We need to define substitutions of effects on effects, effects on types, effects on terms, terms on terms.

0.1 Effect Substitutions

Define a substitution, σ as

$$\sigma ::= \diamond \mid \sigma, \alpha := \epsilon \tag{1}$$

Define the free-effect Variables of σ:

$$\begin{aligned} fev(\diamond) &= \emptyset \\ fev(\sigma, \alpha := \epsilon) &= fev(\sigma) \cup fev(\epsilon) \end{aligned}$$

We define the property:

$$\alpha \# \sigma \Leftrightarrow \alpha \notin (\mathtt{dom}(\sigma) \cup fev(\sigma)) \tag{2}$$

0.1.1 Action of Effect Substitution on Effects

Define the action of applying an effect substitution to an effect symbol:

$$\sigma(\epsilon) \tag{3}$$

$$\sigma(e) = e \tag{4}$$

$$\sigma(\epsilon_1 \cdot \epsilon_2) = (\sigma(\epsilon_1)) \cdot (\sigma(\epsilon_2)) \tag{5}$$

$$\diamond(\alpha) = \alpha \tag{6}$$

$$(\sigma, \beta := \epsilon)(\alpha) = \sigma(\alpha) \tag{7}$$

$$(\sigma, \alpha := \epsilon)(\alpha) = \epsilon \tag{8}$$

0.1.2 Action of Effect Substitution on Types

Define the action of applying an effect substitution, σ to a type τ as:

$$\tau\,[\sigma]$$

Defined as so

$$\gamma\,[\sigma] = \gamma \tag{9}$$

$$(A \to \mathtt{M}_\epsilon B)\,[\sigma] = (A\,[\sigma]) \to \mathtt{M}_{\sigma(\epsilon)}(B\,[\sigma]) \tag{10}$$

$$(\mathtt{M}_\epsilon A)\,[\sigma] = \mathtt{M}_{\sigma(\epsilon)}(A\,[\sigma]) \tag{11}$$

$$(\forall \alpha.A)\,[\sigma] = \forall \alpha.(A\,[\sigma]) \quad \text{If } \alpha \# \sigma \tag{12}$$

0.1.3 Action of Effect-Substitution on Type Environments

Define the action of effect substitution on type environments:

$$\Gamma\,[\sigma]$$

Defined as so:

$$\begin{aligned} \diamond\,[\sigma] &= \diamond \\ (\Gamma, x : A)\,[\sigma] &= (\Gamma\,[\sigma], x : (A\,[\sigma])) \end{aligned}$$

0.1.4 Action of Effect Substitution on Terms

Define the action of effect-substitution on terms:

$$
\begin{aligned}
x\,[\sigma] &= x && (13)\\
\mathtt{C}^{A}\,[\sigma] &= \mathtt{C}^{(A[\sigma])} && (14)\\
(\lambda x : A.C)\,[\sigma] &= \lambda x : (A\,[\sigma]).(C\,[\sigma]) && (15)\\
(\mathtt{if}_{\epsilon,A}\ v\ \mathtt{then}\ C_1\ \mathtt{else}\ C_2)\,[\sigma] &= \mathtt{if}_{\sigma(\epsilon),(A[\sigma])}\ v\,[\sigma]\ \mathtt{then}\ C_1\,[\sigma]\ \mathtt{else}\ C_2\,[\sigma] && (16)\\
(v_1\ v_2)\,[\sigma] &= (v_1\,[\sigma])\ v_2\,[\sigma] && (17)\\
(\mathtt{do}\ x \leftarrow C_1\ \mathtt{in}\ C_2) &= \mathtt{do}\ x \leftarrow (C_1\,[\sigma])\ \mathtt{in}\ (C_2\,[\sigma]) && (18)\\
(\Lambda\alpha.v)\,[\sigma] &= \Lambda\alpha.(v\,[\sigma]) \quad \text{If } \alpha\#\sigma && (19)\\
(v\ \epsilon)\,[\sigma] &= (v\,[\sigma])\ \sigma(\epsilon) && (20)\\
& && (21)
\end{aligned}
$$

0.1.5 Well-Formed-ness

For any two effect-environments, and a substitution, define the well-formed-ness relation:

$$\Phi' \vdash \sigma \colon \Phi \tag{22}$$

- (Nil) $\frac{\Phi' \mathtt{Ok}}{\Phi' \vdash \diamond : \diamond}$
- (Extend) $\frac{\Phi' \vdash \sigma : \Phi \quad \Phi' \vdash \epsilon \quad \alpha \notin \Phi}{\Phi' \vdash \sigma, \alpha := \epsilon : (\Phi, \alpha)}$

0.1.6 Property 1

If $\Phi' \vdash \sigma \colon \Phi$ then $\Phi' \mathtt{Ok}$ (By the Nil case) and $\Phi \mathtt{Ok}$ Since each use of the extend case preserves $\mathtt{Ok}$.

