2.1 #4 $y' + \frac{1}{t}y = 3\cos(2t), \quad t > 0$

$\mu y' + \mu\left(\frac{1}{t}\right)y = 3\mu\cos(2t)$

$(\mu y)' = \mu y' + \mu' y$ Need $\mu\left(\frac{1}{t}\right) = \mu'$

$\mu' = \mu \cdot \frac{1}{t}$

$ty' + y = 3t\cos(2t)$

$\frac{\mu'}{\mu} = \frac{1}{t} \Rightarrow (\ln\mu)' = \frac{1}{t}$

$\Rightarrow \ln\mu = \ln t$

$\Rightarrow \underline{\mu = t}$

$(ty)' = 3t\cos(2t)$

integrate: $ty = \int 3t\cos(2t)\,dt$

$u = 3t \quad dv = \cos(2t)\,dt$

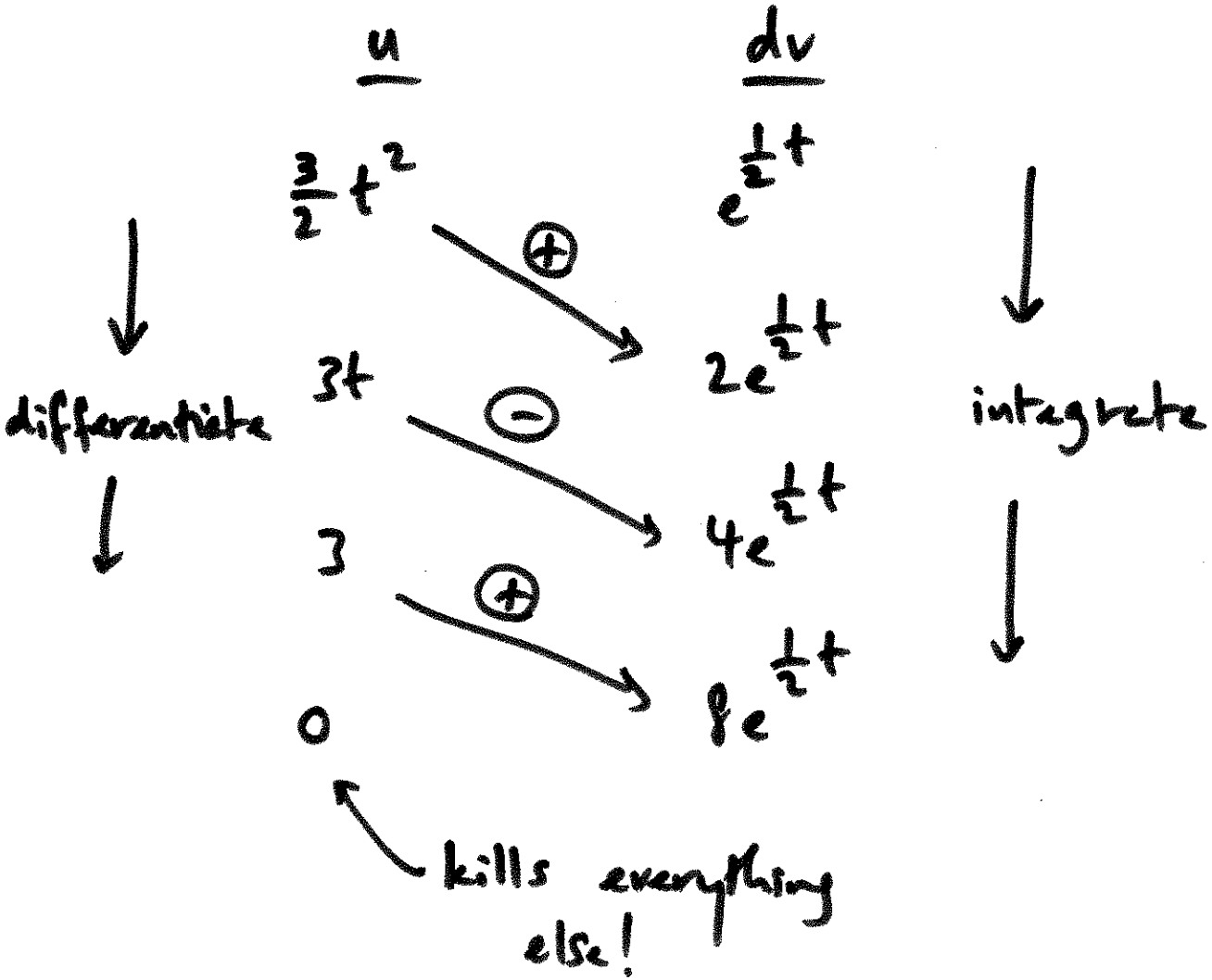
$du = 3\,dt \quad v = \frac{1}{2}\sin(2t)$

