Quiz 8

Name: SOLUTIONS

---

1. Linearize the function $f(x) = \arctan x$ at the point $x_0 = \sqrt{3}$ to obtain the tangent line to the graph of $f(x)$ at this point.

$$f(\sqrt{3}) = \arctan(\sqrt{3}) = \frac{\pi}{3} \qquad \left(\begin{array}{l}\text{since } \sin(\pi/3) = \frac{\sqrt{3}}{2} \\ \text{and } \cos(\pi/3) = 1/2\end{array}\right)$$

$$f'(x) = \frac{1}{1+x^2}, \quad \text{so the slope is } f'(\sqrt{3}) = \frac{1}{1+3} = \frac{1}{4}$$

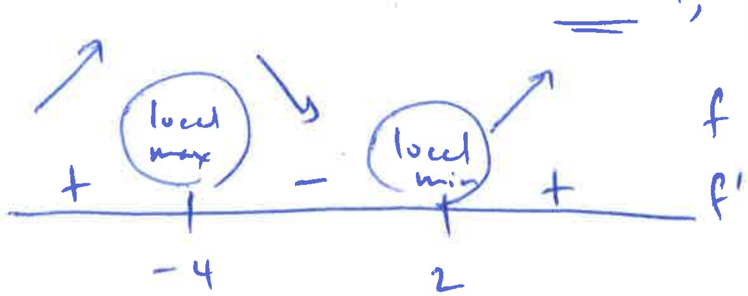
The linearization (a.k.a. tangent line) is thus

$$\boxed{y = \underbrace{\frac{\pi}{3}}_{f(x_0)} + \underbrace{\frac{1}{4}}_{f'(x_0)}\underbrace{(x-\sqrt{3})}_{(x-x_0)}}$$

2. Determine the critical points of the function $f(x) = (x^2-8)\,e^x$, and classify each critical point as a local maximum, local minimum, or neither.

$$f'(x) = 2xe^x + (x^2-8)e^x = e^x(x^2+2x-8)$$

critical points: $f'(x) = 0$ when
$$x^2+2x-8 = 0$$
$$(x+4)(x-2) = 0$$
$$\underline{x = -4}, \quad \underline{x = 2}$$



There is a local maximum at $x = -4$; there is a local minimum at $x = 2$.