Math 212
Quiz 3 2/4/20 Name: SOLUTIONS

1. The logistic population model

$$x' = x(1-x)$$

is a Bernoulli equation. Use this fact to determine $x(t)$ explicitly.

$x' = x - x^2$

Divide by x^2: $x^{-2}x' = x^{-1} - 1$

Let $y = x^{-1} \Longrightarrow y' = -x^{-2}x'$

$-y' = x^{-2}x'$

Then the ODE becomes $-y' = y - 1$

$\Longrightarrow y' + y = 1$ integrating factor $\mu = e^{t}$

$\Longrightarrow (e^{t}y)' = e^{t}$

$\Longrightarrow e^{t}y = e^{t} + c$

$\Longrightarrow y(t) = 1 + ce^{-t}$

$\Longrightarrow \dfrac{1}{x(t)} = 1 + ce^{-t}$

$\Longrightarrow x(t) = \dfrac{1}{1+ce^{-t}} = \dfrac{e^{t}}{e^{t}+c}$

2. Determine the general solution (in implicit form) of the exact ODE

$$\left(\frac{y}{x}+6x\right)+(\ln x-2)\,y' \;=\; 0.$$

We need to find $H(x,y)$ such that

$$\frac{\partial H}{\partial x} = \frac{y}{x} + 6x \quad \& \quad \frac{\partial H}{\partial y} = \ln x - 2$$

$$\Longrightarrow H(x,y) = y\ln x + 3x^2 + ? \qquad \Longrightarrow H(x,y) = y\ln x - 2y + ?$$

$$H(x,y) = y\ln x + 3x^2 - 2y$$

The ODE then says that $\frac{dH}{dx} = 0$, so the general solution is $H(x,y) = c$, i.e.,

$$y\ln x + 3x^2 - 2y = c.$$