

Quiz 2

Name: _____

1. Use elementary row operations to transform the augmented matrix for the following system of linear equations into reduced row echelon form, then solve the system using vector notation. Be sure to clearly identify the row operations you use at each step.

$$\begin{array}{rrrrrr} x & + & 2y & + & 4z & - & 7w & = & 3 \\ x & & & & - & 2z & + & w & = & -1 \\ 2x & + & 3y & + & 5z & - & 10w & = & 4 \\ x & + & y & + & z & - & 3w & = & 1 \end{array}$$

Here's one possible sequence of row operations leading to the reduced row echelon form:

$$\begin{aligned} & \left[\begin{array}{cccc|c} 1 & 2 & 4 & -7 & 3 \\ 1 & 0 & -2 & 1 & -1 \\ 2 & 3 & 5 & -10 & 4 \\ 1 & 1 & 1 & -3 & 1 \end{array} \right] \xrightarrow[\substack{-\rho_1 + \rho_2 \\ -2\rho_1 + \rho_3 \\ -\rho_1 + \rho_4}]{} \left[\begin{array}{cccc|c} 1 & 2 & 4 & -7 & 3 \\ 0 & -2 & -6 & 8 & -4 \\ 0 & -1 & -3 & 4 & -2 \\ 0 & -1 & -3 & 4 & -2 \end{array} \right] \\ & \xrightarrow{-\frac{1}{2}\rho_2} \left[\begin{array}{cccc|c} 1 & 2 & 4 & -7 & 3 \\ 0 & 1 & 3 & -4 & 2 \\ 0 & -1 & -3 & 4 & -2 \\ 0 & -1 & -3 & 4 & -2 \end{array} \right] \xrightarrow[\substack{\rho_2 + \rho_3 \\ \rho_2 + \rho_4}]{} \left[\begin{array}{cccc|c} 1 & 2 & 4 & -7 & 3 \\ 0 & 1 & 3 & -4 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \\ & \xrightarrow{-2\rho_2 + \rho_1} \left[\begin{array}{cccc|c} 1 & 0 & -2 & 1 & -1 \\ 0 & 1 & 3 & -4 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \end{aligned}$$

Now that we have the system in reduced row echelon form, we can read off the set of solutions:

$$\left\{ \begin{bmatrix} -1 \\ 2 \\ 0 \\ 0 \end{bmatrix} + z \begin{bmatrix} -2 \\ 3 \\ 1 \\ 0 \end{bmatrix} + w \begin{bmatrix} 1 \\ -4 \\ 0 \\ 1 \end{bmatrix} : z, w \in \mathbb{R} \right\}.$$