0.1.7 Property 2

If $\Phi' \vdash \sigma \colon \Phi$ then $\omega : \Phi' \triangleright \Phi' \implies \Phi'' \vdash \sigma \colon \Phi$ since $\Phi' \vdash \epsilon \implies \Phi'' \vdash \epsilon$ and $\Phi' \mathtt{Ok} \implies \Phi'' \mathtt{Ok}$

0.1.8 Property 3

If $\Phi' \vdash \sigma \colon \Phi$ then

$$\alpha \notin \Phi \wedge \alpha \notin \Phi' \implies (\Phi', \alpha) \vdash (\sigma, \alpha := \alpha) \colon (\Phi, \alpha) \tag{23}$$

Since $\iota\pi : \Phi', \alpha \triangleright \Phi'$ so $\Phi', \alpha \vdash \sigma \colon \Phi$ and $\Phi', \alpha \vdash \alpha$

0.2 Substitution Preserves the Well-formed-ness of Effects

I.e.

$$\Phi \vdash \epsilon \wedge \Phi' \vdash \iota \colon \Phi \implies \Phi' \vdash \sigma(\epsilon) \tag{24}$$

Proof:

Case Ground: $\sigma(e) = e$, so $\Phi' \vdash \sigma(\epsilon)$ holds.

Case Multiply: By inversion, $\Phi \vdash \epsilon_1$ and $\Phi \vdash \epsilon_2$ so $\Phi' \vdash \sigma(\epsilon_1)$ and $\Phi' \vdash \sigma(\epsilon_2)$ by induction and hence $\Phi' \vdash \sigma(\epsilon_1 \cdot \epsilon_2)$

Case Var: By inversion, $\Phi = \Phi'', \alpha$ and $\Phi'', \alpha\mathtt{Ok}$. Hence byu case splitting on ι, we see that $\sigma = \sigma', \alpha := \epsilon$.

So by inversion, $\sigma \vdash \epsilon$ so $\Phi' \vdash \sigma(\alpha) = \epsilon$

Case Weaken: By inversion$\Phi = \Phi'', \beta$ and $\Phi'' \vdash \alpha$, so $\sigma = \sigma'\beta := \epsilon$.

So $\Phi' \vdash \sigma' \colon \Phi''$.

hence by induction, $\Phi' \vdash \sigma'(a)$, so $\Phi' \vdash \sigma(\alpha)$ since $\alpha \neq \beta$)

0.2.1 Effect Subsitution preserves the sub-effect relation

If $\Phi' \vdash \sigma \colon \Phi$ and $\epsilon_1 \leq_\Phi \epsilon_2$, then $\epsilon_1 [\sigma] \leq_{\Phi'} \epsilon_2 [\sigma]$.

Proof: For any ground substitution σ' of Φ', then $\sigma\sigma'$ (the substitution σ' applied after σ) is also a ground substitution.

So $\epsilon_1 [\sigma] [\sigma'] \leq \epsilon_2 [\sigma] [\sigma']$.

So $\epsilon_1 [\sigma] \leq_{\Phi'} \epsilon_2 [\sigma]$.

0.2.2 Substitution preserves well-formed-ness of Types

$$\Phi' \vdash \sigma \colon \Phi \wedge \Phi \vdash A \implies \Phi' \vdash A [\sigma] \tag{25}$$

Proof:

Case Ground: $\Phi'\mathtt{Ok}$ so $\Phi' \vdash \gamma$ and $\gamma [\sigma] = \gamma$.

Hence $\Phi' \vdash \gamma [\sigma]$.

Case Lambda: By inversion$\Phi \vdash A$ and $\Phi \vdash B$.

So by induction, $\Phi' \vdash A [\sigma]$ and $\Phi' \vdash B [\sigma]$.

