

GSERM - St. Gallen 2019

Survival Model Extensions

June 21, 2019

- Duration Dependence
- Competing Risks
- Repeated Events

Duration Dependence

Duration Dependence

1. *State Dependence*

- E.g., Institutionalization / Degradation

Positive State Dependence \longrightarrow Negative Duration Dependence

Negative State Dependence \longrightarrow Positive Duration Dependence

Duration Dependence

2. *Unobserved / Unmodeled Heterogeneity*

- $h(t|\mathbf{X}_i) \neq h(t|\mathbf{X}_j)$ for $\mathbf{X}_i = \mathbf{X}_j$
- Adverse selection in the sample / data
- Result: “Spurious” duration dependence

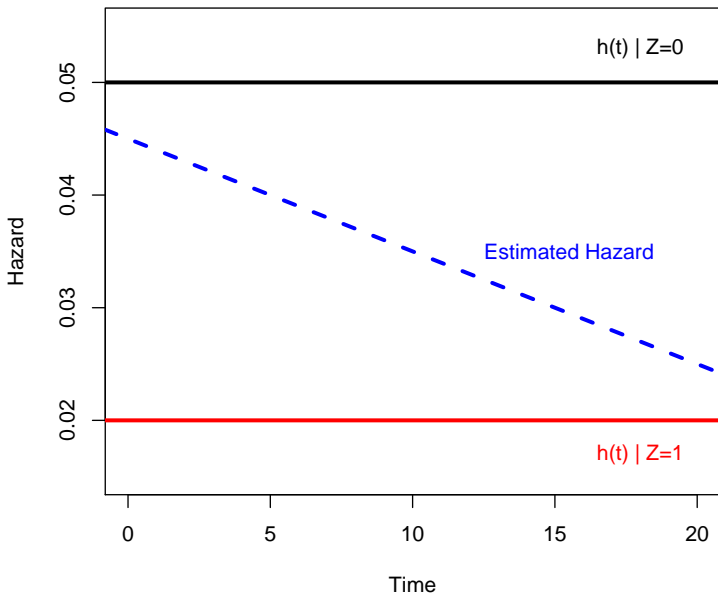
Suppose we have an unobserved Z , with

$$h_i(t|\mathbf{X}_i, Z_i = 0) = 0.05$$

and

$$h_i(t|\mathbf{X}_i, Z_i = 1) = 0.02.$$

Unobserved Heterogeneity Illustrated



Unobserved Heterogeneity: A Simulation

```
> set.seed(7222009)
> W<-rnorm(500)
> X<-rnorm(500)
> Z<-rnorm(500)
> T<-rexp(500,rate=(exp(0+0.5*W+0.5*X-0.6*Z))) # exponential hazard
> C<-rep(1,times=500)
> S<-Surv(T,C)
> summary(survreg(S~W,dist="weibull"))
```

Call:

```
survreg(formula = S ~ W, dist = "weibull")
```

	Value	Std. Error	z	p
(Intercept)	-0.0101	0.0629	-0.16	8.73e-01
W	-0.6339	0.0610	-10.40	2.47e-25
Log(scale)	0.2833	0.0333	8.52	1.62e-17

```
Scale= 1.33 # implies p = 1/Scale = 0.753
```

Weibull distribution

```
Loglik(model)= -568.1   Loglik(intercept only)= -615.3
```

```
Chisq= 94.47 on 1 degrees of freedom, p= 0
```

```
Number of Newton-Raphson Iterations: 5
```

```
n= 500
```

Unobserved Heterogeneity: A Simulation

```
> summary(survreg(S~W+X,dist="weibull"))
```

Call:

```
survreg(formula = S ~ W + X, dist = "weibull")
```

	Value	Std. Error	z	p
(Intercept)	-0.0511	0.0591	-0.865	3.87e-01
W	-0.5907	0.0581	-10.160	2.98e-24
X	-0.4750	0.0556	-8.549	1.24e-17
Log(scale)	0.2202	0.0329	6.689	2.24e-11

```
Scale= 1.25 # implies p = 1/Scale = 0.802
```

