

1. Consider the homogeneous differential equation

$$(2 + x^2) y'' - xy' + 4y = 0 .$$

- (a) Determine the recurrence relation for the coefficients of a power series solution centered at  $x_0 = 0$ .
- (b) Find the first four nonzero terms for each of two independent solutions  $y_1(x)$  and  $y_2(x)$  (unless the series terminates sooner).

2. Consider the homogeneous differential equation

$$(1 + x^2) y'' - 4xy' + 6y = 0 .$$

- (a) Determine the recurrence relation for the coefficients of a power series solution centered at  $x_0 = 0$ .
- (b) Find the first four nonzero terms for each of two independent solutions  $y_1(x)$  and  $y_2(x)$  (unless the series terminates sooner).

3. Consider the first-order linear system

$$\begin{cases} x' &= -3x + 4y \\ y' &= -2x + 3y \end{cases}$$

for trajectories  $(x(t), y(t))$  in the  $xy$ -plane.

- (a) Write this system in matrix form and determine its general solution.
- (b) Sketch the phase portrait, i.e., plot some representative trajectories in the  $xy$ -plane.
- (c) How do solutions behave as  $t \rightarrow \infty$ ?

4. Consider the first-order linear system

$$\begin{cases} x' &= -5x - 10y \\ y' &= x + y \end{cases}$$

for trajectories  $(x(t), y(t))$  in the  $xy$ -plane.

- (a) Write this system in matrix form and determine its general solution.
- (b) Sketch the phase portrait, i.e., plot some representative trajectories in the  $xy$ -plane.
- (c) How do solutions behave as  $t \rightarrow \infty$ ?