32. $y = \frac{1}{2}(1 - e^{-2t})$ for $0 \le t \le 1$; $y = \frac{1}{2}(e^2 - 1)e^{-2t}$ for $t > 1$
33. $y = e^{-2t}$ for $0 \le t \le 1$; $y = e^{-(t+1)}$ for $t > 1$

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1. $y = 0$ is unstable
2. $y = -a/b$ is asymptotically stable, $y = 0$ is unstable
3. $y = 1$ is asymptotically stable, $y = 0$ and $y = 2$ are unstable
4. $y = 0$ is unstable
5. $y = 0$ is asymptotically stable
6. $y = 0$ is asymptotically stable
7. (c) $y = [y_0 + (1 - y_0)kt]/[1 + (1 - y_0)kt]$
8. $y = 1$ is semistable
9. $y = -1$ is asymptotically stable, $y = 0$ is semistable, $y = 1$ is unstable
10. $y = -1$ and $y = 1$ are asymptotically stable, $y = 0$ is unstable
11. $y = 0$ is asymptotically stable, $y = b^2/a^2$ is unstable
12. $y = 2$ is asymptotically stable, $y = 0$ is semistable, $y = -2$ is unstable
13. $y = 0$ and $y = 1$ are semistable
15. (a) $\tau = (1/r)\ln 4$; 55.452 yr
    (b) $T = (1/r)\ln[\beta(1 - \alpha)/(1 - \beta)\alpha]$; 175.78 yr
16. (a) $y = 0$ is unstable, $y = K$ is asymptotically stable
    (b) Concave up for $0 < y \le K/e$, concave down for $K/e \le y < K$
17. (a) $y = K\exp\{[\ln(y_0/K)]e^{-rt}\}$
    (b) $y(2) \cong 0.7153K \cong 57.6 \times 10^6$ kg
    (c) $\tau \cong 2.215$ yr
18. (b) $(h/a)\sqrt{k/\alpha\pi}$; yes
    (c) $k/\alpha \le \pi a^2$
19. (b $k^2/2g(\alpha a)^2$
20. (c) $Y = Ey_2 = KE[1 - (E/r)]$
    (d) $Y_m = Kr/4$ for $E = r/2$
21. (a) $y_{1,2} = K[1 \mp \sqrt{1 - (4h/rK)}]/2$
22. (a) $y = 0$ is unstable, $y = 1$ is asymptotically stable
    (b) $y = y_0/[y_0 + (1 - y_0)e^{-\alpha t}]$
23. (a) $y = y_0e^{-\beta t}$ (b) $x = x_0\exp[-\alpha y_0(1 - e^{-\beta t})/\beta]$ (c) $x_0\exp(-\alpha y_0/\beta)$
24. (b) $z = 1/[\nu + (1 - \nu)e^{\beta t}]$
    (c) 0.0927
25. (a,b) $a = 0$: $y = 0$ is semistable.
    $a > 0$: $y = \sqrt{a}$ is asymptotically stable and $y = -\sqrt{a}$ is unstable.
26. (a) $a \le 0$: $y = 0$ is asymptotically stable.
    $a > 0$: $y = 0$ is unstable; $y = \sqrt{a}$ and $y = -\sqrt{a}$ are asymptotically stable.
27. (a) $a < 0$: $y = 0$ is asymptotically stable and $y = a$ is unstable.
    $a = 0$: $y = 0$ is semistable.
    $a > 0$: $y = 0$ is unstable and $y = a$ is asymptotically stable.
28. (a) $\lim_{t\to\infty} x(t) = \min(p, q)$; $x(t) = \dfrac{pq[e^{\alpha(q-p)t} - 1]}{qe^{\alpha(q-p)t} - p}$

    (b) $\lim_{t\to\infty} x(t) = p$; $x(t) = \dfrac{p^2\alpha t}{p\alpha t + 1}$

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1. $x^2 + 3x + y^2 - 2y = c$
2. Not exact
3. $x^3 - x^2y + 2x + 2y^3 + 3y = c$
4. $x^2y^2 + 2xy = c$
5. $ax^2 + 2bxy + cy^2 = k$
6. Not exact
7. $e^x\sin y + 2y\cos x = c$; also $y = 0$
8. Not exact
9. $e^{xy}\cos 2x + x^2 - 3y = c$
10. $y\ln x + 3x^2 - 2y = c$
11. Not exact
12. $x^2 + y^2 = c$
13. $y = \left[x + \sqrt{28 - 3x^2}\right]/2$, $|x| < \sqrt{28/3}$
14. $y = [x - (24x^3 + x^2 - 8x - 16)^{1/2}]/4$, $x > 0.9846$

