Algebra: → x y z f

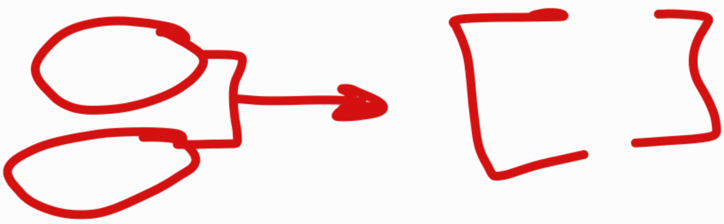
$x=1$, $y=5$, $z=00011$

linear Equation: $2x+5=25$

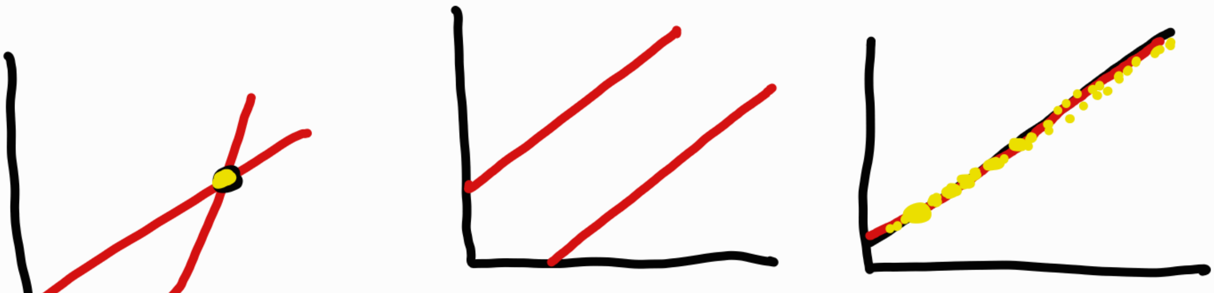
system of linear Equation

$$\begin{bmatrix} 4x+5y=20 \\ x+3y=30 \end{bmatrix} \rightarrow$$

linear Algebra



◀ linear Algebra ▶

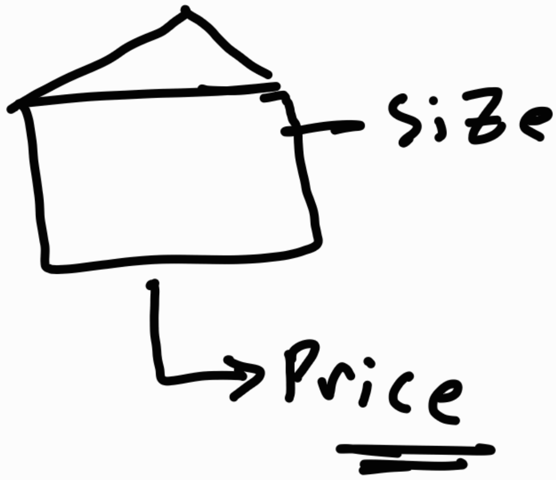


One Solution

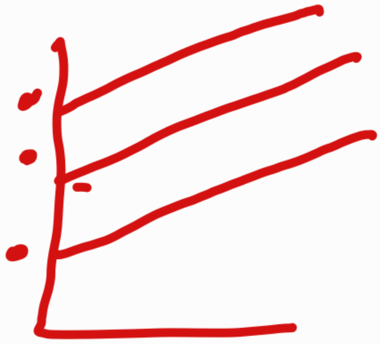
No Solution

Infinitely many Solution

$$x + y = 0$$



$$\boxed{y = Wx + b}$$



$x = 5$

Scalar: [9] [5]

