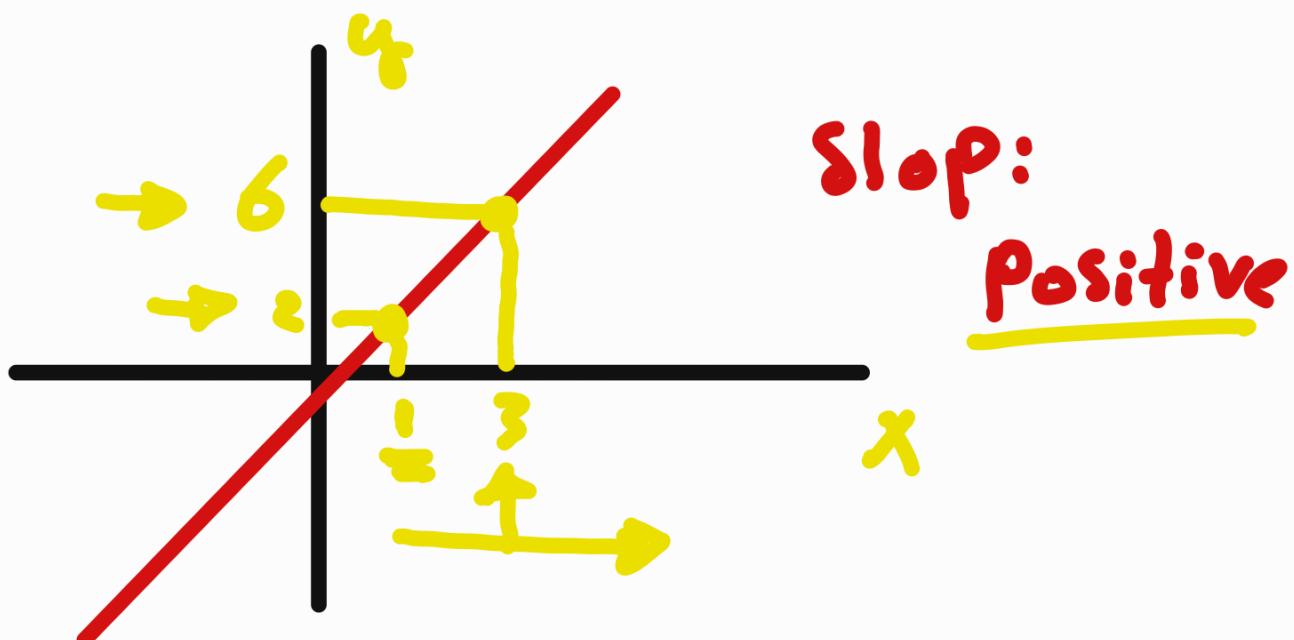


Multivariate Calculus for Data Science

◀ Derivative ▶



Slope: $\frac{P}{N}$

Rise over Run

Rise → مقدار التخوم

Run → مقدار التخوم

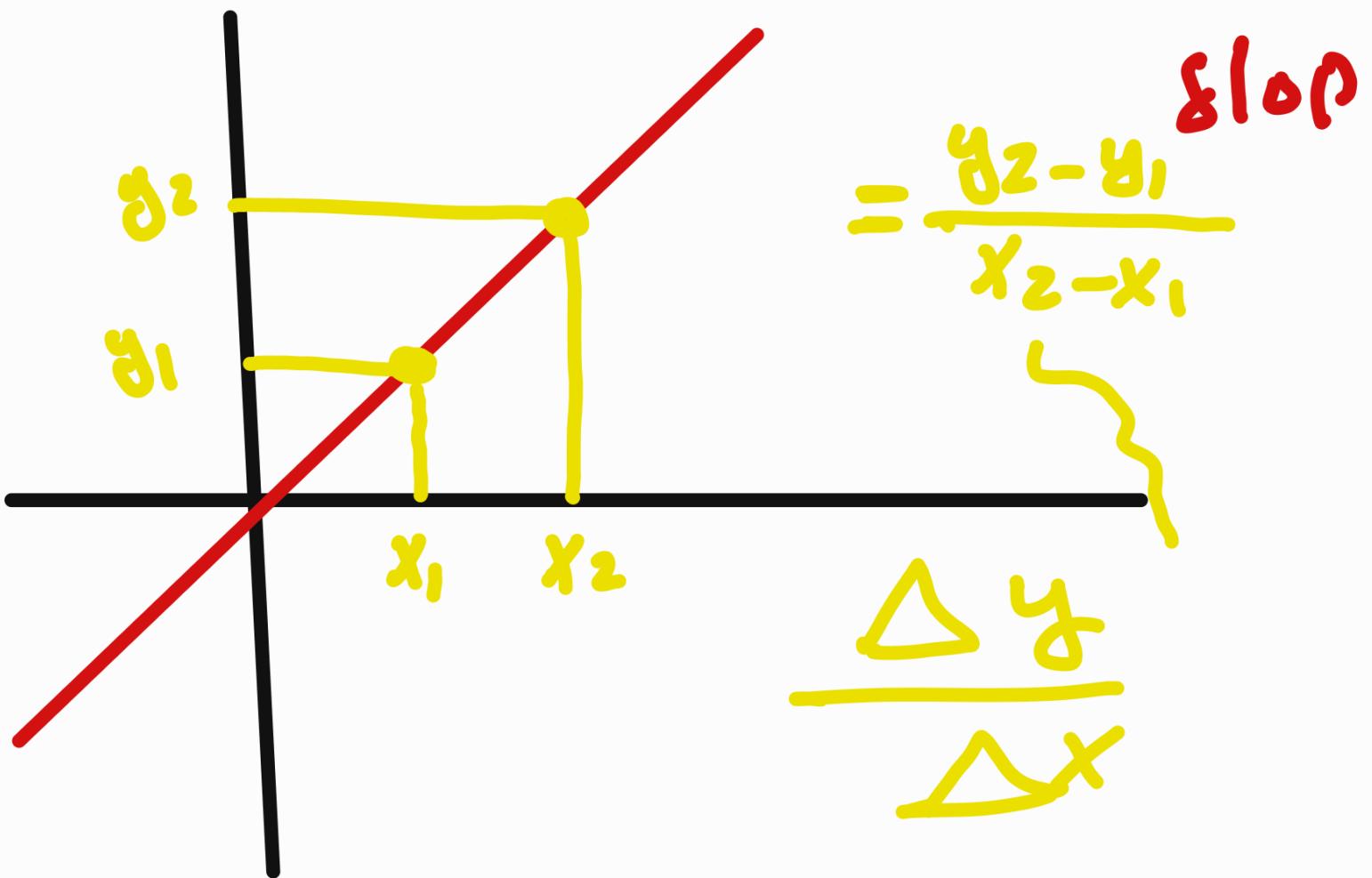
