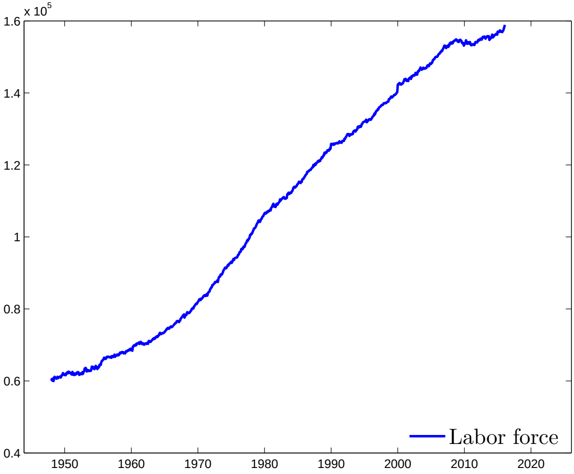
Lake Model: Employment and Unemployment

Quantitative Economics with Python

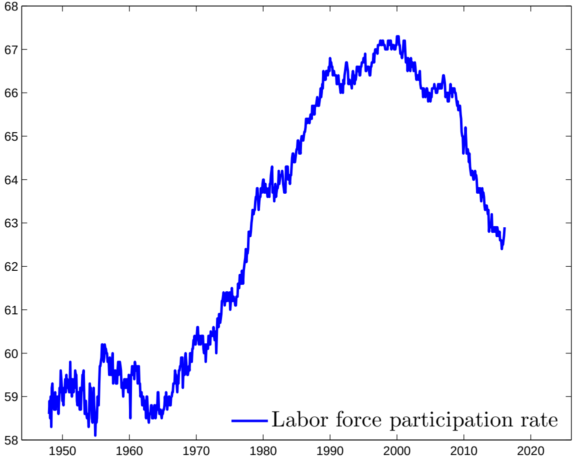
New York University

Thursday 10^{th} March, 2016

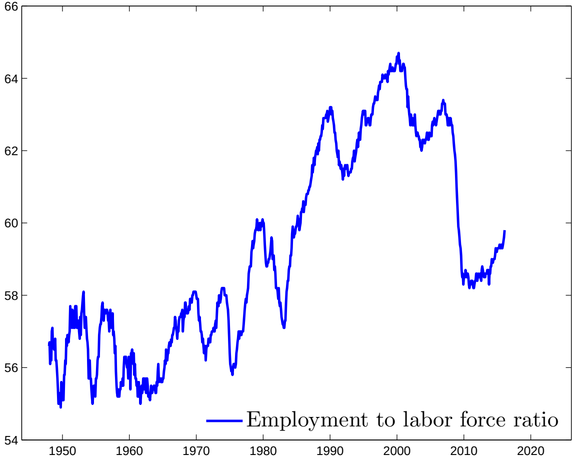
Facts


x 10^5
1.6
1.4
1.2
1
0.8
0.6
0.4
1950
1960
1970
1980
1990
2000
2010
2020
Labor force

Facts

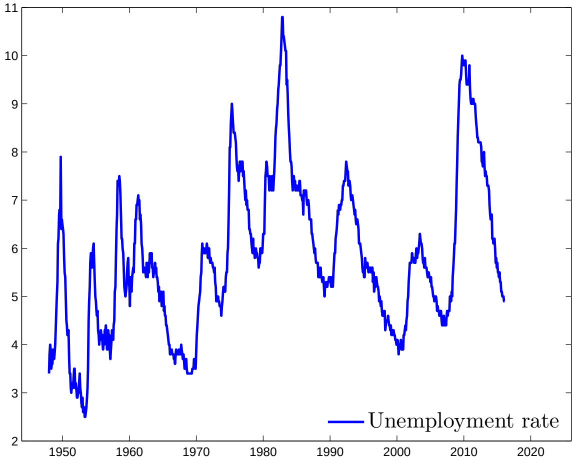

68
67
66
65
64
63
62
61
60
59
58
1950
1960
1970
1980
1990
2000
2010
2020
Labor force participation rate

Facts



66
64
62
60
58
56
54
1950
1960
1970
1980
1990
2000
2010
2020
Employment to labor force ratio

Facts



11
10
9
8
7
6
5
4
3
2
1950
1960
1970
1980
1990
2000
2010
2020
Unemployment rate

Overview

- The lake model is a basic tool for modeling unemployment
- It is a tool for analyzing
  - flows between unemployment and employment
  - how they influence steady state employment and unemployment rates
- It is a good model for interpreting monthly labor department reports on gross and net jobs created and jobs destroyed

