

# Home Work 1

## Question (1) Math Review (25 points)

**Question (1\_1):**  $f(x, y, z) = 3x^2 + \sin(y)z$ , I need to find partial derivatives for each  $x, y$ , and  $z$ :

Solution:

With respect to  $x$ :  $\frac{\partial f(x, y, z)}{\partial x} = 6x$ , everything else is constant as they don't have  $x$  in their representation.

With respect to  $y$ :  $\frac{\partial f(x, y, z)}{\partial y} = z \cos(y)$ , the first term derivative is zero in this case.

With respect to  $z$ :  $\frac{\partial f(x, y, z)}{\partial z} = \sin(y)$ , the first term derivative is also zero in this case.

**Question (1\_2):** In this question, I will need to find the  $\nabla f(x, y, z)$ :

Using the results from the previous question,  $\nabla f(x, y, z) = [6x, z \cos(y), \sin(y)]$

**Question (1\_3):** Here we are replicating 1 and 2 but  $f(x) = 3x_1^2 + \sin(x_2)x_3$ :

$\nabla f(x_1, x_2, x_3) = [6x_1, x_3 \cos(x_2), \sin(x_2)]$

**Question (1\_4\_A):** In this part, I will need to get the derivative for  $\|x\|_2^2$ :

$$\|x\|_2^2 = \sum_{i=1}^n x_i^2 = x_1^2 + x_2^2 + \dots + x_n^2$$

In this case:  $\frac{\partial \|x\|_2^2}{\partial x} = \frac{\partial x_1^2 + x_2^2 + \dots + x_n^2}{\partial x_1} + \frac{\partial x_1^2 + x_2^2 + \dots + x_n^2}{\partial x_2} + \dots + \frac{\partial x_1^2 + x_2^2 + \dots + x_n^2}{\partial x_n} = [2x_1, 2x_2, \dots, 2x_n]$ , This represents the partial derivative for each  $x$ .

**Question (1\_4\_B):**In this part, I will need to get the derivative for  $\|x\|_2$ :

$$\|x\|_2 = \sqrt{(\sum_{i=1}^n x_i^2)} = \sqrt{(x_1^2)} + \sqrt{(x_2^2)} + \dots + \sqrt{(x_n^2)}$$

To get this derivative,  $\frac{\partial \|x\|_2}{\partial x}$ , I will be using the chain rule assuming that a new function  $h(z) = \sqrt{z}$ , where  $z(x) = \sum_{i=1}^n x_i^2$ .

$$\frac{\partial h(z)}{\partial x} = \frac{\partial h(z)}{\partial z} * \frac{\partial z(x)}{\partial x}$$

$$\frac{\partial h(z)}{\partial x} = \frac{1}{2\sqrt{(\sum_{i=1}^n x_i^2)}} * \sum_{i=1}^n 2x_i \frac{\partial h(z)}{\partial x} = \frac{x}{\|x\|_2}$$

**Question (1\_4\_C):**In this part, I will need to get the derivative for  $\|x\|_1$ :

$$\|x\|_1 = \sum_{i=1}^n |x_i| = |x_1| + |x_2| + \dots + |x_n|$$

$\frac{\partial \|x\|_1}{\partial x}$ : depends on the value of  $X_i$ . If positive, it would be 1, if negative, it would be -1, and undefined when  $x$  equals zero. To represent the derivative of this function, the sign function can be used in this case:

$$\frac{\partial \|x\|_1}{\partial x} = [\text{sign}(x_1), \text{sign}(x_2), \dots, \text{sign}(x_n)]^T$$

**Question (1\_4\_D):**In this part, I will need to get the derivative for  $\|x\|_\infty$ :

$\|x\|_\infty = \max |X_i|$ , This entails many cases under the hood and can be represented using the sign function:

$\frac{\partial \|x\|_\infty}{\partial x_i} = \text{sign}(x_i)$  Accordingly,  $\frac{\partial \|x\|_\infty}{\partial x} = [0, \dots, \text{sign}(x_i), \dots, 0]^T$ . Those components that are not achieving the maximum, the derivative is 0.

**Question (1\_5):** In this part, I will need to get the derivative of  $f(x) = e^{\frac{-1}{2}\|x\|_2^2}$ .

For this function, the chain rule will be used assuming a new function  $h(z) = e^z$ , where  $z = \frac{-1}{2}\|x\|_2^2$ . In this case, we would be looking for:

$$\frac{\partial h(z)}{\partial x} = \frac{\partial h(z)}{\partial z} * \frac{\partial z(x)}{\partial x}$$

$$\frac{\partial h(z)}{\partial x} = e^z * -x_i$$

$$\frac{\partial h(z)}{\partial x} = -x_i * e^{\frac{-1}{2}\|x\|_2^2} = [-x_1 e^{\frac{-1}{2}\|x_1\|_2^2}, -x_2 e^{\frac{-1}{2}\|x_2\|_2^2}, \dots, -x_n e^{\frac{-1}{2}\|x_n\|_2^2}]$$

**Question (1\_6):** In this part, I will need to get the two components of  $f(A,x)$ :

$$\text{In this case } A \text{ is a } 2 \times 3 \text{ matrix} = A_{2 \times 3} = \begin{bmatrix} A_{1,1} & A_{1,2} & A_{1,3} \\ A_{2,1} & A_{2,2} & A_{2,3} \end{bmatrix}$$

$$\text{And } x_{1 \times 3} = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$$

$$f(A,x) = Ax = \begin{bmatrix} A_{1,1} & A_{1,2} & A_{1,3} \\ A_{2,1} & A_{2,2} & A_{2,3} \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$f(A,x) = Ax = \begin{bmatrix} A_{1,1} * x_1 + A_{1,2} * x_2 + A_{1,3} * x_3 \\ A_{2,1} * x_1 + A_{2,2} * x_2 + A_{2,3} * x_3 \end{bmatrix}$$