15. $b = 3$; $x^2y^2 + 2x^3y = c$
16. $b = 1$; $e^{2xy} + x^2 = c$
19. $x^2 + 2\ln|y| - y^{-2} = c$; also $y = 0$
20. $e^x \sin y + 2y\cos x = c$
21. $xy^2 - (y^2 - 2y + 2)e^y = c$
22. $x^2e^x \sin y = c$
24. $\mu(t) = \exp\int R(t)\,dt$, where $t = xy$
25. $\mu(x) = e^{3x}$; $(3x^2y + y^3)e^{3x} = c$
26. $\mu(x) = e^{-x}$; $y = ce^x + 1 + e^{2x}$
27. $\mu(y) = y$; $xy + y\cos y - \sin y = c$
28. $\mu(y) = e^{2y}/y$; $xe^{2y} - \ln|y| = c$; also $y = 0$
29. $\mu(y) = \sin y$; $e^x \sin y + y^2 = c$
30. $\mu(y) = y^2$; $x^4 + 3xy + y^4 = c$
31. $\mu(x, y) = xy$; $x^3y + 3x^2 + y^3 = c$

Section 2.7, page 109

1. (a) 1.2, 1.39, 1.571, 1.7439
   (b) 1.1975, 1.38549, 1.56491, 1.73658
   (c) 1.19631, 1.38335, 1.56200, 1.73308
   (d) 1.19516, 1.38127, 1.55918, 1.72968
2. (a) 1.1, 1.22, 1.364, 1.5368
   (b) 1.105, 1.23205, 1.38578, 1.57179
   (c) 1.10775, 1.23873, 1.39793, 1.59144
   (d) 1.1107, 1.24591, 1.41106, 1.61277
3. (a) 1.25, 1.54, 1.878, 2.2736
   (b) 1.26, 1.5641, 1.92156, 2.34359
   (c) 1.26551, 1.57746, 1.94586, 2.38287
   (d) 1.2714, 1.59182, 1.97212, 2.42554
4. (a) 0.3, 0.538501, 0.724821, 0.866458
   (b) 0.284813, 0.513339, 0.693451, 0.831571
   (c) 0.277920, 0.501813, 0.678949, 0.815302
   (d) 0.271428, 0.490897, 0.665142, 0.799729
5. Converge for $y \geq 0$; undefined for $y < 0$
6. Converge for $y \geq 0$; diverge for $y < 0$
7. Converge
8. Converge for $|y(0)| < 2.37$ (approximately); diverge otherwise
9. Diverge
10. Diverge
11. (a) 2.30800, 2.49006, 2.60023, 2.66773, 2.70939, 2.73521
    (b) 2.30167, 2.48263, 2.59352, 2.66227, 2.70519, 2.73209
    (c) 2.29864, 2.47903, 2.59024, 2.65958, 2.70310, 2.73053
    (d) 2.29686, 2.47691, 2.58830, 2.65798, 2.70185, 2.72959
12. (a) 1.70308, 3.06605, 2.44030, 1.77204, 1.37348, 1.11925
    (b) 1.79548, 3.06051, 2.43292, 1.77807, 1.37795, 1.12191
    (c) 1.84579, 3.05769, 2.42905, 1.78074, 1.38017, 1.12328
    (d) 1.87734, 3.05607, 2.42672, 1.78224, 1.38150, 1.12411
13. (a) −1.48849, −0.412339, 1.04687, 1.43176, 1.54438, 1.51971
    (b) −1.46909, −0.287883, 1.05351, 1.42003, 1.53000, 1.50549
    (c) −1.45865, −0.217545, 1.05715, 1.41486, 1.52334, 1.49879
    (d) −1.45212, −0.173376, 1.05941, 1.41197, 1.51949, 1.49490
14. (a) 0.950517, 0.687550, 0.369188, 0.145990, 0.0421429, 0.00872877
    (b) 0.938298, 0.672145, 0.362640, 0.147659, 0.0454100, 0.0104931
    (c) 0.932253, 0.664778, 0.359567, 0.148416, 0.0469514, 0.0113722
    (d) 0.928649, 0.660463, 0.357783, 0.148848, 0.0478492, 0.0118978
15. (a) −0.166134, −0.410872, −0.804660, 4.15867
    (b) −0.174652, −0.434238, −0.889140, −3.09810
16. A reasonable estimate for y at $t = 0.8$ is between 5.5 and 6. No reliable estimate is possible at $t = 1$ from the specified data.
17. A reasonable estimate for y at $t = 2.5$ is between 18 and 19. No reliable estimate is possible at $t = 3$ from the specified data.
18. (b) $2.37 < \alpha_0 < 2.38$
19. (b) $0.67 < \alpha_0 < 0.68$