Quiz 1

SOLUTION (Many others are possible!)

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1. Use elementary row operations to transform the following system of linear equations into echelon form, then solve the system. Be sure to clearly identify the row operations you use at each step.

$$\begin{array}{rcrcrcr} & & y & + & 2z & = & 5 \\ 3x & - & 4y & - & z & = & 7 \\ 4x & - & 2y & - & z & = & 7 \end{array}$$

Here's one possible sequence of elementary row operations:

$$\xrightarrow[\rho_1 \leftrightarrow \rho_3]{} \begin{array}{rcrcrcr} 4x & - & 2y & - & z & = & 7 \\ 3x & - & 4y & - & z & = & 7 \\ & & y & + & 2z & = & 5 \end{array} \quad \xrightarrow[-\rho_2+\rho_1]{} \begin{array}{rcrcrcr} x & + & 2y & & & = & 0 \\ 3x & - & 4y & - & z & = & 7 \\ & & y & + & 2z & = & 5 \end{array}$$

$$\xrightarrow[-3\rho_1+\rho_2]{} \begin{array}{rcrcrcr} x & + & 2y & & & = & 0 \\ & - & 10y & - & z & = & 7 \\ & & y & + & 2z & = & 5 \end{array} \quad \xrightarrow[10\rho_3+\rho_2]{} \begin{array}{rcrcrcr} x & + & 2y & & & = & 0 \\ & & & & 19z & = & 57 \\ & & y & + & 2z & = & 5 \end{array}$$

$$\xrightarrow[\rho_2 \leftrightarrow \rho_3]{} \begin{array}{rcrcrcr} x & + & 2y & & & = & 0 \\ & & y & + & 2z & = & 5 \\ & & & & 19z & = & 57 \end{array}$$

The system is now in echelon form. Solve the bottom equation to find that $z = 3$, substitute into the second equation to get

$$y + 6 \ = \ 5 \quad \Longrightarrow \quad y \ = \ -1 \ ,$$

then substitute into the first equation to get

$$x - 2 \ = \ 0 \quad \Longrightarrow \quad x = 2 \ .$$

This system therefore has the unique solution

$$x = 2, \ y = -1, \ z = 3 \ .$$

To see the augmented matrix version of this solution, turn the page.

Here is the same solution written in *augmented matrix* form:

$$\left[\begin{array}{rrr|r} 0 & 1 & 2 & 5 \\ 3 & -4 & -1 & 7 \\ 4 & -2 & -1 & 7 \end{array}\right] \xrightarrow[\rho_1 \leftrightarrow \rho_3]{} \left[\begin{array}{rrr|r} 4 & -2 & -1 & 7 \\ 3 & -4 & -1 & 7 \\ 0 & 1 & 2 & 5 \end{array}\right]$$

$$\xrightarrow[-\rho_2+\rho_1]{} \left[\begin{array}{rrr|r} 1 & 2 & 0 & 0 \\ 3 & -4 & -1 & 7 \\ 0 & 1 & 2 & 5 \end{array}\right] \xrightarrow[-3\rho_1+\rho_2]{} \left[\begin{array}{rrr|r} 1 & 2 & 0 & 0 \\ 0 & -10 & -1 & 7 \\ 0 & 1 & 2 & 5 \end{array}\right]$$

$$\xrightarrow[10\rho_3+\rho_2]{} \left[\begin{array}{rrr|r} 1 & 2 & 0 & 0 \\ 0 & 0 & 19 & 57 \\ 0 & 1 & 2 & 5 \end{array}\right] \xrightarrow[\rho_2 \leftrightarrow \rho_3]{} \left[\begin{array}{rrr|r} 1 & 2 & 0 & 0 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 19 & 57 \end{array}\right]$$

The system is again in echelon form and, as before, we get the unique solution

$$x = 2, \; y = -1, \; z = 3 \; .$$