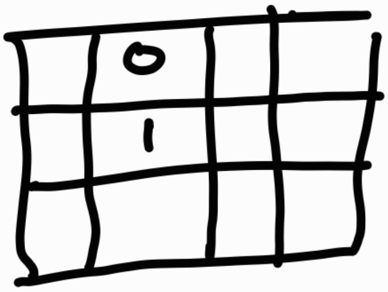
Vector → $[x_1, x_2, x_3]$

Matrix $\begin{bmatrix} x & b \\ c & d \end{bmatrix}$ 2-dimensional array

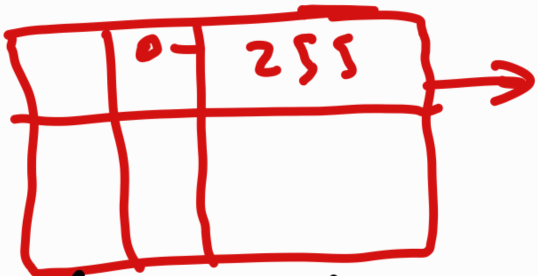
Tensor

N-dimensional array

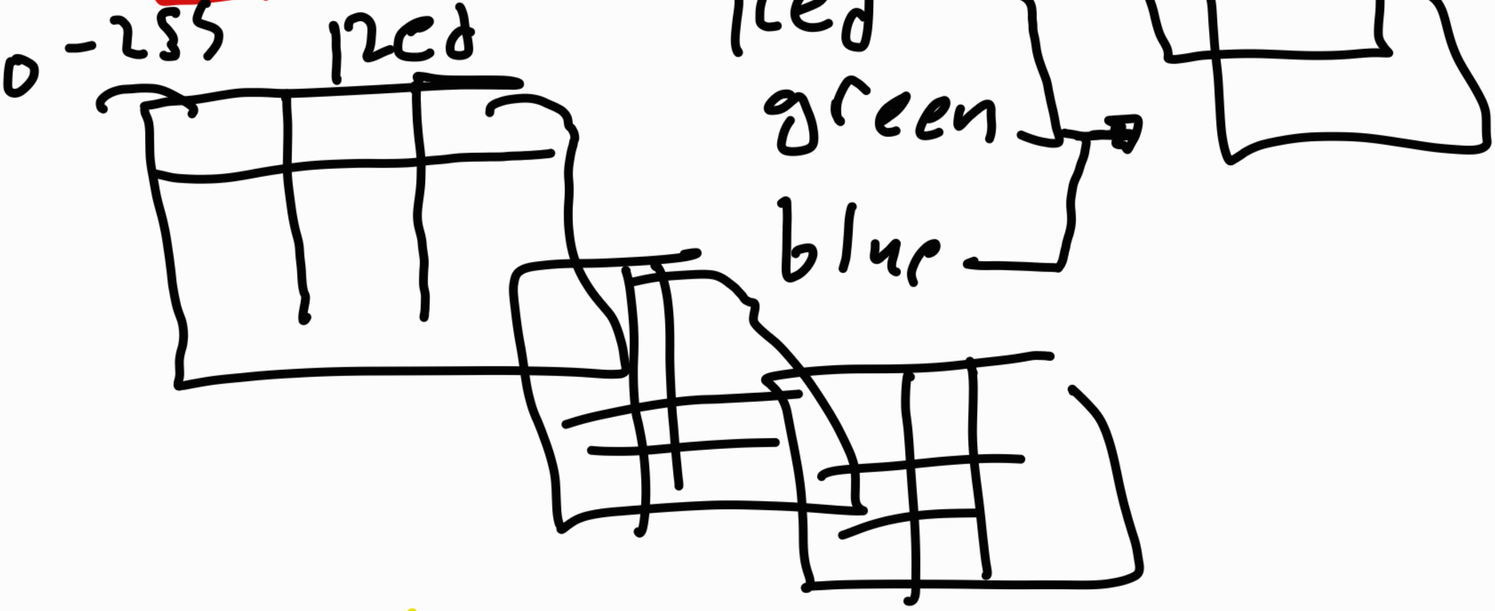
RGb

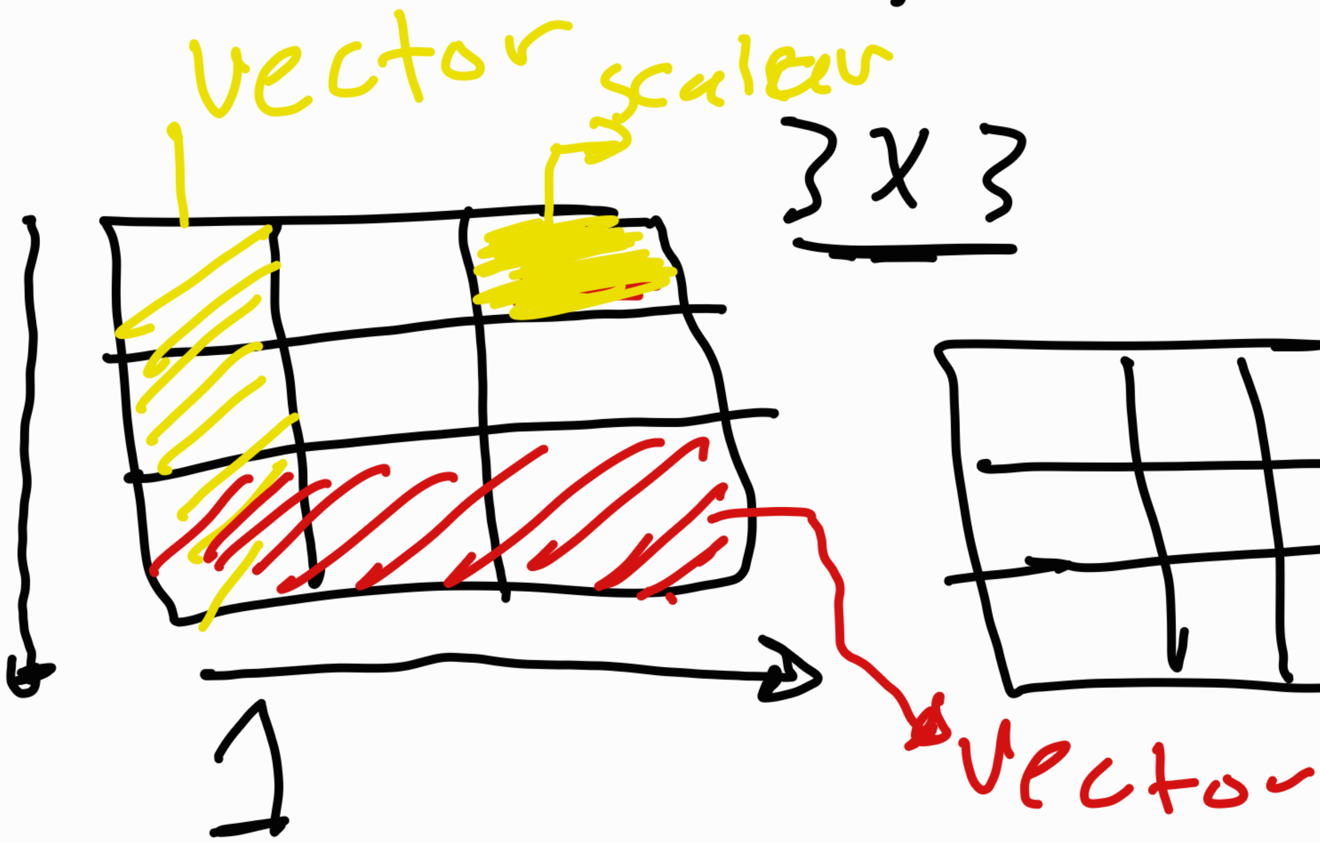


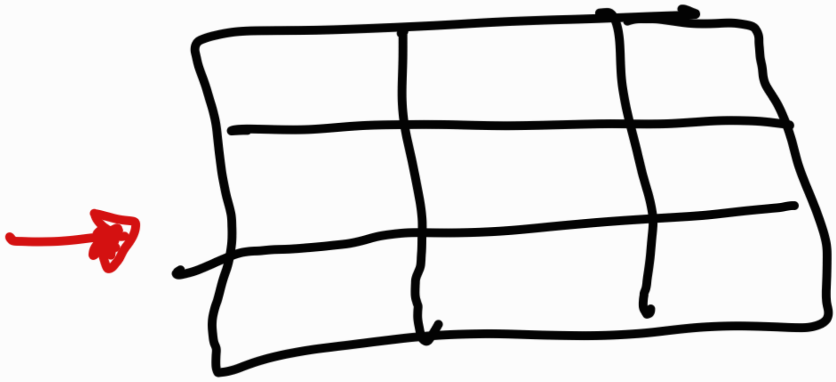
1 - black
0 - w



Gray Image







X = 5 → scalar

print(x)

5

list

$$\begin{array}{ccccc} & 0 & 1 & 2 & 3 \\ x = [& 5, & 6, & 7, & 8] \end{array} \rightarrow \text{array}$$

vector

Print(x)

print(X[0]) 5

print(X[3]) 8

X = [[1,2,3], [4,5,6],
[7,8,9]]

List

[ ] [ ] [ ]

$$X = \begin{array}{cc} \begin{array}{ccc} 0 & 1 & 2 \end{array} & \\ \begin{bmatrix} \boxed{1}, & 2, & 3, \\ 4, & 5, & 6, \\ 7, & 8, & 9 \end{bmatrix} & \begin{array}{c} 0 \rightarrow \\ 1 \\ 2 \end{array} \end{array}$$