$$\frac{\Delta y}{\Delta x}$$

$$\frac{\Delta}{\Delta} \quad y_2 - y_1$$

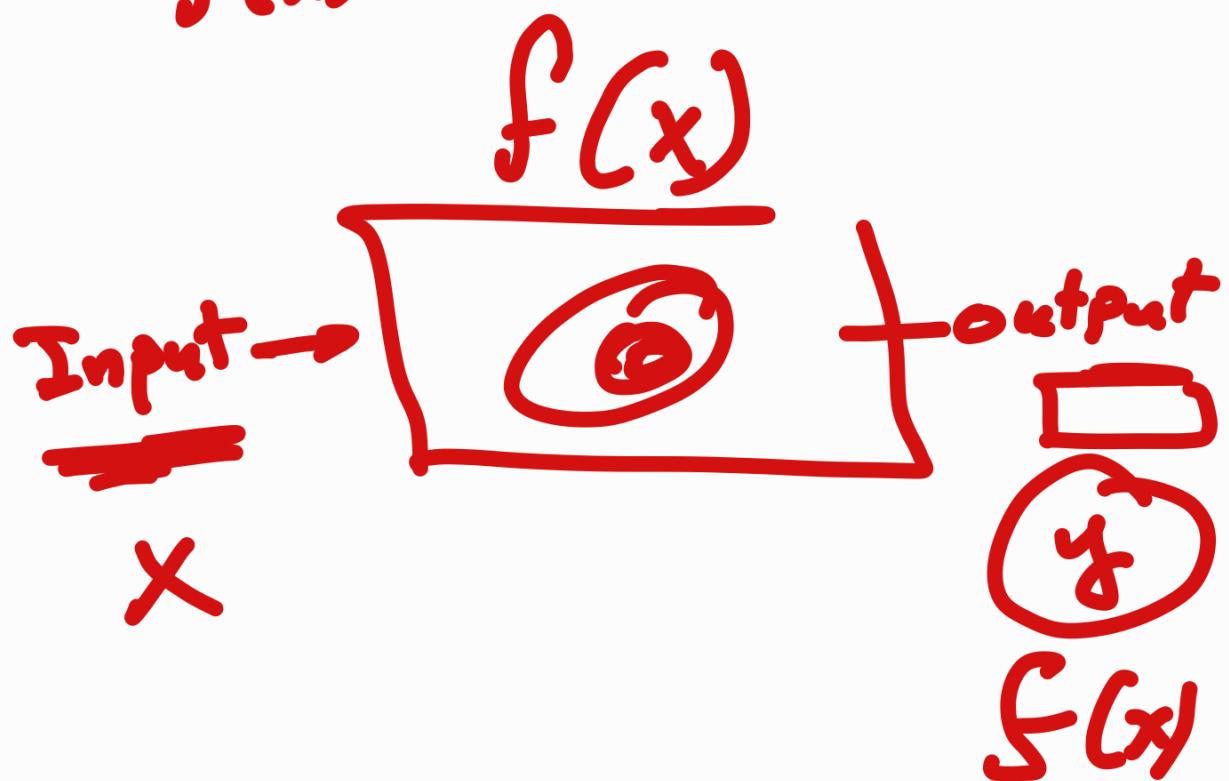
$$\begin{matrix} \sum y_2 & \dots & \dots \\ \sum y_1 & \dots & \dots \\ x_1 & \dots & x_2 \end{matrix}$$

Gradient $\equiv \frac{\text{Rise}}{\text{Run}} \equiv$ Slope $\equiv \frac{\Delta y}{\Delta x}$
 $\equiv \tan \theta \equiv$ Derivative \equiv
 Diff. \equiv التماضي \equiv الإخراج \equiv إخراج

