

0.1 Beta and Eta Equivalence

0.1.1 Beta-Eta conversions

- (Lambda-Beta) $\frac{\Phi|\Gamma, x:A \vdash v:B \quad \Phi|\Gamma \vdash v:A}{\Phi|\Gamma \vdash (\lambda x:A.v) \ v =_{\beta\eta} v[x/v]:B}$
- (Lambda-Eta) $\frac{\Phi|\Gamma \vdash v:A \rightarrow B}{\Phi|\Gamma \vdash \lambda x:A.(v \ x) =_{\beta\eta} v:A \rightarrow B}$
- (Left Unit) $\frac{\Phi|\Gamma \vdash v:A \quad \Phi|\Gamma, x:A \vdash v:\mathbf{M}_\epsilon B}{\Phi|\Gamma \vdash \mathbf{do} \ x \leftarrow \mathbf{return} \ v \ \mathbf{in} \ v =_{\beta\eta} v[V/x]:\mathbf{M}_\epsilon B}$
- (Right Unit) $\frac{\Phi|\Gamma \vdash v:\mathbf{M}_\epsilon A}{\Phi|\Gamma \vdash \mathbf{do} \ x \leftarrow v \ \mathbf{in} \ \mathbf{return} \ x =_{\beta\eta} v:\mathbf{M}_\epsilon A}$
- (Associativity) $\frac{\Phi|\Gamma \vdash v_1:\mathbf{M}_{\epsilon_1} A \quad \Phi|\Gamma, x:A \vdash v_2:\mathbf{M}_{\epsilon_2} B \quad \Phi|\Gamma, y:B \vdash v_3:\mathbf{M}_{\epsilon_3} C}{\Phi|\Gamma \vdash \mathbf{do} \ x \leftarrow v_1 \ \mathbf{in} \ (\mathbf{do} \ y \leftarrow v_2 \ \mathbf{in} \ v_3) =_{\beta\eta} \mathbf{do} \ y \leftarrow (\mathbf{do} \ x \leftarrow v_1 \ \mathbf{in} \ v_2) \ \mathbf{in} \ v_3:\mathbf{M}_{\epsilon_1 \cdot \epsilon_2 \cdot \epsilon_3} C}$
- (Unit) $\frac{\Phi|\Gamma \vdash v:\mathbf{Unit}}{\Phi|\Gamma \vdash v =_{\beta\eta} ():\mathbf{Unit}}$
- (if-true) $\frac{\Phi|\Gamma \vdash v_1:A \quad \Phi|\Gamma \vdash v_2:A}{\Phi|\Gamma \vdash \mathbf{if}_{A, \ \mathbf{true}} \ \mathbf{then} \ v_1 \ \mathbf{else} \ v_2 =_{\beta\eta} v_1:A}$
- (if-false) $\frac{\Phi|\Gamma \vdash v_2:A \quad \Phi|\Gamma \vdash v_1:A}{\Phi|\Gamma \vdash \mathbf{if}_{A, \ \mathbf{false}} \ \mathbf{then} \ v_1 \ \mathbf{else} \ v_2 =_{\beta\eta} v_2:A}$
- (If-Eta) $\frac{\Phi|\Gamma, x:\mathbf{Bool} \vdash v:A \quad \Phi|\Gamma \vdash v:\mathbf{Bool}}{\Phi|\Gamma \vdash \mathbf{if}_{A, \ v} \ \mathbf{then} \ v[\mathbf{true}/x] \ \mathbf{else} \ v[\mathbf{false}/x] =_{\beta\eta} v[V/x]:A}$
- (Effect-beta) $\frac{\Phi \vdash \epsilon \quad \Phi, \alpha|\Gamma \vdash v:A}{\Phi|\Gamma \vdash (\Lambda \alpha.v \ \epsilon) =_{\beta\eta} v[\epsilon/\alpha]:A[\epsilon/\alpha]}$
- (Effect-eta) $\frac{\Phi|\Gamma \vdash v:\forall \alpha.A}{\Phi|\Gamma \vdash \Lambda \alpha.(v \ \alpha) =_{\beta\eta} v:\forall \alpha.A}$