$$X[0][0]$$

$$= 1$$

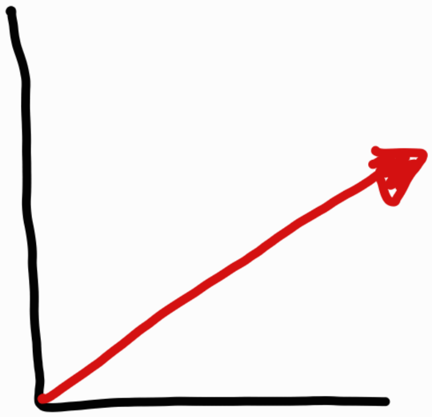
$$X = [[[1,2,3],[4,5,6]],[[7,8,9],[10,11,12]]]$$

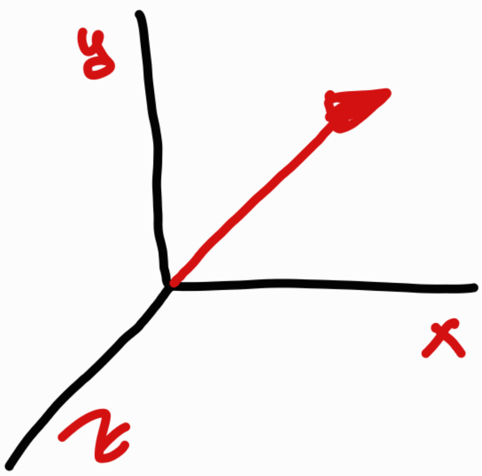
$$X[0][0][1]$$

$$0 \quad \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 1 & 2 & 3 \\ 1 & \boxed{4} & 5 & 6 \end{array}$$

$$1 \quad \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 7 & 8 & 9 \\ 1 & 10 & \boxed{11} & 12 \end{array}$$

Vector



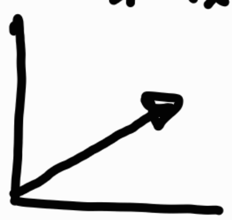


one-dimension:

$$X=[x_1]$$

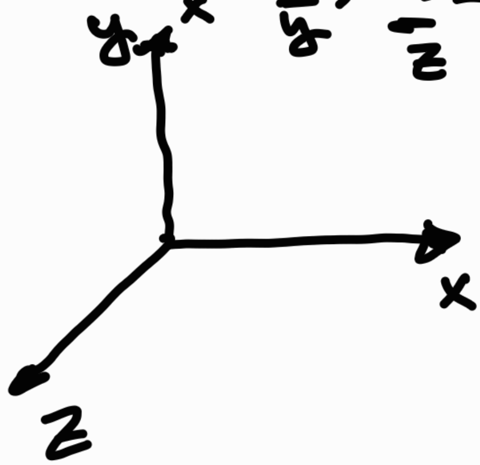
Two-dimension:

$$X=[\underbrace{x_1}_{x\text{-axis}}, \underbrace{x_2}_{y\text{-axis}}]$$



Three-dimension:

