#8 $y'' + 6y' + 13y = 0$

$y = e^{rt} \implies r^2 + 6r + 13 = 0 \implies r = \dfrac{-6 \pm \sqrt{36 - 52}}{2}$

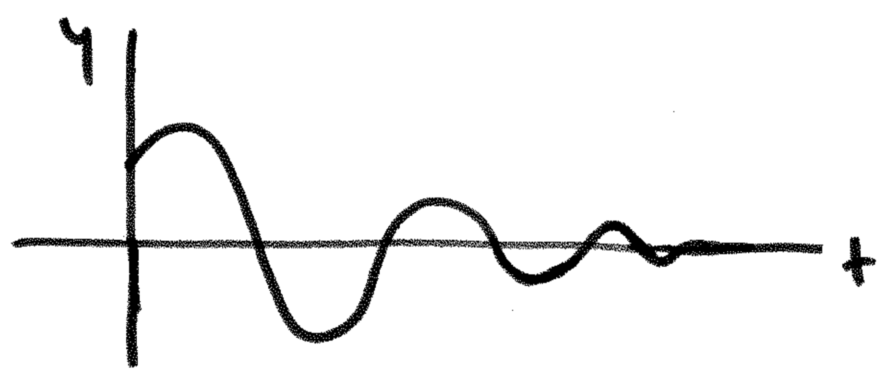
$\implies r = -3 \pm 2i$

$\implies y_1 = e^{-3t}\cos(2t), \quad y_2 = e^{-3t}\sin(2t)$

$\implies$ General solution: $y(t) = c_1 y_1 + c_2 y_2$

i.e., $y(t) = c_1 e^{-3t}\cos(2t) + c_2 e^{-3t}\sin(2t)$

Typical solution:



#12 $y'' + 4y = 0$, $\cancel{y(0) = y'(0) = 1}$

$y(0) = 0$; $y'(0) = 1$

$r^2 + 4 = 0 \Longrightarrow r = \pm 2i \Longrightarrow y_1 = \cos(2t)$,

$y_2 = \sin(2t)$

$\Longrightarrow$ general sol'n : $y(t) = c_1 \cos(2t) + c_2 \sin(2t)$

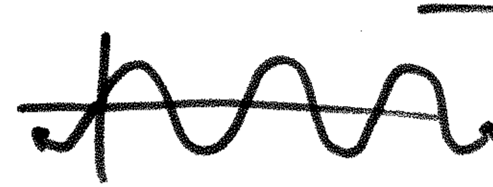
$y(0) = 0 \Longrightarrow 0 = c_1 + 0 \Longrightarrow \underline{\underline{c_1 = 0}}$

$y(t) = c_2 \sin(2t)$

$y'(t) = 2c_2 \cos(2t)$

$y'(0) = 1 \Longrightarrow 1 = 2c_2 \Longrightarrow \underline{\underline{c_2 = \frac{1}{2}}}$

$\Longrightarrow y(t) = \frac{1}{2}\sin(2t)$



#13 $y'' - 2y' + 5y = 0 \,, \quad y\left(\frac{\pi}{2}\right) = 0, \quad y'\left(\frac{\pi}{2}\right) = 2$

$r^2 - 2r + 5 = 0 \implies r = \frac{2 \pm \sqrt{4 - 20}}{2} = \underline{\underline{1 \pm 2i}}$

$\implies$ general sol'n : $y(t) = c_1 e^t \cos(2t) + c_2 e^t \sin(2t)$

$y\left(\frac{\pi}{2}\right) = 0 \implies 0 = c_1 e^{\frac{\pi}{2}} \cos(\pi) + c_2 e^{\frac{\pi}{2}} \sin(\pi)$

$0 = -c_1 e^{\frac{\pi}{2}} + 0 \implies c_1 = 0$

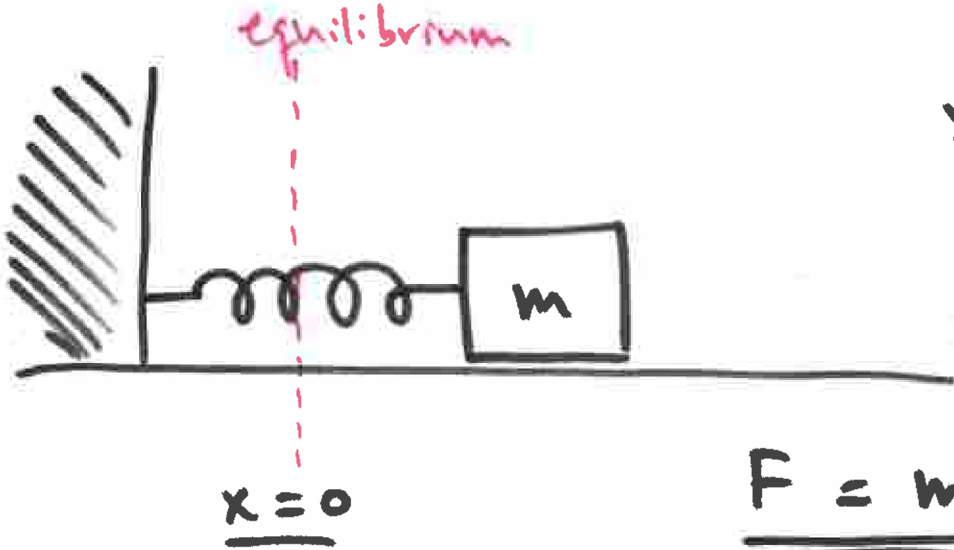
$y(t) = c e^t \sin(2t)$ $\quad \boxed{y(t) = -e^{-\pi/2} \sin 2t}$