0.1.2 Equivalence Relation

- (Reflexive) $\frac{\Phi|\Gamma \vdash v:A}{\Phi|\Gamma \vdash v =_{\beta\eta} v:A}$
- (Symmetric) $\frac{\Phi|\Gamma \vdash v_1 =_{\beta\eta} v_2:A}{\Phi|\Gamma \vdash v_2 =_{\beta\eta} v_1:A}$
- (Transitive) $\frac{\Phi|\Gamma \vdash v_1 =_{\beta\eta} v_2:A \quad \Phi|\Gamma \vdash v_2 =_{\beta\eta} v_3:A}{\Phi|\Gamma \vdash v_1 =_{\beta\eta} v_3:A}$

0.1.3 Congruences

- (Effect-Abs) $\frac{\Phi, \alpha|\Gamma \vdash v_1 =_{\beta\eta} v_2:A}{\Phi|\Gamma \vdash \Lambda \alpha.v_1 =_{\beta\eta} \Lambda \alpha.v_2:\forall \alpha.A}$
- (Effect-Apply) $\frac{\Phi|\Gamma \vdash v_1 =_{\beta\eta} v_2:\forall \alpha.A \quad \Phi \vdash \epsilon}{\Phi|\Gamma \vdash v_1 \ \epsilon =_{\beta\eta} v_2 \ \epsilon:A[\epsilon/\alpha]}$
- (Lambda) $\frac{\Phi|\Gamma, x:A \vdash v_1 =_{\beta\eta} v_2:B}{\Phi|\Gamma \vdash \lambda x:A.v_1 =_{\beta\eta} \lambda x:A.v_2:A \rightarrow B}$
- (Return) $\frac{\Phi|\Gamma \vdash v_1 =_{\beta\eta} v_2:A}{\Phi|\Gamma \vdash \mathbf{return} \ v_1 =_{\beta\eta} \mathbf{return} \ v_2:\mathbf{M}_1 A}$
- (Apply) $\frac{\Phi|\Gamma \vdash v_1 =_{\beta\eta} v'_1:A \rightarrow B \quad \Phi|\Gamma \vdash v_2 =_{\beta\eta} v'_2:A}{\Phi|\Gamma \vdash v_1 \ v_2 =_{\beta\eta} v'_1 \ v'_2:B}$
- (Bind) $\frac{\Phi|\Gamma \vdash v_1 =_{\beta\eta} v'_1:\mathbf{M}_{\epsilon_1} A \quad \Phi|\Gamma, x:A \vdash v_2 =_{\beta\eta} v'_2:\mathbf{M}_{\epsilon_2} B}{\Phi|\Gamma \vdash \mathbf{do} \ x \leftarrow v_1 \ \mathbf{in} \ v_2 =_{\beta\eta} \mathbf{do} \ c \leftarrow v'_1 \ \mathbf{in} \ v'_2:\mathbf{M}_{\epsilon_1 \cdot \epsilon_2} B}$
- (If) $\frac{\Phi|\Gamma \vdash v =_{\beta\eta} v':\mathbf{Bool} \quad \Phi|\Gamma \vdash v_1 =_{\beta\eta} v'_1:A \quad \Phi|\Gamma \vdash v_2 =_{\beta\eta} v'_2:A}{\Phi|\Gamma \vdash \mathbf{if}_{A, \ v} \ \mathbf{then} \ v_1 \ \mathbf{else} \ v_2 =_{\beta\eta} \mathbf{if}_{A, \ v'} \ \mathbf{then} \ v'_1 \ \mathbf{else} \ v'_2:A}$
- (Subtype) $\frac{\Phi|\Gamma \vdash v =_{\beta\eta} v':A \quad A \leq B}{\Phi|\Gamma \vdash v =_{\beta\eta} v':B}$

0.2 Beta-Eta Equivalence Implies Both Sides Have the Same Type

0.2.1 Equivalence Relations

Case Symmetric

Case Transitive

0.2.2 Beta conversions

Case Lambda

Case Associativity

Case Eta

Case If-True

0.2.3 Congruences

Case Lambda

Case Return

Case Apply

Case Bind

Case If

Case Subtype

Case subeffect

0.3 Beta-Eta equivalent terms have equal denotations

0.3.1 Equivalence Relation

Case Reflexive

Case Symmetric

Case Transitive

0.3.2 Beta Conversions

Case Lambda

Case Left Unit

Case Right Unit

Case Associative

Case Eta

Case If-True

Case If-False

0.3.3 Case If-Eta

0.3.4 Congruences

Case Lambda

Case Return

Case Apply

Case Bind

Case If

Case Subtype

Case subeffect