PLSC 504 - Fall 2018
Survival Model Extensions, II

October 10, 2018

Topics

- Duration Dependence
- Competing Risks
- Repeated Events

Duration Dependence

Duration Dependence

1. *State Dependence*
   - E.g., Institutionalization / Degradation

Positive State Dependence $\longrightarrow$ Negative Duration Dependence

Negative State Dependence $\longrightarrow$ Positive Duration Dependence

Duration Dependence

2. *Unobserved / Unmodeled Heterogeneity*
   - $h(t|\mathbf{X}_i) \neq h(t|\mathbf{X}_j)$ for $\mathbf{X}_i = \mathbf{X}_j$
   - Adverse selection in the sample / data
   - Result: "Spurious" duration dependence

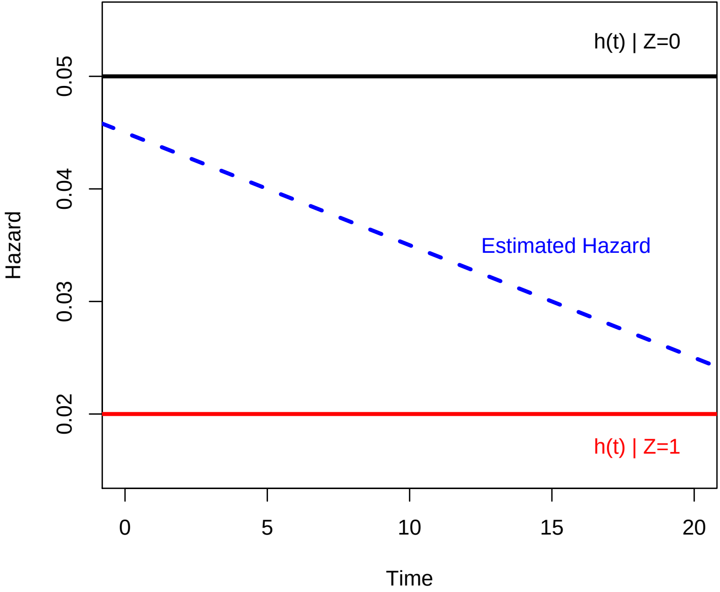
Suppose we have an unobserved Z, with

$$h_i(t|\mathbf{X}_i, Z_i = 0) = 0.05$$

and

$$h_i(t|\mathbf{X}_i, Z_i = 1) = 0.02.$$

Unobserved Heterogeneity Illustrated



Unobserved Heterogeneity: A Simulation

```
> set.seed(7222009)
> W<-rnorm(500)
> X<-rnorm(500)
> Z<-rnorm(500)
> T<-rexp(500,rate=(exp(0+0.5*W+0.5*X-0.6*Z))) # exponential hazard
> C<-rep(1,times=500)
> S<-Surv(T,C)
> summary(survreg(S~W,dist="weibull"))

Call:
survreg(formula = S ~ W, dist = "weibull")
              Value Std. Error      z        p
(Intercept) -0.0101     0.0629  -0.16 8.73e-01
W           -0.6339     0.0610 -10.40 2.47e-25
Log(scale)   0.2833     0.0333   8.52 1.62e-17

Scale= 1.33 # implies p = 1/Scale = 0.753

Weibull distribution
Loglik(model)= -568.1   Loglik(intercept only)= -615.3
Chisq= 94.47 on 1 degrees of freedom, p= 0
Number of Newton-Raphson Iterations: 5
n= 500
```

