Newton's Law of Cooling

$y(t)$ = temperature @ time t ; T = ambient (constant) temp.

$$y'(t) = \frac{dy}{dt} = -k(y(t) - T)$$

$$\boxed{\begin{aligned} y' &= -k(y - T) \\ y(0) &= y_0 \end{aligned}}$$

Example: $y' = -2(y-5)$

$$\frac{y'}{y-5} = -2$$

Now integrate:

$$\left(\ln(|y-5|)\right)' = -2$$

$$\Longrightarrow \ln|y-5| = -2t + c$$

$$\Longrightarrow y - 5 = e^{-2t} e^{c} = c e^{-2t}$$

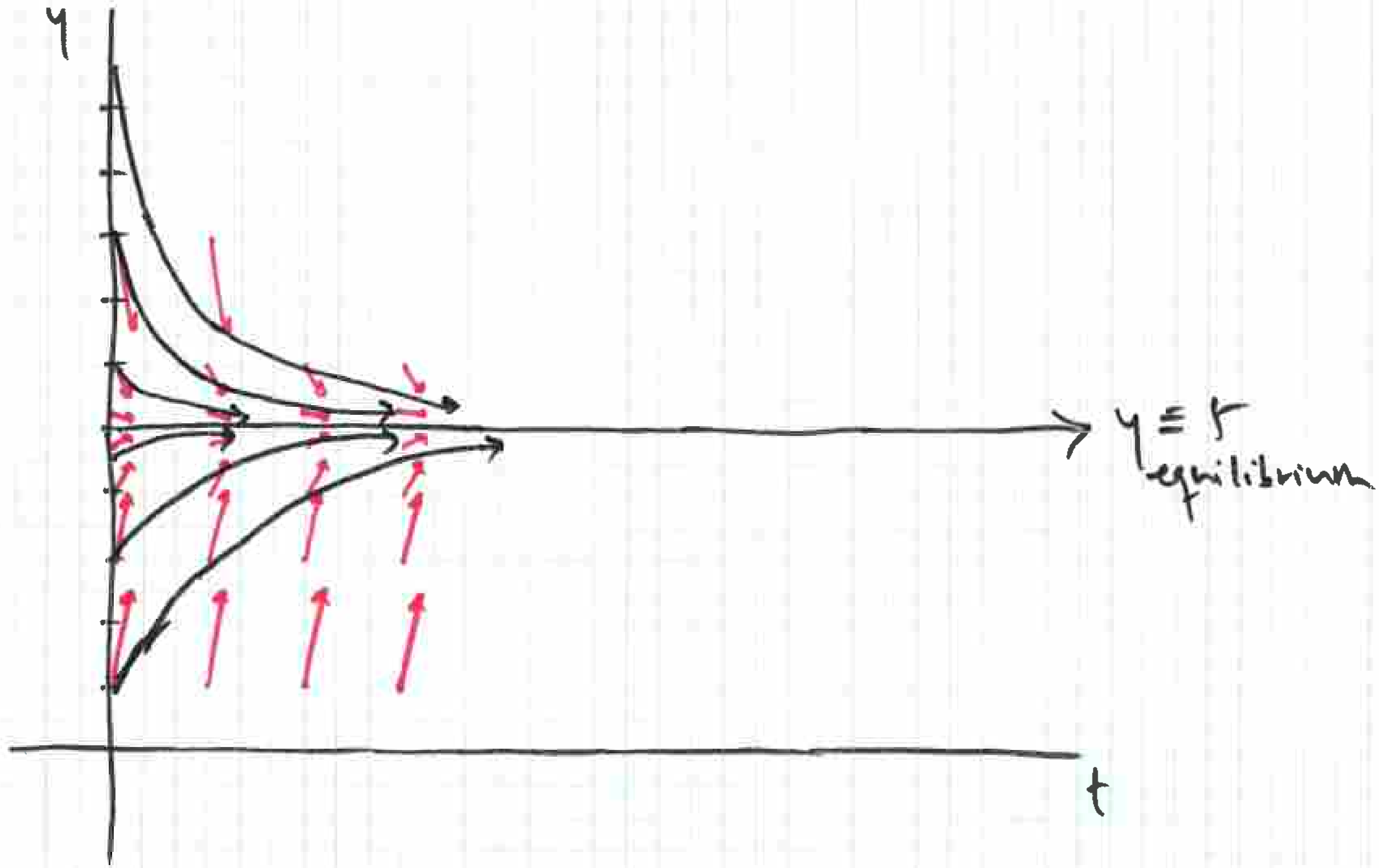
$$\Longrightarrow y = 5 + c e^{-2t}$$

$t = 0$: $y_0 = 5 + c \Longrightarrow c = y_0 - 5$

Thus:

$$\boxed{y(t) = 5 + (y_0 - 5) e^{-2t}}$$

$y' = -2(y-5)$



$$\begin{cases} y' = -k(y-T) \\ y(0) = y_0 \end{cases} \Longrightarrow \underline{\underline{y(t) = T + (y_0 - T)e^{-kt}}}$$

$$\frac{y'}{y-T} = -k$$

$$\left(\ln(|y-T|)\right)' = -k$$

Integrate:

$$\ln(|y-T|) = -kt + c$$

$$y - T = e^c e^{-kt} = ce^{-kt}$$

$$y = T + ce^{-kt}$$

$$t = 0 \Longrightarrow y_0 = T + c$$

$$\Longrightarrow c = y_0 - T$$

The same technique can be used to solve any ODE of the form

$$y' = a + by,$$ for constants a & b.

$$y' = y^2 \implies \frac{y'}{y^2} = 1$$

Integrate: $\quad (-y^{-1})' = 1$

$$\implies -\frac{1}{y} = t + c \qquad t = 0 \implies -\frac{1}{y_0} = c$$

$$\implies y = \frac{-1}{t+c} = \frac{-1}{t - \frac{1}{y_0}}$$

$$\implies \underline{\underline{y(t) = \frac{1}{\frac{1}{y_0} - t}}}$$

e.g.: $y_0 = 2 \implies y(t) = \dfrac{1}{\frac{1}{2} - t}$ blows up @ $\underline{t = \frac{1}{2}}$!