$$X=[\underbrace{x_1}_{x}, \underbrace{x_2}_{y}, \underbrace{x_3}_{z}]$$



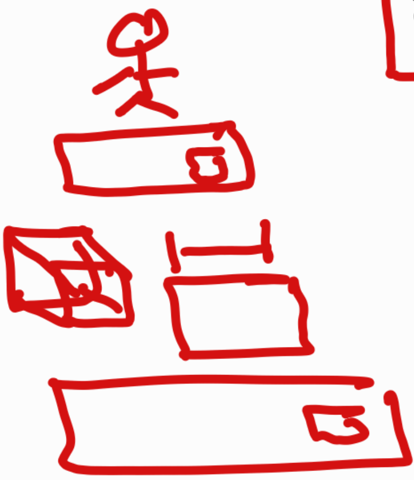
z-

$$X=[x_1, x_2, x_3, x_4]$$

$$x=[5, 7, 20, 30]$$

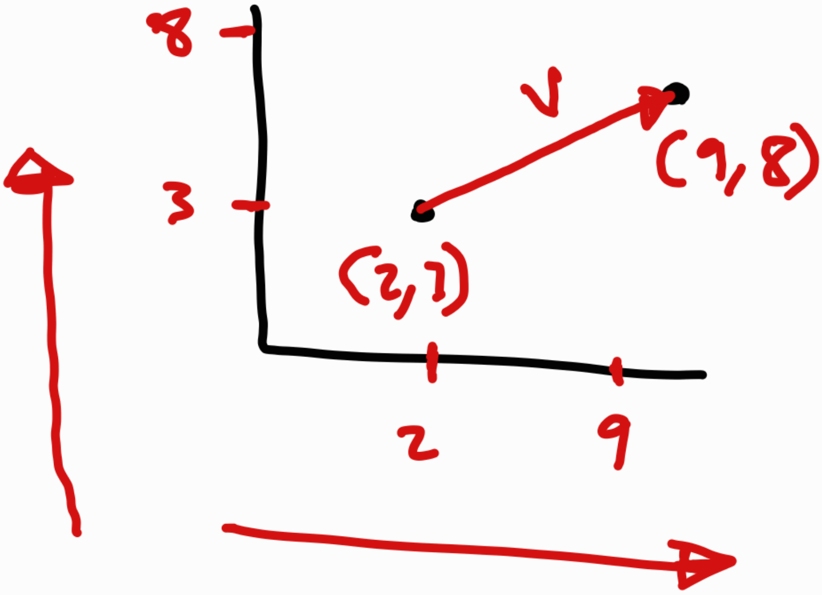
قياسية | متجهة

- مساحة
- كتلة
- حجم



السرعة

25 K/H



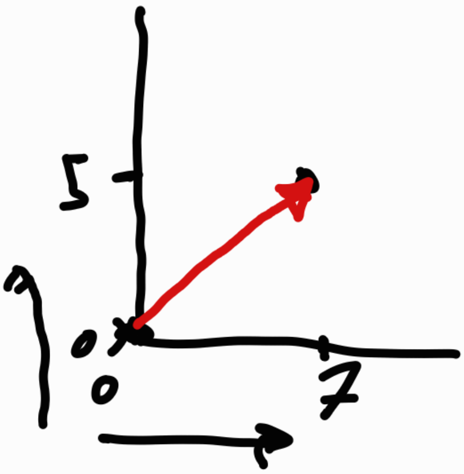
$$(9,8)-(2,3)=\boxed{(7,5)}$$

v

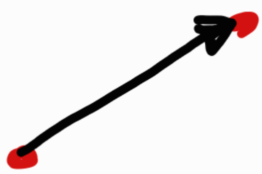
1- Cartesion form:

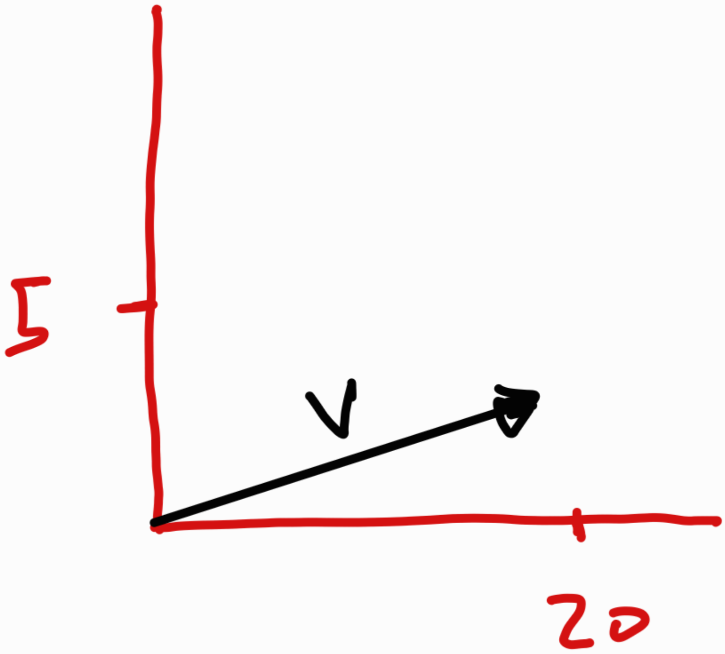
$$V = 7\underset{x}{i} + 5\underset{y}{j}$$