$y'(t) = c e^t \sin(2t) + c e^t \cdot 2\cos(2t)$

$= c e^t (\sin(2t) + 2\cos(2t))$

$y'\left(\frac{\pi}{2}\right) = c e^{\frac{\pi}{2}} \left(\cancel{\sin(\pi)} + 2\cos(\pi) \right) = 2$

$\implies -2ce^{\pi/2} = 2 \implies ce^{\pi/2} = -1 \implies c = -e^{-\pi/2}$



$x(t)$ = position of mass at time t

x' = Velocity, x'' = acceleration

$F = ma$ Newton's 2nd Law

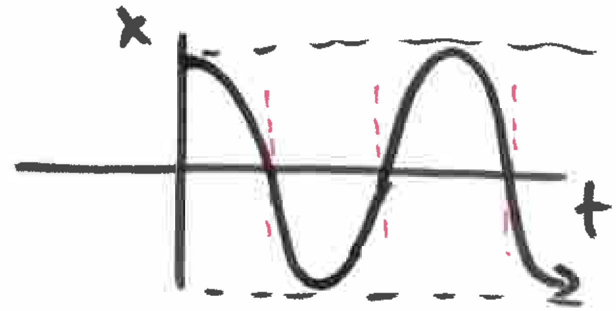
$F = mx''$ Hooke's Law: $F = -kx$, $k > 0$ (spring constant)

$$\Longrightarrow \quad -kx = mx''$$

$$\Longrightarrow \quad mx'' + kx = 0 \Longrightarrow x'' + \frac{k}{m}x = 0$$

$$\Longrightarrow \quad \boxed{x'' + \omega^2 x = 0 \qquad \omega^2 = \frac{k}{m} \quad \left(\omega = \sqrt{\frac{k}{m}}\right)}$$

$x'' + \omega^2 x = 0$, $x = x(t)$



$$\Longrightarrow r^2 + \omega^2 = 0 \Longrightarrow \underline{\underline{r = \pm i\omega}}$$

$$x(t) = c_1 \cos(\omega t) + c_2 \sin(\omega t)$$

Harmonic motion / harmonic oscillator

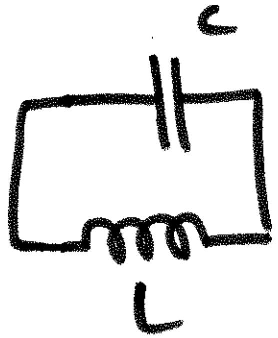
Friction:

$$F = mx'', \quad F = \underbrace{-kx}_{\text{Hooke}} + \underbrace{\gamma x'}_{\text{friction}} \qquad \underline{\underline{\gamma < 0}}$$

$$\Longrightarrow -kx + \gamma x' = mx'' \qquad \underline{\underline{\gamma^2 - 4km < 0}}$$

$$mx'' - \gamma x' + kx = 0$$

$$mr^2 - \gamma r + k = 0 \Longrightarrow r = \frac{\gamma \pm \sqrt{\gamma^2 - 4km}}{2m}$$

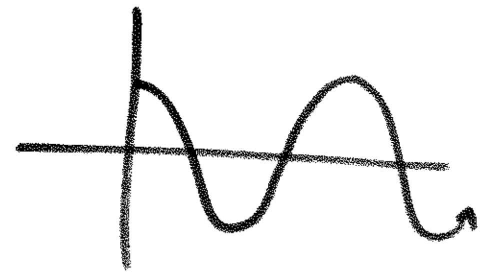


capacitor: like a battery; charges & discharges; $\underline{\underline{I = CV_c'}}$

inductor: coil of wire;

$$\underline{\underline{V_L = LI'}}$$

Kirchhoff: $V_c + V_L = 0$



$$\Longrightarrow \quad V_c + LCV_c'' = 0$$

$$\Longrightarrow \quad V_c'' + \frac{1}{LC} V_c = 0$$

$$\Longrightarrow \quad \boxed{V_c'' + \omega^2 V_c = 0 \,, \quad \omega^2 = \frac{1}{LC}}$$

$$\Longrightarrow \quad V_c = c_1 \cos(\omega t) + c_2 \sin(\omega t) \qquad \begin{cases} V_c(0) = V_{max} \\ V_c'(0) = 0 \end{cases}$$

$$\Longrightarrow \quad \boxed{V_c = V_{max} \cos(\omega t)}$$

#1 $y'' - 2y' + y = 0$

$$e^{rt} \Longrightarrow r^2 - 2r + 1 = 0 \Longrightarrow r = 1$$
$$(r-1)^2 = 0$$
$$\Longrightarrow \boxed{y_1(t) = e^t}$$

General sol'n:
$$\boxed{y(t) = c_1 e^t + c_2 t e^t}$$

$$y_2 = ue^t \Longrightarrow y_2' = u'e^t + ue^t$$
$$y_2'' = u''e^t + u'e^t + u'e^t + ue^t$$
$$= u''e^t + 2u'e^t + ue^t$$

$$\Longrightarrow u''e^t + \cancel{2u'e^t} + \cancel{ue^t} - \cancel{2u'e^t} - \cancel{2ue^t} + \cancel{ue^t} = 0$$

$$\Longrightarrow u''e^t = 0 \Longrightarrow u'' = 0 \Longrightarrow u(t) = at + \cancel{b}$$

$$\Longrightarrow \boxed{y_2 = te^t}$$