

Quiz 5

Name: SOLUTIONS

Let \mathcal{P}_2 be the vector space of polynomials of degree 2 (i.e., quadratic polynomials), and let S_1 and S_2 be the following subsets of \mathcal{P}_2 :

$$S_1 = \{ 5x^2 - 4x + 1, 1 + 2x - x^2, 3x^2 - 3x \}, \quad S_2 = \{ 5 - 2x + 3x^2, 1 + x^2, 2x^2 - x + 2 \}.$$

For each of these sets, explain carefully why the set is or is not a basis for \mathcal{P}_2 ; one of these sets is a basis, and the other is not.

$$\begin{aligned} \underline{S_1}: & \begin{bmatrix} 1 & 1 & 0 \\ -4 & 2 & -3 \\ 5 & -1 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 6 & -3 \\ 0 & -6 & 3 \end{bmatrix} \\ & \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

For S_1 , the RREF is not the identity — not a basis!

$$\begin{aligned} \underline{S_2}: & \begin{bmatrix} 5 & 1 & 2 \\ -2 & 0 & -1 \\ 3 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 5 & 1 & 2 \\ -2 & 0 & -1 \\ 5 & 1 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 5 & 1 & 2 \\ -2 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix} \\ & \rightarrow \begin{bmatrix} 10 & 2 & 4 \\ -10 & 0 & -5 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 10 & 2 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix} \\ & \rightarrow \begin{bmatrix} 10 & 2 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 10 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

Here, the RREF is the identity — S_2 is a basis for \mathcal{P}_2 .