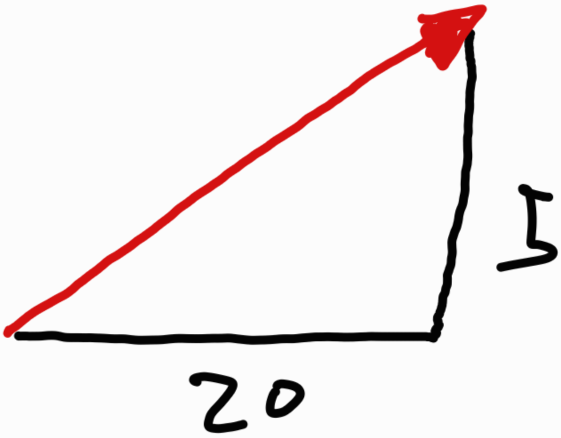
0 – 5

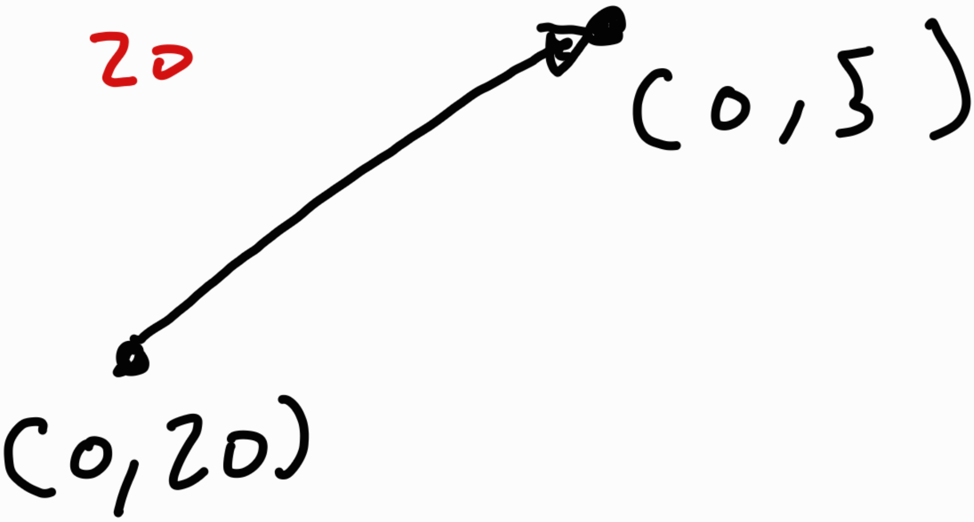


0 – 7

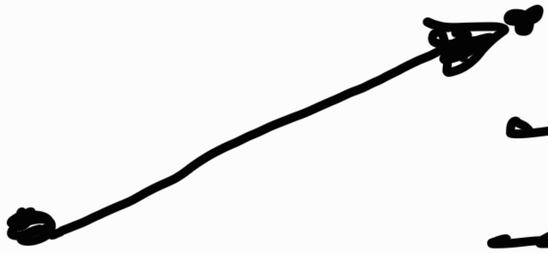



5
v
20


5
20


(0,5)
(0,20)

Polar form

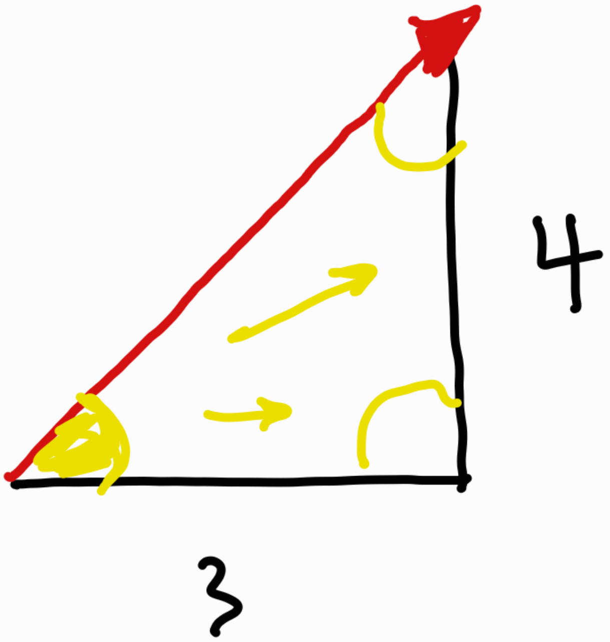


→ Magnitude

→ direction
(Angle)

* Magnitude:

ℓ2- Norm:



