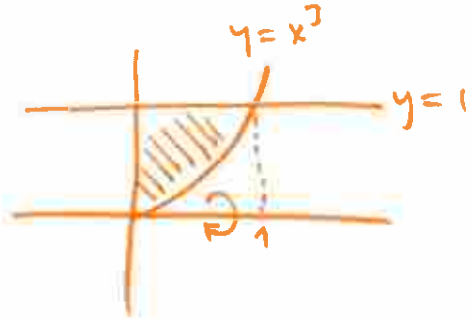
Math 102
Quiz 2 9/13/19 **Name:** SOLUTIONS

1. Compute the volume of the solid generated when the area bounded by

$$x = 0, \quad y = 0, \quad y = 1, \quad \text{and} \quad y = x^3$$

is rotated about the x-axis.



$x^3 = 1$ when $x = 1$

$$\text{Volume} = \int_0^1 [\pi(1)^2 - \pi(x^3)^2]\,dx$$

$$= \int_0^1 (\pi - \pi x^6)\,dx$$

$$= \pi x - \frac{\pi}{7} x^7 \Big|_0^1$$

$$= \pi - \frac{\pi}{7} = \frac{6\pi}{7}$$

3 pts

2. Evaluate the following integrals.

(a) $\int (e^x + e^{-x})\,dx$ $= e^x - e^{-x} + c$ 2 pts

(b) $\int \frac{\cos x}{2 - \sin x}\,dx$ $= -\int \frac{1}{u}\,du = -\ln|u| + c =$

$u = 2 - \sin x$

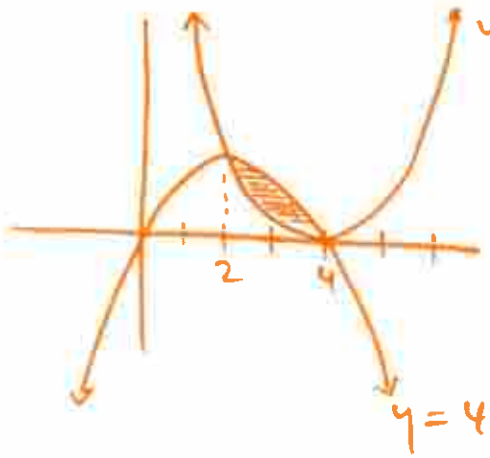
$du = -\cos x\,dx$

$-du = \cos x\,dx$

$-\ln|2 - \sin x| + c$

2 pts

3. Compute the exact area of the region bounded by the curves $y = (x-4)^2$ and $y = 4x - x^2$.



$y = (x-4)^2$

$y = 4x - x^2$

intersections:

$(x-4)^2 = 4x - x^2$

$(x-4)^2 = x(4-x)$

$x - 4 = 0$ or $x - 4 = -x$

$x = 4$ $\quad 2x = 4$

$x = 2$

$$\text{area} = \int_2^4 \left[(4x - x^2) - (x-4)^2 \right] dx$$

$$= 2x^2 - \frac{1}{3}x^3 - \frac{1}{3}(x-4)^3 \Big|_2^4$$

$$= \left(32 - \frac{64}{3} - 0\right) - \left(8 - \frac{8}{3} - \frac{1}{3}(-8)\right)$$

$$= 32 - \frac{64}{3} - 8$$

$$= \frac{8}{3}$$

3 points