0.1 Introduce Substitutions

0.1.1 Substitutions as SNOC lists

$$\sigma ::= \diamond \mid \sigma, x := v \tag{1}$$

Definition of σ

0.1.2 Trivial Properties of substitutions

$\mathtt{fv}(\sigma)$

$$\mathtt{fv}(\diamond) = \emptyset \tag{2}$$
$$\mathtt{fv}(\sigma, x := v) = \mathtt{fv}(\sigma) \cup \mathtt{fv}(v) \tag{3}$$

$\mathtt{dom}(\sigma)$

$$\mathtt{dom}(\diamond) = \emptyset \tag{4}$$
$$\mathtt{dom}(\sigma, x := v) = \mathtt{dom}(\sigma) \cup \{x\} \tag{5}$$

$x\#\sigma$

$$x\#\sigma \Leftrightarrow x \notin (\mathtt{fv}(\sigma) \cup \mathtt{dom}(\sigma`)) \tag{6}$$

0.1.3 Effect of substitutions

0.1.4 Well Formedness

0.1.5 Simple Properties Of Substitution

If $\Gamma' \vdash \sigma\colon \Gamma$ then: **TODO: Number these**

- $\Gamma\mathtt{Ok}$ and $\Gamma'\mathtt{Ok}$
- $\omega : \Gamma'' \triangleright \Gamma'$ implies $\Gamma'' \vdash \sigma\colon \Gamma$

0.2 Substitution Preserves Typing

TODO: State property TODO: Proof by induction overtype relation

0.3 Semantics of Substitution

0.3.1 Denotation of Substitutions

TODO: Fill in from p98

0.3.2 Lemma

TODO: Fill in from p98

0.3.3 Substitution Theorem

TODO: There is Tikz code here to draw the Substitution Theorem diagram, but it compiles v slowly If $\Gamma \vdash t\colon \tau$ and $\Gamma' \vdash \sigma\colon \Gamma$ then

0.4 Single Substitution