$V = 3i + 4j$

ℓ_1-Norm = 7

ℓ_2-Norm

$$\sqrt{3^2 + 4^2}$$

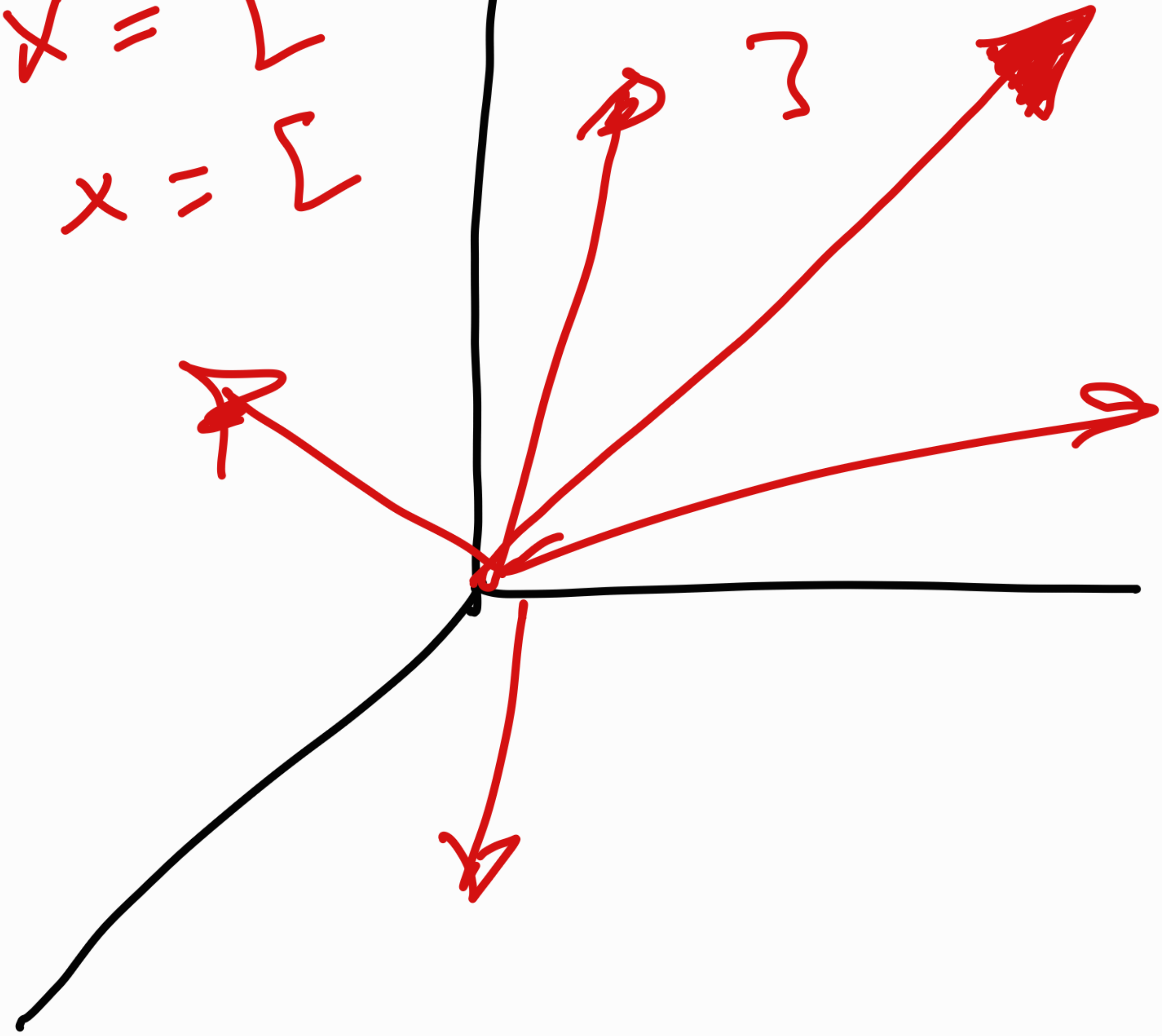
$$\|V\| = 5$$

$$V = \tan^{-1}\frac{4}{3} = 53.13$$

$X = [0 \quad 0 \quad 0]$

$x = [\quad]$

$x = [\quad]$



Vector:

Vector Transposition:

$$[x_1, x_2, x_3]^T \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

row

---

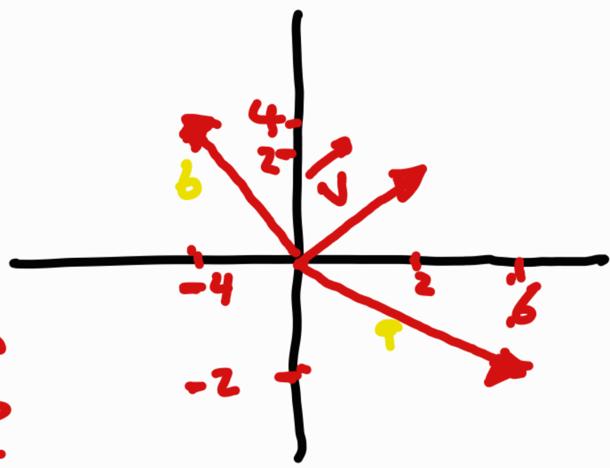
* Vector operations:

1- Addition: $\vec{a} + \vec{b}$

$$\vec{v} = \vec{a}\begin{bmatrix} 6 \\ -2 \end{bmatrix} + \vec{b}\begin{bmatrix} -4 \\ 4 \end{bmatrix}$$

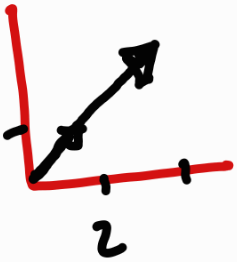
$$\vec{v} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

ذيل التاني
في راس الأول



2- Muliplying avector by a Scalar:

$\vec{a}\begin{bmatrix}2\\1\end{bmatrix} \sim 3\vec{a} = \begin{bmatrix}6\\3\end{bmatrix}$



$\boxed{\text{* linear combination:}}$

$$S = \{\underbrace{v_1, \overset{3}{v_2}}_{+}, \overset{4v?}{v_3}, \dots\}$$

Ex: $\vec{a}\begin{bmatrix}1\\2\end{bmatrix}$ $\quad b = \begin{bmatrix}0\\3\end{bmatrix}$

what is l.c between them!

