Homework #2

Math 334

Presentations to begin on 10/3/17

1. Solve the 1–dimensional wave equation for a string of infinite length,

$$u_{tt} \;=\; c^2\, u_{xx}\,, \quad \text{for} \quad -\infty < x < \infty \quad \text{and} \quad t > 0\,,$$

with zero initial velocity, $\psi(x) \;\equiv\; 0$, and with the initial profile

$$\varphi(x) \;=\; \begin{cases} 1 - |x|\,, & \text{for} \quad |x| \;\le\; 1\,, \\ 0\,, & \text{otherwise}\,. \end{cases}$$

In particular, plot the solution at the times

$$0, \quad \frac{1}{2c}\,, \quad \frac{1}{c}\,, \quad \frac{2}{c}\,, \quad \text{and} \quad \frac{3}{c}\,.$$

2. Solve the 1–dimensional wave equation for a semi-infinite string with one fixed end,

$$u_{tt} \;=\; c^2\, u_{xx}\,, \quad \text{for} \quad 0 < x < \infty \quad \text{and} \quad t > 0\,,$$

$$u(0,t) \;=\; 0\,, \quad \text{for} \quad t \ge 0\,,$$

with a trivial initial profile, $\varphi \;\equiv\; 0$, and with the initial velocity

$$\psi(x) \;=\; \begin{cases} 1 - |x-2|\,, & \text{for} \quad 1 \;\le\; x \;\le\; 3\,, \\ 0\,, & \text{otherwise}\,. \end{cases}$$

You will need to adapt what we discussed in class to this problem; in other words, you will have to use the boundary condition at $x \;=\; 0$ to determine how to handle negative arguments in D'Alembert's formula. When evaluating the integrals that appear, it might simplify calculations to remember the connection between integration and area. Finally, plot the solution at several representative times.

3. Adapt the methods you used for a semi-infinite string to solve the 1-dimensional wave equation for a string of finite length with pinned ends,

$$u_{tt} \;=\; c^2\, u_{xx}\,, \quad \text{for} \quad 0 < x < 4 \quad \text{and} \quad t > 0\,,$$

$$u(0,t) \;=\; 0 \;=\; u(4,t)\,, \quad \text{for} \quad t \ge 0\,,$$

with zero initial velocity, $\psi(x) \;\equiv\; 0$, and with the initial profile

$$\varphi(x) \;=\; \begin{cases} 0\,, & \text{for} \quad 0 \le x \le 1\,, \\ (x-1)\,(3-x)\,, & \text{for} \quad 1 \le x \le 3\,, \\ 0\,, & \text{for} \quad 3 \le x \le 4\,. \end{cases}$$

Use characteristics in the xt-plane to understand the solution, and plot the solution at several representative times.

4. A *spherical wave* is a solution of the 3-dimensional wave equation that only depends on time, t, and distance from the origin, r. A spherical wave therefore has the form $u(r,t)$, where u satisfies the *spherical wave equation*,

$$u_{tt} \;=\; c^2 \left(u_{rr} + \frac{2}{r} u_r \right) , \quad \text{for} \quad r,t > 0 \,.$$

a. Change variables using $v \;=\; ru$ to get the equation for v : $v_{tt} \;=\; c^2 v_{rr}$.
b. Use D'Alembert's formula to solve for v , hence for u .
c. Determine the solution u corresponding to the initial data

$$u(r,0) \;=\; \begin{cases} 1\,, & \text{for} \quad r \le 1\,, \\ 0\,, & \text{otherwise} \end{cases} \qquad \text{and} \qquad u_t(r,0) \;=\; e^{-r^2} \,.$$

What's a good way to visualize the solution at different times?