Tabular Integration

We have seen that integrals of the form $\int f(x)\, g(x)\, dx$, in which f can be differentiated repeatedly to become zero and g can be integrated repeatedly without difficulty, are natural candidates for integration by parts. However, if many repetitions are required, the calculations can be cumbersome. In situations like this, there is a way to organize the calculations that saves a great deal of work. It is called **tabular integration** and is illustrated in the following examples.

EXAMPLE 7 Evaluate $\displaystyle\int x^2 e^x\, dx$ by tabular integration.

Solution With $f(x) = x^2$ and $g(x) = e^x$, we list

$f(x)$ and its derivatives		$g(x)$ and its integrals
x^2	$(+)$	e^x
$2x$	$(-)$	e^x
2	$(+)$	e^x
0		e^x.

We add the products of the functions connected by the arrows, with the middle sign changed, to obtain

$$\int x^2 e^x\, dx = x^2 e^x - 2xe^x + 2e^x + C.$$

EXAMPLE 8 Evaluate $\displaystyle\int x^3 \sin x\, dx$ by tabular integration.

Solution With $f(x) = x^3$ and $g(x) = \sin x$, we list

$f(x)$ and its derivatives		$g(x)$ and its integrals
x^3	$(+)$	$\sin x$
$3x^2$	$(-)$	$-\cos x$
$6x$	$(+)$	$-\sin x$
6	$(-)$	$\cos x$
0		$\sin x$.

Again we add the products of the functions connected by the arrows, with every other sign changed, to obtain

$$\int x^3 \sin x\, dx = -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x + C.$$