Weibull distribution

```
Loglik(model)= -534.5   Loglik(intercept only)= -615.3
```

```
Chisq= 161.6 on 2 degrees of freedom, p= 0
```

```
Number of Newton-Raphson Iterations: 5
```

```
n= 500
```


Unobserved Heterogeneity: A Simulation

```
> summary(survreg(S~W+X+Z,dist="weibull"))
```

Call:

```
survreg(formula = S ~ W + X + Z, dist = "weibull")
```

	Value	Std. Error	z	p
(Intercept)	-0.0777	0.0494	-1.57	1.16e-01
W	-0.5665	0.0468	-12.11	9.17e-34
X	-0.5041	0.0473	-10.66	1.58e-26
Z	0.5923	0.0446	13.29	2.73e-40
Log(scale)	0.0423	0.0345	1.22	2.21e-01

Scale= 1.04 # implies $p = 1/\text{Scale} = 0.959$

Weibull distribution

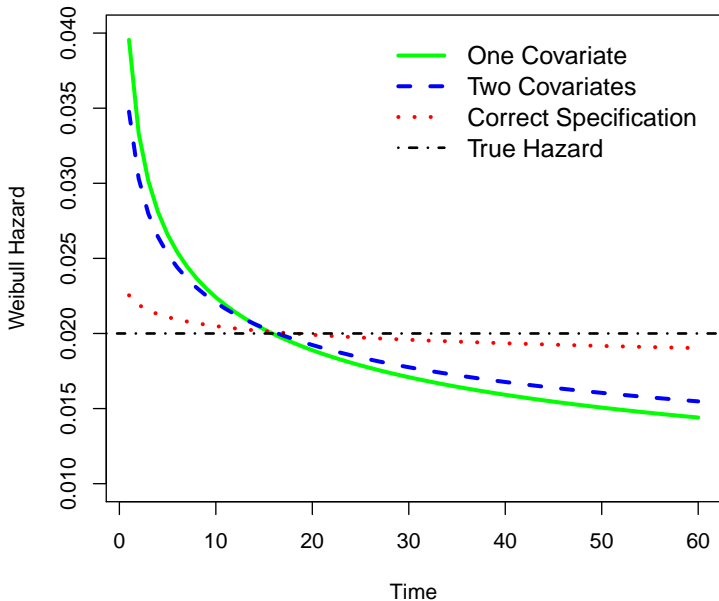
Loglik(model)= -464.3 Loglik(intercept only)= -615.3

Chisq= 302.01 on 3 degrees of freedom, $p= 0$

Number of Newton-Raphson Iterations: 5

n= 500

Unobserved Heterogeneity: A Simulation



Duration Dependence: What To Do?

(At least) Three Options:

1. Model Specification
2. Unit-Level Effects
3. Model the Duration Dependence

Modeling Duration Dependence

Weibull with:

$$p = \exp(\mathbf{Z}_i\gamma)$$

Gives:

$$h_i(t) = \exp(\mathbf{X}_i\beta) \exp(\mathbf{Z}_i\gamma) [\exp(\mathbf{X}_i\beta)t]^{\exp(\mathbf{Z}_i\gamma)-1}$$

and (more usefully):

$$S(t) = \exp(-\exp(\mathbf{X}_i\beta)t)^{\exp(\mathbf{Z}_i\gamma)}$$

Example: SCOTUS Departures

```
> library(flexsurv)
> ct.weib<-flexsurvreg(scotus.S~age+pension+pagree,
                      data=scotus,dist="weibull")
> ct.weib
```

Estimates:

	data mean	est	L95%	U95%	exp(est)
shape	NA	0.999	0.637	1.570	NA
scale	NA	942.000	13.700	64800.000	NA
age	62.100	-0.041	-0.102	0.019	0.959
pension	0.199	-1.310	-2.360	-0.265	0.269
pagree	0.616	-0.113	-0.673	0.447	0.893

	L95%	U95%
shape	NA	NA
scale	NA	NA
age	0.903	1.020
pension	0.095	0.767
pagree	0.510	1.560

