Math 102 **Problem Set 1** **Due 9/9/19**

1. Do problem 4 at the end of Lesson 4 (page 31).

For each of the following problems, solve the 1–dimensional heat equation,

$$u_t = u_{xx}\,, \quad \text{for} \quad 0 < x < \pi\,,$$

with the given auxiliary conditions.

2. (a) Determine the general solution satisfying the mixed homogeneous BCs

$$u_x(0,t) = 0, \quad u(\pi,t) = 0\,.$$

(b) Determine the solution satisfying these boundary conditions and the IC

$$u(x,0) = \begin{cases} \pi\,, & \text{for} \quad 0 \le x \le \frac{\pi}{2}\,, \\ 2\pi - 2x\,, & \text{for} \quad \frac{\pi}{2} \le x \le \pi\,. \end{cases}$$

(c) How does the solution behave as $t \to \infty$?

3. (a) Determine the general solution satisfying the no-flux homogeneous BCs

$$u_x(0,t) = 0, \quad u_x(\pi,t) = 0\,.$$

(b) Determine the solution satisfying these boundary conditions and the IC

$$u(x,0) = \begin{cases} 1\,, & \text{for} \quad 0 \le x < \frac{\pi}{2}\,, \\ -1\,, & \text{for} \quad \frac{\pi}{2} < x \le \pi\,. \end{cases}$$

(c) The initial condition is discontinuous at $x = \frac{\pi}{2}$. Is this true of the solution when $t > 0$?

(d) Using the Fourier series you found in (b), evaluate the solution when $x = 0$ and $t = 0$ to compute the sum of the series

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\,.$$

4. (a) Determine the general solution satisfying the periodic boundary conditions

$$u(0,t) = u(\pi,t), \quad u_x(0,t) = u_x(\pi,t)\,.$$

(b) Determine the solution satisfying these boundary conditions and the IC

$$u(x,0) = x\,(\pi - x)\;.$$

(c) Using the Fourier series you just found, evaluate the solution when $x = 0$ and $t = 0$ to compute the sum of an interesting series.