0.1 Introduce Substitutions

0.1.1 Substitutions as SNOC lists

Definition of σ

0.1.2 Trivial Properties of substitutions

$\texttt{fv}(\sigma)$

$\texttt{dom}(\sigma)$

$x\#\sigma$

0.1.3 Effect of substitutions

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0.1.4 Well Formedness

0.1.5 Simple Properties Of Substitution

If $\Gamma' \vdash \sigma\colon \Gamma$ then: **TODO: Number these**

- $\Gamma\texttt{Ok}$ and $\Gamma'\texttt{Ok}$
- $\omega : \Gamma'' \triangleright \Gamma'$ implies $\Gamma'' \vdash \sigma\colon \Gamma$
- $\times \notin (\texttt{dom}(\Gamma)\texttt{dom}(\Gamma''))$ implies $(\Gamma', x : A) \vdash (\sigma, x := x)\colon (\Gamma, x : A)$

0.2 Substitution Preserves Typing

TODO: State property TODO: Proof by induction overtype relation

0.3 Semantics of Substitution

0.3.1 Denotation of Substitutions

TODO: Fill in from p98

0.3.2 Lemma

TODO: Fill in from p98

0.3.3 Substitution Theorem

If $\Gamma \vdash t\colon \tau$ and $\Gamma' \vdash \sigma\colon \Gamma$ then

$$\Gamma' \xrightarrow{[\![\Gamma' \vdash \sigma:\Gamma]\!]_M} \Gamma$$

$$[\![\Gamma' \vdash t$$

$$\sigma : \tau]\!]_M pgfextrapgf@stop \qquad \downarrow [\![\Gamma \vdash t:\tau]\!]_M$$

$$[\![T]\!]_M$$

0.4 Single Substitution