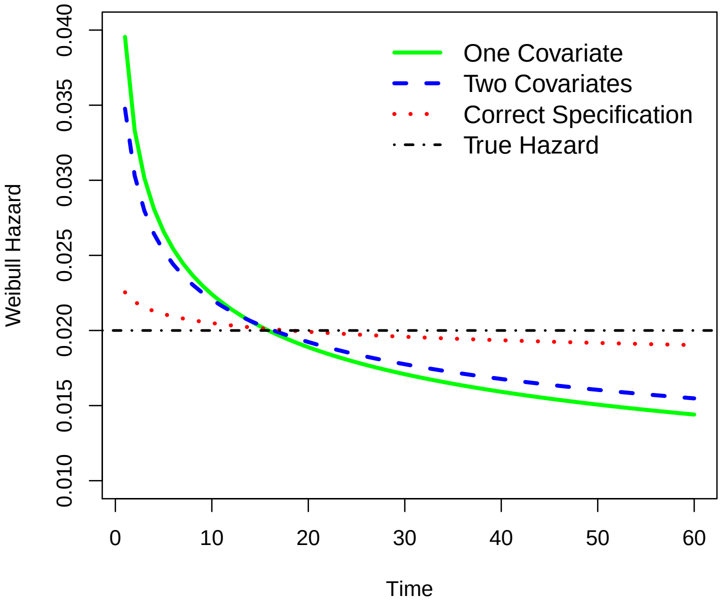
Unobserved Heterogeneity: A Simulation

```
> summary(survreg(S~W+X,dist="weibull"))

Call:
survreg(formula = S ~ W + X, dist = "weibull")
              Value Std. Error       z        p
(Intercept) -0.0511     0.0591  -0.865 3.87e-01
W           -0.5907     0.0581 -10.160 2.98e-24
X           -0.4750     0.0556  -8.549 1.24e-17
Log(scale)   0.2202     0.0329   6.689 2.24e-11

Scale= 1.25 # implies p = 1/Scale = 0.802

Weibull distribution
Loglik(model)= -534.5   Loglik(intercept only)= -615.3
Chisq= 161.6 on 2 degrees of freedom, p= 0
Number of Newton-Raphson Iterations: 5
n= 500
```

Unobserved Heterogeneity: A Simulation

```
> summary(survreg(S~W+X+Z,dist="weibull"))

Call:
survreg(formula = S ~ W + X + Z, dist = "weibull")
              Value Std. Error      z        p
(Intercept) -0.0777     0.0494  -1.57 1.16e-01
W           -0.5665     0.0468 -12.11 9.17e-34
X           -0.5041     0.0473 -10.66 1.58e-26
Z            0.5923     0.0446  13.29 2.73e-40
Log(scale)   0.0423     0.0345   1.22 2.21e-01

Scale= 1.04 # implies p = 1/Scale = 0.959

Weibull distribution
Loglik(model)= -464.3   Loglik(intercept only)= -615.3
Chisq= 302.01 on 3 degrees of freedom, p= 0
Number of Newton-Raphson Iterations: 5
n= 500
```

Unobserved Heterogeneity: A Simulation



Duration Dependence: What To Do?

(At least) Three Options:

1. Model Specification
2. Unit-Level Effects
3. <u>Model</u> the Duration Dependence

Modeling Duration Dependence

Weibull with:

$$p = \exp(\mathbf{Z}_i \gamma)$$

Gives:

$$h_i(t) = \exp(\mathbf{X}_i \beta) \exp(\mathbf{Z}_i \gamma) [\exp(\mathbf{X}_i \beta) t]^{[\exp(\mathbf{Z}_i \gamma)] - 1}$$

and (more usefully):

