Quiz 9

Name: SOLUTIONS

1. Optimize the function $f(x) = 12x - 4x^2 + \frac{1}{3}x^3$ over the interval $0 \le x \le 3$.

$f'(x) = 12 - 8x + x^2$

$f' = 0$ when $x^2 - 8x + 12 = 0$

$(x-2)(x-6) = 0$

$x = 2$ or $x = 6$

in interval — NOT in interval!

x	$f(x)$	
0	0	→ min. value
2	$24 - 16 + 8/3 = \frac{32}{3}$	→ max. value
3	$36 - 36 + 9 = 9$	

2. Optimize the function $f(x) = 2x\sqrt{4-x^2}$ over the interval $0 \le x \le 2$.

$= 2x(4-x^2)^{1/2}$

$f'(x) = 2(4-x^2)^{1/2} + (2x)(\frac{1}{2})(4-x^2)^{-1/2}(-2x)$

$= 2\sqrt{4-x^2} - \frac{2x^2}{\sqrt{4-x^2}}$

$f' = 0$ when $2\sqrt{4-x^2} = \frac{2x^2}{\sqrt{4-x^2}}$

i.e., when $4 - x^2 = x^2 \iff 4 = 2x^2 \iff x = \pm\sqrt{2}$

* only $\sqrt{2}$ is in this interval *

x	$f(x)$	
0	0	→ min.
$\sqrt{2}$	4	→ max.
2	0	→ min.