

1. Consider the following linear system:

$$\begin{array}{rclcl} x_1 + 5x_2 - 2x_3 & = & 0 \\ -2x_2 + x_3 - x_4 & = & -1 \\ x_4 & = & 5 \end{array}$$

(a) Is this system in *triangular form*, *echelon form*, or neither?

echelon form

(b) Which variables are *leading variables*?

x_1, x_2, x_4

(c) Which variables are *free variables*?

x_3

(d) Find the set of solutions of the system.

$$x_4 = 5, \quad x_3 = s_1 \text{ (free parameter)}$$

Then (2nd equation):

$$\begin{aligned} -2x_2 + s_1 - 5 &= -1 \implies -2x_2 = 4 - s_1 \\ \implies x_2 &= \frac{s_1 - 4}{2} \end{aligned}$$

And (1st equation)

$$x_1 + 5\left(\frac{s_1 - 4}{2}\right) - 2s_1 = 0$$

$$\begin{aligned} \implies x_1 &= 2s_1 - 5\left(\frac{s_1 - 4}{2}\right) = \frac{4s_1 - 5s_1 + 20}{2} \\ &= \frac{20 - s_1}{2} \end{aligned}$$