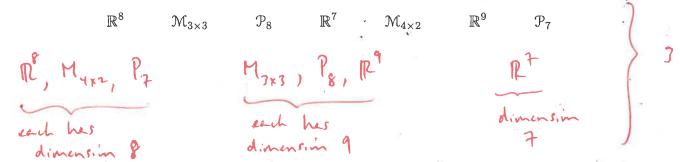
## Quiz 6

## Name:

1. Which of the following vector spaces are isomorphic? Group them accordingly and provide your rationale.



- 2. Define the map  $T \colon \mathbb{R}^3 \to \mathbb{R}^2$  by  $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ .
- (a) Verify that T is a homomorphism:

$$T\left(\left(\begin{array}{c} x_{1} \\ x_{2} \\ x_{3} \end{array}\right) + \beta \begin{bmatrix} y_{1} \\ y_{2} \\ y_{3} \end{bmatrix}\right) = T\left(\begin{bmatrix} x_{1} + \beta y_{1} \\ x_{2} + \beta y_{2} \\ x_{3} + \beta y_{3} \end{bmatrix}\right)$$

$$= \begin{bmatrix} ax_{1} + \beta y_{1} \\ ax_{2} + \beta y_{2} \end{bmatrix} = a\begin{bmatrix} x_{1} \\ x_{2} \end{bmatrix} + \beta \begin{bmatrix} y_{1} \\ y_{2} \end{bmatrix}$$

$$= aT\left(\begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \end{bmatrix}\right) + \beta T\left(\begin{bmatrix} y_{1} \\ y_{2} \\ y_{3} \end{bmatrix}\right).$$

(b) Why is T not an isomorphism?

T is not 1-1; for example, 
$$T(\begin{bmatrix} 1 \\ 2 \end{bmatrix}) = T(\begin{bmatrix} 1 \\ 3 \end{bmatrix})$$
,  $\begin{cases} 2 \\ 4 \end{cases}$  while  $\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ .

(c) Identify one easy way to change T to make it an isomorphism.

Ophim 1: redefine as

$$T: \mathbb{R}^2 \to \mathbb{R}^2$$

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$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \mapsto \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

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