Algebra I
Winter 2020

WAYNE STATE
UNIVERSITY

1.1 Divisors

1. Let $m, n.r.s \in \mathbb{Z}$. If $m^2 + n^2 = r^2 + s^2 = mr + ns$, prove that $m = r$ and $n = r$.

Joe Starr

We select $m, n.r.s \in \mathbb{Z}$, given $m^2 + n^2 = r^2 + s^2 = mr + ns$ which can write as $m^2 + n^2 - mr - ns = r^2 + s^2 - mr - ns$. From here we can simplify:

$$
\begin{aligned}
m^2 + n^2 - mr - ns = r^2 + s^2 - mr - ns &\Rightarrow m(m-r) + n(n-s) = r(r-m) + s(s-n) \\
&\Rightarrow m(m-r) + n(n-s) - r(r-m) - s(s-n) = 0 \\
&\Rightarrow m(m-r) + r(m-r) + n(n-s) + s(n-s) = 0 \\
&\Rightarrow (m-r)(m+r) + (n-s)(n+s) = 0
\end{aligned}
$$

from here we can see that in order for $(m-r)(m+r) + (n-s)(n+s) = 0$ to be true $m = r$ and $n = s$

3. Find the quotient and reminder when a id divided by b.

a $a = 99$, $b = 17$

b $a = -99$, $b = 17$

c $a = 17$, $b = 99$

d $a = -1017$, $b = 99$

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a $99 = 17q + r \Rightarrow q = 5, r = 14$

b $-99 = 17q + r \Rightarrow q = -6, r = 3$

c $17 = 99q + r \Rightarrow q = 0, r = 17$

d $-1017 = 99q + r \Rightarrow q = -11, r = 72$

5. Use the Euclidean algorithm to find the following greatest common divisors

a $(6643, 2873)$

b $(7684, 4148)$

c $(26460, 12600)$

d $(6540, 1206)$

e $(12091, 8439)$

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a $(6643, 2873)$

$$6643 = 2873 * 2 + 897$$

$$2873 = 897 * 3 + 182$$

$$897 = 182 * 4 + 169$$

$$182 = 169 * 1 + 13$$

$$169 = 13 * 13$$

b $(7684, 4148)$

$$7684 = 4148 * 1 + 3536$$

$$4148 = 3536 * 1 + 612$$

$$3536 = 612 * 5 + 476$$

$$612 = 476 * 1 + 136$$

$$476 = 136 * 3 + 68$$

$$136 = 68 * 68$$

c $(26460, 12600)$

$$26460 = 12600 * 2 + 1260$$

$$12600 = 1260 * 10$$

d $(6540, 1206)$

$$6540 = 1206 * 5 + 510$$

$$1206 = 510 * 2 + 186$$

$$510 = 186 * 2 + 138$$

$$186 = 138 * 1 + 48$$

$$138 = 48 * 2 + 42$$

$$48 = 42 * 1 + 6$$

$$42 = 6 * 7$$

e $(12091, 8439)$

$$12091 = 8439 * 1 + 3652$$

$$8439 = 3652 * 2 + 1135$$

$$3652 = 1135 * 3 + 247$$

$$1135 = 247 * 4 + 147$$

$$247 = 147 * 1 + 100$$

$$147 = 100 * 1 + 47$$

$$100 = 47 * 2 + 6$$

$$47 = 6 * 7 + 5$$

$$6 = 5 * 1 + 1$$

$$5 = 1 * 5$$

CHAPTER 2 SECTION

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2.1 A Subsection

CHAPTER 3 SECTION

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3.1 A Subsection

CHAPTER 4 SECTION

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4.1 A Subsection

CHAPTER 5 SECTION

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6.1 A Subsection

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7.1 A Subsection

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8.1 A Subsection

CHAPTER 9 SECTION

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9.1 A Subsection

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10.1 A Subsection