$\Rightarrow ty = \frac{3}{2}t\sin(2t) - \int \frac{3}{2}\sin(2t)\,dt = \frac{3}{2}t\sin(2t) + \frac{3}{4}\cos(2t) + c$

$\Rightarrow \boxed{y(t) = \frac{3}{2}\sin(2t) + \frac{3}{4t}\cos(2t) + \frac{c}{t}}$ General solution

TABULAR INTEGRATION - Example

$$\int \frac{3}{2}t^2 e^{\frac{1}{2}t}\,dt = 3t^2 e^{\frac{1}{2}t} - 12t e^{\frac{1}{2}t} + 24 e^{\frac{1}{2}t} + c$$



2.1 #18 $y' + \frac{2}{3}y = 1 - \frac{1}{2}t, \quad y(0) = y_0$

General solution:

integrating factor: $\mu y' + \frac{2}{3}\mu y = \mu(1 - \frac{1}{2}t)$

$(\mu y)' = \mu y' + \mu' y \Longrightarrow$ need $\mu' = \frac{2}{3}\mu$

$\longrightarrow \mu = e^{\frac{2}{3}t}$

Then have $(e^{\frac{2}{3}t} y)' = e^{\frac{2}{3}t}(1 - \frac{1}{2}t)$

integrate: $e^{\frac{2}{3}t} y = \int e^{\frac{2}{3}t}(1 - \frac{1}{2}t)\,dt$

$= (1 - \frac{1}{2}t)(\frac{3}{2}e^{\frac{2}{3}t})$

$+ \frac{1}{2}(\frac{9}{4}e^{\frac{2}{3}t}) + c$

tabular integration:

u	dv
$1 - \frac{1}{2}t$	$e^{\frac{2}{3}t}$
$-\frac{1}{2}$	$\frac{3}{2}e^{\frac{2}{3}t}$
0	$\frac{9}{4}e^{\frac{2}{3}t}$

$\longrightarrow \quad y(t) = \frac{3}{2}(1 - \frac{1}{2}t) + \frac{9}{8} + ce^{-\frac{2}{3}t}$

For y to touch, but not cross, the t-axis, we need $y' = 0$ when $y = 0$; the ODE then forces this to happen at time $t = 2$, since $y' = 0$ & $y = 0 \Rightarrow 0 = 1 - \frac{1}{2}t \Rightarrow t = 2$.

Thus, $y(2) = 0 \Rightarrow 0 = \frac{3}{2}(0) + \frac{9}{8} + ce^{-4/3} \Rightarrow ce^{-4/3} = -\frac{9}{8}$

$$\Rightarrow c = -\frac{9}{8}e^{4/3} = -4.27\ldots$$

$$\Rightarrow y = \frac{3}{2}(1 - \frac{1}{2}t) + \frac{9}{8} + ce^{-\frac{2}{3}t}, \quad c = -4.27\ldots$$

$$\Rightarrow y(0) = \frac{3}{2} + \frac{9}{8} + c = \underline{\underline{-1.64287\ldots}}.$$

2.2 #1 $y' = \frac{x^2}{y}$

$\Rightarrow y \frac{dy}{dx} = x^2$

$\Rightarrow \left(\frac{1}{2}y^2\right)' = x^2$

integrate: $\frac{1}{2}y^2 = \frac{1}{3}x^3 + c$

2.2 SEPARABLE ODEs

#5 $\frac{dy}{dx} = \frac{x - e^{-x}}{y + e^y}$

$(y + e^y)y' = x - e^{-x}$

$\left(\frac{1}{2}y^2 + e^y\right)' = x - e^{-x} \xrightarrow{\text{integrate}} \frac{1}{2}y^2 + e^y = \frac{1}{2}x^2 + e^{-x} + c$

General solution

#9 $y' = (1-2x)y^2$, $y(0) = -\frac{1}{6}$

$$\frac{1}{y^2}y' = (1-2x)$$

$$\left(-\frac{1}{y}\right)' = (1-2x) \xrightarrow{\text{integrate}} -\frac{1}{y} = x - x^2 + c$$

$$\Longrightarrow \quad y = \frac{-1}{x - x^2 + c}$$

$$y(0) = -\frac{1}{6} \Longrightarrow \quad 6 = 0 + c \Longrightarrow c = 6,$$

$$y(x) = \frac{-1}{x - x^2 + 6} = \frac{1}{x^2 - x - 6}$$

$$= \frac{1}{(x-3)(x+2)}$$