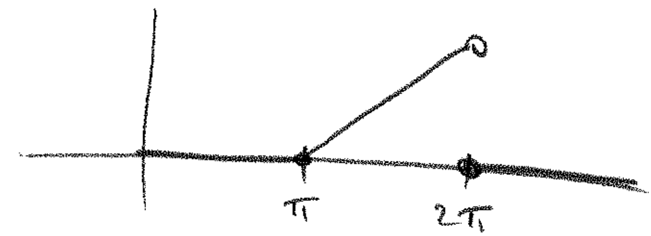
#10 $f(t) = \begin{cases} 0, & t < \pi \\ t-\pi, & \pi \le t < 2\pi \\ 0, & t \ge 2\pi \end{cases}$



$$= u_{\pi}(t)(t-\pi) - u_{2\pi}(t)(t-\pi)$$

$$= \underbrace{u_{\pi}(t)\cdot(t-\pi)} - \underbrace{u_{2\pi}(t)\cdot(t-2\pi)} - \underbrace{\pi u_{2\pi}(t)}$$

since $\mathcal{L}(u_c(t)\cdot f(t-c)) = e^{-cs}F(s)$

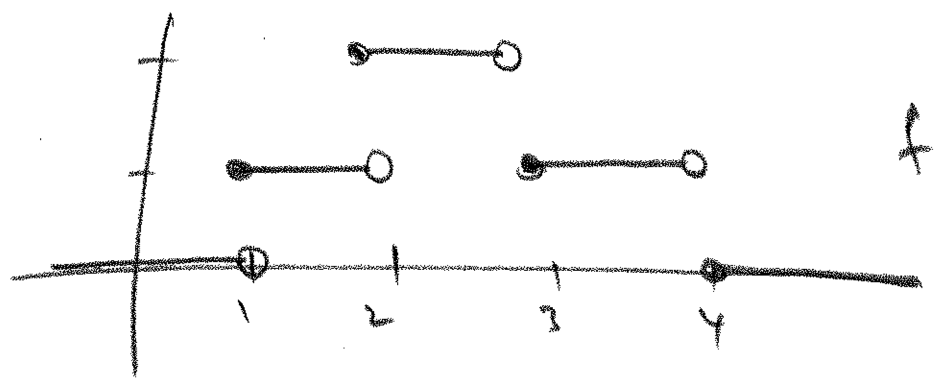
$$\mathcal{L}(f) = \mathcal{L}(\underbrace{u_{\pi}\cdot(t-\pi)} - u_{2\pi}(t-2\pi) - u_{2\pi}\cdot\pi)$$

$$= e^{-\pi s}\cdot\frac{1}{s^2} - e^{-2\pi s}\cdot\frac{1}{s^2} - e^{-2\pi s}\cdot\frac{\pi}{s}$$

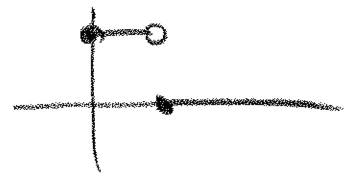
$$= \frac{e^{-\pi s} - e^{-2\pi s}}{s^2} - \frac{\pi e^{-2\pi s}}{s}$$

#16 $F(s) = \dfrac{e^{-s} + e^{-2s} - e^{-3s} - e^{-4s}}{s}$

$f(t) = u_1(t)\cdot 1 + u_2(t)\cdot 1 - u_3(t) - u_4(t)$



#21 $f(t) = \begin{cases} 1, & 0 \le t < 1 \\ 0, & t \ge 1 \end{cases}$



$= 1 - u_1(t)$

$\Longrightarrow F(s) = \frac{1}{s} - e^{-s} \cdot \frac{1}{s} = \frac{1 - e^{-s}}{s}$

#22 $f(t) = 1 - u_1(t) + u_2(t) - u_3(t)$

$F(s) = \frac{1 - e^{-s} + e^{-2s} - e^{-3s}}{s}$

#23 $f(t) = 1 + \sum_{k=1}^{\infty} (-1)^k u_k(t)$

$F(s) = \frac{1 - e^{-s} + e^{-2s} - e^{-3s} + \dots}{s} = \frac{\frac{1}{1-(-e^{-s})}}{s} = \frac{\frac{1}{1+e^{-s}}}{s}$

