McCall Search Model

Quantitative Economics with Python

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Overview

- The lake model is a basic tool for modeling unemployment
- It is a tool for analyzing
  - flows between unemployment and employment
  - how they influence steady state employment and unemployment rates
- It is a good model for interpreting monthly labor department reports on gross and net jobs created and jobs destroyed

Overview

- The "lakes" in the model are the pools of employed and unemployed
- The "flows" between the lakes are caused by
  - firing and hiring
  - entry and exit from the labor force
- First, the parameters governing transitions into and out of unemployment and employment are exogenous
- Later, some of the transition rates are endogenously: McCall search model

Model

- The economy is inhabited by a large number of **ex-ante** identical workers.
  - live forever
  - spend their lives moving between unemployment and employment
- Transition rate between being unemployed and employed are
  - λ: job finding rate for currently unemployed workers
  - α: dismissal rate for currently employed workers
  - Assume there is no death or birth, the population size is fixed

Aggregates

- We want the dynamics of the following aggregates
  - E_t: total number of employed workers
  - U_t: total number of unemployed workers
  - $N_t = 1$: number of workers in the labor force
- We also want to know the values of the following objects
  - $e_t = E_t$: employment rate E_t/N_t
  - $u_t = U_t$: unemployment rate U_t/N_t

Laws of Motion

- The number of workers who will be employed at $t+1$

$$e_{t+1} = (1-\alpha)e_t + \lambda u_t$$

- The number of workers who will be unemployed at $t+1$

$$u_{t+1} = \alpha e_t + (1-\lambda)u_t$$

- More compactly

$$\begin{pmatrix} e_{t+1} \\ u_{t+1} \end{pmatrix} = \begin{pmatrix} 1-\alpha & \lambda \\ \alpha & 1-\lambda \end{pmatrix} \begin{pmatrix} e_t \\ u_t \end{pmatrix}$$

- In steady state

$$\begin{pmatrix} e \\ u \end{pmatrix} = \begin{pmatrix} 1-\alpha & \lambda \\ \alpha & 1-\lambda \end{pmatrix} \begin{pmatrix} e \\ u \end{pmatrix}$$

McCall Search Model

- The model is about the life an infinitely lived worker and
  - he has the opportunities to work at different wages
  - exogenous events that destroy his current job
  - decide whether to take a job while unemployed
- Key: endogenous decision whether to take a job
  - benefit: earn higher income
  - cost: could wait for a better job

Utility

- Workers try to maximize the lifetime utility

$$\sum_{t=0}^{\infty} \beta^t u(y_t)$$

  - y_t is his wage w_t when employed
  - y_t is unemployment compensation c when he is unemployed
  - $u(\cdot)$ is a utility function, for example, $u(y) = \log y$

Choice

- Wage offers are drawn from a vector of possible wages
  - **w** $= \{w_1, w_2, \ldots, w_n\}$ with probabilities in vector **p** $= \{p_1, p_2, \ldots, p_n\}$.
- When employed with wage $w \in \{w_1, w_2, \ldots, w_n\}$
  - consumes w, and with prob α of becoming unemployed at the end of the period
- The unemployed worker consumes unemployment benefit b. At the end of a period, with probability λ a worker receives an offer to work next period.
  - conditional on receiving an offer, he receives an offer w_s with prob p_s
  - given offer w_s, the unemployed worker chooses to accept or reject the offer
  - if accepts, he consumes w_s today and enters next period employed with w_s
  - if rejects, he consumes benefit c and enters next period unemployed

Lifetime utility

- The only decision is when employed, whether to accept a wage offer or not

$$d(s) = 1 \text{ or } d(s) = 0$$

- Let $d^*(s)$ denote the optimal choice of the worker

- Given $d^*(s)$, we can calculate the expected lifetime utility of a worker
  - when the worker is unemployed

$$U = u(c) + \mathbb{E} \sum_{t=1}^{\infty} \beta^t u(y_t)$$

  - when the worker is employed with wage w_s

$$V_s = u(w_s) + \mathbb{E} \sum_{t=1}^{\infty} \beta^t u(y_t)$$

- There are $n+1$ numbers we need to solve, U and $\{V_1, V_2, \dots, V_n\}$

Value function

- Alternative way to write the lifetime utility

$$V_s = u(w_s) + \beta \left[(1-\alpha)V_s + \alpha U\right]$$

$$U = u(c) + \beta(1-\lambda)U + \beta\lambda \sum_s p_s \max\{U, V_s\}$$

- How to solve this problem? Iteration.

- Imagine we know the U and $\{V_1, V_2, \ldots, V_n\}$

$$V_s^{(j+1)} = u(w_s) + \beta \left[(1-\alpha)V_s^{(j)} + \alpha U^{(j)}\right]$$

$$U^{(j+1)} = u(c) + \beta(1-\lambda)U^{(j)} + \beta\lambda \sum_s p_s \max\left\{U^{(j)}, V_s^{(j)}\right\}$$

- Stop if the values converge

Reservation wage

- Imagine we have solved the value function

$$V_s = u(w_s) + \beta \left[(1-\alpha)V_s + \alpha U\right]$$

$$U = u(c) + \beta(1-\gamma)U + \beta\gamma \sum_s p_s \max\{U, V_s\}$$

- The optimal choice of the worker is $d^*(s)$

$$d^*(s) = 1 \quad \text{if} \quad V_s > U$$

- Workers will have a reservation wage $\widehat{w}_s$

- The probability that a worker transits from unemployment to employment is

$$\alpha \sum_s d^*(s) p_s$$