in Chapter 8, that proceed directly from the differential equation and need no expression for the solution. Software packages such as Maple and Mathematica readily execute such procedures and produce graphs of solutions of differential equations.

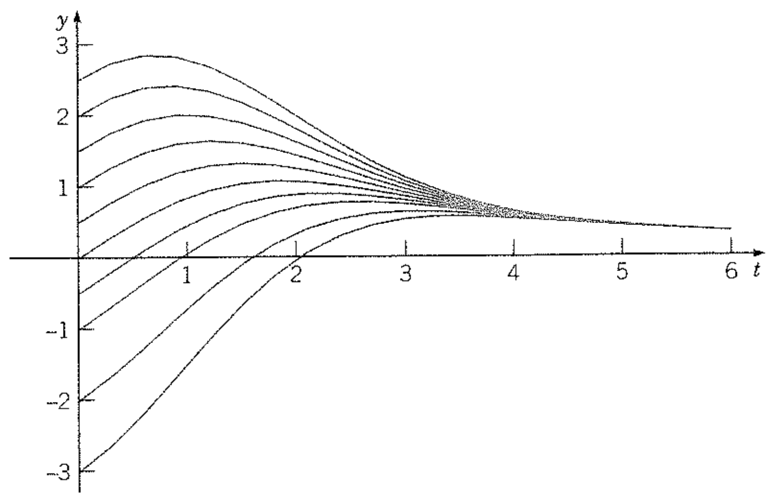

FIGURE 2.1.4 Integral curves of $2y' + ty = 2$.

Figure 2.1.4 displays graphs of the solution (47) for several values of c. From the figure it may be plausible to conjecture that all solutions approach a limit as $t \to \infty$. The limit can be found analytically (see Problem 32).

PROBLEMS

In each of Problems 1 through 12:

(a) Draw a direction field for the given differential equation.

(b) Based on an inspection of the direction field, describe how solutions behave for large t.

(c) Find the general solution of the given differential equation, and use it to determine how solutions behave as $t \to \infty$.

1. $y' + 3y = t + e^{-2t}$
2. $y' - 2y = t^2 e^{2t}$
3. $y' + y = te^{-t} + 1$
4. $y' + (1/t)y = 3\cos 2t, \quad t > 0$
5. $y' - 2y = 3e^t$
6. $ty' + 2y = \sin t, \quad t > 0$
7. $y' + 2ty = 2te^{-t^2}$
8. $(1 + t^2)y' + 4ty = (1 + t^2)^{-2}$
9. $2y' + y = 3t$
10. $ty' - y = t^2 e^{-t}, \quad t > 0$
11. $y' + y = 5\sin 2t$
12. $2y' + y = 3t^2$

In each of Problems 13 through 20 find the solution of the given initial value problem.

13. $y' - y = 2te^{2t}, \quad y(0) = 1$
14. $y' + 2y = te^{-2t}, \quad y(1) = 0$
15. $ty' + 2y = t^2 - t + 1, \quad y(1) = \frac{1}{2}, \quad t > 0$
16. $y' + (2/t)y = (\cos t)/t^2, \quad y(\pi) = 0, \quad t > 0$
17. $y' - 2y = e^{2t}, \quad y(0) = 2$

18. $ty' + 2y = \sin t, \qquad y(\pi/2) = 1, \quad t > 0$
19. $t^3y' + 4t^2y = e^{-t}, \qquad y(-1) = 0, \quad t < 0$
20. $ty' + (t + 1)y = t, \qquad y(\ln 2) = 1, \quad t > 0$

In each of Problems 21 through 23:

(a) Draw a direction field for the given differential equation. How do solutions appear to behave as t becomes large? Does the behavior depend on the choice of the initial value a? Let a_0 be the value of a for which the transition from one type of behavior to another occurs. Estimate the value of a_0.

(b) Solve the initial value problem and find the critical value a_0 exactly.

(c) Describe the behavior of the solution corresponding to the initial value a_0.

21. $y' - \frac{1}{2}y = 2\cos t, \qquad y(0) = a$
22. $2y' - y = e^{t/3}, \qquad y(0) = a$
23. $3y' - 2y = e^{-\pi t/2}, \qquad y(0) = a$

In each of Problems 24 through 26:

(a) Draw a direction field for the given differential equation. How do solutions appear to behave as $t \to 0$? Does the behavior depend on the choice of the initial value a? Let a_0 be the value of a for which the transition from one type of behavior to another occurs. Estimate the value of a_0.

(b) Solve the initial value problem and find the critical value a_0 exactly.

(c) Describe the behavior of the solution corresponding to the initial value a_0.

24. $ty' + (t + 1)y = 2te^{-t}, \qquad y(1) = a, \quad t > 0$
25. $ty' + 2y = (\sin t)/t, \qquad y(-\pi/2) = a, \quad t < 0$
26. $(\sin t)y' + (\cos t)y = e^t, \qquad y(1) = a, \quad 0 < t < \pi$
27. Consider the initial value problem

$$y' + \tfrac{1}{2}y = 2\cos t, \qquad y(0) = -1.$$

Find the coordinates of the first local maximum point of the solution for $t > 0$.

28. Consider the initial value problem

$$y' + \tfrac{2}{3}y = 1 - \tfrac{1}{2}t, \qquad y(0) = y_0.$$

Find the value of y_0 for which the solution touches, but does not cross, the t-axis.

29. Consider the initial value problem

$$y' + \tfrac{1}{4}y = 3 + 2\cos 2t, \qquad y(0) = 0.$$

(a) Find the solution of this initial value problem and describe its behavior for large t.

(b) Determine the value of t for which the solution first intersects the line $y = 12$.

30. Find the value of y_0 for which the solution of the initial value problem

$$y' - y = 1 + 3\sin t, \qquad y(0) = y_0$$

remains finite as $t \to \infty$.

31. Consider the initial value problem

$$y' - \tfrac{3}{2}y = 3t + 2e^t, \qquad y(0) = y_0.$$