$= \frac{1}{s(1+e^{-s})}$

$$F(s) = \frac{2e^{-2s}}{s^2-4}$$

$$\Rightarrow f(t) = u_2(t)\sinh(2(t-2))$$

$$\mathcal{L}(u_c f(t-c)) = e^{-cs}F(s)$$

Aside:

$$\sinh(at) = \frac{e^{at} - e^{-at}}{2}$$

$$\cosh(at) = \frac{e^{at} + e^{-at}}{2}$$

$$\mathcal{L}(\sinh(at)) = \mathcal{L}\left(\frac{1}{2}e^{at} - \frac{1}{2}e^{-at}\right)$$

$$= \frac{1}{2}\frac{1}{s-a} - \frac{1}{2}\frac{1}{s+a}$$

$$= \frac{1}{2}\left(\frac{1}{s-a} - \frac{1}{s+a}\right)$$

$$= \frac{1}{2}\left(\frac{(s+a)-(s-a)}{s^2-a^2}\right)$$

$$= \frac{1}{2}\left(\frac{2a}{s^2-a^2}\right) = \frac{a}{s^2-a^2}$$

#1 $y'' + y = \begin{cases} 1, & 0 \le t < 3\pi \\ 0, & 3\pi \le t < \infty \end{cases} \Longrightarrow y'' + y = 1 - u_{3\pi}(t)$

$y(0) = 0, \quad y'(0) = 1$

Apply $\mathcal{L}$:

$$-y'(0) - \cancel{sy(0)} + s^2 Y + Y = \frac{1}{s} - \frac{e^{-3\pi s}}{s}$$

$$-1 + (s^2+1)Y = \frac{1 - e^{-3\pi s}}{s}$$

$$(s^2+1)Y = 1 + \frac{1}{s} - \frac{e^{-3\pi s}}{s}$$

$$Y = \frac{1}{s^2+1} + \frac{1}{s(s^2+1)} - \frac{e^{-3\pi s}}{s(s^2+1)}$$

$$\frac{1}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1}$$

$$1 = A(s^2+1) + s(Bs+C)$$

$$\underbrace{A = 1, \quad B = -1, \quad C = 0}$$

$$\frac{1}{s(s^2+1)} = \frac{1}{s} - \frac{s}{s^2+1}$$

$$Y = \frac{1}{s^2+1} + \frac{1}{s} - \frac{s}{s^2+1} - e^{-3\pi s}\left(\frac{1}{s} - \frac{s}{s^2+1}\right)$$

$$\Longrightarrow \quad y(t) = \sin(t) + 1 - \cos t - u_{3\pi}(t)\left(1 - \cos(t - 3\pi)\right)$$

$$= \sin t + 1 - \cos t - u_{3\pi}(t)\left(1 + \cos t\right)$$

#2 $y'' + 2y' + 2y = u_\pi(t) - u_{2\pi}(t)$

$\cancel{y(0)} = 0, \quad y'(0) = 1$

$$-y'(0) - s\cancel{y(0)} + s^2 Y + 2(-\cancel{y(0)} + sY) + 2Y = \frac{e^{-\pi s} - e^{-2\pi s}}{s}$$

$$-1 + (s^2+2s+2)Y = \frac{e^{-\pi s} - e^{-2\pi s}}{s}$$

$$(s^2+2s+2)Y = 1 + \frac{e^{-\pi s} - e^{-2\pi s}}{s}$$

$$\frac{1}{s(s^2+2s+2)} = \frac{A}{s} + \frac{Bs+C}{s^2+2s+2}$$

$$1 = A(s^2+2s+2) + s(Bs+C)$$

$A = \frac{1}{2}, \quad B = -\frac{1}{2}, \quad C = -1$

$$Y = \underbrace{\frac{1}{s^2+2s+2}} + \frac{e^{-\pi s}}{s(s^2+2s+2)} - \frac{e^{-2\pi s}}{s(s^2+2s+2)}$$

$$= \frac{1}{(s+1)^2+1} + e^{-\pi s}\left(\frac{\frac{1}{2}}{s} - \frac{\frac{1}{2}s+1}{(s+1)^2+1}\right) - e^{-2\pi s}\left(\frac{\frac{1}{2}}{s} - \frac{\frac{1}{2}s+1}{(s+1)^2+1}\right)$$

$$= \frac{1}{(s+1)^2+1} + e^{-\pi s}\left(\frac{\frac{1}{2}}{s} - \frac{\frac{1}{2}(s+1)+\frac{1}{2}}{(s+1)^2+1}\right) - e^{-2\pi s}\left(\ '' \ \right)$$

$$\Longrightarrow y(t) = e^{-t}\sin t + u_\pi(t)\left(\frac{1}{2} - \frac{1}{2}e^{-(t-\pi)}\cos(t-\pi) - \frac{1}{2}e^{-(t-\pi)}\sin(t-\pi)\right)$$
$$- u_{2\pi}(t)\left(\frac{1}{2} - \frac{1}{2}e^{-(t-2\pi)}\cos(t-2\pi) - \frac{1}{2}e^{-(t-2\pi)}\sin(t-2\pi)\right)$$