Quiz 4

Name:

Consider the following set of vectors in $\mathbb{R}^3$:

$$S = \left\{ \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \right\}.$$

1. Does the vector $\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$ belong to the span of S?

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 1 \\ -1 & 3 & 2 & 3 \\ 2 & -2 & 1 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 3 & 1 \\ 0 & 4 & 5 & 4 \\ 0 & -4 & -5 & -1 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 3 & 1 \\ 0 & 4 & 5 & 4 \\ 0 & 0 & 0 & 3 \end{array}\right]$$

The last row shows that there is no solution. Thus, $\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$ is not in the span.

2. Does the span of S equal $\mathbb{R}^3$? If so, prove it; if not, describe the span of S as succinctly as you can.

#1 shows that the span of S is not $\mathbb{R}^3$. To determine the span:

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & x \\ -1 & 3 & 2 & y \\ 2 & -2 & 1 & z \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 3 & x \\ 0 & 4 & 5 & x+y \\ 0 & -4 & -5 & z-2x \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 3 & x \\ 0 & 4 & 5 & x+y \\ 0 & 0 & 0 & -x+y+z \end{array}\right].$$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ is in the span of S if and only if $-x+y+z=0$.

Parametrization of span S: $\begin{pmatrix} y+z \\ y \\ z \end{pmatrix} = y\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + z\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$.