Math 210
Quiz 3 9/13/19 Name: SOLUTIONS

1. If possible, express $\vec{b}$ as a linear combination of $\vec{a}_1$ and $\vec{a}_2$.

$$\vec{a}_1 = \begin{bmatrix} -2 \\ 5 \end{bmatrix}, \quad \vec{a}_2 = \begin{bmatrix} 7 \\ -3 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 8 \\ 9 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} -2 & 7 & 8 \\ 5 & -3 & 9 \end{array}\right] \underset{2R_1+R_2 \to R_2}{\sim} \left[\begin{array}{cc|c} -2 & 7 & 8 \\ 1 & 11 & 25 \end{array}\right]$$

$$\underset{R_1 \leftrightarrow R_2}{\sim} \left[\begin{array}{cc|c} 1 & 11 & 25 \\ -2 & 7 & 8 \end{array}\right] \underset{2R_1+R_2 \to R_2}{\sim} \left[\begin{array}{cc|c} 1 & 11 & 25 \\ 0 & 29 & 58 \end{array}\right]$$

$\Longrightarrow x_2 = 2, \; x_1 = 3$ (via back-substitution)

Thus, $\begin{bmatrix} 8 \\ 9 \end{bmatrix} = 3\begin{bmatrix} -2 \\ 5 \end{bmatrix} + 2\begin{bmatrix} 7 \\ -3 \end{bmatrix}$.

4 points

2. Determine if the columns of $A = \begin{bmatrix} 1 & 3 & 4 \\ 1 & 4 & 6 \\ 2 & -1 & -6 \end{bmatrix}$ span $\mathbb{R}^3$. If they don't, find a vector outside their span.

$$\left[\begin{array}{ccc|c} 1 & 3 & 4 & 0 \\ 1 & 4 & 6 & 0 \\ 2 & -1 & -6 & 1 \end{array}\right] \underset{\substack{-R_1+R_2 \to R_2 \\ -2R_1+R_3 \to R_3}}{\sim} \left[\begin{array}{ccc|c} 1 & 3 & 4 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -7 & -14 & 1 \end{array}\right]$$

$$\underset{7R_2+R_3 \to R_3}{\sim} \left[\begin{array}{ccc|c} 1 & 3 & 4 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

No pivot in 3rd row $\Longrightarrow$ columns don't span $\mathbb{R}^3$!

4 points

work back to see that $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ is not in the span of the columns of $A$.

2 points