Math 210
Quiz 2 9/6/19 **Name:** SOLUTION

1. Solve the following linear system by transforming its augmented matrix into reduced echelon form.

$$\begin{array}{rcrcrcrcr} 2x_1 & + & 4x_2 & + & x_3 & - & 3x_4 & = & -2 \\ x_1 & + & 2x_2 & - & x_3 & & & = & 2 \\ & & x_2 & + & x_3 & - & 2x_4 & = & -3 \\ x_1 & - & 4x_2 & - & 7x_3 & - & x_4 & = & -19 \end{array}$$

$$\left[\begin{array}{cccc|c} 2 & 4 & 1 & -3 & -2 \\ 1 & 2 & -1 & 0 & 2 \\ 0 & 1 & 1 & -2 & -3 \\ 1 & -4 & -7 & -1 & -19 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 0 & 2 \\ 2 & 4 & 1 & -3 & -2 \\ 0 & 1 & 1 & -2 & -3 \\ 1 & -4 & -7 & -1 & -19 \end{array}\right] \quad R_1 \longleftrightarrow R_2$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 0 & 2 \\ 0 & 0 & 3 & -3 & -6 \\ 0 & 1 & 1 & -2 & -3 \\ 0 & -6 & -6 & -1 & -21 \end{array}\right] \quad -2R_1 + R_2 \to R_2, \; -R_1 + R_4 \to R_4$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 0 & 2 \\ 0 & 1 & 1 & -2 & -3 \\ 0 & 0 & 3 & -3 & -6 \\ 0 & -6 & -6 & -1 & -21 \end{array}\right] \quad R_2 \longleftrightarrow R_3$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 0 & 2 \\ 0 & 1 & 1 & -2 & -3 \\ 0 & 0 & 3 & -3 & -6 \\ 0 & 0 & 0 & -13 & -39 \end{array}\right] \quad 6R_2 + R_4 \to R_4$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 0 & 2 \\ 0 & 1 & 1 & -2 & -3 \\ 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \quad \frac{1}{3}R_3 \to R_3, \; -\frac{1}{13}R_4 \to R_4$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 0 & 2 \\ 0 & 1 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \quad R_4 + R_3 \to R_3, \; 2R_4 + R_2 \to R_2$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \quad -R_3 + R_2 \to R_2, \; R_3 + R_1 \to R_1$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \quad -2R_2 + R_1 \to R_1$$

Solution: $x_1 = -1$, $x_2 = 2$, $x_3 = 1$, $x_4 = 3$