Sol: $1\vec{a} + 1\vec{b}$

$\vdots$

$5\vec{a} + 20\vec{b}$

The vector Space:

Set of vectors

Subspace

شرط ۳

- $\vec{0} \to \begin{bmatrix}0\\0\end{bmatrix}$

Subspace:

- $\vec{0}$
- scalar
- Addition

$v \to \vec{0} \begin{bmatrix}0\\0\end{bmatrix}$

*Vector span:

span($\vec{a}$, $\vec{b}$)

a $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$ b: $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$

Vector Norms

L1 Norms

L2 Norms

$\boxed{\text{ML}}$ →  6 ← size

size [30 40 100 20 10 15]

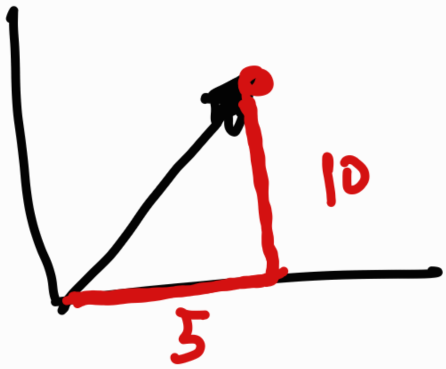
Norms

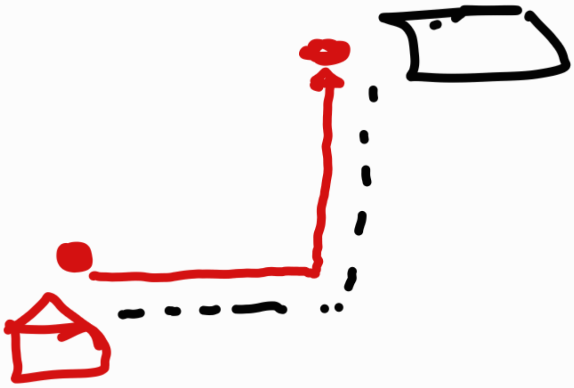
* L1 Norm (Manhattan Norm):

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \|X\| = |x_1| + |x_2|$$

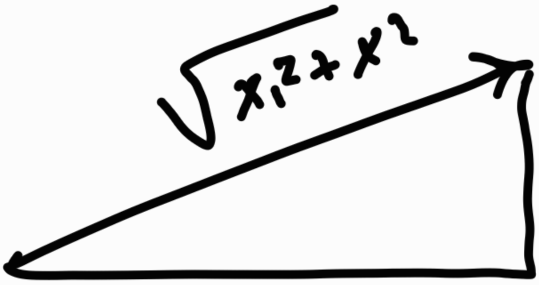
$$X = \begin{bmatrix} 5 \\ 10 \end{bmatrix} \quad \|X\| = 15$$

$$X \begin{bmatrix} -5 \\ 10 \end{bmatrix} = 15 \quad - 5$$





L2-Norm: Eclidean Norm



$\|V\|$

* Dot product:

Vector * vector = [ ]

Cartesion form

$V = 7i + 5j$

Polar form

$V = (5, 53.13)$

$\|V\|$ θ

Column form

$\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ $\begin{bmatrix} x \\ y \end{bmatrix}$

* product: Vector * Vector!

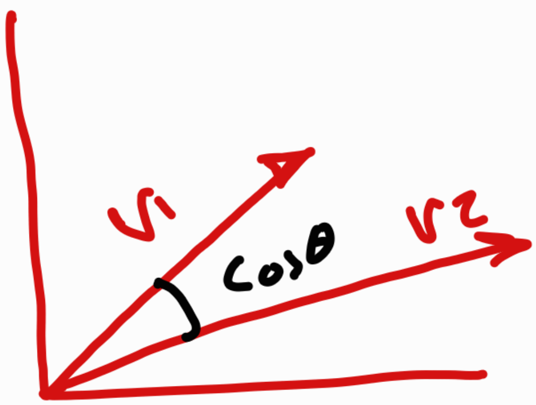
- dot product →
- cross product

* dot product: Inner product:

*Polar form

$V_1 * V_2 =$ direction, Mag

$V_1 \cdot V_2 = |V_1| \cdot |V_2| \cdot \cos\theta$



$\|V_1\| \cdot \|V_2\| \cdot \cos\theta$

Ex: $\|V_1\| = 4$, $\|V_2\| = 4$

$\theta = 60^\circ$

$V_1 \cdot V_2 = 4 \cdot 4 \cdot \cos 60 = \boxed{10}$

$[10]$

* Dot. product: Cartesion form:

$V_1 = 2i + 3j$

$V_2 = 4i + 5j$

$V_1 \cdot V_2 =$

$(x_1 \cdot x_2) + (y_1 \cdot y_2)$

$= 23$

* Column form:

$V_1 \cdot V2 = V_1^T \cdot V2$

Ex: $V_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, $V_2 = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

$V_1 \cdot V_2 = \begin{bmatrix} 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 5 \end{bmatrix} =$

$2 \cdot 4 + 3 \cdot 5 = 23$

* Linear Independence

$$X [x_1, x_2, x_3 \dots]$$

Correlated

dependent

Independent of each other

bakery → price of donut —
→ bread —
→ bagel —

العجين

أجرة العاملين

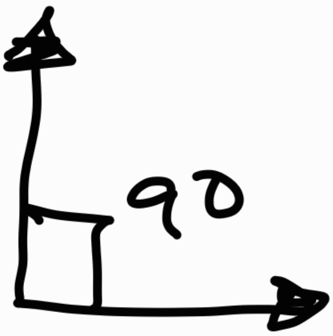
Type	عجين	العاملين	vector
donut	1u	2u	[1 2]
bread	2u	3u	[2 3]
bagel	1u	1u	[1 1]

dependent

$\begin{bmatrix}1\\0\end{bmatrix}$ donut $\rightarrow$ $\begin{bmatrix}\text{chocolate cost}\\\text{sun} \ldots\end{bmatrix}$

