#22 $y' = -2 + t - y \,, \quad y(0) = y_0$

$y' + y = t - 2$ Multiply both sides by $\mu(t)$, the integrating factor TBD.

$$\underbrace{\mu y' + \mu y} = \mu (t-2)$$

$$(\mu y)' = \mu y' + \mu' y \implies \text{Require } \mu' = \mu \implies \underline{\mu(t) = e^t}$$

Now use μ: $\quad e^t y' + e^t y = e^t (t-2)$

$$(e^t y)' = e^t (t-2)$$

Integrate: $\quad e^t y = \int e^t (t-2)\,dt = e^t (t-2) - \int e^t\,dt$

$u = t-2 \quad dv = e^t\,dt$

$du = dt \quad v = e^t$

$$\implies e^t y = e^t (t-2) - e^t + c \implies y = t - 2 - 1 + ce^{-t}$$

$$\implies \boxed{y(t) = t - 3 + ce^{-t}}$$

#2 $y' - 2y = t^2 e^{2t}$

$$\mu y' - 2\mu y = \mu(t^2 e^{2t})$$

$$\frac{\mu'}{\mu} = -2$$

$$\ln \mu = -2t$$

$$\mu = e^{-2t}$$

$$(\mu y)' = \mu y' + \mu' y \Longrightarrow \text{Need } \mu' = -2\mu$$

$$\mu(t) = e^{-2t}$$

$$e^{-2t} y' - 2e^{-2t} y = e^{-2t}(t^2 e^{2t})$$

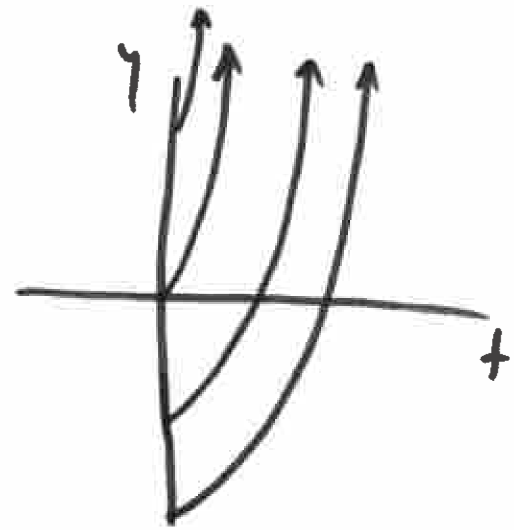
$$(e^{-2t} y)' = t^2$$

Integrate: $e^{-2t} y = \frac{1}{3}t^3 + c$

$$\boxed{y(t) = e^{2t}\left(\frac{1}{3}t^3 + c\right)}$$

$c = y_0$

$$y(t) = e^{2t}\left(\frac{1}{3}t^3 + y_0\right)$$



#6 $ty' - y = t^2 e^{-t}, \quad t > 0$

$$\Longrightarrow \; y' - \frac{1}{t} y = t e^{-t}$$

$$\underbrace{\mu y' - \frac{1}{t}\mu y}_{(\mu y)' = \mu y' + \mu' y} = \mu t e^{-t}$$

$$(\mu y)' = \mu y' + \mu' y \Longrightarrow \mu' = -\frac{1}{t}\mu$$

$$\underline{\underline{\mu(t) = \frac{1}{t}}}$$

$$\begin{cases} \frac{\mu'}{\mu} = -\frac{1}{t} \\ \ln|\mu| = -\ln t \\ \qquad = +\ln\left(\frac{1}{t}\right) \end{cases}$$

$$\underbrace{\frac{1}{t} y' - \frac{1}{t^2} y}_{\left(\frac{1}{t} y\right)'} = \frac{1}{t}(t e^{-t})$$

$$\left(\frac{1}{t} y\right)' = e^{-t} \Longrightarrow \frac{1}{t} y = -e^{-t} + c$$

$$\Longrightarrow \boxed{y(t) = -t e^{-t} + ct}$$

#11 $y' + \frac{2}{t} y = \frac{\cos t}{t^2}, \quad y(\pi) = 0, \quad t > 0$

$$\underbrace{\mu y' + \frac{2}{t}\mu y} = \mu\left(\frac{\cos t}{t^2}\right)$$

$$(\mu y)' = \mu y' + \mu' y \Longrightarrow \mu' = \frac{2}{t}\mu$$

$$\left\{ \begin{aligned} \frac{\mu'}{\mu} &= \frac{2}{t} \\ \ln \mu &= 2 \ln t \\ &= \ln(t^2) \end{aligned} \right.$$

$$\underline{\underline{\mu(t) = t^2}}$$

$$\underbrace{t^2 y' + 2t y} = t^2\left(\frac{\cos t}{t^2}\right) = \cos t$$

$$\underline{\underline{(t^2 y)' = \cos t}} \Longrightarrow t^2 y = \sin t + c$$

$$\boxed{y(t) = \frac{\sin t + c}{t^2} \quad \text{General } \underline{\text{sol'n}}}$$

$$y(\pi) = 0 \Longrightarrow 0 = \frac{c}{\pi^2} \Longrightarrow c = 0$$

$$\boxed{y(t) = \frac{\sin t}{t^2} \quad \text{sol'n of IVP}}$$