

Homework #1

Math 334

Due 9/14/17

For each of the following problems, solve the 1-dimensional heat equation,

$$u_t = u_{xx}, \quad \text{for } 0 < x < \pi,$$

with the given auxiliary conditions.

1. (a) Determine the general solution satisfying the ***mixed*** homogeneous boundary conditions

$$u_x(0, t) = 0, \quad u(\pi, t) = 0.$$

- (b) Determine the solution satisfying these boundary conditions and the initial condition

$$u(x, 0) = \begin{cases} \pi, & \text{for } 0 \leq x \leq \frac{\pi}{2}, \\ 2\pi - 2x, & \text{for } \frac{\pi}{2} \leq x \leq \pi. \end{cases}$$

- (c) How does the solution behave as $t \rightarrow \infty$?

2. (a) Determine the general solution satisfying the ***no-flux*** homogeneous boundary conditions

$$u_x(0, t) = 0, \quad u_x(\pi, t) = 0.$$

- (b) Determine the solution satisfying these boundary conditions and the initial condition

$$u(x, 0) = \begin{cases} 1, & \text{for } 0 \leq x < \frac{\pi}{2}, \\ -1, & \text{for } \frac{\pi}{2} < x \leq \pi. \end{cases}$$

- (c) The initial condition is discontinuous at $x = \frac{\pi}{2}$. Is this true of the solution when $t > 0$?

- (d) Using the Fourier series you found in (b), evaluate the solution when $x = 0$ and $t = 0$ to compute the sum of the series

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots.$$

3. (a) Determine the general solution satisfying the ***periodic*** boundary conditions

$$u(0, t) = u(\pi, t), \quad u_x(0, t) = u_x(\pi, t).$$

- (b) Determine the solution satisfying these boundary conditions and the initial condition

$$u(x, 0) = x(\pi - x).$$

- (c) Using the Fourier series you just found, evaluate the solution when $x = 0$ and $t = 0$ to compute the sum of an interesting series.