Overview

- The "lakes" in the model are the pools of employed and unemployed
- The "flows" between the lakes are caused by
  - firing and hiring
  - entry and exit from the labor force
- First, the parameters governing transitions into and out of unemployment and employment are exogenous
- Later, some of the transition rates are endogenously: McCall search model

Overview

- The **only** knowledge required for this lecture is
  - basic linear algebra
  - elementary concepts of Markov Chains
  - dynamic program
- We'll use some nifty concepts like ergodicity, which provides a fundamental link between cross sectional and long run time series distributions
- These concepts will help us build an equilibrium model of ex ante homogeneous workers whose different luck generates variations in their ex post experiences

Model

- The economy is inhabited by a large number of **ex-ante** identical workers.
  - live forever
  - spend their lives moving between unemployment and employment
- Transition rate between being unemployed and employed are
  - λ: job finding rate for currently unemployed workers
  - α: dismissal rate for currently employed workers
  - b: entry rate into the labor force
  - d: exit rate from the labor force
- The growth rate of the labor force evidently equals $g = b - d$

Aggregates

- We want the dynamics of the following aggregates
  - E_t: total number of employed workers
  - U_t: total number of unemployed workers
  - N_t: number of workers in the labor force
- We also want to know the values of the following objects
  - e_t: employment rate E_t/N_t
  - u_t: unemployment rate U_t/N_t

Laws of Motion

- Of the mass of workers E_t who are employed
    - $(1-d)E_t$ will remain in the labor force
    - $(1-\alpha)(1-d)E_t$ will remain employed
- Of the mass of workers U_t workers who are currently unemployed
    - $(1-d)U_t$ will remain in the labor force
    - $\lambda(1-d)U_t$ will become employed
- The number of workers who will be employed at $t+1$

$$E_{t+1} = (1-d)(1-\alpha)E_t + (1-d)\lambda U_t$$

- The number of workers who will be unemployed at $t+1$

$$U_{t+1} = (1-d)\alpha E_t + (1-d)(1-\lambda)U_t + b(E_t + U_t)$$

Laws of Motion

- The total stock of workers $N_t = E_t + U_t$ evolves as

$$N_{t+1} = (1 + b - d)N_t = (1 + g)N_t$$

- Linear state space

$$X_t = \begin{pmatrix} E_t \\ U_t \end{pmatrix}$$

- Law of motion for X is

$$X_{t+1} = \begin{pmatrix} E_{t+1} \\ U_{t+1} \end{pmatrix} = A \begin{pmatrix} E_t \\ U_t \end{pmatrix} = AX_t$$

where

$$A = \begin{bmatrix} (1-d)(1-\alpha) & (1-d)\lambda \\ (1-d)\alpha + b & (1-d)(1-\lambda) + b \end{bmatrix}$$

Laws of Motion

- Laws of Motion for Rates of Employment and Unemployment

$$\begin{pmatrix} E_{t+1}/N_{t+1} \\ U_{t+1}/N_{t+1} \end{pmatrix} = \frac{1}{1+g} \begin{pmatrix} (1-d)(1-\alpha) & (1-d)\lambda \\ (1-d)\alpha + b & (1-d)(1-\lambda) + b \end{pmatrix} \begin{pmatrix} E_t/N_t \\ U_t/N_t \end{pmatrix}$$

- Define x_t as

$$x_t = \begin{pmatrix} e_t \\ u_t \end{pmatrix} = \begin{pmatrix} E_t/N_t \\ U_t/N_t \end{pmatrix}$$

or

$$x_{t+1} = \hat{A} x_t \quad \text{where} \quad \hat{A} := \frac{1}{1+g} A$$

- Evidently, $e_t + u_t = 1$ implies that $e_{t+1} + u_{t+1} = 1$

Steady States

- The aggregates E_t and U_t won't converge to steady states because their sum $E_t + U_t$ grows at gross rate $1 + g$

- The vector of employment and unemployment rates x_t can be in a steady state $\bar{x}$ provided that we can find a solution to the matrix equation

$$\bar{x} = \hat{A}\bar{x}$$

where the components satisfy

$$\bar{e} + \bar{u} = 1$$

- A steady state $\bar{x}$ is an eigenvector of $\hat{A}$ associated with a unit eigenvalue

- We also have $x_t \to \bar{x}$ as $t \to \infty$ provided that the remaining eigenvalues of $\hat{A}$ are in modulus less that 1

Steady States

Let us go to Python notebook...