Use the Laplace transform to solve each of the following initial-value problems.

1. $y'' - y' - 6y = 0\,, \quad y(0) = 1, \quad y'(0) = -1$

2. $y'' + 2y' + 5y = 0\,, \quad y(0) = 2, \quad y'(0) = -1$

3. $y'' + 2y' + y = 4e^{-t}\,, \quad y(0) = 2, \quad y'(0) = -1$

4. $y'' + y = h(t)\,, \quad y(0) = 0, \quad y'(0) = 1, \quad \text{where} \quad h(t) = \begin{cases} 1, & 0 \le t < 3\pi \\ 0, & 3\pi \le t < \infty \end{cases}$

5. $y'' + 3y' + 2y = h(t)\,, \quad y(0) = 0, \quad y'(0) = 0, \quad \text{where} \quad h(t) = \begin{cases} 1, & 0 \le t < 10 \\ 0, & t \ge 10 \end{cases}$