$$S(t) = \exp(-\exp(\mathbf{X}_i \beta) t)^{\exp(\mathbf{Z}_i \gamma)}$$

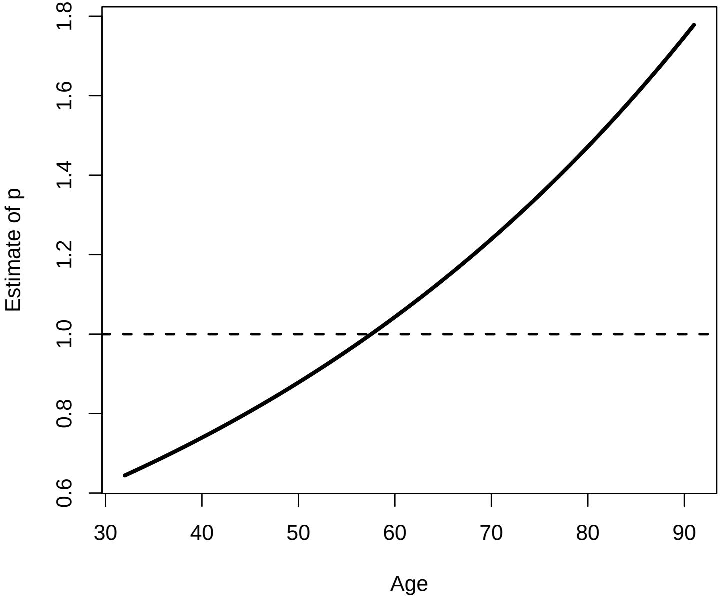
Example: SCOTUS Departures

```
> library(flexsurv)
> ct.weib<-flexsurvreg(scotus.S~age+pension+pagree,
                       data=scotus,dist="weibull")
> ct.weib

Estimates:
        data mean  est         L95%        U95%        exp(est)
shape          NA      0.999       0.637       1.570          NA
scale          NA    942.000      13.700   64800.000          NA
age        62.100     -0.041      -0.102       0.019       0.959
pension     0.199     -1.310      -2.360      -0.265       0.269
pagree      0.616     -0.113      -0.673       0.447       0.893
        L95%       U95%
shape          NA         NA
scale          NA         NA
age         0.903      1.020
pension     0.095      0.767
pagree      0.510      1.560

N = 1765,  Events: 51,  Censored: 1714
Total time at risk: 1765
Log-likelihood = -209, df = 5
AIC = 429
```

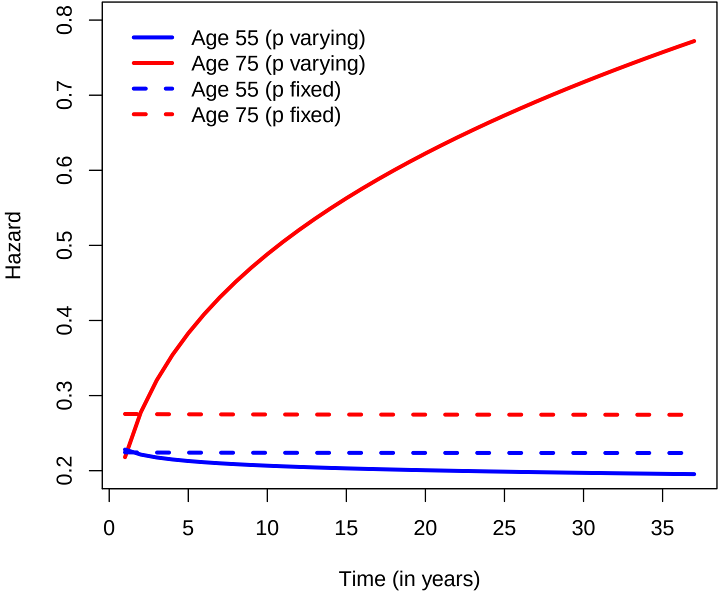
Example: SCOTUS Departures

```
> ct.weib.DD<-flexsurvreg(scotus.S~age+pension+pagree+shape(age),
                        data=scotus,dist="weibull")
> ct.weib.DD

Estimates:
           data mean   est         L95%        U95%
shape             NA      0.3710      0.1260      1.0900
scale             NA    491.0000     16.7000  14500.0000
age          62.1000     -0.0307     -0.0779      0.0164
pension       0.1990     -1.0900     -1.9700     -0.2190
pagree        0.6160     -0.0328     -0.4840      0.4180
shape(age)   62.1000      0.0172     -0.0011      0.0356
           exp(est)    L95%        U95%
shape             NA          NA          NA
scale             NA          NA          NA
age           0.9700      0.9250      1.0200
pension       0.3350      0.1400      0.8030
pagree        0.9680      0.6160      1.5200
shape(age)    1.0200      0.9990      1.0400

N = 1765,  Events: 51,  Censored: 1714
Total time at risk: 1765
Log-likelihood = -208, df = 6
AIC = 427
```