N = 1765, Events: 51, Censored: 1714

Total time at risk: 1765

Log-likelihood = -209, df = 5

AIC = 429

Example: SCOTUS Departures

```
> ct.weib.DD<-flexsurvreg(scotus.S~age+pension+pagree+shape(age),  
                           data=scotus,dist="weibull")
```

```
> ct.weib.DD
```

Estimates:

	data mean	est	L95%	U95%
shape	NA	0.3710	0.1260	1.0900
scale	NA	491.0000	16.7000	14500.0000
age	62.1000	-0.0307	-0.0779	0.0164
pension	0.1990	-1.0900	-1.9700	-0.2190
pagree	0.6160	-0.0328	-0.4840	0.4180
shape(age)	62.1000	0.0172	-0.0011	0.0356

	exp(est)	L95%	U95%
shape	NA	NA	NA
scale	NA	NA	NA
age	0.9700	0.9250	1.0200
pension	0.3350	0.1400	0.8030
pagree	0.9680	0.6160	1.5200
shape(age)	1.0200	0.9990	1.0400

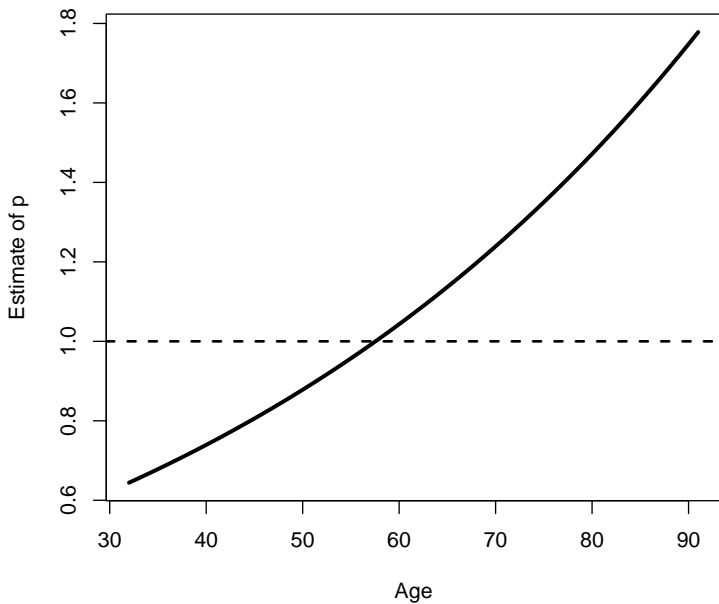
N = 1765, Events: 51, Censored: 1714

Total time at risk: 1765

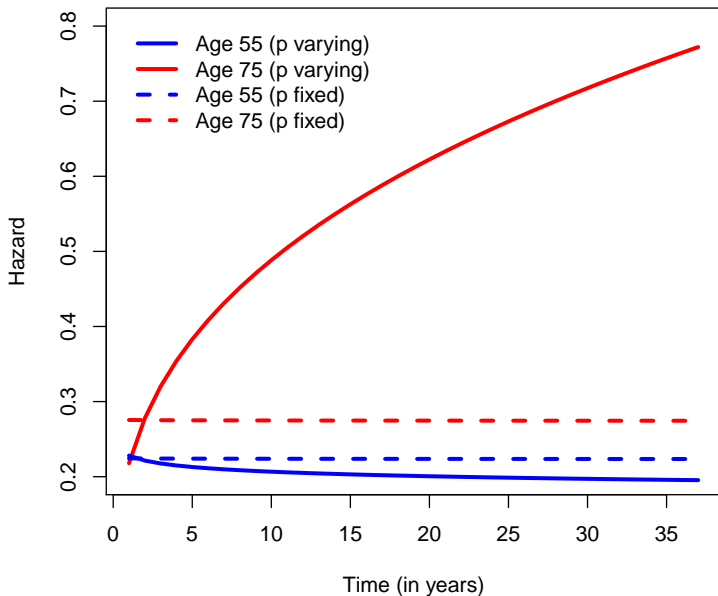
Log-likelihood = -208, df = 6

AIC = 427

\hat{p} by Age



$\widehat{h(t)}$ s by Age and Model



Competing Risks

Competing Risks

R multiple kinds of events:

