

Explain that we implicitly carry around a derivation in the denotation

- Denotation for each typing relation derivation
  - (Unit)  $\frac{}{\llbracket \Gamma \vdash () : \mathbf{Unit} \rrbracket_M = \llbracket () \rrbracket_M \circ \langle \rangle_{\Gamma} : \Gamma \rightarrow \llbracket \mathbf{Unit} \rrbracket_M}$
  - (Const)  $\frac{}{\llbracket \Gamma \vdash \mathbf{C}^A : A \rrbracket_M = \llbracket \mathbf{C}^A \rrbracket_M \circ \langle \rangle_{\Gamma} : \Gamma \rightarrow \llbracket A \rrbracket_M}$
  - (True)  $\frac{}{\llbracket \Gamma \vdash \mathbf{true} : \mathbf{Bool} \rrbracket_M = \llbracket \mathbf{true} \rrbracket_M \circ \langle \rangle_{\Gamma} : \Gamma \rightarrow \llbracket \mathbf{Bool} \rrbracket_M}$
  - (False)  $\frac{}{\llbracket \Gamma \vdash \mathbf{false} : \mathbf{Bool} \rrbracket_M = \llbracket \mathbf{false} \rrbracket_M \circ \langle \rangle_{\Gamma} : \Gamma \rightarrow \llbracket \mathbf{Bool} \rrbracket_M}$
  - (Lambda)  $\frac{f = \llbracket \Gamma, x : A \rrbracket_M \mathbf{C} \mathbf{M}_{\epsilon} B : \Gamma \times A \rightarrow T_{\epsilon} B}{\llbracket \Gamma \vdash \lambda x : A. C : A \rightarrow \mathbf{M}_{\epsilon} B \rrbracket_M = \mathbf{cur}(f) : \Gamma \rightarrow (T_{\epsilon} B)^A}$
  - (Return)  $\frac{f = \llbracket \Gamma \vdash v : A \rrbracket_M}{\llbracket \Gamma \vdash \mathbf{return} v : \mathbf{M}_1 A \rrbracket_M = \eta_A \circ f}$
  - (Subtype)  $\frac{f = \llbracket \Gamma \vdash v : A \rrbracket_M : \Gamma \rightarrow A \text{ and } g = \llbracket A \leq B \rrbracket_M}{\llbracket \Gamma \vdash v : B \rrbracket_M = g \circ f : \Gamma \rightarrow B}$
  - (Subeffect)  $\frac{f = \llbracket \Gamma \vdash c : \mathbf{M}_{\epsilon_1} A \rrbracket_M : \Gamma \rightarrow T_{\epsilon_1} A \text{ and } g = \llbracket A \leq B \rrbracket_M \text{ and } h = \llbracket \epsilon_1 \leq \epsilon_2 \rrbracket_M}{\llbracket \Gamma \vdash C : \mathbf{M}_{\epsilon_2} B \rrbracket_M = h_B \circ T_{\epsilon_1} g \circ f}$
  - (If)  $\frac{f = \llbracket \Gamma \vdash v : \mathbf{Bool} \rrbracket_M \text{ and } g = \llbracket \Gamma \vdash C_1 : \mathbf{M}_{\epsilon} A \rrbracket_M \text{ and } h = \llbracket \Gamma \vdash C_2 : \mathbf{M}_{\epsilon} A \rrbracket_M}{\llbracket \Gamma \vdash \mathbf{if}_{\epsilon, A} v \mathbf{then} C_1 \mathbf{else} C_2 : \mathbf{M}_{\epsilon} A \rrbracket_M = \mathbf{if}_{\epsilon, A} \circ \langle f, \langle g, h \rangle \rangle : \Gamma \rightarrow T_{\epsilon} A}$
  - (Bind)  $\frac{f = \llbracket \Gamma \vdash C_1 : \mathbf{M}_{\epsilon_1} A : \Gamma \rightarrow T_{\epsilon_1} A \text{ and } g = \llbracket \Gamma, x : A \vdash C_2 : \mathbf{M}_{\epsilon_2} B \rrbracket_M : \Gamma \times A \rightarrow T_{\epsilon_2} B}{\llbracket \Gamma \vdash \mathbf{do} x \leftarrow C_1 \mathbf{in} C_2 : \mathbf{M}_{\epsilon_1 \cdot \epsilon_2} B \rrbracket_M = \mu_{\epsilon_1, \epsilon_2, B} \circ T_{\epsilon_1} g \circ \mathbf{t}_{\Gamma, A, \epsilon_1} \circ \langle \mathbf{Id}_{\Gamma}, f \rangle : \Gamma \rightarrow T_{\epsilon_1 \cdot \epsilon_2} B}$
- Denotations of Types
  - **Fill in from notebook**
  - For each ground type  $g \in \gamma$
  - morphism  $\llbracket A \leq B \rrbracket_M : \llbracket A \rrbracket_M \rightarrow \llbracket B \rrbracket_M$  for each  $A \leq B$
  - Natural Transformation  $\llbracket \epsilon_1 \leq \epsilon_2 \rrbracket_M : T_{\epsilon_1} \rightarrow T_{\epsilon_2}$  for each  $\epsilon_1 \leq \epsilon_2$