0.1 Weakening Definition

0.1.1 Relation

We define the ternary weaking relation $w : \Gamma' \triangleright \Gamma$ using the following rules.

- (Id) $\frac{\Gamma \texttt{Ok}}{\iota : \Gamma \triangleright \Gamma}$
- (Project) $\frac{\omega : \Gamma' \triangleright \Gamma x \notin \texttt{dom}(\Gamma')}{\omega\pi : \Gamma, x : A \triangleright \Gamma}$
- (Extend) $\frac{\omega : \Gamma' \triangleright \Gamma \times \notin \texttt{dom}(\Gamma') A \leq : B}{w \times : \Gamma', x : A \triangleright \Gamma, x : B}$

0.1.2 `Ok` definition

0.1.3 `Dom` definition

0.1.4 Weakening Denotations

0.2 Weakening Theorems

0.2.1 Theorem 1

If $\omega : \Gamma' \triangleright \Gamma$ and $\Gamma \texttt{Ok}$ then $\Gamma' \texttt{Ok}$

Proof **TODO: this**

0.2.2 Theorem 2

If $\Gamma \vdash t\colon \tau$ and $\omega : \Gamma' \triangleright \Gamma$ then $\Gamma' \vdash t\colon \tau$

Proof Proved in parallel with theorem 3 below

0.2.3 Theorem 3

If $\omega : \Gamma' \triangleright \Gamma$ and $\Delta = [\![\Gamma \vdash t\colon \tau]\!]_M$ and $\Delta' = [\![\Gamma' \vdash t\colon \tau]\!]_M$ then

$$\Delta \circ [\![\omega]\!]_M = \Delta' : \Gamma' \to [\![\tau]\!]_M$$

Proof **TODO: this, induct over typing relation/definition of Denotations**