1 Semantics

1.1 Grammar and definitions

Value expressions

$$
\begin{aligned}
V \to{} & C^\gamma \text{ Typed contants} \\
& |\ () \text{ Unit constructor} \\
& |\ \lambda x : \tau.M \text{ term lambda expressions} \\
& |\ x \text{ Variables} \\
& |\ \Lambda\phi.V \text{ Effect lambda expressions} \\
& |\ V\epsilon \text{ Effect specialisation}
\end{aligned}
\tag{1}
$$

Computation expressions

$$
\begin{aligned}
M \to{} & V\ V \text{Application} \\
& |\ do\ x\ \leftarrow\ M\ in\ M \text{ Sequencing} \\
& |\ return\ V \text{ Unit computation} \\
& |\ Cons^{\tau_1 \to T_\xi \tau_2}(V) \text{ Computation Constructors} \\
& |\ Op^{T_\xi \tau_1 \to T_\xi \tau_2}(M) \text{ Computation Combinators}
\end{aligned}
\tag{2}
$$

Value types

$$
\begin{aligned}
\tau \to{} & \gamma \text{ Ground Types} \\
& |\ \tau \to \Theta \text{ Functions are kleisli arrows} \\
& |\ \forall\phi.\tau \text{ Effect quantification}
\end{aligned}
\tag{3}
$$

Computation (Monad) types

$$
\Theta \to T_\epsilon \tau \text{ Monad type constructor}
\tag{4}
$$

Effects

$$
\epsilon \to \phi \mid \xi.. \text{ Effect variables or ground effects}
$$

1.2 typing

Value typing rules

$$\text{(Const)}\frac{\Gamma\ OK}{\Gamma \vdash C^\gamma\colon \gamma}$$

$$\text{(Unit)}\frac{\Gamma\ OK}{\Gamma \vdash ()\colon unit}$$

$$\text{(Weaken)}\frac{\Gamma \vdash x\colon A}{\Gamma, x'\colon A' \vdash x\colon A}(x \neq x')$$

$$\text{(Var)}\frac{\Gamma\ OK}{\Gamma, x\colon A \vdash x\colon A}$$

$$\text{(Eff-Gen)}\frac{\Gamma \vdash V\colon A}{\Gamma \vdash \Lambda\phi.V\colon\forall\phi.A}\phi \notin fev(\Gamma)$$

$$\text{(Eff-Spec)}\frac{\Gamma \vdash V\colon\forall\phi.A}{\Gamma \vdash V\epsilon\colon A[\epsilon\phi]}$$

$$\text{(Lambda)}\frac{\Gamma, x\colon A \vdash M\colon\Theta}{\Gamma \vdash \lambda x\colon A.M\colon A \to \Theta}x \notin fv(\Gamma)$$

$$\text{(SubType)}\frac{\Gamma \vdash V\colon A}{\Gamma \vdash V\colon B}A \leq B$$

Computation Typing Rules

$$\text{(Application)}\frac{\Gamma \vdash V_1\colon A \to \Theta\ \ \Gamma \vdash V_2\colon A}{\Gamma \vdash V_1V_2\colon\Theta}$$

$$\text{(Sequencing)}\frac{\Gamma \vdash M_1\colon T_{\epsilon_1}A\ \ \Gamma, x\colon A \vdash M_2\colon T_{\epsilon_2}B}{\Gamma \vdash do\ x \leftarrow M_1\ in\ M_2\colon T_{\epsilon_1 \circ \epsilon_2}B}$$

$$\text{(Unit Computation)}\frac{\Gamma \vdash V\colon A}{\Gamma \vdash return(V)\colon T_1A}$$

$$\text{(Constructor)}\frac{\Gamma \vdash V\colon\tau_1}{\Gamma \vdash Cons^{\tau_1 \to T_\xi\tau_2}(V)\colon T_\xi\tau_2}$$

$$\text{(Combinator)}\frac{\Gamma \vdash M\colon T_{\xi_1}\tau_1}{\Gamma \vdash Op^{T_\xi\tau_1 \to T_\xi\tau_2}(M)\colon T_{\xi_2}\tau_2}$$

$$\text{(SubTypeC)}\frac{\Gamma \vdash M\colon\Theta_1}{\Gamma \vdash M\colon\Theta_2}\Theta_2 \leq \Theta_1$$

Subtyping Rules A ranges over value types

$$\text{(Reflexive)} \frac{}{A \leq A}$$

$$\text{(Transitive)} \frac{A \leq B \quad B \leq C}{A \leq C}$$

$$\text{(Function)} \frac{A' \leq A \quad \Theta \leq \Theta'}{A \to \Theta \leq A' \to \Theta'}$$

$$\text{(Quantification)} \frac{\tau_1 \leq \tau_2}{\Lambda\phi.\tau_1 \leq \Lambda\phi.\tau_2}$$

$$\text{(Computation)} \frac{\epsilon_1 \sqsubseteq \epsilon_2}{\Theta_{\epsilon_1} \leq \Theta_{\epsilon_2}}$$