Math 102

Quiz 1 9/6/19 Name: SOLUTIONS

1. Evaluate the following integrals:

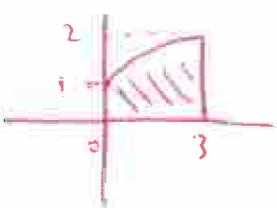
3 (a) $\int (x^3 + 2x + 1)\, dx = \frac{1}{4}x^4 + x^2 + x + c$

3 (b) $\int 8x^3 (x^4 - 1)^3\, dx = \frac{1}{2}(x^4 - 1)^4 + c$

(can use the substitution $u = x^4 - 1$, $du = 4x^3 dx$, $2du = 8x^3 dx$; get $\int 2u^3\, du = \frac{1}{2}u^4 + c = \frac{1}{2}(x^4-1)^4 + c$)

4 2. Compute the exact area bounded by the x-axis, the given curve $y = f(x)$, and the given vertical lines. Be sure to show your work clearly.

(a) $y = \sqrt{x+1}$, $x = 0$, $x = 3$



$$\text{area} = \int_0^3 \sqrt{x+1}\, dx = \frac{2}{3}(x+1)^{3/2} \Big|_{x=0}^{x=3}$$

$$= \frac{2}{3}(4)^{3/2} - \frac{2}{3}(1)^{3/2} = \frac{16}{3} - \frac{2}{3} = \underline{\underline{\frac{14}{3}}}$$

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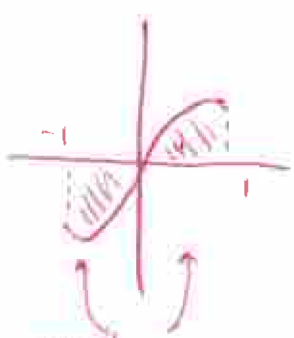
2 BONUS (b) $y = \dfrac{x}{(1+x^2)^2}$, $x = -1$, $x = 1$



equal areas by symmetry

$$\text{area} = 2\int_0^1 \frac{x}{(1+x^2)^2}\, dx = 2\left[-\frac{1}{2} \cdot \frac{1}{1+x^2} \Big|_0^1 \right]$$

$$= 2\left[-\frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \right]$$

$$= 2\left[\frac{1}{4} \right] = \frac{1}{2}.$$