Midterm Examination
Quantitative Economics with Python

March 29, 2016

Instructions: This is a closed book examination except that you may look at the Python command sheet that we will have handed out with this exam. Please answer all questions. Prospective points are indicated beside each problem.

(30 points) Programming and Computation

Q1. (15 points) Consider the three python implementations of a maximum function. Given a `list` of values they will return the maximum value from the list.

A

```
def maximum(data):
    return sorted(data)[-1]
```

B

```
def maximum(data):
    max_item = data[0]
    for item in data:
        if item > max_item:
            max_item = item
    return max_item
```

C

```
def maximum(data):
    max_item = data[0]
    for item in data:
        if not str(item).isnumeric():
            continue
        if item > max_item:
            max_item = item
    return max_item
```

a. What is the output from **A**, **B**, and **C** given the following input:

1. `data = [-4,2,3,1,0,9]`
2. `data = [2,100,3,'A',4]`
3. `data = ['C','Z','A','D']`

b. Please briefly explain which **you** think is the best implementation and why. (**Hint:** What criteria are you using to make your assesment?)

Q2. (15 points) Consider the following python variable that contains text from a recent NYTimes article:

```
data = "The early land vertebrates, known as tetrapods, evolved \
adaptations that enabled them to move efficiently over solid ground. \
A pelvis joined their hind limbs to their spines, for example. \
Their vertebrae grew flanges so that they interlocked, helping the \
spine hold itself stiff and straight even when being pulled down \
by gravity.\n These adaptations led tetrapods to walk in a \
distinctive fashion, moving their forelegs and hind legs together \
in a cycle. Early tetrapods probably walked much the way \
salamanders do today, bending their trunk from side to side as \
they traveled."
```

a. Write a short python program that parses this text and builds a **histogram** of the words used in the text. You may assume that the text is contained as a single string in a variable named `data` (as shown above). You do **not** need to write the code to **plot** the histogram, just compute the data for a histogram. Explain what data structures you use.

(35 points) Linear State Space Models

Q3. Consider a Linearized Stochastic Solow Growth Model with fixed saving rate. Production function:

$$y_t = \exp(z_t) k_t^{\alpha}$$

The law of motion of capital is

$$k_{t+1} = (1-\delta)k_t + s y_t$$

The process for the technology shock follows an AR(1) process:

$$z_t = \rho z_{t-1} + w_t$$

where w_t is a random variable with standard normal distribution. The log-linearized capital and output follows the following processes:

$$\widehat{k}_{t+1} = \phi_1 \widehat{k}_t + \phi_2 \widehat{k}_{t-2} + \delta w_t$$

$$\widehat{y}_t = \phi_1 \widehat{y}_{t-1} + \phi_2 \widehat{y}_{t-1} + w_t + \theta w_{t-1}$$

where

$$\begin{aligned} \phi_1 &= (1-\delta+\alpha\delta)+\rho \\ \phi_2 &= -(1-\delta+\alpha\delta)\rho \end{aligned}$$

a. Write down the state space representation for capital and output, respectively. (**Hint:** in the form of $X_t = AX_{t-1} + Cw_t$ and $Y_t = GX_t$.).

b. Assume that α, δ, ρ are all between 0 and 1. Determine whether the process for capital is stationary or not. (**Hint:** check determinant.).

c. Suppose the unconditional mean of $X_0 = [\widehat{k}_1, \widehat{k}_0]$ is μ. Write down the unconditional mean of $X_1 = [\widehat{k}_2, \widehat{k}_1]$, $X_2 = [\widehat{k}_3, \widehat{k}_2]$, and $X_t = [\widehat{k}_{t+1}, \widehat{k}_t]$. (**Hint:** you may want to use matrix form.)

d. Now let us focus on the process of the technology shock. Suppose we observe current technology shock z_t. What is the conditional forecast of $\mathbb{E}[z_{t+1}|z_t]$,$\mathbb{E}[z_{t+2}|z_t]$, and $\mathbb{E}[z_{t+j}|z_t]$? What is the conditional forecast of the following geometric sum?

$$z_t + \beta\mathbb{E}[z_{t+1}|z_t] + \beta^2\mathbb{E}[z_{t+2}|z_t] + \beta^3 z_{t+3}|z_t] + \dots$$

where $\beta \in (0,1)$.

e. Write down a Python code to simulate the process for capital for n periods. The initial capital is given by $X_0 = [\widehat{k}_1, \widehat{k}_0] = [0,0]$.

(35 points) Lake Model

Q4. Consider the lake model. The transition probability for different groups of workers are given by

- λ: job finding rate for currently unemployed workers
- α: dismissal rate for currently employed workers
- b: entry rate into the labor force
- d: exit rate from the labor force

Let E_t denote the total amount of employed workers, U_t denote the total amount of unemployed workers, and $N_t = E_t + U_t$ denote the total amount of labor force. Assume that new born workers become unemployed initially.

a. Given E_t and U_t, write down the law of motion for E_{t+1}, U_{t+1}. Also write down the law of motion for N_{t+1} given N_t.

b. Find the A matrix such that

$$\begin{pmatrix} E_{t+1} \\ U_{t+1} \end{pmatrix} = A \begin{pmatrix} E_t \\ U_t \end{pmatrix}$$

c. Define $e_t = \frac{E_t}{N_t}$ as the employment rate, and $u_t = \frac{U_t}{N_t}$ as the unemployed rate. Write down the matrix $\widehat{A}$ such that

$$\begin{pmatrix} e_{t+1} \\ u_{t+1} \end{pmatrix} = \widehat{A} \begin{pmatrix} e_t \\ u_t \end{pmatrix}$$

d. Assume $d = 0$ and $b = 0$. What is $\widehat{A}$ now?

e. Still assume $d = 0$ and $b = 0$. Find the steady state employment rate e and unemployment rate u such that

$$\begin{pmatrix} e \\ u \end{pmatrix} = \widehat{A} \begin{pmatrix} e \\ u \end{pmatrix}$$

f. When $d = 0$ and $b = 0$, there are two possible states of a worker: $s = 1$, employed and $s = 2$, unemployed. Write down the stochastic matrix for these two states.

g. Suppose a worker is employed at time t. What is the probability the worker is employed at time $t + 1$? What is the probability the worker is employed at time $t + 2$?