So

$$\Phi' \vdash (A [\sigma]) \to (B [\sigma]) \tag{26}$$

So

$$\Phi' \vdash (A \to B) [\sigma] \tag{27}$$

Case Computation: By inversion, $\Phi \vdash \epsilon$ and $\Phi \vdash A$ so by induction and substitution of effect preserving effect-well-formed-ness,

$\Phi' \vdash \sigma(\epsilon)$ and $\Phi' \vdash A [\sigma]$ so $\Phi \vdash \mathtt{M}_{\sigma(\epsilon)} A [\sigma]$ so $\Phi' \vdash (\mathtt{M}_\epsilon A) [\sigma]$

Case For All: By inversion, $\Phi, \alpha \vdash A$. So by picking $\alpha \notin \Phi \wedge \alpha \notin \Phi'$ using α-equivalence, we have $(\Phi', \alpha) \vdash (\sigma\alpha := \alpha) \colon (\Phi, \alpha)$.

So by induction $(\Phi, \alpha) \vdash A [\sigma, \alpha := \alpha]$

So $(\Phi', \alpha) \vdash A [\sigma]$

So $\Phi' \vdash (\forall \alpha . A) [\sigma]$

0.2.3 Subtitution of effects preserves Sub-Typing Relation

If $\Phi' \vdash \sigma \colon \Phi$ and $A \leq:_\Phi B$ then $A [\sigma] \leq:_{\Phi'} B [\sigma]$

Proof: By induction on the sub-typing relation

Case Ground: By inversion, $A \leq:_\gamma B$, so A, B are ground types. Hence $A [\sigma] = A$ and $B [\sigma] = B$. So $A [\sigma] \leq:_{\Phi'} B [\sigma]$

Case Fn: By inversion, $A' \leq:_\Phi A$ and $B \leq:_\Phi B'$.
So by induction, $A'[\sigma] \leq:_{\Phi'} A[\sigma]$ and $B[\sigma] \leq:_{\Phi'} B'[\sigma]$.
So $(A[\sigma]) \rightarrow (B[\sigma]) \leq:_{\Phi'} (A'[\sigma]) \rightarrow (B'[\sigma])$
So $(A \rightarrow B)[\sigma] \leq:_{\Phi'} (A' \rightarrow B')[\sigma]$

Case Computation: By inversion, $A \leq:_\Phi B$, $\epsilon_1 \leq_\Phi \epsilon_2$.
So by induction and substitution preserving the sub-effect relation,
$A[\sigma] \leq:_{\Phi'} B[\sigma]$ and $\sigma(\epsilon_1) \leq_{\Phi'} \sigma(\epsilon_2)$
So $\mathtt{M}_{\sigma(\epsilon_1)}(A[\sigma]) \leq:_{\Phi'} \mathtt{M}_{\sigma(\epsilon_2)}(B[\sigma])$
So $(\mathtt{M}_{\epsilon_1} A)[\sigma] \leq:_{\Phi'} (\mathtt{M}_{\epsilon_2} B)[\sigma]$

0.2.4 Substitution preserves well-formed-ness of Type Environments

If $\Phi \vdash \Gamma \mathtt{Ok}$ and $\Phi' \vdash \sigma \colon \Phi$ then $\Phi' \vdash \Gamma[\sigma]\,\mathtt{Ok}$

Proof:

Case Nil: $\Phi\mathtt{Ok} \implies \Phi'\mathtt{Ok}$ so $\Phi' \vdash \diamond\mathtt{Ok}$ and $\diamond[\sigma] = \diamond$

Case Var: By inversion, $\Phi \vdash \Gamma\mathtt{Ok}$ and $\Phi \vdash A$.
By induction and substitution preserving well-formed-ness of types, $\Phi' \vdash \Gamma'[\sigma]\,\mathtt{Ok}$ and $\Phi' \vdash A[\sigma]$.
So $\Phi' \vdash (\Gamma'[\sigma], x : A[\sigma])\mathtt{Ok}$.
Hence $\Phi' \vdash \Gamma, x : A[\sigma]\,\mathtt{Ok}$.

0.2.5 Effect-Polymorphism Preserves the Typing Relation

If $\Phi' \vdash \sigma \colon \Phi$ and $\Phi \mid \Gamma \vdash v \colon A$, then $\Phi' \mid \Gamma[\sigma] \vdash v[\sigma] : A[\sigma]$

Proof:

Case Const: By inversion, $\Phi \vdash \Gamma\mathtt{Ok}$.
So $\Phi' \vdash \Gamma\mathtt{Ok}$
So $\Phi' \mid \Gamma[\sigma] \vdash \mathtt{C}^{A[\sigma]} \colon A[\sigma]$

Case True, False, Unit: The logic is the same for each of these cases, so we look at the case `true` only.
By inversion, $\Phi \vdash \Gamma\mathtt{Ok}$.
So $\Phi' \vdash \Gamma\mathtt{Ok}$
So $\Phi' \mid \Gamma[\sigma] \vdash \mathtt{true} \colon \mathtt{Bool}$
Since $\mathtt{true}[\sigma] = \mathtt{true}$ and $\mathtt{Bool}[\sigma] = \mathtt{Bool}$.

