U-74	boundary condition, U-136
WM_MPLIB	zipUpMesh utility, U-90