

1. Determine the general solution of each of the following differential equations.

(a)  $y'' + 2y' + 5y = e^{-x} \sec 2x$

(b)  $2y'' + 3y' + y = x^2 + 3 \sin x$

(c)  $y'' + 2y' + y = e^{-x} \ln x$

2. Given that  $y_1(x) = \sin(x^2)$  is one solution of

$$xy'' - y' + 4x^3y = 0 ,$$

determine the general solution of this ODE.

3. Consider the ODE

$$y'' - xy = 0 .$$

- (a) Determine the Maclaurin series for two independent solutions of this ODE. For each of these solutions, obtain a nice, compact formula for the coefficients.
- (b) Using the formulas obtained in part (a), apply the ratio test to determine the values of  $x$  for which these Maclaurin series converge.
4. Determine the first 4 nonzero terms of the Maclaurin series of two independent solutions of the ODE

$$(3 - x^2) y'' - 3xy' - y = 0 .$$

5. Apply the method of Frobenius to determine the first four nonzero terms of the power series expansions of two independent solutions of the ODE

$$2xy'' + (x + 1)y' + 3y = 0 .$$