

# Quiz 9

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1. Define the homomorphisms  $g: \mathbb{R}^3 \longrightarrow \mathcal{P}_1$  and  $h: \mathcal{M}_{2 \times 2} \longrightarrow \mathbb{R}^3$  as follows:

$$g \begin{pmatrix} a \\ b \\ c \end{pmatrix} = (2a + b)x + (b - c) \quad \text{and} \quad h \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = \begin{pmatrix} \alpha + 3\delta \\ \beta - \gamma \\ 2\delta - \beta \end{pmatrix}$$

- (a) Compute the matrix representations of  $g$  and  $h$  using the standard bases of  $\mathbb{R}^3$ ,  $\mathcal{P}_1$ , and  $\mathcal{M}_{2 \times 2}$ .
- (b) Use the matrix representations from part (a) to compute the matrix representation of the composition of  $g$  and  $h$ . (There is only one way to compose these two homomorphisms!)
- (c) Compute the matrix representations of  $g$  and  $h$  using the standard basis for  $\mathcal{M}_{2 \times 2}$  and the following bases for  $\mathbb{R}^3$  and  $\mathcal{P}_1$ .

$$\mathbb{R}^3 : \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\} \quad \mathcal{P}_1 : \{ 1 - x, 1 + x \}$$

- (d) Use the matrix representations from part (c) to compute the matrix representation of the composition of  $g$  and  $h$  with respect to the given bases.