April 21

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$$\boxed{v(\lambda_t) = p_t / d_t}$$

$$v(\lambda_t) = \beta E_t \left(\lambda_{t+1} + v(\lambda_{t+1}) \lambda_{t+1} \right)$$

$$\lambda_t \in [s_1, s_2, \ldots, s_n]$$

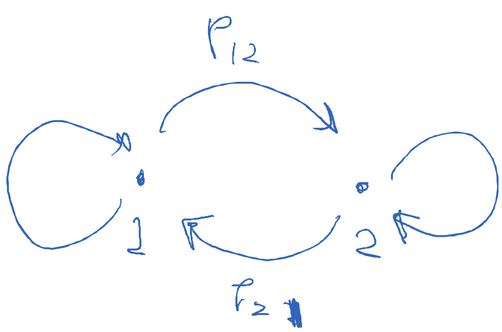
$$v(s_1), v(s_2), \ldots, v(s_n)$$

$$v_1 \quad v_2 \quad \ldots \quad v_n$$

$$\begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} \sim v(\lambda_t)$$

$$\lambda_t = \begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ \vdots \\ s_n \end{pmatrix}, \quad \lambda_{t+1} = \begin{pmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{pmatrix}$$

$$\pi_t = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \pi_{t+1} = \begin{pmatrix} \pi_{1,t+1} \\ \pi_{2,t+1} \\ \vdots \\ \pi_{n,t+1} \end{pmatrix}$$



$$v(\lambda_t) = \beta E_t \left[\lambda_{t+1} + v(\lambda_{t+1}) \lambda_{t+1} \right]$$

$n \times 1$ $\qquad$ $n \times 1$

$$\overset{n\times 1}{v} = \beta * Pa\,[\overset{n\times 1}{s} + v * s]$$

to be solved for v

$$P_t = E_t\, m_{t+1}\,[d_{t+1} + P_{t+1}]$$

m_{t+1} ~ stochastic discount factor.

$$v_i = \sum_{j=1}^{n} P_{ij}\,\beta_j\,[s_j + s_j v_j], \quad i = 1, \dots n$$

$\frac{P}{d}$

$$d_{t+1} = \lambda_{t+1}\, d_t$$

$$\boxed{\lambda_{t+1} = d_{t+1} / d_t}$$

$$v(\lambda_t) = \sum_j P_{ij}\,\beta\,[\lambda_{t+1} + \lambda_{t+1} v(\lambda_{t+1})]$$

v_i

Stochastic discount factor

$$P_t = E_t [m_{t+1} (d_{t+1} + P_{t+1})]$$

m

Lucas - Prescott - Mehra model.

$d_{t+1} = c_{t+1}$ ~ aggregate consumption

$\underbrace{c_{t+1} = \lambda_{t+1} c_t}$, $\lambda_{t+1} \sim$ Markov

$P, s_1, \ldots, s_n$

$m_{t+1} = \beta \lambda_{t+1}^{-\gamma}$, where γ is a measure of risk aversion

$\gamma \geqslant 1$

Mehra & Prescott

$$P_t = E_t [m_{t+1} (c_{t+1} + P_{t+1})]$$

$$\boxed{m_{t+1} = \beta \lambda_{t+1}^{-\gamma}, \quad c_{t+1} = \lambda_{t+1} c_t}$$

$\lambda_{t+1} \sim$ Markov

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guess: $P_t = v(\lambda_t)\, c_t$

$$v(\lambda_t)\cancel{c_t} = E_t\left[\beta \lambda_{t+1}^{-\gamma}\left(c_{t+1} + P_{t+1}\right)\right]$$

$$= E_t\left[\beta \lambda_{t+1}^{-\gamma}\left(\lambda_{t+1}\cancel{c_t} + \lambda_{t+1} v(\lambda_{t+1})\cancel{c_t}\right)\right]$$

$$v(\lambda_t) = E_t\left[\beta \lambda_{t+1}^{1-\gamma}\left(1 + v(\lambda_{t+1})\right)\right]$$

$$v_i = \beta \sum_j \underbrace{P_{ij}\, s_j^{1-\gamma}}\,(1 + v_j)$$

$$\hat{P}_{ij} = P_{ij}\, s_j^{1-\gamma}$$

$$v = \beta \tilde{P} \mathbb{1} + \beta \tilde{P} v$$

$$\boxed{v = \beta (I - \beta \tilde{P})^{-1} \tilde{P} \cdot \mathbb{1}}$$

if eigenvalues of $\beta \tilde{P}$ are strictly less than 1 strictly less than one in absolute value

$\gamma \leftarrow$ risk aversion

β

$s_1, \ldots, s_n$, ρ