**Question (1\_7):** In this part, I would need to get  $\frac{\partial f(A,x)_1}{\partial x}$  and  $\frac{\partial f(A,x)_2}{\partial x}$ .

$$\frac{\partial f(A,x)_1}{\partial x} = \begin{bmatrix} A_{1,1} \\ A_{1,2} \\ A_{1,3} \end{bmatrix}$$

$$\frac{\partial f(A,x)_2}{\partial x} = \begin{bmatrix} A_{2,1} \\ A_{2,2} \\ A_{2,3} \end{bmatrix}$$

**Question (1\_8):** In this part, I would need to get

$\frac{\partial f(A,x)}{\partial x}$ : Using results from the previous question, The result will be:

$$\frac{\partial f(A,x)}{\partial x} = \begin{bmatrix} A_{1,1} & A_{2,1} \\ A_{1,2} & A_{2,2} \\ A_{1,3} & A_{2,3} \end{bmatrix}$$

**Question (1\_9):** In this part, I would need to get the derivative of

$$\mathbb{E}[f(x)].$$

Given that:  $\mathbb{E}[f(x)] = \sum \Pr(X = x)f(x)$ , then:

$$\frac{\partial \mathbb{E}[f(x)]}{\partial x} = \frac{\partial}{\partial x} \sum \Pr(X = x)f(x), \text{ note that } \Pr(X = x) \text{ is constant with respect to } x.$$

$$\frac{\partial \mathbb{E}[f(x)]}{\partial x} = \sum \Pr(X = x) \frac{\partial}{\partial x} f(x).$$

Question (2): Linear Algebra

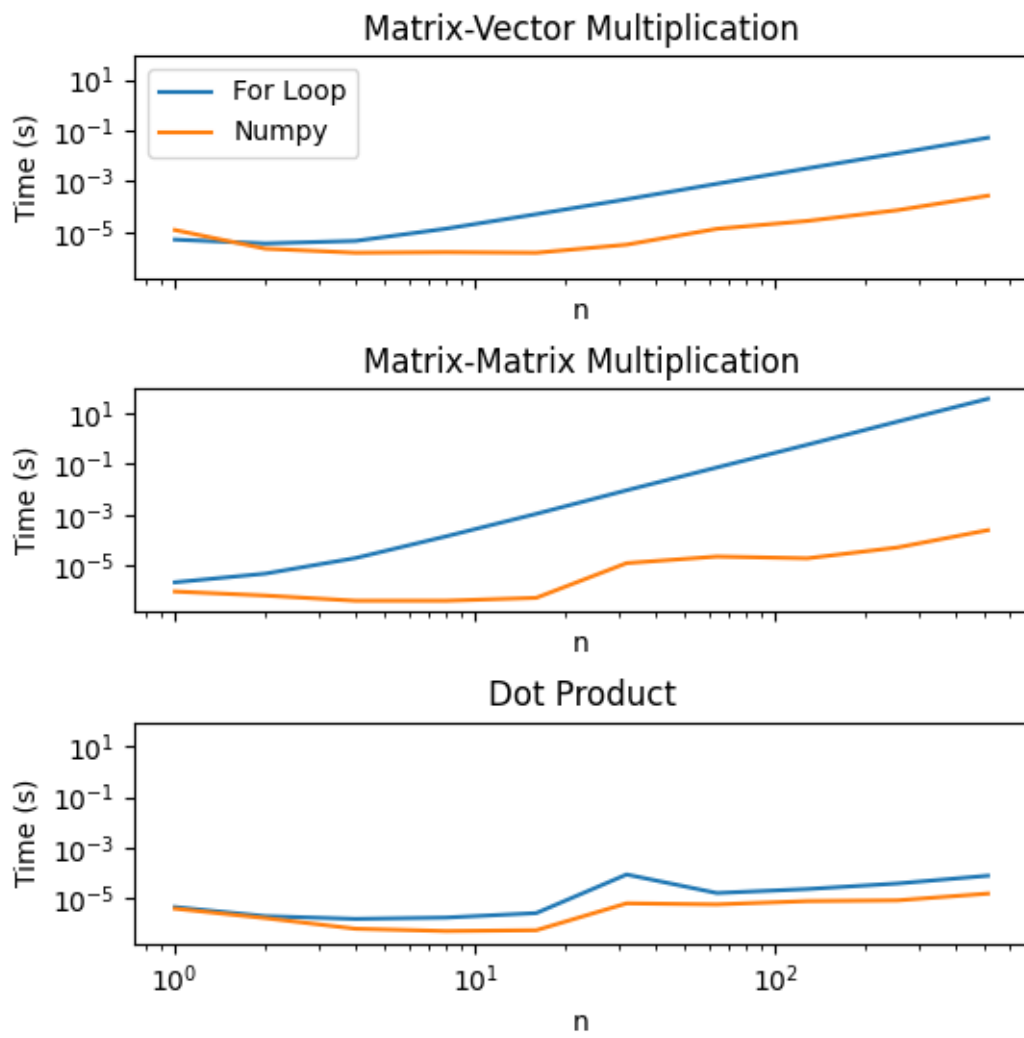


Figure 1: Performance comparison