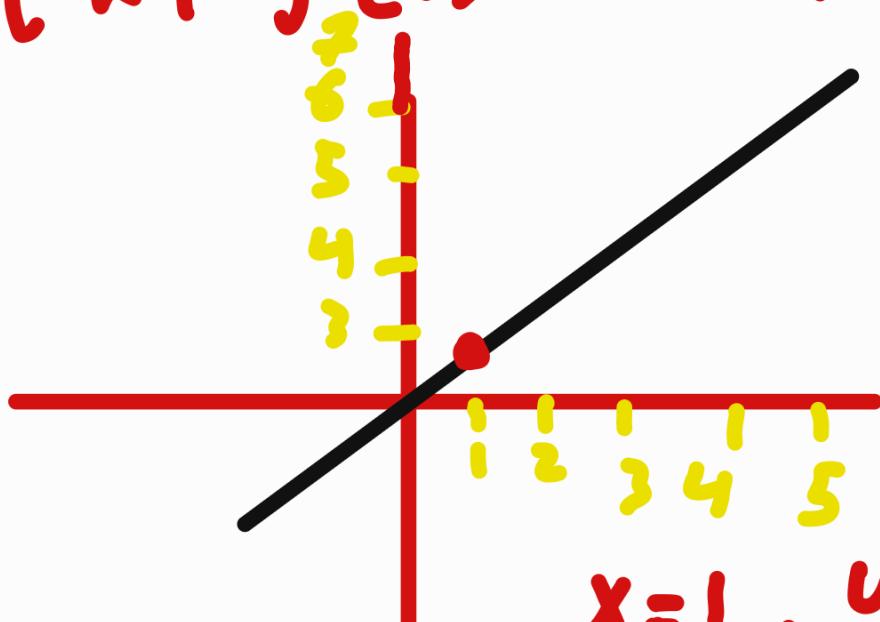




Ex: $f(x) =$



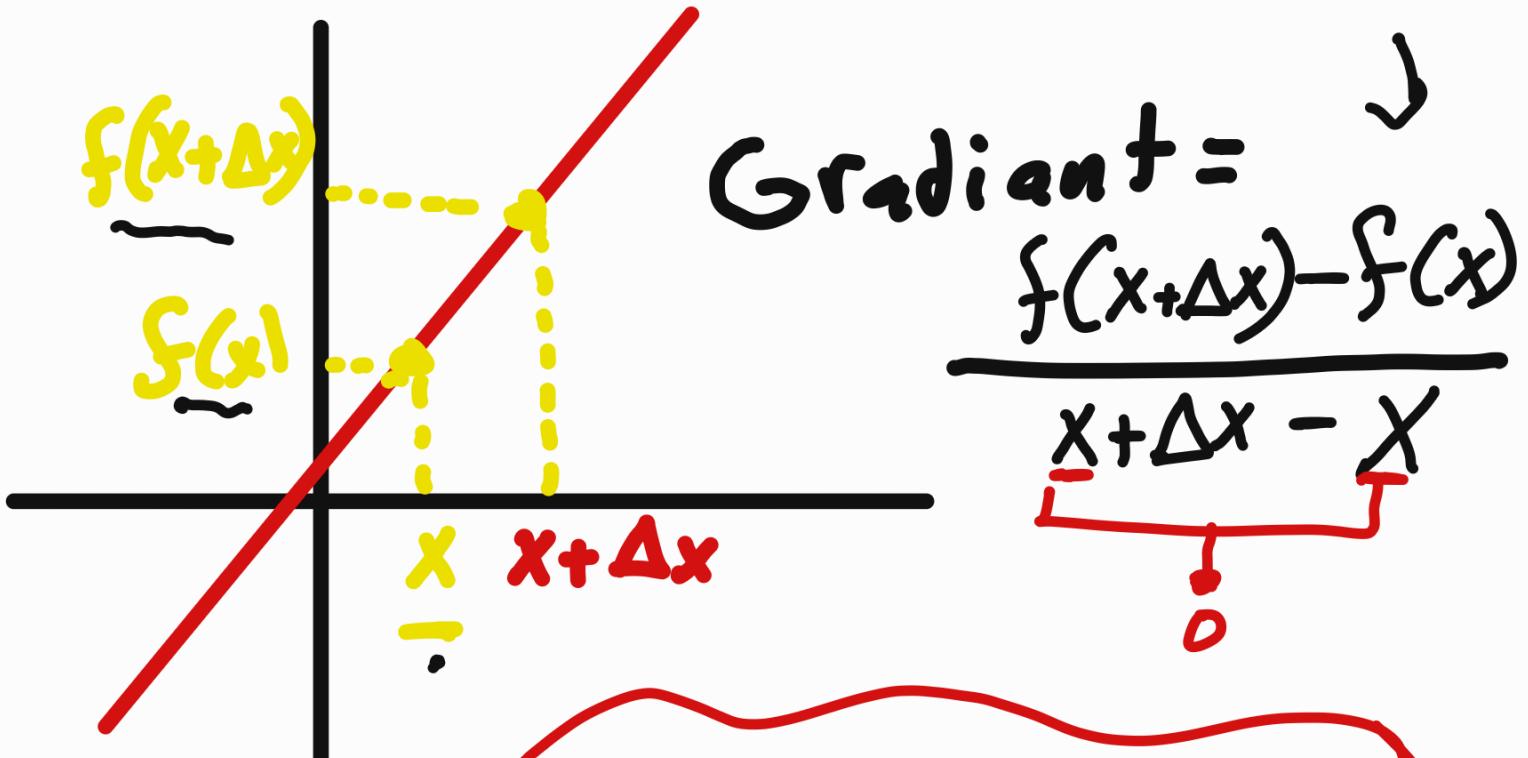
Ex: $f(x) = x + 2$



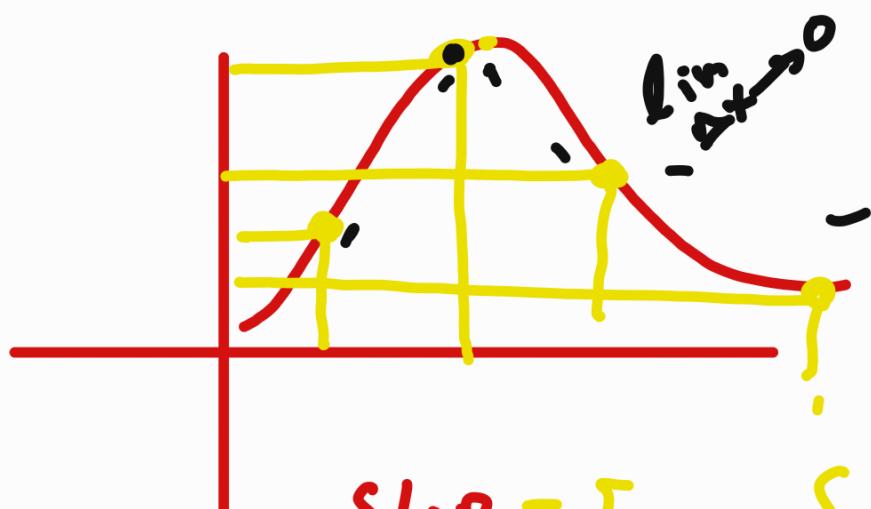
$$x=1, y=3$$

2
x = 1.0001
y = 3.001
ابد مبتدا

$$* \text{Gradient} = \frac{f(x + \Delta x) - f(x)}{(x + \Delta x) - x}$$