$$T_i \in T_{i1}, \dots, T_{iR}$$

Observed duration:

$$T_i = \min(T_{i1}, \dots, T_{iR})$$

Event indicator:

$$D_i = r \text{ iff } T_i = T_{ri}$$

R censoring indicators:

$$C_{ir} = \begin{cases} 1 & \text{if observation } i \text{ experienced event } r \\ 0 & \text{otherwise} \end{cases}$$

$$L_i = f_r(T_i|\mathbf{X}_{ir}, \beta_r) \prod_{r \neq D_i} S_r(T_i|\mathbf{X}_{ir}, \beta_r)$$

$$\begin{aligned} L &= \prod_{i=1}^N \left\{ f_r(T_i|\mathbf{X}_{ir}, \beta_r) \prod_{r \neq D_i} S_r(T_i|\mathbf{X}_{ir}, \beta_r) \right\} \\ &= \prod_{r=1}^R \prod_{i=1}^{N_r} \{ f_r(T_i|\mathbf{X}_{ir}, \beta_r) S_r(T_i|\mathbf{X}_{ir}, \beta_r) \} \\ &= \prod_{r=1}^R \prod_{i=1}^N [f_r(T_i|\mathbf{X}_{ir}, \beta_r)]^{C_{ir}} [S_r(T_i|\mathbf{X}_{ri}, \beta_r)]^{1-C_{ir}} \end{aligned}$$

Practical Estimation

- Independent risks = separate models
- Otherwise identical estimation, interpretation, etc.
- No identification problem
- Discrete-Time \rightarrow MNL
- See (e.g.) Diermeier and Stevenson 1999; Zorn and Van Winkle 2000; Goemans 2008

- Key: Conditional independence
- \rightsquigarrow Model specification
- Dependent risks:
 - Using frailties ([Gordon 2002](#))
 - Discrete-time: strategic ([Fukumoto 2009](#))
 - Discrete-time: bivariate probit ([Quiros Flores 2012](#))
 - SUR?

Example: SCOTUS Vacancies

- Supreme Court Vacancies, 1789-1992
($NT = 1783$)
- Departures $\in \{\text{Retirement, Mortality}\}$
- Independent competing risks models: Cox + MNL

SCOTUS Data

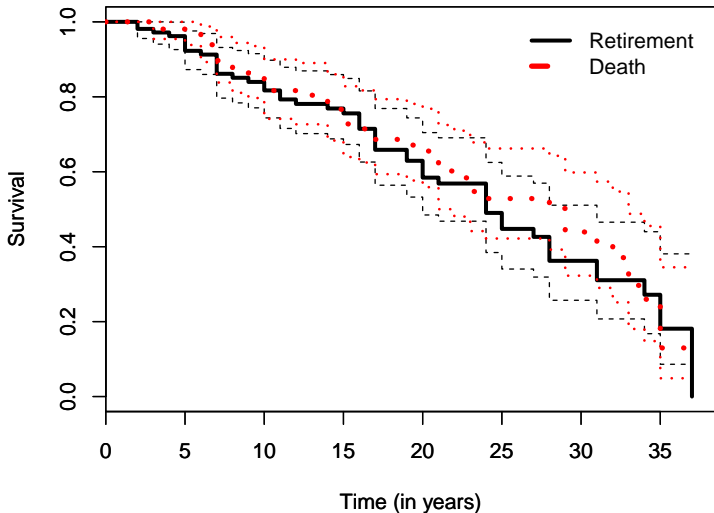
```
> summary(scotus)
```

justice	svcstart	service	retire
Min. : 1	Min. : 0	Min. : 1	Min. :0.00
1st Qu.: 26	1st Qu.: 4	1st Qu.: 5	1st Qu.:0.00
Median : 51	Median : 9	Median :10	Median :0.00
Mean : 53	Mean :11	Mean :12	Mean :0.03
3rd Qu.: 79	3rd Qu.:16	3rd Qu.:17	3rd Qu.:0.00
Max. :109	Max. :36	Max. :37	Max. :1.00

