Math 212 – Problem Set #5 SOLUTIONS

① $y'' - 5y' + 6y = 0$

characteristic equation: $m^2 - 5m + 6 = 0$

$\Rightarrow (m-2)(m-3) = 0$

$\Rightarrow m = 2, \; m = 3$

General solution: $y(x) = c_1 e^{2x} + c_2 e^{3x}$

② $y''' - 3y'' - y' + 3y = 0$

characteristic equation: $m^3 - 3m^2 - m + 3 = 0$

$\Rightarrow m^2(m-3) - (m-3) = 0$

$\Rightarrow (m^2 - 1)(m-3) = 0$

$\Rightarrow m = 1, \; m = -1, \; m = 3$

General solution: $y(x) = c_1 e^{x} + c_2 e^{-x} + c_3 e^{3x}$

③ $y'' - 4y' + 13y = 0$

characteristic equation: $m^2 - 4m + 13 = 0$

$$\Rightarrow m = \frac{4 \pm \sqrt{16 - 4(13)}}{2} = \frac{4 \pm 6i}{2} = 2 \pm 3i$$

$$m = 2 + 3i, \quad m = 2 - 3i$$

General solution: $y(x) = c_1 e^{2x} \cos(3x) + c_2 e^{2x} \sin(3x)$

④ $y''' - 5y'' + 7y' - 3y = 0$

characteristic equation: $m^3 - 5m^2 + 7m - 3 = 0$

$$\Rightarrow (m-1)(m^2 - 4m + 3) = 0$$

$$\Rightarrow (m-1)^2 (m-3) = 0$$

$$\Rightarrow m = 1 \text{ (double root)}, \quad m = 3$$

General solution: $y(x) = (c_1 + c_2 x) e^x + c_3 e^{3x}$

(5) $y''' - 6y'' + 12y' - 8y = 0$

characteristic equation: $m^3 - 6m^2 + 12m - 8 = 0$

$\Longrightarrow (m-2)^3 = 0 \Rightarrow \underline{m = 2}$ (triple root)

General solution: $y(x) = (c_1 + c_2 x + c_3 x^2) e^{2x}$

(6) $y''' - y'' + y' - y = 0$

characteristic equation: $m^3 - m^2 + m - 1 = 0$

$\Longrightarrow m^2(m-1) + m - 1 = 0$

$\Longrightarrow (m^2+1)(m-1) = 0$

$\Longrightarrow \underline{m = \pm i}, \ \underline{m = 1}$

General solution: $y(x) = c_1 \cos x + c_2 \sin x + c_3 e^x$