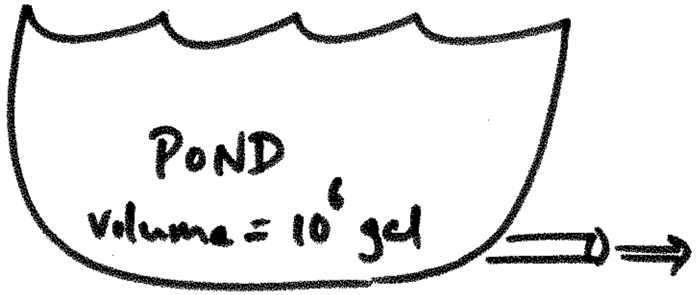
#14

$.01$ g/gal @ 300 gal/hr $= 3$ g/hr $\underline{\underline{\text{in}}}$



out @
300 gal/hr

$Q(t) =$ amount of chemical in pond at time t (grams)

$$\frac{dQ}{dt} = \underset{\text{in}}{\text{rate}} - \underset{\text{out}}{\text{rate}} = \left(3 \ \text{g/hr}\right) - \left(300 \ \text{gal/hr} \times \frac{Q(t) \text{ grams}}{10^6 \text{ gal}}\right)$$

$$= 3 \ \text{g/hr} - \frac{3}{10^4} Q$$

$\Longrightarrow$ $\underline{\text{ODE model}}$: $Q' = 3 - .0003\,Q = 3(1 - .0001\,Q)$

$Q(0) = 0$ since initially free of chemical

Alternative form (easier to integrate): $\underline{\underline{Q' = \frac{3}{10^4}(10^4 - Q)}}$.

Solution: $Q' = \frac{3}{10^4}(10^4 - Q)$, $Q(0) = 0$

$$\Longrightarrow \frac{Q'}{10^4 - Q} = \frac{3}{10^4}$$

$$\Longrightarrow -\ln|10^4 - Q| = \frac{3t}{10^4} + c \quad \text{after integrating}$$

$$\Longrightarrow \ln|10^4 - Q| = -\frac{3t}{10^4} + c$$

$$\Longrightarrow 10^4 - Q = c\,e^{-3t/10^4} \quad \text{after exponentiation}$$

$$\Longrightarrow Q(t) = 10^4 - c\,e^{-3t/10^4} \quad \text{(General solution of ODE)}$$

$$Q(0) = 0 \Longrightarrow c = 10^4, \quad \underline{\underline{Q(t) = 10^4\left(1 - e^{-3t/10^4}\right)}}$$

One year later, $t = 365 \times 24 = 8760$ hours and

$$Q(t) = 10^4\left(1 - e^{-.0003 \times 8760}\right) = 9277.8 \text{ grams}.$$

Now clean water goes into the pond, so the new model is

$$Q' = -.0003\,Q, \quad Q(0) = 9277.8.$$

$$\Longrightarrow Q(t) = 9277.8\,e^{-.0003t}.$$

One more year goes by, then $Q(t) = 9277.8\,e^{-.0003 \times 8760}$

$= 670$ grams.

To reduce $Q(t)$ to 10 grams:

$$10 = 9277.8\,e^{-.0003t} \Longrightarrow e^{-.0003t} = \frac{10}{9277.8}$$

$$\Longrightarrow -.0003t = \ln\left(\frac{10}{9277.8}\right) \Longrightarrow t = 22{,}775.98 \text{ hrs}$$

$\Longrightarrow$ 2.6 years $\Longrightarrow$ 3.6 years since start.