Algebra I Winter 2020



CHAPTER 1 INTEGERS

1.1 Divisors

1. Let $m, n.r.s \in \mathbb{Z}$. If $m^2 + n^2 = r^2 + s^2 = mr + ns$, prove that m = r and n = s.

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We select $m, n.r.s \in \mathbb{Z}$, given $m^2 + n^2 = r^2 + s^2 = mr + ns$ which can write as $m^2 + n^2 - mr - ns = r^2 + s^2 - mr - ns$. From here we can simplify:

$$m^{2} + n^{2} - mr - ns = r^{2} + s^{2} - mr - ns \Rightarrow m(m - r) + n(n - s) = r(r - m) + s(s - n)$$

$$\Rightarrow m(m - r) + n(n - s) - r(r - m) - s(s - n) = 0$$

$$\Rightarrow m(m - r) + r(m - r) + n(n - s) + s(n - s) = 0$$

$$\Rightarrow (m - r)(m + r) + (n - s)(n + s) = 0$$

from here we can see that in order for (m-r)(m+r)+(n-s)(n+s)=0 to be true m=r and n=s.

3. Find the quotient and reminder when a id divided by b.

a
$$a = 99, b = 17$$

b
$$a = -99, b = 17$$

c
$$a = 17, b = 99$$

d
$$a = -1017$$
, $b = 99$

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a
$$99 = 17q + r \Rightarrow q = 5, r = 14$$

b
$$-99 = 17q + r \Rightarrow q = -6, r = 3$$

c
$$17 = 99q + r \Rightarrow q = 0, r = 17$$

d
$$-1017 = 99q + r \Rightarrow q = -11, r = 72$$

- 5. Use the Euclidean algorithm to find the following greatest common divisors
 - a (6643, 2873)
 - b (7684, 4148)
 - c (26460, 12600)
 - d (6540, 1206)
 - e (12091, 8439)

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a (6643, 2873)

$$6643 = 2873 * 2 + 897$$

$$2873 = 897 * 3 + 182$$

$$897 = 182 * 4 + 169$$

$$182 = 169 * 1 + 13$$

$$169 = 13 * 13$$

b (7684, 4148)

$$7684 = 4148 * 1 + 3536$$

$$4148 = 3536 * 1 + 612$$

$$3536 = 612 * 5 + 476$$

$$612 = 476 * 1 + 136$$

$$476 = 136 * 3 + 68$$

$$136 = 68 * 68$$

c (26460, 12600)

$$26460 = 12600 * 2 + 1260$$

$$12600 = 1260 * 10$$

d (6540, 1206)

$$6540 = 1206 * 5 + 510$$

$$1206 = 510 * 2 + 186$$

$$510 = 186 * 2 + 138$$

$$186 = 138 * 1 + 48$$

$$138 = 48 * 2 + 42$$

$$48 = 42 * 1 + 6$$

$$42 = 6 * 7$$

$$12091 = 8439 * 1 + 3652$$

$$8439 = 3652 * 2 + 1135$$

$$3652 = 1135 * 3 + 247$$

$$1135 = 247 * 4 + 147$$

$$247 = 147 * 1 + 100$$

$$147 = 100 * 1 + 47$$

$$100 = 47 * 2 + 6$$

$$47 = 6 * 7 + 5$$

$$6 = 5 * 1 + 1$$

$$5 = 1 * 5$$

7. For each part of Exercise 5, find integers m and n such that (a,b) is expressed in the form ma+nb.

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a (6643, 2873)

b (7684, 4148)

c (26460, 12600)

d (6540, 1206)

e (12091, 8439)

9. let a, b, c be integers such that a + b + c = 0. Show that if n is an integer which is a divisor of two of the three integers, then it is also a divisor of the third.

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Select $a,b,c\in\mathbb{Z}$ to satisfy a+b+c=0, WLOG let $n\in\mathbb{Z}$ such that n|a and n|b. Since (a+b)+c=0 it must be that (a+b)=-c. From here we must show n|(a+b), or a+b=nq. Since n|a and n|b we may write $a=nq_1$ and $b=nq_2$, yielding, $nq_1+nq_2=n$ $(q_1+q_2)=nq$ thus n|c, as desired.

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