Problem #2

a) Yes! PDF: $f(y;\lambda) = \lambda . e^{-\lambda . y}$

$$f(y|\lambda) = \exp\left\{\log \lambda . e^{-\lambda . y}\right\} = \exp\left\{-\lambda . y + \log \lambda\right\}$$

$$\boxed{\theta = -\lambda}$$

$$-b(\theta) = \log \lambda \Rightarrow \boxed{b(\theta) = -\log(-\theta)}$$

b) Yes! PMF: $f(y;\pi) = \binom{n}{y} . \pi^{y} . (1-\pi)^{n-y}$

$$f(y|\pi) = \exp\left\{\log \pi^{y} . (1-\pi)^{n-y}\right\} . \binom{n}{y} = \exp\left\{\log \pi . y + \log(1-\pi) . (n-y)\right\} \binom{n}{y}$$

$$= \binom{n}{y} . \exp\left\{\log\left(\frac{\pi}{1-\pi}\right) . y + n . \log(1-\pi)\right\}$$

$$\boxed{\theta = \log\left(\frac{\pi}{1-\pi}\right)}$$

$$-b(\theta) = n . \log(1-\pi) \Rightarrow \boxed{b(\theta) = n . \log(1+e^{\theta})}$$

c) Yes! PDF: $f(y;c) = 1/c$

$$f(y|c) = \exp\left\{\log 1/c\right\} = \exp\left\{-\log(c)\right\}$$

D) Yes! PDF. $f(y;\pi,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y-\pi)^2}{2\sigma^2}\right)$

$$\theta . y - b(\theta) = -\frac{y^2}{2\sigma^2} + \frac{y . \pi}{\sigma^2} - \frac{\pi^2}{2\sigma^2}$$

$$\boxed{\theta = \left(\frac{\pi}{\sigma^2}, -\frac{1}{2\sigma^2}\right)}$$

$$-b(\theta) = -\log\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right) - \frac{\pi^2}{2\sigma^2} \Rightarrow \boxed{b(\theta) = \frac{1}{2}\left(\log(2\pi\sigma^2) + \frac{\pi^2}{\sigma^2}\right)}$$