

## 0.1 Terms

### 0.1.1 Value Terms

$$\begin{aligned}
 v ::= & x \\
 & | \lambda x : A. C \\
 & | \mathbf{c}^A \\
 & | () \\
 & | \mathbf{true} \mid \mathbf{false}
 \end{aligned} \tag{1}$$

### 0.1.2 Computation Terms

$$\begin{aligned}
 C ::= & \mathbf{if}_{\epsilon, A} \quad v \quad \mathbf{then} \quad C_1 \quad \mathbf{else} \quad C_2 \\
 & | v_1 \quad v_2 \\
 & | \mathbf{do} \quad x \leftarrow C_1 \quad \mathbf{in} \quad C_2 \\
 & | \mathbf{return} v
 \end{aligned} \tag{2}$$

## 0.2 Type System

### 0.2.1 Effects

The effects should form a monotonous, pre-ordered monoid  $(E, \cdot, 1, \leq)$  with elements  $\epsilon$

### 0.2.2 Types

**Ground Types** There exists a set  $\gamma$  of ground types, including `Unit`, `Bool`

**Value Types**

$$A, B, C ::= \gamma \mid A \rightarrow \mathbf{M}_\epsilon B$$

**Computation Types** Computation types are of the form  $\mathbf{M}_\epsilon A$

### 0.2.3 Sub-typing

There exists a sub-typing pre-order relation  $\leq_\gamma$  over ground types that is:

- (Reflexive)  $\overline{A \leq_\gamma A}$
- (Transitive)  $\frac{A \leq_\gamma B \quad B \leq_\gamma C}{A \leq_\gamma C}$

We extend this relation with the function sub-typing rule to yield the full sub-typing relation  $\leq$ :

- (ground)  $\frac{A \leq_\gamma B}{A \leq B}$
- (Fn)  $\frac{A \leq_\gamma A' \quad B' \leq_\gamma B \quad \epsilon \leq \epsilon'}{A' \rightarrow \mathbf{M}_{\epsilon'} B' \leq A \rightarrow \mathbf{M}_\epsilon B}$

### 0.2.4 Type Environments

An environment,  $G ::= \diamond \mid \Gamma, x : A$

### Domain Function

- $\text{dom}(\diamond) = \emptyset$
- $\text{dom}(\Gamma, x : A) = \text{dom}(\Gamma) \cup \{x\}$

### Ok Predicate

- $(\text{Atom}) \frac{}{\text{Ok}}$
- $(\text{Var}) \frac{\Gamma \text{Ok} \not\in \text{dom}(\Gamma)}{\Gamma, x : A \text{Ok}}$

## 0.2.5 Type Rules

### Value Typing Rules

- $(\text{Const}) \frac{\Gamma \text{Ok}}{\Gamma \vdash C^A : A}$
- $(\text{Unit}) \frac{\Gamma \text{Ok}}{\Gamma \vdash () : \text{Unit}}$
- $(\text{True}) \frac{\Gamma \text{Ok}}{\Gamma \vdash \text{true} : \text{Bool}}$
- $(\text{False}) \frac{\Gamma \text{Ok}}{\Gamma \vdash \text{false} : \text{Bool}}$
- $(\text{Var}) \frac{\Gamma, x : A \text{Ok}}{\Gamma, x : A \vdash X : A}$
- $(\text{Weaken}) \frac{\Gamma \vdash x : A}{\Gamma, y : B \vdash X : A} \text{ (if } x \neq y \text{)}$
- $(\text{Fn}) \frac{\Gamma, x : A \vdash C : M_\epsilon B}{\Gamma \vdash \lambda x : A. C : A \rightarrow M_\epsilon B}$
- $(\text{Sub}) \frac{\Gamma \vdash v : A \leq B}{\Gamma \vdash v : B}$

### Computation typing rules

- $(\text{Return}) \frac{\Gamma \vdash v : A}{\Gamma \vdash \text{return } v : M_1 A}$
- $(\text{Apply}) \frac{\Gamma \vdash v_1 : A \rightarrow M_\epsilon B \quad \Gamma \vdash v_2 : A}{\Gamma \vdash v_1 v_2 : M_\epsilon B}$
- $(\text{if}) \frac{\Gamma \vdash v : \text{Bool} \quad \Gamma \vdash C_1 : M_\epsilon A \quad \Gamma \vdash C_2 : M_\epsilon A}{\Gamma \vdash \text{if}_{\epsilon, A} v \text{ then } C_1 \text{ else } C_2 : M_\epsilon A}$
- $(\text{Do}) \frac{\Gamma \vdash C_1 : M_{\epsilon_1} A \quad \Gamma, x : A \vdash C_2 : M_{\epsilon_2} B}{\Gamma \vdash \text{do } x \leftarrow C_1 \text{ in } C_2 : M_{\epsilon_1 \cdot \epsilon_2} B}$
- $(\text{Subeffect}) \frac{\Gamma \vdash C : M_{\epsilon_1} A \leq B \leq \epsilon_2}{\Gamma \vdash C : M_{\epsilon_2} B}$

## 0.2.6 Ok Lemma

If  $\Gamma \vdash t : \tau$  then  $\Gamma \text{Ok}$ .

**Proof** If  $\Gamma, x : A \text{Ok}$  then by inversion  $\Gamma \text{Ok}$ . Only the type rule **Weaken** adds terms to the environment from its preconditions to its post-condition and it does so in an **Ok** preserving way. Any type derivation tree has at least one leaf. All leaves are axioms which require  $\Gamma \text{Ok}$ . And all non-axiom derivations preserve the **Ok** property.