Case Var: By inversion$\Gamma = \Gamma', x : A$ and $\Phi \vdash \Gamma', x : A\mathtt{Ok}$.
So since substitution preserves well-formed-ness of type environments, $\Phi' \vdash \Gamma'[\sigma], x : A[\sigma]\,\mathtt{Ok}$
So $\Phi' \mid \Gamma[\sigma] \vdash x \colon A[\sigma]$
Since $x[\sigma] = x$

Case Weaken: By inversion$\Gamma = \Gamma', y : B$, $\Phi \vdash B$, and $\Phi \mid \Gamma' \vdash x \colon A$. $x \neq y$
By induction and the theorem that effect-substitution preserves type well-formed-ness, we have: $\Phi' \mid \Gamma'[\sigma] \vdash x \colon A[\sigma]$ and $\Phi' \vdash B[\sigma]$
So $\Phi' \mid \Gamma[\sigma] \vdash x[\sigma] : A[\sigma]$
Since $x[\sigma] = x$, $\Gamma[\sigma] = (\Gamma'[\sigma], y : B[\sigma])$

Case Lambda: By inversion$\Phi \mid \Gamma, x : A \vdash v\!: B$.

So, by induction $\Phi' \mid (\Gamma, x : A)\,[\sigma] \vdash v\,[\sigma] : B\,[\sigma]$.

So, $\Phi \mid \Gamma\,[\sigma]\,, x : A\,[\sigma] \vdash v\,[\sigma] : B\,[\sigma]$.

Hence by the lambda type rule,

$\Phi' \mid \Gamma\,[\sigma] \vdash \lambda x : A\,[\sigma]\,.v\,[\sigma] : (A\,[\sigma]) \to (B\,[\sigma])$

So

$\Phi' \mid \Gamma\,[\sigma] \vdash (\lambda x : A.v)\,[\sigma] : (A \to B)\,[\sigma])$

Case Apply: By inversion, $\Phi \mid \Gamma \vdash v_1\!: A \to B$, $\Phi \mid \Gamma \vdash V_2\!: A$.

So by induction, $\Phi' \mid \Gamma\,[\sigma] \vdash v_1\,[\sigma] : (A\,[\sigma]) \to (B\,[\sigma])$.

So $\Phi' \mid \Gamma\,[\sigma] \vdash (v_1\,[\sigma])\ (v_2\,[\sigma])\!: B\,[\sigma]$.

So $\Phi' \mid \Gamma\,[\sigma] \vdash (v_1\ v_2)\,[\sigma] : (A \to B)\,[\sigma]$

Case Subtype: By inversion, $\Phi \mid \Gamma \vdash v\!: A$ and $\Phi \vdash A \leq: B$

So by induction and effect-substitution preserving subtyping, $\Phi' \mid \Gamma\,[\sigma] \vdash v\,[\sigma] : A\,[\sigma]$ and $\Phi' \vdash A\,[\sigma] \leq: B\,[\sigma]$

So $\Phi' \mid \Gamma\,[\sigma] \vdash v\,[\sigma] : B\,[\sigma]$

Case Return: By inversion, $\Phi \mid \Gamma \vdash v\!: A$

So by induction, $\Phi' \mid \Gamma\,[\sigma] \vdash v\,[\sigma] : A\,[\sigma]$

So $\Phi' \mid \Gamma\,[\sigma] \vdash \texttt{return}(v\,[\sigma])\!: \texttt{M}_{\mathbf{1}}(A\,[\sigma])$

Hence $\Phi' \mid \Gamma\,[\sigma] \vdash (\texttt{return}v)\,[\sigma] : (\texttt{M}_{\mathbf{1}}A)\,[\sigma]$

Case Bind: By inversion, $\Phi \mid \Gamma \vdash v_1\!: \texttt{M}_{\epsilon_1}A$ and $\Phi \mid \Gamma, x : A \vdash v_2\!: \texttt{M}_{\epsilon_2}B$.

