0.1 Terms

0.1.1 Value Terms

$$
\begin{aligned}
v ::= & x \\
& \mid \lambda x : A.C \\
& \mid \mathtt{C}^A \\
& \mid \mathtt{()} \\
& \mid \mathtt{true} \mid \mathtt{false}
\end{aligned} \tag{1}
$$

0.1.2 Computation Terms

$$
\begin{aligned}
C ::= & \mathtt{if}_{\epsilon,A}\ v\ \mathtt{then}\ C_1\ \mathtt{else}\ C_2 \\
& \mid v_1\ v_2 \\
& \mid \mathtt{do}\ x \leftarrow C_1\ \mathtt{in}\ C_2 \\
& \mid \mathtt{return}\ v
\end{aligned} \tag{2}
$$

0.2 Type System

0.2.1 Effects

The effects should form a monotonous, pre-ordered monoid $(E, \cdot, \mathtt{1}, \leq)$ with elements ϵ

0.2.2 Types

Ground Types There exists a set γ of ground types, including `Unit`, `Bool`

Value Types

$$A, B, C ::= \gamma \mid A \rightarrow \mathtt{M}_\epsilon B$$

Computation Types Computation types are of the form $\mathtt{M}_\epsilon A$

0.2.3 Sub-typing

There exists a sub-typing pre-order relation $\leq:_\gamma$ over ground types that is:

- (Reflexive)$\dfrac{}{A \leq:_\gamma A}$
- (Transitive)$\dfrac{A \leq:_\gamma B \qquad B \leq:_\gamma C}{A \leq:_\gamma C}$

We extend this relation with the function sub-typing rule to yield the full sub-typing relation $\leq:$

- (ground)$\dfrac{A \leq:_\gamma B}{A \leq: B}$
- (Fn)$\dfrac{A \leq: A' \qquad B' \leq: B}{\epsilon \leq \epsilon'} A' \rightarrow \mathtt{M}_{\epsilon'} B' \leq: A \rightarrow \mathtt{M}_\epsilon B$

0.2.4 Type Environments

An environment, $G ::= \diamond \mid \Gamma, x : A$

Domain Function

- $\mathtt{dom}(\diamond) = \emptyset$
- $\mathtt{dom}(\Gamma, x : A) = \mathtt{dom}(\Gamma) \cup \{x\}$

$\mathtt{Ok}$ Predicate

- (Atom) $\dfrac{}{\diamond\mathtt{Ok}}$
- (Var) $\dfrac{\Gamma\mathtt{Ok}}{x \notin \mathtt{dom}(\Gamma)}\Gamma, x : A\mathtt{Ok}$

0.2.5 Type Rules

Value Typing Rules

- (Const) $\dfrac{\Gamma\mathtt{Ok}}{\Gamma \vdash \mathtt{C}^A \colon A}$
- (Unit) $\dfrac{\Gamma\mathtt{Ok}}{\Gamma \vdash \mathtt{()} \colon \mathtt{Unit}}$
- (True) $\dfrac{\Gamma\mathtt{Ok}}{\Gamma \vdash \mathtt{true} \colon \mathtt{Bool}}$
- (False) $\dfrac{\Gamma\mathtt{Ok}}{\Gamma \vdash \mathtt{false} \colon \mathtt{Bool}}$
- (Var) $\dfrac{\Gamma, x : A\mathtt{Ok}}{\Gamma, x : A \vdash X \colon A}$
- (Weaken) $\dfrac{\Gamma \vdash x \colon A}{\Gamma, y : B \vdash X \colon A}$(if $x \neq y$)
- (Fn) $\dfrac{\Gamma, x : A \vdash C \colon \mathtt{M}_\epsilon B}{\Gamma \vdash \lambda x : A.C : A \to \mathtt{M}_\epsilon B}$
- (Sub) $\dfrac{\Gamma \vdash v \colon A \qquad A \leq \colon B}{\Gamma \vdash v \colon B}$

Computation typing rules

- (Return) $\dfrac{\Gamma \vdash v \colon A}{\Gamma \vdash \mathtt{return}\ v : \mathtt{M_1} A}$

- (Apply) $\dfrac{\Gamma \vdash v_1 \colon A \to \mathtt{M}_\epsilon B \qquad \Gamma \vdash v_2 \colon A}{\Gamma \vdash v_1\ v_2 \colon \mathtt{M}_\epsilon B}$

- (if) $\dfrac{\Gamma \vdash v \colon \mathtt{Bool} \qquad \Gamma \vdash C_1 \colon \mathtt{M}_\epsilon A \qquad \Gamma \vdash C_2 \colon \mathtt{M}_\epsilon A}{\Gamma \vdash \mathtt{if}_{\epsilon,A}\ V\ \mathtt{then}\ C_1\ \mathtt{else}\ C_2 : \mathtt{M}_\epsilon A}$

- (Do) $\dfrac{\Gamma \vdash C_1 \colon \mathtt{M}_{\epsilon_1} A \qquad \Gamma, x : A \vdash C_2 \colon \mathtt{M}_{\epsilon_2} B}{\Gamma \vdash \mathtt{do}\ x \leftarrow C_1\ \mathtt{in}\ C_2 : \mathtt{M}_{\epsilon_1 \cdot \epsilon_2} B}$

- (Subeffect) $\dfrac{\Gamma \vdash C \colon \mathtt{M}_{\epsilon_1} A \qquad A \leq : B}{\epsilon_1 \leq \epsilon_2} \Gamma \vdash C \colon \mathtt{M}_{e_2} B$

0.2.6 Ok Lemma

If $\Gamma \vdash t \colon \tau$ then $\Gamma \mathtt{Ok}$.

Proof If $\Gamma, x : A \mathtt{Ok}$ then by inversion $\Gamma \mathtt{Ok}$ Only the type rule `Weaken` adds terms to the environment from its preconditions to its post-condition and it does so in an `Ok` preserving way. Any type derivation tree has at least one leaf. All leaves are axioms which require $\Gamma \mathtt{Ok}$. And all non-axiom derivations preserve the `Ok` property.