$\hat{p}$ by Age



$\widehat{h(t)}$s by Age and Model



Competing Risks

Competing Risks

R multiple kinds of events:

$$T_i \in T_{i1}, ..., T_{iR}$$

Observed duration:

$$T_i = \min(T_{i1}, ... T_{iR})$$

Event indicator:

$$D_i = r \text{ iff } T_i = T_{ri}$$

R censoring indicators:

$$C_{ir} = \begin{cases} 1 \text{ if observation } i \text{ experienced event } r \\ 0 \text{ otherwise} \end{cases}$$

Likelihoods

$$L_i = f_r(T_i|\mathbf{X}_{ir}, \beta_r) \prod_{r \neq D_i} S_r(T_i|\mathbf{X}_{ir}, \beta_r)$$

$$\begin{aligned}
L &= \prod_{i=1}^{N} \left\{ f_r(T_i|\mathbf{X}_{ir}, \beta_r) \prod_{r \neq D_i} S_r(T_i|\mathbf{X}_{ir}, \beta_r) \right\} \\
&= \prod_{r=1}^{R} \prod_{i=1}^{N_r} \{ f_r(T_i|\mathbf{X}_{ir}, \beta_r)\, S_r(T_i|\mathbf{X}_{ir}, \beta_r) \} \\
&= \prod_{r=1}^{R} \prod_{i=1}^{N} [f_r(T_i|\mathbf{X}_{ir}, \beta_r)]^{C_{ir}} \, [S_r(T_i|\mathbf{X}_{ri}, \beta_r)]^{1-C_{ir}}
\end{aligned}$$

Practical Estimation

- Independent risks = separate models
- Otherwise identical estimation, interpretation, etc.
- No identification problem
- Discrete-Time $\rightarrow$ MNL
- See (e.g.) Diermeier and Stevenson 1999; Zorn and Van Winkle 2000; Goemans 2008

Independent Risks

- Key: Conditional independence
- $\rightsquigarrow$ Model specification
- Dependent risks:
  - Using frailties (Gordon 2002)
  - Discrete-time: strategic (Fukumoto 2009)
  - Discrete-time: bivariate probit (Quiros Flores 2012)
  - SUR?

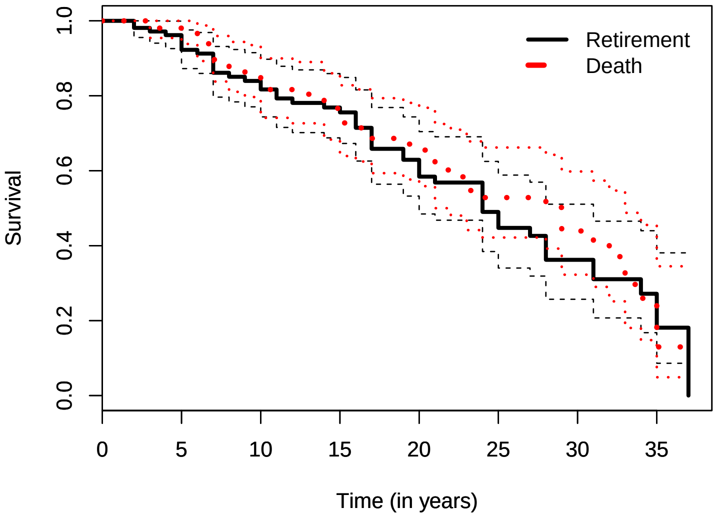
Example: SCOTUS Vacancies

- Supreme Court Vacancies, 1789-1992 ($NT = 1783$)
- Departures $\in$ {Retirement, Mortality}
- Independent competing risks models: Cox + MNL