death	chief	south	age
Min. :0.00	Min. :0.00	Min. :0.00	Min. :32
1st Qu.:0.00	1st Qu.:0.00	1st Qu.:0.00	1st Qu.:56
Median :0.00	Median :0.00	Median :0.00	Median :62
Mean :0.03	Mean :0.12	Mean :0.31	Mean :62
3rd Qu.:0.00	3rd Qu.:0.00	3rd Qu.:1.00	3rd Qu.:69
Max. :1.00	Max. :1.00	Max. :1.00	Max. :91

pension	pagree	threecat
Min. :0.0	Min. :0.00	Min. :0.00
1st Qu.:0.0	1st Qu.:0.00	1st Qu.:0.00
Median :0.0	Median :1.00	Median :0.00
Mean :0.2	Mean :0.61	Mean :0.08
3rd Qu.:0.0	3rd Qu.:1.00	3rd Qu.:0.00
Max. :1.0	Max. :1.00	Max. :2.00

SCOTUS: Death and Retirement



Independent Risks (Cox) Models

	Combined	Retirement	Death
Age	0.06 (0.02)	0.07 (0.03)	0.04 (0.02)
Chief	-0.03 (0.30)	-0.23 (0.44)	0.09 (0.40)
South	0.29 (0.23)	0.06 (0.34)	0.45 (0.33)
Pension Eligibility	0.59 (0.28)	2.04 (0.55)	-0.48 (0.41)
Party Agreement	-0.01 (0.21)	0.10 (0.29)	-0.10 (0.31)
AIC	713.26	356.70	348.83
Num. events	99	52	47

Multinomial Logit

	Retirement	Death
Intercept	-7.77 (1.45)	-8.28 (1.28)
Age	-0.29 (0.45)	0.00 (0.42)
Chief	0.06 (0.34)	0.48 (0.32)
South	0.07 (0.03)	0.06 (0.02)
Pension Eligibility	1.40 (0.42)	-0.56 (0.41)
Party Agreement	0.03 (0.30)	-0.26 (0.31)
log(Time)	-0.30 (0.27)	0.51 (0.29)
AIC	847.51	847.51
BIC	924.31	924.31
Log Likelihood	-409.75	-409.75

Multiple / Repeated Events

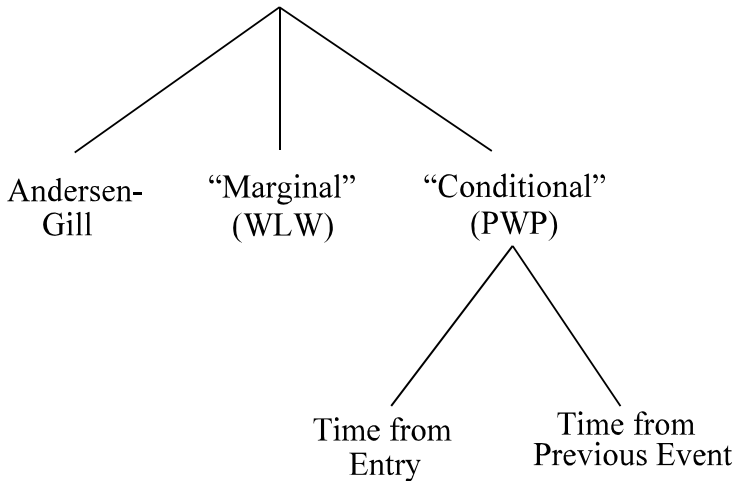
Multiple / Repeated Events

Events are not “absorbing” → capable of repetition

Raises (at least) two issues:

