CHAPTER 2   **Section 2.1, page 39**

1. (c) $y = ce^{-3t} + (t/3) - (1/9) + e^{-2t}$; y is asymptotic to $t/3 - 1/9$ as $t \to \infty$
2. (c) $y = ce^{2t} + t^3 e^{2t}/3$; $y \to \infty$ as $t \to \infty$
3. (c) $y = ce^{-t} + 1 + t^2 e^{-t}/2$; $y \to 1$ as $t \to \infty$
4. (c) $y = (c/t) + (3\cos 2t)/4t + (3\sin 2t)/2$; y is asymptotic to $(3\sin 2t)/2$ as $t \to \infty$
5. (c) $y = ce^{2t} - 3e^t$; $y \to \infty$ or $-\infty$ as $t \to \infty$
6. (c) $y = (c - t\cos t + \sin t)/t^2$; $y \to 0$ as $t \to \infty$
7. (c) $y = t^2 e^{-t} + ce^{-t^2}$; $y \to 0$ as $t \to \infty$
8. (c) $y = (\arctan t + c)/(1 + t^2)^2$; $y \to 0$ as $t \to \infty$
9. (c) $y = ce^{-t/2} + 3t - 6$; y is asymptotic to $3t - 6$ as $t \to \infty$
10. (c) $y = -te^{-t} + ct$; $y \to \infty, 0$, or $-\infty$ as $t \to \infty$
11. (c) $y = ce^{-t} + \sin 2t - 2\cos 2t$; y is asymptotic to $\sin 2t - 2\cos 2t$ as $t \to \infty$
12. (c) $y = ce^{-t/2} + 3t^2 - 12t + 24$; y is asymptotic to $3t^2 - 12t + 24$ as $t \to \infty$
13. $y = 3e^t + 2(t - 1)e^{2t}$
14. $y = (t^2 - 1)e^{-2t}/2$
15. $y = (3t^4 - 4t^3 + 6t^2 + 1)/12t^2$
16. $y = (\sin t)/t^2$
17. $y = (t + 2)e^{2t}$
18. $y = t^{-2}[(\pi^2/4) - 1 - t\cos t + \sin t]$
19. $y = -(1 + t)e^{-t}/t^4$, $\quad t \neq 0$
20. $y = (t - 1 + 2e^{-t})/t$, $\quad t \neq 0$
21. (b) $y = -\frac{4}{5}\cos t + \frac{8}{5}\sin t + (a + \frac{4}{5})e^{t/2}$; $a_0 = -\frac{4}{5}$
    (c) y oscillates for $a = a_0$
22. (b) $y = -3e^{t/3} + (a + 3)e^{t/2}$; $a_0 = -3$
    (c) $y \to -\infty$ for $a = a_0$
23. (b) $y = [2 + a(3\pi + 4)e^{2t/3} - 2e^{-\pi t/2}]/(3\pi + 4)$; $a_0 = -2/(3\pi + 4)$
    (c) $y \to 0$ for $a = a_0$
24. (b) $y = te^{-t} + (ea - 1)e^{-t}/t$; $a_0 = 1/e$
    (c) $y \to 0$ as $t \to 0$ for $a = a_0$
25. (b) $y = -(\cos t)/t^2 + \pi^2 a/4t^2$; $a_0 = 4/\pi^2$
    (c) $y \to \frac{1}{2}$ as $t \to 0$ for $a = a_0$
26. (b) $y = (e^t - e + a\sin 1)/\sin t$; $a_0 = (e - 1)/\sin 1$
    (c) $y \to 1$ for $a = a_0$
27. $(t, y) = (1.364312, 0.820082)$
28. $y_0 = -1.642876$
29. (b) $y = 12 + \frac{8}{65}\cos 2t + \frac{64}{65}\sin 2t - \frac{788}{65}e^{-t/4}$; y oscillates about 12 as $t \to \infty$
    (c) $t = 10.065778$
30. $y_0 = -5/2$
31. $y_0 = -16/3$; $y \to -\infty$ as $t \to \infty$ for $y_0 = -16/3$
39. See Problem 2.
40. See Problem 4.
41. See Problem 6.
42. See Problem 12.

Section 2.2, page 47

1. $3y^2 - 2x^3 = c$; $\quad y \neq 0$
2. $3y^2 - 2\ln|1 + x^3| = c$; $\quad x \neq -1, y \neq 0$
3. $y^{-1} + \cos x = c$ if $y \neq 0$; also $y = 0$; everywhere
4. $3y + y^2 - x^3 + x = c$; $\quad y \neq -3/2$
5. $2\tan 2y - 2x - \sin 2x = c$ if $\cos 2y \neq 0$; also $y = \pm(2n + 1)\pi/4$ for any integer n; everywhere
6. $y = \sin[\ln|x| + c]$ if $x \neq 0$ and $|y| < 1$; also $y = \pm 1$
7. $y^2 - x^2 + 2(e^y - e^{-x}) = c$; $\quad y + e^y \neq 0$
8. $3y + y^3 - x^3 = c$; everywhere
9. (a) $y = 1/(x^2 - x - 6)$ (c) $-2 < x < 3$
10. (a) $y = -\sqrt{2x - 2x^2 + 4}$ (c) $-1 < x < 2$
11. (a) $y = [2(1 - x)e^x - 1]^{1/2}$ (c) $-1.68 < x < 0.77$ approximately
12. (a) $r = 2/(1 - 2\ln\theta)$ (c) $0 < \theta < \sqrt{e}$
13. (a) $y = -[2\ln(1 + x^2) + 4]^{1/2}$ (c) $-\infty < x < \infty$

