1. Determine the values of m for which the given function solves the given ODE.
   (a) $y(x) = e^{mx}$; $y''' - 3y'' - 4y' + 12y = 0$.
   (b) $x(t) = t^m$; $t^3 \dfrac{d^3x}{dt^3} + 2\,t^2 \dfrac{d^2x}{dt^2} - 10\,t \dfrac{dx}{dt} - 8\,x = 0$.

2. Consider the ODE $y' + y = 2x\,e^{-x}$.
   (a) Show that, for any value of c, the function $y(x) = (x^2 + c)\,e^{-x}$ is a solution.
   (b) You will soon learn how to obtain $y(x) = (x^2 + c)\,e^{-x}$ as the general solution of this ODE. Armed with that information, determine the particular solution of each of the following problems:
      i. $y' + y = 2x\,e^{-x}$ with $y(0) = 2$.
      ii. $y' + y = 2x\,e^{-x}$ with $y(-1) = e + 3$.

3. Consider the ODE $x'' + x = 0$.
   (a) Show that, for any constants c_1 and c_2, $x(t) = c_1 \sin t + c_2 \cos t$ is a solution.
   (b) You will soon learn how to obtain $x(t) = c_1 \sin t + c_2 \cos t$ as the general solution of this ODE. Given that information, consider the following initial-value problems; solve the one(s) you can, and determine which one(s) cannot be solved.
      i. $x'' + x = 0$ with $x(0) = 0$ and $x(\frac{\pi}{2}) = 1$.
      ii. $x'' + x = 0$ with $x(0) = 1$ and $x'(\frac{\pi}{2}) = -1$.
      iii. $x'' + x = 0$ with $x(0) = 0$ and $x(\pi) = 1$.

4. The size $p(t)$ of a mouse population changes according to the ODE
$$p' = 2p - 300\,, \quad \text{with} \quad p(0) = p_0\ .$$
   (a) Sketch the direction field for this ODE for $t \geq 0$ and $p_0 \geq 0$, and plot a few representative integral curves for different values of p_0.
   (b) For what value(s) of p_0 does the mouse population remain constant? How do solutions behave for other values of p_0?
   (c) Solve the ODE to obtain an explicit formula for $p(t)$. (Note: Your formula should be consistent with your answers above!)

5. Newton's Law of Cooling says that the temperature of an object changes at a rate proportional to the difference between the object's temperature and the temperature of its surroundings (often the ambient air temperature). Mathematically,
$$y' = -\,k\,(y - T) \quad \text{and} \quad y(0) = y_0\,,$$
where $y = y(t)$ is the temperature of the object at time t, y_0 is its initial temperature, k is an appropriate positive constant, and T is the constant ambient temperature.
   (a) Since $k > 0$, the coefficient of $(y - T)$ is negative. Explain why this makes sense.
   (b) Solve the ODE to determine an explicit formula for $y(t)$.
   (c) Let t^* be the time at which the initial temperature difference $y_0 - T$ has been reduced by half. Determine the relationship between t^* and k.