- Dependence across events
- Parameter variability

Variance-Correction Models



Variance Correction Model Properties

Model Property	Andersen-Gill (AG)	Marginal (WLW)	Conditional (PWP), Elapsed Time	Conditional (PWP), Gap Time
Risk Set for Event k at Time t	Independent Events	All Subjects that Haven't Experienced Event k at Time t	All Subjects that Have Experienced Event $k - 1$, and Haven't Experienced Event k , at Time t	
Time Scale	Duration Since Starting Observation	Duration Since Starting Observation	Duration Since Starting Observation	Duration Since Previous Event
Robust standard errors?	Yes	Yes		Yes
Stratification by Event?	No	Yes		Yes

Data Organization

```
> OR<-OR[order(OR$dyadid,OR$year),]  
> OR$one<-rep(1,times=nrow(OR))  
> OR<-ddply(OR,"dyadid",mutate,eventno=cumsum(dispute)+1,  
  altstart=cumsum(one)-1,altstop=cumsum(one))
```

	dyadid	year	start	stop	altstart	altstop	dispute	eventno
461	2130	1951	0	1	0	1	0	1
462	2130	1952	1	2	1	2	1	1
463	2130	1953	0	1	2	3	0	2
464	2130	1954	1	2	3	4	1	2
465	2130	1956	0	1	4	5	0	3
466	2130	1957	1	2	5	6	0	3
467	2130	1958	2	3	6	7	0	3
468	2130	1959	3	4	7	8	0	3
469	2130	1960	4	5	8	9	0	3
470	2130	1961	5	6	9	10	0	3
471	2130	1962	6	7	10	11	0	3
472	2130	1963	7	8	11	12	1	3
473	2130	1964	0	1	12	13	0	4
474	2130	1965	1	2	13	14	0	4
.								
.								
.								

First Events

```
> OR1st<-OR[OR$eventno==1,]  
> OR.1st<-Surv(OR1st$saltstart,OR1st$saltstop,OR1st$dispute)  
> OR.Cox.1st<-coxph(OR.1st~allies+contig+capratio+growth+democracy+  
  trade+cluster(dyadid),data=OR1st,method="efron")  
  
> OR.Cox.1st  
Call:  
coxph(formula = OR.1st ~ allies + contig + capratio + growth +  
  democracy + trade + cluster(dyadid), data = OR1st, method = "efron")
```

	coef	exp(coef)	se(coef)	robust se	z	p
allies	-0.448	0.6389	0.1585	0.1640	-2.732	0.0063000000
contig	1.070	2.9167	0.1681	0.1767	6.059	0.0000000014
capratio	-0.196	0.8223	0.0603	0.0779	-2.510	0.0120000000
growth	-2.198	0.1110	1.7195	1.9005	-1.157	0.2500000000
democracy	-0.424	0.6547	0.1298	0.1259	-3.365	0.0007600000
trade	-6.728	0.0012	12.3255	13.9025	-0.484	0.6300000000

Likelihood ratio test=121 on 6 df, p=0 n= 17158, number of events= 205


```
> OR.AGS<-Surv(OR$saltstart,OR$saltstop,OR$dispute)
> OR.Cox.AG<-coxph(OR.AGS~allies+contig+capratio+growth+democracy+
  trade+cluster(dyadid),data=OR,method="efron")

> OR.Cox.AG
Call:
coxph(formula = OR.AGS ~ allies + contig + capratio + growth +
  democracy + trade + cluster(dyadid), data = OR, method = "efron")
```

	coef	exp(coef)	se(coef)	robust se	z	p
allies	-0.414	0.66090755	0.1107	0.1703	-2.431	1.5e-02
contig	1.213	3.36515975	0.1209	0.1782	6.811	9.7e-12
capratio	-0.214	0.80717357	0.0514	0.0817	-2.620	8.8e-03
growth	-3.227	0.03967003	1.2279	1.3169	-2.451	1.4e-02
democracy	-0.439	0.64437744	0.0998	0.1231	-3.571	3.6e-04
trade	-13.162	0.00000192	10.3266	13.8188	-0.953	3.4e-01

Likelihood ratio test=272 on 6 df, p=0 n= 20448, number of events= 405

Prentice et al.: Elapsed Time

```
> OR.PWPES<-Surv(OR$altstart,OR$altstop,OR$dispute)
> OR.Cox.PWPE<-coxph(OR.PWPES~allies+contig+capratio+growth+democracy+
  trade+strata(eventno)+cluster(dyadid),data=OR,
  method="efron")
```

```
> OR.Cox.PWPE
```

