

Use trigonometric substitutions to evaluate the integrals in Problems 1–30.

$$1 \int \frac{\sqrt{1-x^2}}{x^2} dx$$

$$2 \int \frac{\sqrt{1+x^2}}{x^2} dx$$

$$3 \int \frac{\sqrt{x^2-1}}{x^2} dx$$

$$4 \int x^3 \sqrt{4-x^2} dx$$

$$5 \int x^3 \sqrt{9+4x^2} dx$$

$$6 \int \frac{x^3 dx}{\sqrt{x^2+25}}$$

$$7 \int \frac{(1-4x^2)^{1/2}}{x} dx$$

$$8 \int \frac{dx}{\sqrt{1+x^2}}$$

$$9 \int \frac{dx}{\sqrt{9+4x^2}}$$

$$10 \int \sqrt{1+4x^2} dx$$

$$11 \int \frac{x^2 dx}{\sqrt{25-x^2}}$$

$$12 \int \frac{x^3 dx}{\sqrt{25-x^2}}$$

$$13 \int \frac{x^2 dx}{\sqrt{1+x^2}}$$

$$14 \int \frac{x^3 dx}{\sqrt{1+x^2}}$$

$$15 \int \frac{x^2 dx}{\sqrt{4+9x^2}}$$

$$16 \int (1-x^2)^{3/2} dx$$

$$17 \int \frac{dx}{(1+x^2)^{3/2}}$$

$$18 \int \frac{dx}{(4-x^2)^2}$$

$$19 \int \frac{dx}{(4-x^2)^3}$$

$$20 \int \frac{dx}{(4x^2+9)^3}$$

$$21 \int \sqrt{9+16x^2} dx$$

$$22 \int (9+16x^2)^{3/2} dx$$

SECTION 9-5 (page 473)

$$\begin{aligned}
 1 & -(1/x)\sqrt{1-x^2} - \sin^{-1}x + C \quad 3 \ln|x + \sqrt{x^2-1}| - (1/x)\sqrt{x^2-1} + C \\
 5 & \frac{1}{80}[(9+4x^2)^{5/2} - 15(9+4x^2)^{3/2}] + C = ((2x^2-3)/40)(9+4x^2)^{3/2} + C \\
 7 & \sqrt{1-4x^2} - \ln|(1+\sqrt{1-4x^2})/2x| + C \quad 9 \frac{1}{2} \ln|2x + \sqrt{9+4x^2}| + C \quad 11 \frac{25}{2} \sin^{-1}(x/5) - (x/2)\sqrt{25-x^2} + C \\
 13 & (x/2)\sqrt{x^2+1} - \frac{1}{2} \ln|x + \sqrt{1+x^2}| + C \quad 15 \frac{1}{18}x\sqrt{4+9x^2} - \frac{2}{27} \ln|3x + \sqrt{4+9x^2}| + C \quad 17 x/\sqrt{1+x^2} + C \\
 19 & \frac{1}{236}(16x/(4-x^2)^2 + 6x/(4-x^2) + 3 \ln|(2+x)/\sqrt{4-x^2}|) + C = (20x-3x^3)/128(4-x^2)^2 + \\
 & \frac{3}{236} \ln|(2+x)/(2-x)| + C
 \end{aligned}$$

Answers to Odd-Numbered Problems

A-45

$$\begin{aligned}
 21 & \frac{1}{2}x\sqrt{9+16x^2} + \frac{9}{8} \ln|4x + \sqrt{9+16x^2}| + C \quad 23 \sqrt{x^2-25} - 5 \sec^{-1}|x/5| + C \\
 25 & \frac{1}{8}x(2x^2+1)\sqrt{x^2-1} - \frac{1}{8} \ln|x + \sqrt{x^2-1}| + C \quad 27 -x/\sqrt{4x^2-1} + C \\
 29 & -(1/x)\sqrt{x^2-5} + \ln|x + \sqrt{x^2-5}| + C \quad 31 \sinh^{-1}(x/5) + C \quad 33 \cosh^{-1}(x/2) - (1/x)\sqrt{x^2-4} + C \\
 35 & \frac{1}{8}[x(1+2x^2)\sqrt{1+x^2} - \sinh^{-1}x] + C \quad 37 (\pi/32)[18\sqrt{5} - \ln(2+\sqrt{5})] \approx 3.8097 \\
 39 & \sqrt{5} - \sqrt{2} + \ln[(2+2\sqrt{2})/(1+\sqrt{5})] \approx 1.222016 \quad 43 2\pi[\sqrt{2} + \ln(1+\sqrt{2})] \approx 14.4236 \quad 47 \$6\frac{2}{3} \text{ million}
 \end{aligned}$$