

# 1 Semantics

## 1.1 Grammar and definitions

### Value expressions

$$\begin{aligned} V \rightarrow C^\gamma & \text{ Typed constants} \\ | () & \text{ Unit constructor} \\ | \lambda x : \tau. M & \text{ term lambda expressions} \\ | x & \text{ Variables} \\ | \Lambda \phi. V & \text{ Effect lambda expressions} \\ | V \epsilon & \text{ Effect specialisation} \end{aligned} \tag{1}$$

### Computation expressions

$$\begin{aligned} M \rightarrow V & \text{ VApplication} \\ | do\ x \leftarrow M\ in\ M & \text{ Sequencing} \\ | return\ V & \text{ Unit computation} \\ | Cons^{\tau_1 \rightarrow T_\xi \tau_2}(V) & \text{ Computation Constructors} \\ | Op^{T_\xi \tau_1 \rightarrow T_\xi \tau_2}(M) & \text{ Computation Combinators} \end{aligned} \tag{2}$$

### Value types

$$\begin{aligned} \tau \rightarrow \gamma & \text{ Ground Types} \\ | \tau \rightarrow \Theta & \text{ Functions are kleisli arrows} \\ | \forall \phi. \tau & \text{ Effect quantification} \end{aligned} \tag{3}$$

### Computation (Monad) types

$$\Theta \rightarrow T_\epsilon \tau \text{ Monad type constructor} \tag{4}$$

### Effects

$$\epsilon \rightarrow \phi \mid \xi.. \text{ Effect variables or ground effects}$$

## 1.2 typing

### Value typing rules

$$\begin{aligned}
& (\text{Const}) \frac{\Gamma \text{ OK}}{\Gamma \vdash C^\gamma : \gamma} \\
& (\text{Unit}) \frac{\Gamma \text{ OK}}{\Gamma \vdash () : \text{unit}} \\
& (\text{Weaken}) \frac{\Gamma \vdash x : A}{\Gamma, x' : A' \vdash x : A} (x \neq x') \\
& (\text{Var}) \frac{\Gamma \text{ OK}}{\Gamma, x : A \vdash x : A} \\
& (\text{Eff-Gen}) \frac{\Gamma \vdash V : A}{\Gamma \vdash \Lambda \phi. V : \forall \phi. A} \phi \notin \text{fv}(\Gamma) \\
& (\text{Eff-Spec}) \frac{\Gamma \vdash V : \forall \phi. A}{\Gamma \vdash V \epsilon : A[\epsilon \phi]} \\
& (\text{Lambda}) \frac{\Gamma, x : A \vdash M : \Theta}{\Gamma \vdash \lambda x : A. M : A \rightarrow \Theta} x \notin \text{fv}(\Gamma) \\
& (\text{SubType}) \frac{\Gamma \vdash V : A}{\Gamma \vdash V : B} A \leq B
\end{aligned}$$

### Computation Typing Rules

$$\begin{aligned}
& (\text{Application}) \frac{\Gamma \vdash V_1 : A \rightarrow \Theta \quad \Gamma \vdash V_2 : A}{\Gamma \vdash V_1 V_2 : \Theta} \\
& (\text{Sequencing}) \frac{\Gamma \vdash M_1 : T_{\epsilon_1} A \quad \Gamma, x : A \vdash M_2 : T_{\epsilon_2} B}{\Gamma \vdash \text{do } x \leftarrow M_1 \text{ in } M_2 : T_{\epsilon_1 \circ \epsilon_2} B} \\
& (\text{Unit Computation}) \frac{\Gamma \vdash V : A}{\Gamma \vdash \text{return}(V) : T_1 A} \\
& (\text{Constructor}) \frac{\Gamma \vdash V : \tau_1}{\Gamma \vdash \text{Cons}^{\tau_1 \rightarrow T_\xi \tau_2}(V) : T_\xi \tau_2} \\
& (\text{Combinator}) \frac{\Gamma \vdash M : T_{\xi_1} \tau_1}{\Gamma \vdash \text{Op}^{T_\xi \tau_1 \rightarrow T_\xi \tau_2}(M) : T_{\xi_2} \tau_2} \\
& (\text{SubTypeC}) \frac{\Gamma \vdash M : \Theta_1}{\Gamma \vdash M : \Theta_2} \Theta_2 \leq \Theta_1
\end{aligned}$$

**Subtyping Rules**  $A$  ranges over value types

$$\text{(Reflexive)} \frac{}{A \leq A}$$

$$\text{(Transitive)} \frac{A \leq B \quad B \leq C}{A \leq C}$$

$$\text{(Function)} \frac{A' \leq A \quad \Theta \leq \Theta'}{A \rightarrow \Theta \leq A' \rightarrow \Theta'}$$

$$\text{(Quantification)} \frac{\tau_1 \leq \tau_2}{\Lambda \phi. \tau_1 \leq \Lambda \phi. \tau_2}$$

$$\text{(Computation)} \frac{\epsilon_1 \sqsubseteq \epsilon_2 \quad A \leq B}{T_{\epsilon_1} A \leq T_{\epsilon_2} B}$$