SCOTUS Data

```
> summary(scotus)
    justice        svcstart      service         retire     
 Min.   :  1   Min.   : 0   Min.   : 1   Min.   :0.00  
 1st Qu.: 26   1st Qu.: 4   1st Qu.: 5   1st Qu.:0.00  
 Median : 51   Median : 9   Median :10   Median :0.00  
 Mean   : 53   Mean   :11   Mean   :12   Mean   :0.03  
 3rd Qu.: 79   3rd Qu.:16   3rd Qu.:17   3rd Qu.:0.00  
 Max.   :109   Max.   :36   Max.   :37   Max.   :1.00  
     death          chief           south            age    
 Min.   :0.00   Min.   :0.00   Min.   :0.00   Min.   :32  
 1st Qu.:0.00   1st Qu.:0.00   1st Qu.:0.00   1st Qu.:56  
 Median :0.00   Median :0.00   Median :0.00   Median :62  
 Mean   :0.03   Mean   :0.12   Mean   :0.31   Mean   :62  
 3rd Qu.:0.00   3rd Qu.:0.00   3rd Qu.:1.00   3rd Qu.:69  
 Max.   :1.00   Max.   :1.00   Max.   :1.00   Max.   :91  
    pension        pagree        threecat   
 Min.   :0.0   Min.   :0.00   Min.   :0.00  
 1st Qu.:0.0   1st Qu.:0.00   1st Qu.:0.00  
 Median :0.0   Median :1.00   Median :0.00  
 Mean   :0.2   Mean   :0.61   Mean   :0.08  
 3rd Qu.:0.0   3rd Qu.:1.00   3rd Qu.:0.00  
 Max.   :1.0   Max.   :1.00   Max.   :2.00  
```

SCOTUS: Death and Retirement



Independent Risks (Cox) Models

	Combined	Retirement	Death
Age	0.06	0.07	0.04
	(0.02)	(0.03)	(0.02)
Chief	−0.03	−0.23	0.09
	(0.30)	(0.44)	(0.40)
South	0.29	0.06	0.45
	(0.23)	(0.34)	(0.33)
Pension Eligibility	0.59	2.04	−0.48
	(0.28)	(0.55)	(0.41)
Party Agreement	−0.01	0.10	−0.10
	(0.21)	(0.29)	(0.31)
AIC	713.26	356.70	348.83
Num. events	99	52	47

Multinomial Logit

	Retirement	Death
Intercept	−7.77	−8.28
	(1.45)	(1.28)
Age	−0.29	0.00
	(0.45)	(0.42)
Chief	0.06	0.48
	(0.34)	(0.32)
South	0.07	0.06
	(0.03)	(0.02)
Pension Eligibility	1.40	−0.56
	(0.42)	(0.41)
Party Agreement	0.03	−0.26
	(0.30)	(0.31)
log(Time)	−0.30	0.51
	(0.27)	(0.29)
AIC	847.51	847.51
BIC	924.31	924.31
Log Likelihood	-409.75	-409.75

Multiple / Repeated Events

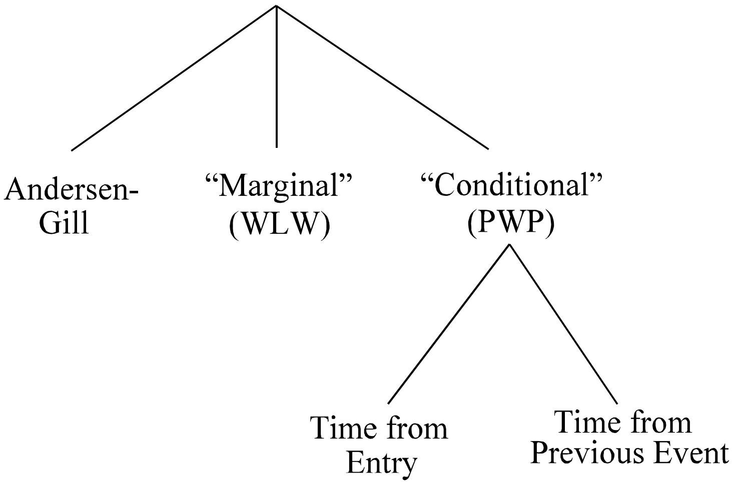
Multiple / Repeated Events

Events are not "absorbing" $\rightarrow$ capable of repetition

Raises (at least) two issues:

- Dependence across events
- Parameter variability

Variance-Correction Models



Variance Correction Model Properties

Model Property	Andersen-Gill (AG)	Marginal (WLW)	Conditional (PWP), Elapsed Time	Conditional (PWP), Gap Time
Risk Set for Event k at Time t	Independent Events	All Subjects that Haven't Experienced Event k at Time t	All Subjects that Have Experienced Event k - 1, and Haven't Experienced Event k, at Time t	
Time Scale	Duration Since Starting Observation	Duration Since Starting Observation	Duration Since Starting Observation	Duration Since Previous Event
Robust standard errors?	Yes	Yes	Yes	
Stratification by Event?	No	Yes	Yes	

Data Organization

```
> OR<-OR[order(OR$dyadid,OR$year),]
> OR$one<-rep(1,times=nrow(OR))
> OR<-ddply(OR,"dyadid",mutate,eventno=cumsum(dispute)+1,
        altstart=cumsum(one)-1,altstop=cumsum(one))

    dyadid year start  stop altstart altstop dispute eventno
461   2130 1951     0     1        0       1       0       1
462   2130 1952     1     2        1       2       1       1
463   2130 1953     0     1        2       3       0       2
464   2130 1954     1     2        3       4       1       2
465   2130 1956     0     1        4       5       0       3
466   2130 1957     1     2        5       6       0       3
467   2130 1958     2     3        6       7       0       3
468   2130 1959     3     4        7       8       0       3
469   2130 1960     4     5        8       9       0       3
470   2130 1961     5     6        9      10       0       3
471   2130 1962     6     7       10      11       0       3
472   2130 1963     7     8       11      12       1       3
473   2130 1964     0     1       12      13       0       4
474   2130 1965     1     2       13      14       0       4
.
.
.
```

First Events

```
> OR1st<-OR[OR$eventno==1,]
> OR.1st<-Surv(OR1st$altstart,OR1st$altstop,OR1st$dispute)
> OR.Cox.1st<-coxph(OR.1st~allies+contig+capratio+growth+democracy+
                  trade+cluster(dyadid),data=OR1st,method="efron")

> OR.Cox.1st
Call:
coxph(formula = OR.1st ~ allies + contig + capratio + growth +
    democracy + trade + cluster(dyadid), data = OR1st, method = "efron")


            coef exp(coef) se(coef) robust se      z            p
allies    -0.448    0.6389   0.1585    0.1640 -2.732 0.0063000000
contig     1.070    2.9167   0.1681    0.1767  6.059 0.0000000014
capratio  -0.196    0.8223   0.0603    0.0779 -2.510 0.0120000000
growth    -2.198    0.1110   1.7195    1.9005 -1.157 0.2500000000
democracy -0.424    0.6547   0.1298    0.1259 -3.365 0.0007600000
trade     -6.728    0.0012  12.3255   13.9025 -0.484 0.6300000000

Likelihood ratio test=121  on 6 df, p=0  n= 17158, number of events= 205
```

Andersen-Gill

```
> OR.AGS<-Surv(OR$altstart,OR$altstop,OR$dispute)
> OR.Cox.AG<-coxph(OR.AGS~allies+contig+capratio+growth+democracy+
                  trade+cluster(dyadid),data=OR,method="efron")

> OR.Cox.AG
Call:
coxph(formula = OR.AGS ~ allies + contig + capratio + growth +
    democracy + trade + cluster(dyadid), data = OR, method = "efron")


             coef  exp(coef) se(coef) robust se      z       p
allies     -0.414 0.66090755   0.1107    0.1703 -2.431 1.5e-02
contig      1.213 3.36515975   0.1209    0.1782  6.811 9.7e-12
capratio   -0.214 0.80717357   0.0514    0.0817 -2.620 8.8e-03
growth     -3.227 0.03967003   1.2279    1.3169 -2.451 1.4e-02
democracy  -0.439 0.64437744   0.0998    0.1231 -3.571 3.6e-04
trade     -13.162 0.00000192  10.3266   13.8188 -0.953 3.4e-01

Likelihood ratio test=272  on 6 df, p=0  n= 20448, number of events= 405
```