Call:

```
coxph(formula = OR.PWPES ~ allies + contig + capratio + growth +
  democracy + trade + strata(eventno) + cluster(dyadid), data = OR,
  method = "efron")
```

	coef	exp(coef)	se(coef)	robust se	z	p
allies	-0.240	0.7865	0.1122	0.1283	-1.872	6.1e-02
contig	0.868	2.3811	0.1223	0.1329	6.526	6.8e-11
capratio	-0.162	0.8506	0.0472	0.0618	-2.618	8.8e-03
growth	-3.625	0.0266	1.2371	1.2032	-3.013	2.6e-03
democracy	-0.273	0.7612	0.1036	0.1074	-2.541	1.1e-02
trade	-2.514	0.0810	9.2934	9.9432	-0.253	8.0e-01

```
Likelihood ratio test=133 on 6 df, p=0 n= 20448, number of events= 405
```

Prentice et al.: Gap Time

```
> OR.PWPGS<-Surv(OR$start,OR$stop,OR$dispute)
> OR.Cox.PWPG<-coxph(OR.PWPGS~allies+contig+capratio+growth+democracy+
  trade+strata(eventno)+cluster(dyadid),data=OR,
  method="efron")
```

```
> OR.Cox.PWPG
```

Call:

```
coxph(formula = OR.PWPGS ~ allies + contig + capratio + growth +
  democracy + trade + strata(eventno) + cluster(dyadid), data = OR,
  method = "efron")
```

	coef	exp(coef)	se(coef)	robust se	z	p
allies	-0.329	0.7193	0.1119	0.1229	-2.68	7.3e-03
contig	0.885	2.4232	0.1222	0.1285	6.89	5.6e-12
capratio	-0.171	0.8431	0.0481	0.0636	-2.68	7.3e-03
growth	-3.459	0.0315	1.2189	1.2102	-2.86	4.3e-03
democracy	-0.284	0.7530	0.1028	0.1016	-2.79	5.2e-03
trade	-4.287	0.0137	9.9352	10.4592	-0.41	6.8e-01

```
Likelihood ratio test=139 on 6 df, p=0 n= 20448, number of events= 405
```

WLW: Data Organization

```
> OR.expand<-OR[rep(1:nrow(OR),each=max(OR$eventno)),]
> OR.expand<-ddply(OR.expand,c("dyadid","year"),mutate,
  eventrisk=cumsum(one))
> OR.expand$dispute<-ifelse(OR.expand$eventno==OR.expand$eventrisk
  & OR.expand$dispute==1,1,0)

> dim(OR.expand)
[1] 163584      17

> head(OR.expand,9)
  dyadid year start stop futime dispute allies contig trade growth
1   2020 1951     0   1    35       0     1     1 0.014 0.0085
2   2020 1951     0   1    35       0     1     1 0.014 0.0085
3   2020 1951     0   1    35       0     1     1 0.014 0.0085
4   2020 1951     0   1    35       0     1     1 0.014 0.0085
5   2020 1951     0   1    35       0     1     1 0.014 0.0085
6   2020 1951     0   1    35       0     1     1 0.014 0.0085
7   2020 1951     0   1    35       0     1     1 0.014 0.0085
8   2020 1951     0   1    35       0     1     1 0.014 0.0085
9   2020 1952     1   2    35       0     1     1 0.015 0.0259
  democracy capratio one eventno altstart altstop eventrisk
1         1      0.20  1         1         0         1         1
2         1      0.20  1         1         0         1         2
3         1      0.20  1         1         0         1         3
4         1      0.20  1         1         0         1         4
5         1      0.20  1         1         0         1         5
6         1      0.20  1         1         0         1         6
7         1      0.20  1         1         0         1         7
8         1      0.20  1         1         0         1         8
9         1      0.19  1         1         1         2         1
```