So by induction: $\Phi' \mid \Gamma\,[\sigma] \vdash v_1\,[\sigma] : \texttt{M}_{\sigma(\epsilon_1)}(A\,[\sigma])$, and $\Phi' \mid \Gamma\,[\sigma]\,, x : A\,[\sigma] \vdash v_2\!: \texttt{M}_{\sigma(\epsilon_2)}(B\,[\sigma])$.

And so $\Phi' \mid \Gamma\,[\sigma] \vdash \texttt{do}\ x \leftarrow (v_1\,[\sigma])\ \texttt{in}\ (v_2\,[\sigma])\!: \texttt{M}_{\sigma(\epsilon_1)\cdot(\epsilon_2[\sigma])}B\,[\sigma]$

Case If: By inversion, $\Phi \mid \Gamma \vdash v\!: \texttt{Bool}$, $\Phi \mid \Gamma \vdash v_1\!: A$, and $\Phi \mid \Gamma \vdash v_2\!: A$

So by induction $\Phi' \mid \Gamma\,[\sigma] \vdash v\,[\sigma] : \texttt{Bool}$, $\Phi' \mid \Gamma\,[\sigma] \vdash v_1\!: A\,[\sigma]$, and $\Phi' \mid \Gamma\,[\sigma] \vdash v\,[\sigma] : \texttt{Bool}$, $\Phi' \mid \Gamma\,[\sigma] \vdash v_2\!: A\,[\sigma]$. (Since $\texttt{Bool}\,[\sigma] = \texttt{Bool}$)

Hence:

$\Phi' \mid \Gamma\,[\sigma] \vdash \texttt{if}_{A[\sigma]}\ v\,[\sigma]\ \texttt{then}\ v_1\,[\sigma]\ \texttt{else}\ v_2\,[\sigma] : A\,[\sigma]$

So $\Phi' \mid \Gamma\,[\sigma] \vdash (\texttt{if}_A\ v\ \texttt{then}\ v_1\ \texttt{else}\ v_2)\,[\sigma] : A\,[\sigma]$

Case Effect-lambda: By inversion, $\Phi, \alpha \mid \Gamma \vdash v\!: A$.

So by the substitution property 3 (**TODO: Is this correct/reference correctly**), pick $\alpha \notin \Phi' \wedge \alpha \notin \Phi$ so we have:

$$(\Phi', \alpha) \vdash (\sigma, \alpha := \alpha)\!: (\Phi, \alpha)$$

So by induction, $\Phi', \alpha \mid \Gamma\,[\sigma, \alpha := \alpha] \vdash v\,[\sigma, \alpha := \alpha] : A\,[\sigma, \alpha := \alpha]$

So $\Phi', \alpha \mid \Gamma\,[\sigma] \vdash v\,[\sigma] : A\,[\sigma]$ since $\alpha \notin \Phi' \wedge \alpha \notin \Phi$.

So $\Phi' \mid \Gamma\,[\sigma] \vdash v\,[\sigma] : (\forall \alpha.A)\,[\sigma]$

Case Effect-Apply: By inversion, $\Phi \mid \Gamma \vdash v\!: \forall \alpha.A$, $\Phi \vdash \epsilon$.

So by induction and effect-substitution preserving well-formed-ness of effects: $\Phi' \mid \Gamma\,[\sigma] \vdash v\,[\sigma] : (\forall \alpha.A)\,[\sigma]$ and $\Phi' \vdash \sigma(\epsilon)$

So $\Phi' \mid \Gamma\,[\sigma] \vdash (v\,[\sigma])\ (\sigma(\epsilon))\!: A\,[\sigma]\,[\sigma(\epsilon)/\alpha]$.

Since $\alpha \# \sigma$, we can commute the applications of substitution. **TODO: Do I need to prove this?**

So, $\Phi' \mid \Gamma\,[\sigma] \vdash (v\ \epsilon)\,[\sigma] : A\,[\epsilon/\alpha]\,[\sigma]$

0.3 Term-Term Substitutions

0.3.1 Substitutions as SNOC lists

$$\sigma ::= \diamond \mid \sigma, x := v \tag{28}$$

0.3.2 Trivial Properties of substitutions

$\texttt{fv}(\sigma)$

$$\texttt{fv}(\diamond) = \emptyset \tag{29}$$
$$\texttt{fv}(\sigma, x := v) = \texttt{fv}(\sigma) \cup \texttt{fv}(v) \tag{30}$$

