$$y'' - 3y' + 2y = 0, \quad y(0) = -1, \quad y'(0) = -3$$

$$(-y'(0) - sy(0) + s^2 Y) - 3(-y(0) + sY) + 2Y = 0$$

$$\cancel{3} + s - \cancel{3} + (s^2 - 3s + 2)Y = 0$$

$$(s^2 - 3s + 2)Y = -s$$

$$Y = \frac{-s}{s^2 - 3s + 2} = \frac{A}{s-1} + \frac{B}{s-2}$$

$$-s = A(s-2) + B(s-1)$$

$$\Longrightarrow B = -2, \quad A = 1$$

$$Y = \frac{1}{s-1} - \frac{2}{s-2}$$

$$\Longrightarrow y(t) = e^{t} - 2e^{2t}$$

$$y'' + 2y' + 2y = \underline{e^{-t}}\tan t$$

Homogeneous: $y'' + 2y' + 2y = 0 \implies \lambda^2 + 2\lambda + 2 = 0$

$$\implies \lambda = \frac{-2 \pm \sqrt{4-8}}{2} = \frac{-2 \pm 2i}{2} = -1 \pm i$$

$\hookrightarrow y_1(t) = e^{-t}\sin t, \quad y_2(t) = e^{-t}\cos t$

particular: $y_p = u_1 y_1 + u_2 y_2$, where u_1 & u_2 satisfy

$$\begin{cases} u_1' y_1 + u_2' y_2 = 0 \\ u_1' y_1' + u_2' y_2' = e^{-t}\tan t \end{cases}$$

$$\implies \begin{cases} u_1' e^{-t}\sin t + u_2' e^{-t}\cos t = 0 \\ u_1'(-\cancel{e^{-t}}\sin t + e^{-t}\cos t) + u_2'(-\cancel{e^{-t}}\cos t - e^{-t}\sin t) = e^{-t}\tan t \end{cases}$$

$$u_1' e^{-t}\cos t - u_2' e^{-t}\sin t = e^{-t}\tan t$$

$$\Longrightarrow \left.\begin{aligned} u_1' \sin t + u_2' \cos t &= 0 \\ u_1' \cos t - u_2' \sin t &= \tan t \end{aligned}\right\}$$

$$\Longrightarrow \left.\begin{aligned} u_1' \sin^2 t + u_2' \sin t \cos t &= 0 \\ u_1' \cos^2 t - u_2' \sin t \cos t &= \sin t \end{aligned}\right\}$$

$$\underline{u_1' = \sin t} \Longrightarrow \underline{\underline{u_1 = -\cos t}}$$

$$\Longrightarrow \sin^2 t + u_2' \cos t = 0 \Longrightarrow u_2' = -\frac{\sin^2 t}{\cos t}$$

$$\Longrightarrow u_2' = \frac{\cos^2 t - 1}{\cos t}$$

$$\Longrightarrow u_2' = \cos t - \sec t$$

$$\Longrightarrow u_2 = \sin t - \ln|\sec t + \tan t|$$

$$\underbrace{y'' - 2y' + 2y}\; = e^{-t}, \quad y(0) = 0, \quad y'(0) = 1$$

$$-y'(0) - \cancel{sy(0)} + s^2 Y - 2(-\cancel{y(0)} + sY) + 2Y = \frac{1}{s+1}$$

$$-1 + (s^2 - 2s + 2)Y = \frac{1}{s+1}$$

$$(s^2 - 2s + 2)Y = 1 + \frac{1}{s+1}$$

$$Y = \frac{1}{s^2 - 2s + 2} + \frac{1}{(s+1)(s^2 - 2s + 2)}$$

$$Y = \frac{1}{(s-1)^2 + 1} + \frac{A}{s+1} + \frac{Bs + C}{s^2 - 2s + 2}$$

$$= \frac{1}{(s-1)^2 + 1} + \frac{\frac{1}{5}}{s+1} + \frac{-\frac{1}{5}s + \frac{3}{5}}{(s-1)^2 + 1}$$

$$= \frac{1}{(s-1)^2 + 1} + \frac{\frac{1}{5}}{s+1} \quad \frac{-\frac{1}{5}(s-1) + \frac{2}{5}}{(s-1)^2 + 1}$$

$$2A + C = 1$$

$$C = 1 - \frac{2}{5} = \frac{3}{5}$$

$$A(s^2 - 2s + 2) + (Bs + C)(s+1) = 1$$

$\underline{s = -1}$: $A = \frac{1}{5}$

$B = -\frac{1}{5}$

$C = \frac{3}{5}$

$$\Longrightarrow Y = \frac{\frac{7}{5}\cdot 1}{(s-1)^2+1} + \frac{\frac{1}{5}}{s+1} - \frac{\frac{1}{5}(s-1)}{(s-1)^2+1}$$

$$\Longrightarrow y(t) = \frac{7}{5}e^{t}\sin t + \frac{1}{5}e^{-t} - \frac{1}{5}e^{t}\cos t$$

$$\begin{cases} y'' + 2y' + 5y = \begin{cases} 1, & 0 < t < \pi \\ 0, & t \geq \pi \end{cases} = 1 - u_\pi(t) \cdot 1 \\ y(0) = 0, \ y'(0) = 0 \end{cases}$$

$$\mathcal{L}(u_c(t) f(t-c)) = e^{-cs} F(s)$$

$$(s^2 + 2s + 5) Y = \frac{1}{s} - e^{-\pi s} \frac{1}{s} = \frac{1 - e^{-\pi s}}{s}$$

$$\Longrightarrow Y = \left(\frac{1}{s(s^2+2s+5)} \right) - \frac{e^{-\pi s}}{s(s^2+2s+5)}$$

$$= \left(\frac{A}{s} + \frac{Bs + C}{s^2+2s+5} \right) - e^{-\pi s} \left(\frac{A}{s} + \frac{Bs+C}{s^2+2s+5} \right)$$

$$A(s^2 + 2s + 5) + s(Bs + C) = 1 \qquad 2A + C = 0$$

$$s = 0 \Longrightarrow A = \frac{1}{5}, \ B = \frac{-1}{5}, \ C = \frac{-2}{5}$$

$$Y = \left(\frac{\frac{1}{5}}{s} - \frac{\frac{1}{5}s + \frac{2}{5}}{(s+1)^2 + 4} \right) - e^{-\pi s} \left(\frac{\frac{1}{5}}{s} - \frac{\frac{1}{5}s + \frac{2}{5}}{(s+1)^2+4} \right)$$

$$\Longrightarrow Y = \left(\frac{\frac{1}{5}}{s} - \frac{\frac{1}{5}(s+1) + \frac{1}{5}}{(s+1)^2 + 4} \right) - e^{-\pi s} \left(\frac{\frac{1}{5}}{s} - \frac{\frac{1}{5}(s+1) + \frac{1}{5}}{(s+1)^2 + 4} \right)$$

$$\Longrightarrow \boxed{\begin{aligned} y(t) = &\left(\frac{1}{5} - \frac{1}{5} e^{-t} \cos(2t) - \frac{1}{10} e^{-t} \sin(2t) \right) \\ &- u_{\pi}(t) \left(\frac{1}{5} - \frac{1}{5} e^{-(t-\pi)} \cos(2(t-\pi)) \right. \\ &\left. - \frac{1}{10} e^{-(t-\pi)} \sin(2(t-\pi)) \right) \end{aligned}}$$