WLW Model

```
> OR.expand.S<-Surv(OR.expand$altstart,OR.expand$altstop,  
                    OR.expand$dispute)  
> OR.Cox.WLW<-coxph(OR.expand.S~allies+contig+capratio+growth+  
                    democracy+trade+strata(eventno)+  
                    cluster(dyadid),data=OR.expand,  
                    method="efron")
```

```
> OR.Cox.WLW
```

Call:

```
coxph(formula = OR.expand.S ~ allies + contig + capratio + growth +  
      democracy + trade + strata(eventno) + cluster(dyadid), data = OR.expand,  
      method = "efron")
```

	coef	exp(coef)	se(coef)	robust se	z	p
allies	-0.230	0.7947	0.1122	0.1248	-1.841	6.6e-02
contig	0.852	2.3435	0.1223	0.1297	6.568	5.1e-11
capratio	-0.160	0.8524	0.0471	0.0609	-2.621	8.8e-03
growth	-3.508	0.0300	1.2370	1.1671	-3.005	2.7e-03
democracy	-0.271	0.7625	0.1037	0.1055	-2.570	1.0e-02
trade	-2.656	0.0702	9.2807	9.6144	-0.276	7.8e-01

Likelihood ratio test=129 on 6 df, p=0 n= 163584, number of events= 405

Models of Repeated Events

	First	AG	PWP-E	PWP-G	WLW
Allies	-0.45 (0.16)	-0.41 (0.17)	-0.24 (0.13)	-0.33 (0.12)	-0.23 (0.12)
Contiguity	1.07 (0.18)	1.21 (0.18)	0.87 (0.13)	0.89 (0.13)	0.85 (0.13)
Capability Ratio	-0.20 (0.08)	-0.21 (0.08)	-0.16 (0.06)	-0.17 (0.06)	-0.16 (0.06)
Growth	-2.20 (1.90)	-3.23 (1.32)	-3.63 (1.20)	-3.46 (1.21)	-3.51 (1.17)
Democracy	-0.42 (0.13)	-0.44 (0.12)	-0.27 (0.11)	-0.28 (0.10)	-0.27 (0.11)
Trade	-6.73 (13.90)	-13.16 (13.82)	-2.51 (9.94)	-4.29 (10.46)	-2.66 (9.61)
AIC	2538.02	5015.77	3892.77	4103.47	5597.54
Num. events	205	405	405	405	405

Parameter Change Across Events

- Values of β differ from k to $k + 1$
- Again: Institutionalization, learning, etc.
- Addressed using *strata by covariate interactions*

Parameter Change Example

```
> OR$capXevent<-OR$capratio*OR$eventno
> OR.Cox.BVary<-coxph(OR.PWPGS~allies+contig+growth+democracy+
  trade+capratio+capXevent+strata(eventno)+
  cluster(dyadid),data=OR,
  method="efron")
```

```
> OR.Cox.BVary
```

Call:

```
coxph(formula = OR.PWPGS ~ allies + contig + growth + democracy +
  trade + capratio + capXevent + strata(eventno) + cluster(dyadid),
  data = OR, method = "efron")
```

	coef	exp(coef)	se(coef)	robust se	z	p
allies	-0.349	0.7053	0.1120	0.1177	-2.967	3.0e-03
contig	0.897	2.4517	0.1221	0.1254	7.150	8.7e-13
growth	-3.519	0.0296	1.2196	1.2129	-2.901	3.7e-03
democracy	-0.305	0.7374	0.1037	0.0972	-3.135	1.7e-03
trade	-3.297	0.0370	9.7624	10.1869	-0.324	7.5e-01
capratio	-0.340	0.7117	0.0997	0.1054	-3.227	1.2e-03
capXevent	0.135	1.1443	0.0631	0.0581	2.321	2.0e-02

Likelihood ratio test=143 on 7 df, p=0 n= 20448, number of events= 405

Conclusions / Recommendations

As a practical matter, estimating these models is simply a function of:

- Setting up the data correctly (so as to define the right risk sets),
- Stratifying when appropriate, and
- Calculating / using robust standard errors...