$\texttt{dom}(\sigma)$

$$\texttt{dom}(\diamond) = \emptyset \tag{31}$$
$$\texttt{dom}(\sigma, x := v) = \texttt{dom}(\sigma) \cup \{x\} \tag{32}$$

$x\#\sigma$

$$x\#\sigma \Leftrightarrow x \notin (\texttt{fv}(\sigma) \cup \texttt{dom}(\sigma`)) \tag{33}$$

0.3.3 Action of substitutions

We define the action of applying a substitution σ as

$$t\,[\sigma]$$

$$x\,[\diamond] = x \tag{34}$$
$$x\,[\sigma, x := v] = v \tag{35}$$
$$x\,[\sigma, x' := v'] = x\,[\sigma] \quad \text{If } x \neq x' \tag{36}$$
$$\texttt{C}^A\,[\sigma] = \texttt{C}^A \tag{37}$$
$$(\lambda x : A.C)\,[\sigma] = \lambda x : A.(C\,[\sigma]) \quad \text{If } x\#\sigma \tag{38}$$
$$(\texttt{if}_{\epsilon,A}\ v\ \texttt{then}\ C_1\ \texttt{else}\ C_2)\,[\sigma] = \texttt{if}_{\epsilon,A}\ v\,[\sigma]\ \texttt{then}\ C_1\,[\sigma]\ \texttt{else}\ C_2\,[\sigma] \tag{39}$$
$$(v_1\ v_2)\,[\sigma] = (v_1\,[\sigma])\ v_2\,[\sigma] \tag{40}$$
$$(\texttt{do}\ x \leftarrow C_1\ \texttt{in}\ C_2) = \texttt{do}\ x \leftarrow (C_1\,[\sigma])\ \texttt{in}\ (C_2\,[\sigma]) \quad \text{If } x\#\sigma \tag{41}$$
$$(\Lambda\alpha.v)\,[\sigma] = \Lambda\alpha.(v\,[\sigma]) \tag{42}$$
$$(v\ \epsilon)\,[\sigma] = (v\,[\sigma])\ \epsilon \tag{43}$$
$$\tag{44}$$

0.3.4 Well-Formed-ness

0.3.5 Simple Properties Of Substitution

If $\Gamma' \vdash \sigma\colon \Gamma$ then:

Property 1: $\Gamma\texttt{Ok}$ and $\Gamma'\texttt{Ok}$ Since $\Gamma'\texttt{Ok}$ holds by the Nil-axiom. $\Gamma\texttt{Ok}$ holds by induction on the well-formed-ness relation.

Property 2: $\omega : \Gamma'' \triangleright \Gamma'$ implies $\Gamma'' \vdash \sigma\colon \Gamma$. By induction over well-formed-ness relation. For each $x := v$ in σ, $\Gamma'' \vdash v\colon A$ holds if $\Gamma' \vdash v\colon A$ holds.

Property 3: $x \notin (\mathtt{dom}(\Gamma) \cup \mathtt{dom}(\Gamma''))$ **implies** $(\Gamma', x : A) \vdash (\sigma, x := x) \colon (\Gamma, x : A)$ Since $\iota\pi : \Gamma', x : A \triangleright \Gamma'$, so by (Property 2) **TODO: Better referencing here**,

$$\Gamma', x : A \vdash \sigma \colon \Gamma$$

In addition, $\Gamma', x : A \vdash x \colon A$ trivially, so by the rule **Extend**, well-formed-ness holds for

$$(\Gamma', x : A) \vdash (\sigma, x := v) \colon (\Gamma, x : A) \tag{45}$$

0.4 Substitution Preserves Typing

0.4.1 Variables

Case Var

Case Weaken

0.4.2 Other Value Terms

Case Lambda

Case Constants

0.4.3 Computation Terms

Case Return

Case Apply

Case If

Case Bind

0.4.4 Sub-typing and Sub-effecting

Case Sub-type

Case Sub-effect

0.5 Semantics of Substitution

0.5.1 Denotation of Substitutions

0.5.2 Extension Lemma

0.5.3 Substitution Theorem

0.5.4 Proof For Value Terms

Case Var

Case Weaken

Case Constants

Case Lambda

Case Sub-type

0.5.5 Proof For Computation Terms

Case Return

Case Apply

Case If

Case Bind

Case Subeffect

0.6 The Identity Substitution

0.6.1 Properties of the Identity Substitution

Property 1

Property 2