Prentice et al.: Elapsed Time

```
> OR.PWPES<-Surv(OR$altstart,OR$altstop,OR$dispute)
> OR.Cox.PWPE<-coxph(OR.PWPES~allies+contig+capratio+growth+democracy+
                     trade+strata(eventno)+cluster(dyadid),data=OR,
                     method="efron")

> OR.Cox.PWPE
Call:
coxph(formula = OR.PWPES ~ allies + contig + capratio + growth +
    democracy + trade + strata(eventno) + cluster(dyadid), data = OR,
    method = "efron")


            coef exp(coef) se(coef) robust se      z       p
allies    -0.240    0.7865   0.1122    0.1283 -1.872 6.1e-02
contig     0.868    2.3811   0.1223    0.1329  6.526 6.8e-11
capratio  -0.162    0.8506   0.0472    0.0618 -2.618 8.8e-03
growth    -3.625    0.0266   1.2371    1.2032 -3.013 2.6e-03
democracy -0.273    0.7612   0.1036    0.1074 -2.541 1.1e-02
trade     -2.514    0.0810   9.2934    9.9432 -0.253 8.0e-01

Likelihood ratio test=133  on 6 df, p=0  n= 20448, number of events= 405
```

Prentice et al.: Gap Time

```
> OR.PWPGS<-Surv(OR$start,OR$stop,OR$dispute)
> OR.Cox.PWPG<-coxph(OR.PWPGS~allies+contig+capratio+growth+democracy+
                     trade+strata(eventno)+cluster(dyadid),data=OR,
                     method="efron")

> OR.Cox.PWPG
Call:
coxph(formula = OR.PWPGS ~ allies + contig + capratio + growth +
    democracy + trade + strata(eventno) + cluster(dyadid), data = OR,
    method = "efron")


            coef exp(coef) se(coef) robust se     z       p
allies    -0.329    0.7193   0.1119    0.1229 -2.68 7.3e-03
contig     0.885    2.4232   0.1222    0.1285  6.89 5.6e-12
capratio  -0.171    0.8431   0.0481    0.0636 -2.68 7.3e-03
growth    -3.459    0.0315   1.2189    1.2102 -2.86 4.3e-03
democracy -0.284    0.7530   0.1028    0.1016 -2.79 5.2e-03
trade     -4.287    0.0137   9.9352   10.4592 -0.41 6.8e-01

Likelihood ratio test=139  on 6 df, p=0  n= 20448, number of events= 405
```

WLW: Data Organization

```
> OR.expand<-OR[rep(1:nrow(OR),each=max(OR$eventno)),]
> OR.expand<-ddply(OR.expand,c("dyadid","year"),mutate,
                  eventrisk=cumsum(one))
> OR.expand$dispute<-ifelse(OR.expand$eventno==OR.expand$eventrisk
                           & OR.expand$dispute==1,1,0)
> dim(OR.expand)
[1] 163584     17

> head(OR.expand,9)
  dyadid year start stop futime dispute allies contig trade growth
1   2020 1951     0    1     35       0      1      1 0.014 0.0085
2   2020 1951     0    1     35       0      1      1 0.014 0.0085
3   2020 1951     0    1     35       0      1      1 0.014 0.0085
4   2020 1951     0    1     35       0      1      1 0.014 0.0085
5   2020 1951     0    1     35       0      1      1 0.014 0.0085
6   2020 1951     0    1     35       0      1      1 0.014 0.0085
7   2020 1951     0    1     35       0      1      1 0.014 0.0085
8   2020 1951     0    1     35       0      1      1 0.014 0.0085
9   2020 1952     1    2     35       0      1      1 0.015 0.0259
  democracy capratio one eventno altstart altstop eventrisk
1         1     0.20   1       1        0       1         1
2         1     0.20   1       1        0       1         2
3         1     0.20   1       1        0       1         3
4         1     0.20   1       1        0       1         4
5         1     0.20   1       1        0       1         5
6         1     0.20   1       1        0       1         6
7         1     0.20   1       1        0       1         7
8         1     0.20   1       1        0       1         8
9         1     0.19   1       1        1       2         1
```

