0.1 Effect Weakening Definition

Introduce a relation $\omega : \Phi' \triangleright \Phi$ relating effect-environments.

0.1.1 Relation

- (Id) $\frac{\Phi\texttt{Ok}}{\iota:\Phi\triangleright\Phi}$
- (Project) $\frac{\omega:\Phi'\triangleright\Phi}{\omega\pi:(\Phi',\alpha)\triangleright\Phi}$
- (Extend) $\frac{\omega:\Phi'\triangleright\Phi}{\omega\times:(\Phi',\alpha)\triangleright(\Phi,\alpha)}$

0.1.2 Weakening Properties

0.1.3 Effect Weakening Preserves Ok

$$\omega : \Phi' \triangleright \Phi \wedge \Phi\texttt{Ok} \Leftarrow \Phi'\texttt{Ok} \tag{1}$$

Proof

Case ι

$$\Phi\texttt{Ok} \wedge \iota : \Phi \triangleright \Phi \Leftarrow \Phi\texttt{Ok}$$

Case $\omega\pi$ By inversion,

$$\omega : \Phi' \triangleright \Phi \wedge \alpha \notin \Phi' \tag{2}$$

So, by induction, $\Phi'\texttt{Ok}$ and hence $(\Phi', \alpha)\texttt{Ok}$

Case $\omega\times$ By inversion,

$$\omega : \Phi' \triangleright \Phi \wedge \alpha \notin \Phi' \tag{3}$$

So

$$\begin{aligned}(\Phi, \alpha)\texttt{Ok} &\Rightarrow \Phi\texttt{Ok} && (4)\\ &\Rightarrow \Phi'\texttt{Ok} && (5)\\ &\Rightarrow (\Phi', \alpha)\texttt{Ok} && (6)\\ & && (7)\end{aligned}$$

0.1.4 Domain Lemma

$$\omega : \Phi' \triangleright \Phi \Rightarrow (\alpha \notin \Phi \Rightarrow \alpha \notin \Phi')$$

Proof By trivial Induction.

0.1.5 Weakening Preserves Effect Well-Formed-Ness

If $\omega : \Phi' \triangleright \Phi$ then $\Phi \vdash \epsilon \implies \Phi' \vdash \epsilon$

Proof By induction over the well-formed-ness of effects

Case Ground By inversion, $\Phi\texttt{Ok} \wedge \epsilon \in E$. Hence by the ok-property, $\Phi'\texttt{Ok}$ So $\Phi' \vdash \epsilon$

Case Var $\Phi = \Phi'', \alpha$
So either:

Case $\Phi' = \Phi''', \alpha$ So $\omega = \omega' \times$ So $\omega' : \Phi''' \triangleright \Phi''$, and hence:

$$(\text{Var})\frac{\Phi''', \alpha \mathtt{Ok}}{\Phi''', \alpha \vdash \alpha} \tag{8}$$

Case $\Phi' = \Phi''', \beta$ and $\beta \neq \alpha$
So $\omega = \omega' \pi$
By induction, $\omega' : \Phi''' \triangleright \Phi$ so

$$(\text{Weaken})\frac{\Phi''' \vdash \alpha}{\Phi' \vdash \alpha} \tag{9}$$

Case Weaken By inversion, $\Phi = \Phi'', \beta$.
So $\omega = \omega' \times$
And, $\Phi' = \Phi''', \beta$ So By inversion$\omega' : \Phi''' \triangleright \pi_1''$
So by induction

$$(\text{weak})\frac{\Phi''' \vdash \alpha}{\Phi' \vdash \alpha} \tag{10}$$

Case Monoid By inversion, $\Phi \vdash \epsilon_1$ and $\Phi \vdash \epsilon_2$. So by induction, $\Phi' \vdash \epsilon_1$ and $\Phi' \vdash \epsilon_2$, and so:

$$\Phi' \vdash \epsilon_1 \cdot \epsilon_2 \tag{11}$$

0.1.6 Weakening Preserves Type-Well-Formed-Ness

If $\omega : \Phi' \triangleright \Phi$ and $\Phi \vdash A$ then $\Phi' \vdash A$.

Proof:

Case Ground: By inversion, $\Phi\mathtt{Ok}$, hence by property 1 of weakening, $\Phi'\mathtt{Ok}$. Hence $\Phi' \vdash \gamma$.

