$$\frac{1-2s}{(s+2)^2+1} = \frac{-2(s+2)}{(s+2)^2+1} + \frac{5}{(s+2)^2+1}$$

$$\xrightarrow{\mathcal{L}^{-1}} \quad -2e^{-2t}\cos(t) + 5e^{-2t}\sin(t)$$

$$\overbrace{\frac{3s+1}{(s+1)(s+2)}}^{Y} = \frac{A}{s+1} + \frac{B}{s+2} \implies \frac{-2}{s+1} + \frac{5}{s+2}$$

$$\xrightarrow{\mathcal{L}^{-1}} \quad -2e^{-t} + 5e^{-2t}$$

$$3s+1 = A(s+2) + B(s+1)$$

$s=-2$: $-5 = -B \implies B = 5$

$s=-1$: $-2 = A$

$$\overbrace{\frac{2s+3}{(s+1)^2+4}}^{Y} = \frac{\overset{2s+2}{2(s+1)}}{(s+1)^2+4} + \frac{\overbrace{\frac{1}{2}\cdot(2)}^{1}}{(s+1)^2+4}$$

$$\Big\downarrow \qquad \mathcal{L}^{-1} \qquad \Big\downarrow$$

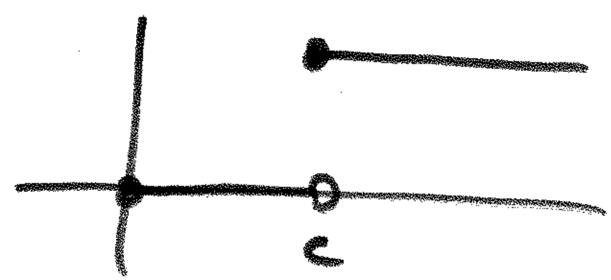
$$\underline{y(t) = 2e^{-t}\cos(2t) + \frac{1}{2}e^{-t}\sin(2t)}$$

$$\underline{\underline{\mathcal{L}(e^{at} f(t))}} = \int_0^\infty e^{-st} e^{at} f(t)\,dt$$

$$= \int_0^\infty e^{-(s-a)t} f(t)\,dt$$

$$= \underline{\underline{F(s-a)}}, \text{ where } \underbrace{F(s)}_{\int_0^\infty e^{-st} f(t)\,dt} = \mathcal{L}(f).$$

<u>Define</u> $u_c(t) := \begin{cases} 0, & 0 \le t < c \\ 1, & t \ge c \end{cases}$



$$\underline{\underline{\mathcal{L}(u_c(t) f(t-c))}} = \int_0^\infty e^{-st} u_c(t) f(t-c)\,dt$$

$$= \int_c^\infty e^{-st} f(t-c)\,dt$$

$\theta = t - c$

$d\theta = dt$

$$= \int_0^\infty e^{-s(\theta+c)} f(\theta)\,d\theta$$

$$= \int_0^\infty e^{-s\theta} e^{-sc} f(\theta)\,d\theta$$

$$= e^{-sc} \int_0^\infty e^{-st} f(t)\,dt$$

$$= \underline{\underline{e^{-sc} F(s)}}$$

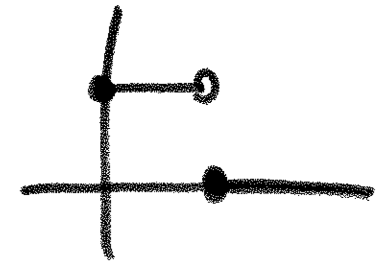
$$\mathcal{L}(e^{at} f(t)) = F(s-a)$$

&

$$\mathcal{L}(u_c(t) f(t-c)) = e^{-sc} F(s)$$

Example $y'' + 2y' + 5y = 1 - u_\pi \quad , \quad y(0) = y'(0) = 0$

$$= \begin{cases} 1, & 0 \le t < \pi \\ 0, & t \ge \pi \end{cases}$$



Apply $\mathcal{L}$:

$$-y'(0) - sy(0) + s^2 Y + 2(-y(0) + sY)$$

$$+ 5Y = \mathcal{L}(1 - u_\pi)$$

$$= \mathcal{L}(1) - \mathcal{L}(u_\pi) \qquad u_\pi(t) \cdot f(t-\pi), \quad f \equiv 1$$

$$= \frac{1}{s} - \frac{e^{-\pi s}}{s}$$

$$\Longrightarrow (s^2 + 2s + 5)Y = \frac{1 - e^{-\pi s}}{s}$$

$$\Longrightarrow Y = \frac{1 - e^{-\pi s}}{s(s^2+2s+5)}$$

$$= \underline{\underline{\frac{1}{s(s^2+2s+5)}}} - \frac{e^{-\pi s}}{s(s^2+2s+5)}$$

$$\frac{1}{s(s^2+2s+5)} = \frac{\frac{1}{5}}{s} + \frac{-\frac{1}{5}(s+1)}{(s+1)^2+4} + \frac{-\frac{1}{10}(2)}{(s+1)^2+4}$$

via

$$\frac{1}{s(s^2+2s+5)} = \frac{A}{s} + \frac{Bs+C}{(s+1)^2+4} = \frac{\frac{1}{5}}{s} - \frac{\frac{1}{5}s+\frac{2}{5}}{''}$$

$$1 = A\left((s+1)^2+4\right) + s(Bs+C)$$

$A = \frac{1}{5}$ $\quad C = \frac{-2}{5}$

$B = -\frac{1}{5}$