14. (a) $y = \left[3 - 2\sqrt{1 + x^2}\right]^{-1/2}$ (c) $|x| < \frac{1}{2}\sqrt{5}$

15. (a) $y = -\frac{1}{2} + \frac{1}{2}\sqrt{4x^2 - 15}$ (c) $x > \frac{1}{2}\sqrt{15}$

16. (a) $y = -\sqrt{(x^2 + 1)/2}$ (c) $-\infty < x < \infty$

17. (a) $y = 5/2 - \sqrt{x^3 - e^x + 13/4}$ (c) $-1.4445 < x < 4.6297$ approximately

18. (a) $y = -\frac{3}{4} + \frac{1}{4}\sqrt{65 - 8e^x - 8e^{-x}}$ (c) $|x| < 2.0794$ approximately

19. (a) $y = [\pi - \arcsin(3\cos^2 x)]/3$ (c) $|x - \pi/2| < 0.6155$

20. (a) $y = \left[\frac{3}{2}(\arcsin x)^2 + 1\right]^{1/3}$ (c) $-1 < x < 1$

21. $y^3 - 3y^2 - x - x^3 + 2 = 0, \quad |x| < 1$

22. $y^3 - 4y - x^3 = -1, \quad |x^3 - 1| < 16/3\sqrt{3}$ or $-1.28 < x < 1.60$

23. $y = -1/(x^2/2 + 2x - 1); \quad x = -2$

24. $y = -3/2 + \sqrt{2x - e^x + 13/4}; \quad x = \ln 2$

25. $y = -3/2 + \sqrt{\sin 2x + 1/4}; \quad x = \pi/4$ 26. $y = \tan(x^2 + 2x); \quad x = -1$

27. (a) $y \to 4$ if $y_0 > 0; \quad y = 0$ if $y_0 = 0; \quad y \to -\infty$ if $y_0 < 0$
    (b) $T = 3.29527$

28. (a) $y \to 4$ as $t \to \infty$ (b) $T = 2.84367$
    (c) $3.6622 < y_0 < 4.4042$

29. $x = \frac{c}{a}y + \frac{ad - bc}{a^2}\ln|ay + b| + k; \quad a \neq 0, \; ay + b \neq 0$

30. (e) $|y + 2x|^3|y - 2x| = c$ 31. (b) $\arctan(y/x) - \ln|x| = c$

32. (b) $x^2 + y^2 - cx^3 = 0$ 33. (b) $|y - x| = c|y + 3x|^5; \quad$ also $y = x$

34. (b) $|y + x||y + 4x|^2 = c$

35. (b) $2x/(x + y) + \ln|x + y| = c; \quad$ also $y = -x$

36. (b) $x/(x + y) + \ln|x| = c; \quad$ also $y = -x$ 37. (b) $|x|^3|x^2 - 5y^2| = c$

38. (b) $c|x|^3 = |y^2 - x^2|$

Section 2.3, page 59

1. $t = 100\ln 100$ min $\cong 460.5$ min   2. $Q(t) = 120\gamma[1 - \exp(-t/60)]; \quad 120\gamma$

3. $Q = 50e^{-0.2}(1 - e^{-0.2})$ lb $\cong 7.42$ lb

4. $Q(t) = 200 + t - [100(200)^2/(200 + t)^2]$ lb, $\quad t < 300; \quad c = 121/125$ lb/gal;
   $\lim\limits_{t \to \infty} c = 1$ lb/gal

5. (a) $Q(t) = \frac{63,150}{2501}e^{-t/50} + 25 - \frac{625}{2501}\cos t + \frac{25}{5002}\sin t$
   (c) level $= 25$; amplitude $= 25\sqrt{2501}/5002 \cong 0.24995$

6. (c) 130.41 s

7. (a) $(\ln 2)/r$ yr   (b) 9.90 yr   (c) 8.66%

8. (a) $k(e^{rt} - 1)/r$ (b) $k \cong \$3930$ (c) 9.77%

9. $k = \$3086.64$/yr; $\$1259.92$ 10. (a) $\$89,034.79$ (b) $\$102,965.21$

11. (a) $t \cong 135.36$ months   (b) $\$152,698.56$

12. (a) 0.00012097 yr^{-1} (b) $Q_0 \exp(-0.00012097t), \quad t$ in yr
    (c) $13,305$ yr

13. $P = 201,977.31 - 1977.31e^{(\ln 2)t}, \quad 0 \leq t \leq t_f \cong 6.6745$ (weeks)

14. (a) $\tau \cong 2.9632$; no   (b) $\tau = 10\ln 2 \cong 6.9315$
    (c) $\tau = 6.3805$

15. (b) $y_c \cong 0.83$ 16. $t = \ln\frac{13}{8}/\ln\frac{13}{12}$ min $\cong 6.07$ min

17. (a) $u(t) = 2000/(1 + 0.048t)^{1/3}$ (c) $\tau \cong 750.77$ s

18. (a) $u(t) = ce^{-kt} + T_0 + kT_1(k\cos\omega t + \omega\sin\omega t)/(k^2 + \omega^2)$
    (b) $R \cong 9.11°$F; $\quad \tau \cong 3.51$ h
    (c) $R = kT_1/\sqrt{k^2 + \omega^2}; \quad \tau = (1/\omega)\arctan(\omega/k)$