Case Function: By inversion, $\Phi \vdash A$, $\Phi \vdash B$. So by induction $\Phi' \vdash A$, $\Phi' \vdash B$, hence,

$$\Phi' \vdash A \to B$$

Case Computation: By inversion$\Phi \vdash A$, and $\Phi \vdash \epsilon$.
So by induction and the effect-well-formed-ness theorem,
$\Phi' \vdash A$ and $\Phi' \vdash \epsilon$
So

$$\Phi' \vdash \mathtt{M}_\epsilon A$$

Case For All: By inversion, $\Phi, \alpha \vdash A$ Picking $\alpha \notin \Phi'$ using α-conversion.
So $\omega\times : (\Phi', \alpha) \triangleright (\Phi, \alpha)$
So $(\Phi', \alpha) \vdash A$
So $\Phi \vdash \forall \alpha. A$

0.1.7 Corollary

$$\omega : \Phi' \triangleright \Phi \wedge \Phi \vdash \Gamma\mathtt{Ok} \implies \Phi' \vdash \Gamma\mathtt{Ok}$$

Case Nil: By inversionΦOk so $\Phi \vdash \diamond\mathtt{Ok}$

Case Var: By inversion$\Phi \vdash \Gamma\mathtt{Ok}$, $x \in \mathtt{dom}(\Gamma)$, $\Phi \vdash A$
So by induction $\Phi' \vdash \Gamma\mathtt{Ok}$, and $\pi_1' \vdash \Gamma\mathtt{Ok}$
So $\Phi' \vdash (\Gamma, x : A)\mathtt{Ok}$

0.1.8 Effect Weakening preserves Type Relations

$$\Phi \mid \Gamma \vdash v\colon A \wedge \omega : \Phi' \triangleright \Phi \implies \Phi' \mid \Gamma \vdash v\colon A \tag{12}$$

Proof:

Case Constants: If $\Phi \vdash \Gamma\mathtt{Ok}$ then $\Phi' \vdash \Gamma\mathtt{Ok}$ so:

$$(\text{Const})\frac{\Phi' \vdash \Gamma\mathtt{Ok}}{\Phi' \mid \Gamma \vdash \mathtt{C}^A\colon A} \tag{13}$$

Case Variables: If $\Phi \vdash \Gamma\mathtt{Ok}$ then $\Phi' \vdash \Gamma\mathtt{Ok}$ so: So, $\Phi' \mid G \vdash x\colon A$, if $\Phi \mid G \vdash x\colon A$

Case Lambda: By inversion, $\Phi \mid \Gamma, x : A \vdash v\colon B$, so by induction $\Phi' \mid \Gamma, x : A \vdash v\colon B$.
So,

$$\Phi' \mid \Gamma \vdash \lambda x : A.v\colon A \to B \tag{14}$$

Case Apply: By inversion$\Phi \mid \Gamma \vdash v_1\colon A \to B$ and $\Phi \mid \Gamma \vdash v_2\colon A$.
Hence by induction, $\Phi \mid \Gamma \vdash v_1\colon A \to B$ and $\Phi \mid \Gamma \vdash v_2\colon A$.
So

$$\Phi \mid \Gamma \vdash \mathtt{app} v_1 v_2\colon B$$

Case Return: By inversion$\Phi \mid \Gamma \vdash v\colon A$
So by induction $\Phi \mid \Gamma \vdash v\colon A$
Hence $\Phi \mid \Gamma \vdash \mathtt{return} v\colon \mathtt{M}_{\mathbf{1}} A$

Case Bind: By inversion$\Phi \mid \Gamma \vdash v_1\colon \mathtt{M}_{\epsilon_1} A$ and $\Phi \mid \Gamma, x : A \vdash \epsilon_2\colon \mathtt{M}_{\epsilon_2} A$.
Hence by induction $\Phi \mid \Gamma \vdash v_1\colon \mathtt{M}_{\epsilon_1} A$ and $\Phi' \mid \Gamma, x : A \vdash v_2\colon \mathtt{M}_{\epsilon_2} A$.
So

$$\Phi \mid \Gamma \vdash \mathtt{do}\ x \leftarrow v_1\ \mathtt{in}\ v_2\colon \mathtt{M}_{\epsilon_1 \cdot \epsilon_2} B \tag{15}$$

Case If: By inversion$\Phi \mid \Gamma \vdash v\colon \mathtt{Bool}$, $\Phi \mid \Gamma \vdash v_1\colon A$, and $\Phi \mid \Gamma \vdash v_2\colon A$.
Hence by induction $\Phi \mid \Gamma \vdash v\colon \mathtt{Bool}$, $\Phi \mid \Gamma \vdash v_1\colon A$, and $\Phi \mid \Gamma \vdash v_2\colon A$.
So

$$\Phi \mid \Gamma \vdash \mathtt{if}_{A,}\ v\ \mathtt{then}\ v_1\ \mathtt{else}\ v_2\colon A \tag{16}$$

Case Subtype:

Case Efect-Lambda:

Case Effect-Apply:

0.2 Type Environment Weakening