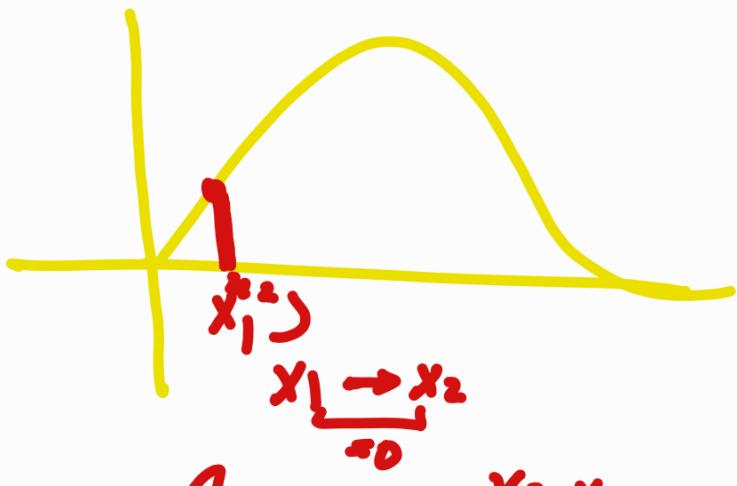


Gradient = $\frac{f(x + \Delta x) - f(x)}{\Delta x}$



Slope = 5
Slope ≠ 5
Slope 0.5

Slope ≡



$$\lim_{\Delta x \rightarrow 0} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$f(x)$
 $f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$f(x) = 3x + 2$$

