

For questions 1–3, determine the orthogonal trajectories for the given family of curves. For the subsequent questions, set up and solve the relevant ODEs.

1. $y^2 = cx$
2. $cx^2 + y^2 = 1$
3. $x = \frac{y^2}{4} + \frac{c}{y^2}$
4. A 4 pound stone falls from rest toward the earth from a great height. As it falls, it is acted upon by air resistance equal to $.5v$, where v is its velocity in feet per second.
 - (a) Determine the velocity and distance fallen at time t seconds.
 - (b) What is the stone's terminal velocity?
5. An object of mass 100 g is dropped from rest toward the earth from a height of 1000 m. As it falls, it encounters air resistance proportional to its velocity. The object's terminal velocity is 245 m/sec.
 - (a) Determine the velocity and distance fallen at time t seconds.
 - (b) Determine the object's velocity when it hits the ground.
6. A Newtonian particle of mass $m = 1$ moves horizontally; its motion is opposed by a frictional force of the form $kv - \lambda v^3$, where v is the particle's velocity and $k > 0$ and λ are constants.
 - (a) Suppose that $\lambda = 0$. Determine $v(t)$. Notice anything weird?
 - (b) Suppose that $\lambda > 0$. Determine $v(t)$. Still weird?
7. An object cools from 60°C to 50°C in 15 minutes when the ambient air temperature is 30°C . How long will it take this object to cool from 100°C to 80°C when the ambient temperature is 50°C ?
8. A tank initially contains 100 gallons of brine in which 10 pounds of salt are dissolved. At $t = 0$, pure water begins flowing into the tank at 5 gallons per minute; the mixture is kept uniform by continual stirring, and the well-stirred mixture flows out of the tank at 2 gallons per minute.
 - (a) How much salt is in the tank at the end of 15 minutes, and what is the concentration then?
 - (b) The tank has a capacity of 250 gallons. What is the concentration at the instant the tank overflows?