Math 212 Final Exam
Due 5/5/20 **Name:** ______________________

1. Consider the homogeneous differential equation

$$\left(2+x^2\right)y'' - xy' + 4y \;=\; 0\ .$$

   (a) Determine the recurrence relation for the coefficients of a power series solution centered at $x_0 = 0$.

   (b) Find the first four nonzero terms for each of two independent solutions $y_1(x)$ and $y_2(x)$ (unless the series terminates sooner).

2. Consider the homogeneous differential equation

$$\left(1+x^2\right)y'' - 4xy' + 6y \;=\; 0\ .$$

   (a) Determine the recurrence relation for the coefficients of a power series solution centered at $x_0 = 0$.

   (b) Find the first four nonzero terms for each of two independent solutions $y_1(x)$ and $y_2(x)$ (unless the series terminates sooner).

3. Consider the first-order linear system

$$\begin{cases} x' &= -3x + 4y \\ y' &= -2x + 3y \end{cases}$$

   for trajectories $(\,x(t),\,y(t)\,)$ in the xy-plane.

   (a) Write this system in matrix form and determine its general solution.

   (b) Sketch the phase portrait, i.e., plot some representative trajectories in the xy-plane.

   (c) How do solutions behave as $t \to \infty$?

4. Consider the first-order linear system

$$\begin{cases} x' &= -5x - 10y \\ y' &= x + y \end{cases}$$

   for trajectories $(\,x(t),\,y(t)\,)$ in the xy-plane.

   (a) Write this system in matrix form and determine its general solution.

   (b) Sketch the phase portrait, i.e., plot some representative trajectories in the xy-plane.

   (c) How do solutions behave as $t \to \infty$?