MATH 210 Assignment 2

Logic, Loops and Functions

INSTRUCTIONS

- ◇ Create a new Jupyter notebook and set the kernel to Python 3
- ◇ Present your solutions in a single Jupyter notebook and clearly label the solutions
- ◇ Your solutions should include clear explanations (including proper use of markdown language and LaTeX) and your functions should include comments
- ◇ There are 30 total points: each question is worth 4 points and 2 points will be awarded for the overall presentation of your notebook
- ◇ Each question is graded out of 4 points according to the rubric:
  - 4 - The function performs its tasks correctly and includes comments which properly explain the code
  - 3 - The function performs its tasks correctly but needs more comments to explain the code
  - 2 - The function performs its tasks somewhat correctly
  - 1 - The solution needs improvement
- ◇ Submit the `.ipynb` file to Connect by **11pm Friday January 22**
- ◇ You may work on these problems with others but you must write your solutions on your own

QUESTIONS

1. Write a function called `roots` which takes 3 numerical inputs `a`, `b` and `c` (which represent the polynomial $ax^2 + bx + c$) and does the following:

   - ◇ if the roots of $ax^2 + bx + c$ are real and distinct, return a Python list consisting of the two roots
   - ◇ if the roots of $ax^2 + bx + c$ are real and repeated, return the single root
   - ◇ if the roots of $ax^2 + bx + c$ are complex, return a list of length 2 such that both entries of the list are lists which give the real part and the imaginary part of each root. In other words, if r_1 and r_2 are the complex roots, then the function returns:

     [[Real part of r_1, Imaginary part of r_1] , [Real part of r_2, Imaginary part of r_2]]

   For example:

   (a) If $a = 1$, $b = 0$ and $c = -1$, then the function returns `[1.0,-1.0]`, the roots of $x^2 - 1$.

(b) If $a = 1$, $b = 2$ and $c = 1$, then the function returns `-1.0`, the only root of $x^2 + 2x + 1$.

(c) If $a = 1$, $b = 0$ and $c = 1$, then the function returns `[[0.0,1.0],[0.0,-1.0]]` which represents i and $-i$, the roots of $x^2 + 1$.

(d) If $a = 1$, $b = 2$ and $c = 2$, then the function returns `[[-1.0,1.0],[-1.0,-1.0]]` which represents $1 + i$ and $1 - i$, the roots of $x^2 + 2x + 2$.

2. Write a function called `fibonacci_less_than` which takes an integer N and computes the largest Fibonacci number less than (or equal to) N. Use your function to find the largest Fibonacci number which is less than 1,000,000.

3. Write a function called `divisors` which takes an integer N and returns a Python list of all its (positive) divisors. For example, if $N = 12$ then the function returns $[1, 2, 3, 4, 6, 12]$.

4. In number theory, the sum of divisors function $\sigma_k(n)$ is

$$\sigma_k(n) = \sum_{d|n} d^k$$

where the sum is taken over the positive divisors of n. For example, $\sigma_2(12)$ is the sum

$$\sigma_2(12) = 1^2 + 2^2 + 3^2 + 4^2 + 6^2 + 12^2 \ .$$

Use the function `divisors` from previous question to write a function called `sum_of_divisors` which takes 2 inputs k and n and returns $\sigma_k(n)$.

5. Write a function called `is_prime` which takes an integer N and returns the Boolean value `True` if N is prime and `False` if N is not prime.

6. Use the function `is_prime` from the previous question to write a function called `primes_up_to` which takes an integer N and returns a Python list of all primes $p \leq N$.

7. Twin primes are a pair of prime numbers whose difference is 2. For example, 3 and 5 are a pair of twin primes and so is the pair 11 and 13. Write a function called `twin_primes` which takes an integer N and returns a list of twin primes (given as a list of length 2) less than (or equal to) N. For example:

(a) if $N = 15$, then the function returns $[[3, 5], [5, 7], [11, 13]]$

(b) if $N = 35$, then the function returns $[[3, 5], [5, 7], [11, 13], [17, 19], [29, 31]]$

(c) if $N = 43$, then the function returns $[[3, 5], [5, 7], [11, 13], [17, 19], [29, 31], [41, 43]]$

(The Twin Prime Conjecture states that there are infinitely many twin primes. The conjecture is still an open problem in number theory.)