Math 212
Quiz 1   1/21/20   **Name:** SOLUTIONS

1. Determine the general solution $x(t)$ of the separable ODE

$$x' = -x^4,$$

then find the unique solution satisfying $x(0) = 1$. Both should be explicit functions of t.

$$\frac{dx}{dt} = -x^4 \implies -\frac{dx}{x^4} = dt$$

Integrate: $\frac{1}{3}x^{-3} = t + c$

$$\implies x^{-3} = 3t + c$$

$$\implies x^3 = \frac{1}{3t+c}$$

$$\implies \underline{\underline{x(t) = \left(\frac{1}{3t+c}\right)^{1/3}}}$$ General solution

solution with $x(0) = 1$: $1 = \left(\frac{1}{c}\right)^{1/3} \implies c = 1,$

$$\underline{\underline{x(t) = \left(\frac{1}{3t+1}\right)^{1/3}}}.$$

2. Determine the general solution $x(t)$ of the separable ODE

$$x' = t^2\, e^{-x},$$

then find the unique solution satisfying $x(0) = \ln 2$. Both should be explicit functions of t.

$$\frac{dx}{dt} = t^2 e^{-x} \implies e^x\, dx = t^2\, dt$$

Integrate: $e^x = \frac{1}{3} t^3 + c$

$$\implies \underline{\underline{x(t) = \ln\left|\frac{1}{3} t^3 + c\right|}} \quad \underline{\underline{\text{General Solution}}}$$

the solution satisfying $x(0) = \ln 2$;

$$\ln 2 = \ln|c| \implies c = 2,$$

$$\underline{\underline{x(t) = \ln\left|\frac{1}{3} t^3 + 2\right|}}$$