Quiz 3

Name:

Complete the following definition by completing the statements of all ten conditions.

Definition A *vector space over* $\mathbb{R}$ is a set V equipped with two operations, $+$ and $\cdot$, that satisfy the following conditions for all vectors $\vec{v}, \vec{w}, \vec{u} \in V$ and all scalars $r, s \in \mathbb{R}$:

(1) the set V is closed under vector addition : $\vec{v} + \vec{w} \in V$

(2) vector addition is commutative : $\vec{v} + \vec{w} \;=\; \vec{w} + \vec{v}$

(3) vector addition is associative : $(\vec{v} + \vec{w}) + \vec{u} \;=\; \vec{v} + (\vec{w} + \vec{u})$

(4) there is a *zero vector* $\vec{0} \in V$ such that $\vec{v} + \vec{0} \;=\; \vec{v}$

(5) each $\vec{v} \in V$ has an *additive inverse* $\vec{w} \in V$ such that $\vec{v} + \vec{w} \;=\; \vec{0}$

(6) the set V is closed under scalar multiplication : $r\,\vec{v} \in V$

(7) addition of scalars distributes over scalar multiplication : $(r + s)\,\vec{v} \;=\; r\,\vec{v} + s\,\vec{v}$

(8) scalar multiplication distributes over vector addition : $r\,(\vec{v} + \vec{w}) \;=\; r\vec{v} + r\vec{w}$

(9) ordinary multiplication of scalars associates with scalar multiplication : $(rs)\,\vec{v} \;=\; r\,(s\vec{v})$

(10) multiplication by the scalar 1 is the identity operation : $1\vec{v} \;=\; \vec{v}$