Quiz 8

Name: SOLUTIONS

1. Define the homomorphism $T\colon \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ by

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2y+z \\ x-4y \end{pmatrix}.$$

(a) Is T an isomorphism? Briefly justify your answer.

No! T cannot be 1-1; its nullity is at least 1.

(b) Compute the matrix representation of T relative to the canonical bases of $\mathbb{R}^3$ and $\mathbb{R}^2$.

$T(e_1) = T\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $T(e_2) = T\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$,

and $T(e_3) = T\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

Thus, $[T] = \begin{bmatrix} 0 & 2 & 1 \\ 1 & -4 & 0 \end{bmatrix}$.

(c) Use the matrix representation you just found to compute $T\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$.

$$T\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = \begin{bmatrix} 0 & 2 & 1 \\ 1 & -4 & 0 \end{bmatrix}\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 7 \end{bmatrix}.$$

(You can check this using the definition above!)