

## 0.1 Weakening Definition

### 0.1.1 Relation

We define the ternary weaking relation  $w : \Gamma' \triangleright \Gamma$  using the following rules.

- (Id)  $\frac{\Gamma \text{Ok}}{\iota : \Gamma \triangleright \Gamma}$
- (Project)  $\frac{\omega : \Gamma' \triangleright \Gamma \times \nexists \text{dom}(\Gamma')}{\omega \pi : \Gamma, x : A \triangleright \Gamma}$
- (Extend)  $\frac{\omega : \Gamma' \triangleright \Gamma \times \nexists \text{dom}(\Gamma') \ A \leq B}{w \times : \Gamma', x : A \triangleright \Gamma, x : B}$

### 0.1.2 Ok definition

### 0.1.3 Dom definition

### 0.1.4 Weakening Denotations

## 0.2 Weakening Theorems

### 0.2.1 Theorem 1

If  $\omega : \Gamma' \triangleright \Gamma$  and  $\Gamma \text{Ok}$  then  $\Gamma' \text{Ok}$

**Proof** TODO: this

### 0.2.2 Theorem 2

If  $\Gamma \vdash t : \tau$  and  $\omega : \Gamma' \triangleright \Gamma$  then  $\Gamma' \vdash t : \tau$

**Proof** Proved in parallel with theorem 3 below

### 0.2.3 Theorem 3

If  $\omega : \Gamma' \triangleright \Gamma$  and  $\Delta = \llbracket \Gamma \vdash t : \tau \rrbracket_M$  and  $\Delta' = \llbracket \Gamma' \vdash t : \tau \rrbracket_M$  then

$$\Delta \circ \llbracket \omega \rrbracket_M = \Delta' : \Gamma' \rightarrow \llbracket \tau \rrbracket_M$$

**Proof** TODO: this, induct over typing relation/definition of Denotations