A Fourier series example

Math 334

9/3/2019

For our first concrete example of a Fourier series, we calculated the coefficients of the Fourier sine series

$$\sum_n A_n \sin(n\pi x)$$

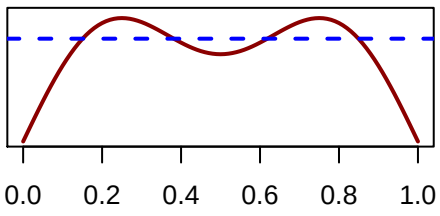
corresponding to the function $\varphi(x) \equiv 1$ on the interval $(0,1)$; in particular, we found that

$$A_n \;=\; 2\int_0^1 \varphi(x)\sin(n\pi x)\,dx \;=\; 2\int_0^1 \sin(n\pi x)\,dx \;=\; \begin{cases} 0\,, & n \quad \text{even} \\ \dfrac{4}{n\pi}\,, & n \quad \text{odd}\,. \end{cases}$$

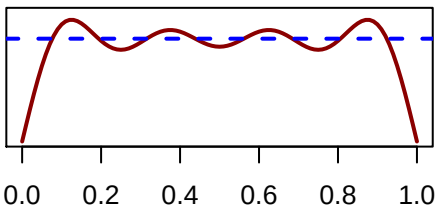
The Fourier sine series for this function φ is therefore

$$\sum_{\text{odd } n} \frac{4}{n\pi}\sin(n\pi x) \tag{1}$$

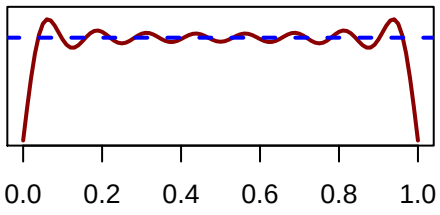
It is instructive to see how the partial sums (plotted in solid red) of (1) behave and how they compare to the function φ (plotted as a dashed blue line) :



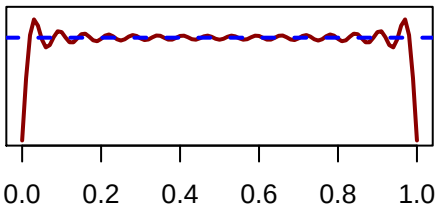
2 terms



4 terms



8 terms



16 terms