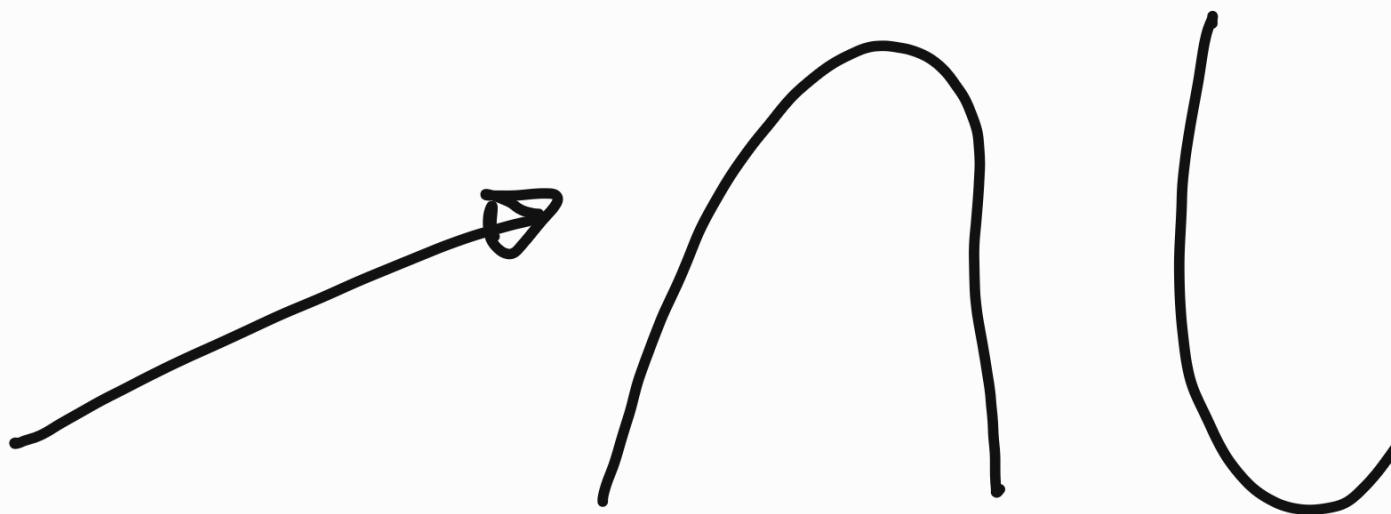
$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{3(x + \Delta x) + 2 - 3x - 2}{\Delta x}$$

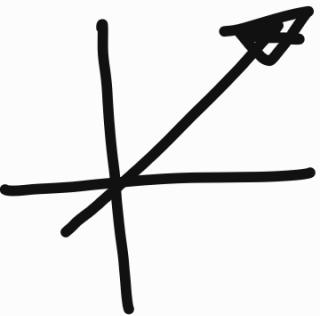
$$\frac{3x + 3\Delta x + 2 - 3x - 2}{\Delta x}$$

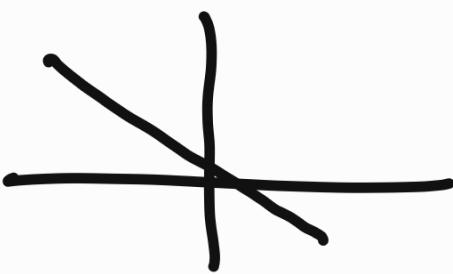
$$\frac{3\cancel{\Delta x}}{\Delta x}$$

$$\underline{f(x)}$$

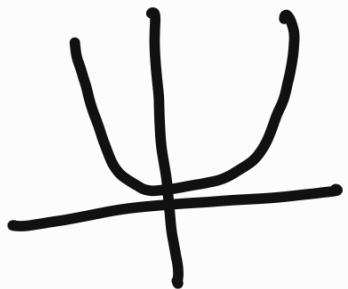
$$\underline{f'(x) = \frac{df}{dx}}$$



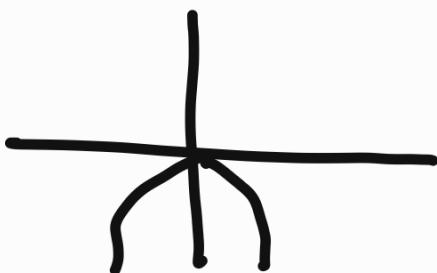
$$f(x) = x$$


$$f(x) = -x$$


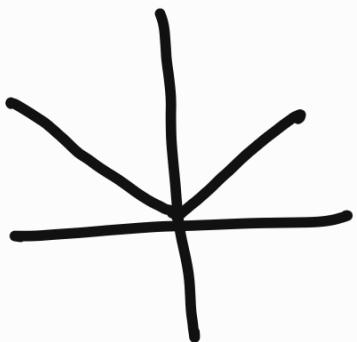
$$f(x) = x^2$$



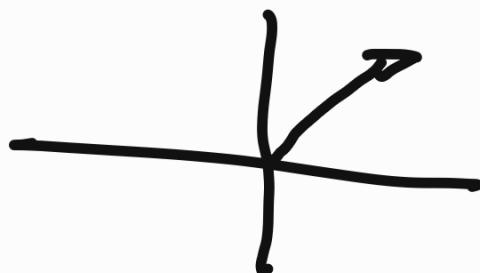
$$f(x) = -x^2$$



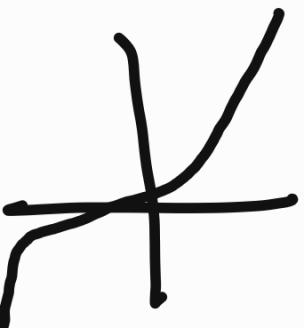
$$y = |x|$$



$$y = \sqrt{x}$$



$$x^3$$



Chain Rule of Partial Differentiation

*Chain Rule = خاصية السلسلة:

$$f(x) \rightarrow h(p(m))$$

$$\rightarrow h(p) = \frac{1}{3}p^2 + p + \frac{1}{5}$$

$$\rightarrow p(m) = e^m - 1$$

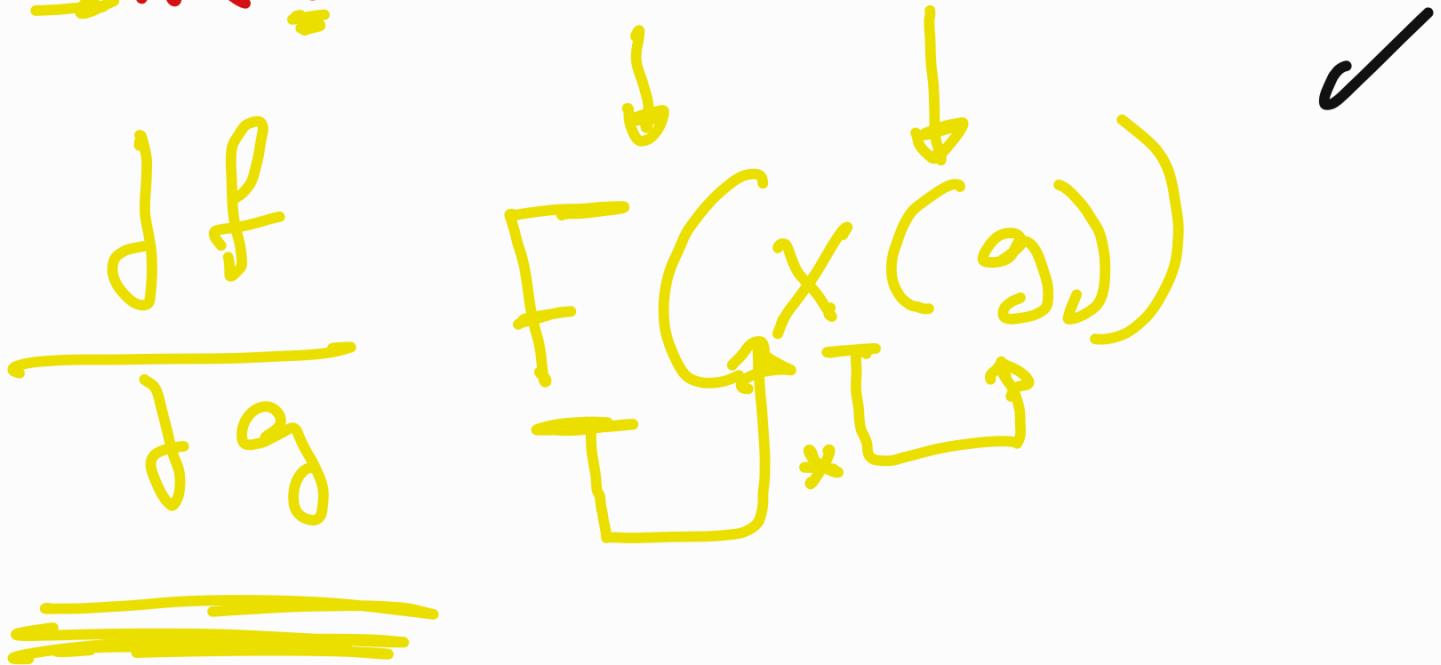
Find $\frac{dh}{dm}$ $h(m)$

$$\frac{\frac{dh}{dp}}{\frac{dp}{dm}}$$

$$\frac{dh}{dm} = \frac{dh}{dp} \times \frac{dp}{dm}$$

$$\rightarrow F(x) = \underline{5x+3} \rightarrow$$

$$\rightarrow x(g) = g+2 \rightarrow$$



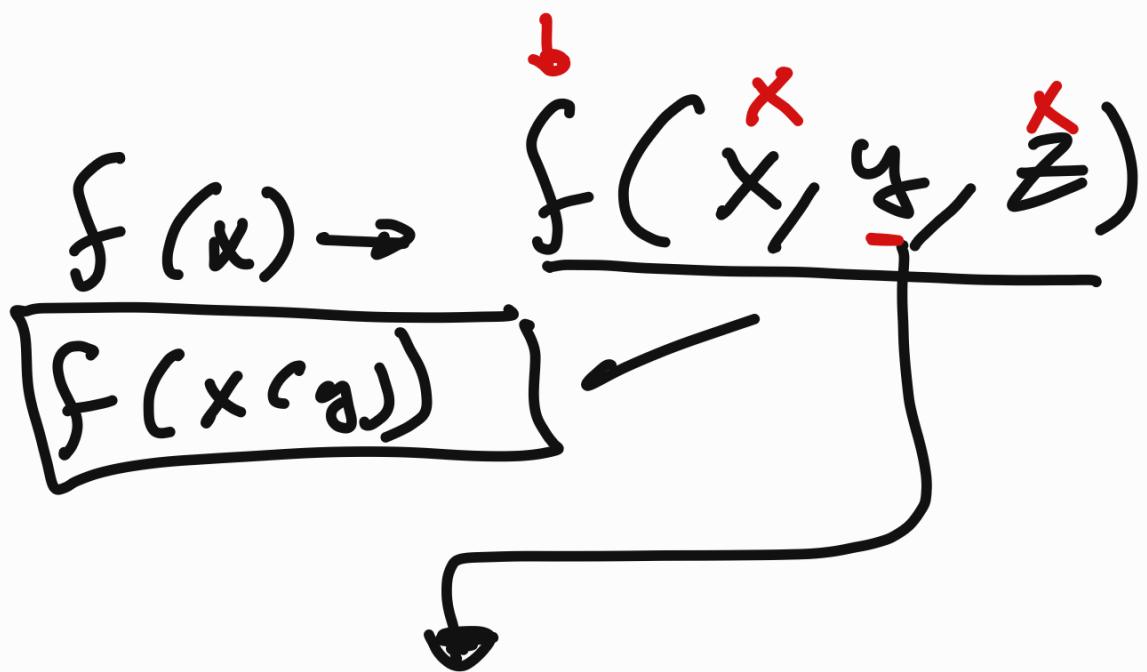
$$\frac{\delta f}{\delta g} = \frac{\delta f}{\delta x} \cdot \frac{\delta x}{\delta g}$$

\downarrow

-

$= \Sigma$

↙



$$f(x, y, z) = \underline{\sin(x)} \cdot e^{yz^2}$$

$$\frac{\partial f}{\partial x} = \cos(x) \cdot e^{yz^2}$$

$$\frac{\partial f}{\partial y} = \sin(x) \cdot e^{\underline{yz^2}} \cdot z^2$$

$$f(\underline{x}, \underline{y}, \underline{z}) = \sin(x) \cdot e^{yz^2}$$

$$\underline{x} = \underline{\delta} - 1, \quad \underline{y} = \delta^2, \quad \underline{z} = \frac{1}{\delta}$$

$$\frac{\partial f}{\partial \delta}$$

$$f(\underline{x(\delta)}, \underline{y(\delta)}, \underline{z(\delta)})$$

$$=$$

$$\frac{\partial f}{\partial \delta}$$

$$x(t) = t - 1, \quad y(t) = t^2$$

$$\underline{z(t)} = \frac{1}{t}$$

$$f(x(t) \ y(t) \ \overset{\downarrow}{z(t)}) =$$

$$\sin(x) \cdot e^{y \cdot z^2}$$

$$\begin{aligned} \frac{\partial f}{\partial t} &= \underline{\frac{\partial f}{\partial x}} \cdot \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial t} \\ &\quad + \frac{\partial f}{\partial z} \cdot \frac{\partial z}{\partial t} \end{aligned}$$