WLW Model

```
> OR.expand.S<-Surv(OR.expand$altstart,OR.expand$altstop,
                    OR.expand$dispute)
> OR.Cox.WLW<-coxph(OR.expand.S~allies+contig+capratio+growth+
                      democracy+trade+strata(eventno)+
                      cluster(dyadid),data=OR.expand,
                      method="efron")

> OR.Cox.WLW
Call:
coxph(formula = OR.expand.S ~ allies + contig + capratio + growth +
    democracy + trade + strata(eventno) + cluster(dyadid), data = OR.expand,
    method = "efron")


            coef exp(coef) se(coef) robust se      z       p
allies    -0.230    0.7947   0.1122    0.1248 -1.841 6.6e-02
contig     0.852    2.3435   0.1223    0.1297  6.568 5.1e-11
capratio  -0.160    0.8524   0.0471    0.0609 -2.621 8.8e-03
growth    -3.508    0.0300   1.2370    1.1671 -3.005 2.7e-03
democracy -0.271    0.7625   0.1037    0.1055 -2.570 1.0e-02
trade     -2.656    0.0702   9.2807    9.6144 -0.276 7.8e-01

Likelihood ratio test=129  on 6 df, p=0  n= 163584, number of events= 405
```

Models of Repeated Events

	First	AG	PWP-E	PWP-G	WLW
Allies	−0.45	−0.41	−0.24	−0.33	−0.23
	(0.16)	(0.17)	(0.13)	(0.12)	(0.12)
Contiguity	1.07	1.21	0.87	0.89	0.85
	(0.18)	(0.18)	(0.13)	(0.13)	(0.13)
Capability Ratio	−0.20	−0.21	−0.16	−0.17	−0.16
	(0.08)	(0.08)	(0.06)	(0.06)	(0.06)
Growth	−2.20	−3.23	−3.63	−3.46	−3.51
	(1.90)	(1.32)	(1.20)	(1.21)	(1.17)
Democracy	−0.42	−0.44	−0.27	−0.28	−0.27
	(0.13)	(0.12)	(0.11)	(0.10)	(0.11)
Trade	−6.73	−13.16	−2.51	−4.29	−2.66
	(13.90)	(13.82)	(9.94)	(10.46)	(9.61)
AIC	2538.02	5015.77	3892.77	4103.47	5597.54
Num. events	205	405	405	405	405

Parameter Change Across Events

- Values of β differ from k to $k + 1$
- Again: Institutionalization, learning, etc.
- Addressed using *strata by covariate interactions*

Parameter Change Example

```
> OR$capXevent<-OR$capratio*OR$eventno
> OR.Cox.BVary<-coxph(OR.PWPGS~allies+contig+growth+democracy+
                      trade+capratio+capXevent+strata(eventno)+
                      cluster(dyadid),data=OR,
                      method="efron")

> OR.Cox.BVary
Call:
coxph(formula = OR.PWPGS ~ allies + contig + growth + democracy +
    trade + capratio + capXevent + strata(eventno) + cluster(dyadid),
    data = OR, method = "efron")


            coef exp(coef) se(coef) robust se      z       p
allies    -0.349    0.7053   0.1120    0.1177 -2.967 3.0e-03
contig     0.897    2.4517   0.1221    0.1254  7.150 8.7e-13
growth    -3.519    0.0296   1.2196    1.2129 -2.901 3.7e-03
democracy -0.305    0.7374   0.1037    0.0972 -3.135 1.7e-03
trade     -3.297    0.0370   9.7624   10.1869 -0.324 7.5e-01
capratio  -0.340    0.7117   0.0997    0.1054 -3.227 1.2e-03
capXevent  0.135    1.1443   0.0631    0.0581  2.321 2.0e-02

Likelihood ratio test=143  on 7 df, p=0  n= 20448, number of events= 405
```

Conclusions / Recommendations

As a practical matter, estimating these models is simply a function of:

- Setting up the data correctly (so as to define the right risk sets),
- Stratifying when appropriate, and
- Calculating / using robust standard errors...