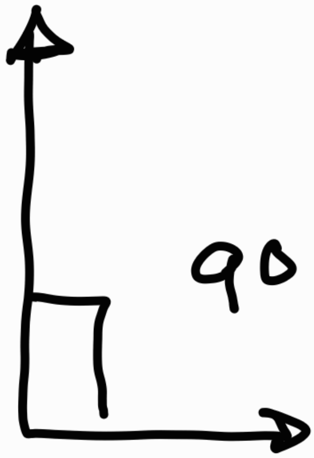
$\begin{bmatrix}0\\1\end{bmatrix}$ bread $\rightarrow$ $\begin{bmatrix}\text{chocolate}\\\text{sunflower cost}\end{bmatrix}$

Indepent

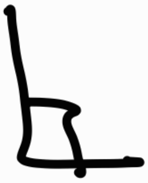


orthogonal

Orthogonal Vector



dot product = 0

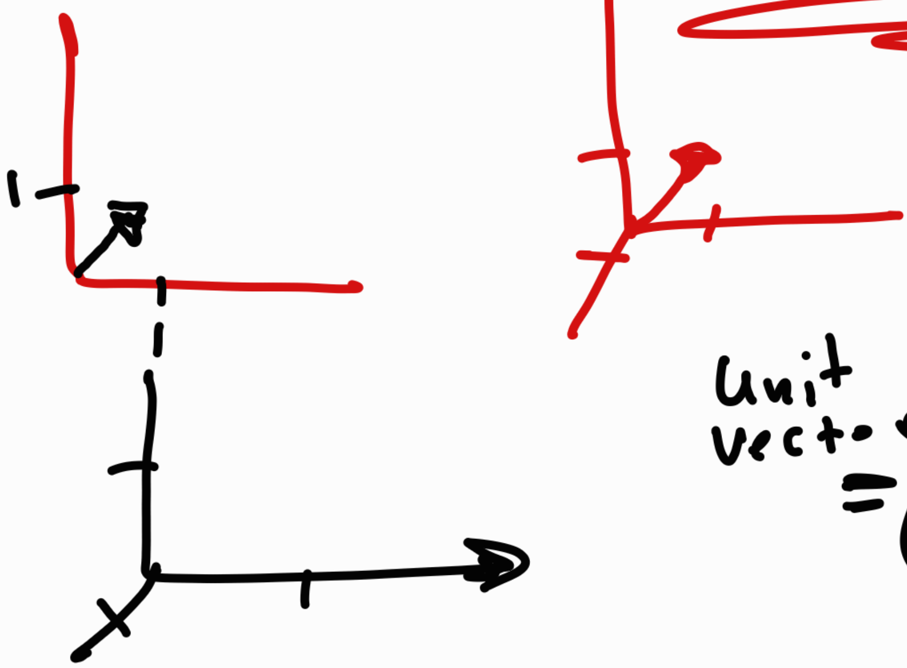


angale = 90

Unit Vector

$\|V\| = 1$

unit vector



unit vector = 1

$\|v\|$

$= 13$

$\to$ convert $\to$ unit vector

$$\hat{V} = \frac{1}{\|V\|} \cdot V = \text{unit vector}$$

Ex! $\vec{a}$ $\begin{bmatrix} 4 \\ 2 \\ -1 \end{bmatrix}$ $\to$ convert this vector to unit vector

$$\hat{a} = \frac{1}{\|\vec{a}\|} \cdot a$$

$$\frac{1}{\sqrt{4^2 + 2^2 + (-1)^2}} \cdot \begin{bmatrix} 4 \\ 2 \\ -1 \end{bmatrix}$$

$$\frac{1}{\sqrt{21}}$$

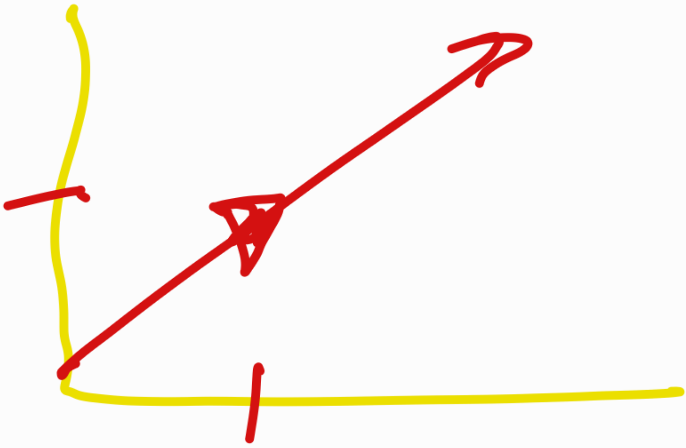
$$\begin{bmatrix} \frac{4}{\sqrt{21}} \\ \frac{2}{\sqrt{21}} \\ \frac{-1}{\sqrt{21}} \end{bmatrix} =$$

$$\hat{a} = \begin{bmatrix} \frac{4}{\sqrt{21}} \\ \frac{2}{\sqrt{21}} \\ \frac{-1}{\sqrt{21}} \end{bmatrix} \qquad \|\hat{a}\| = 1$$

1

Unit vector

→ unit vector

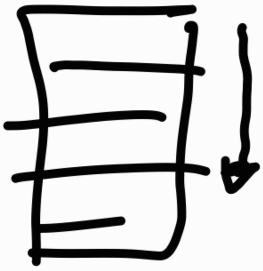


??

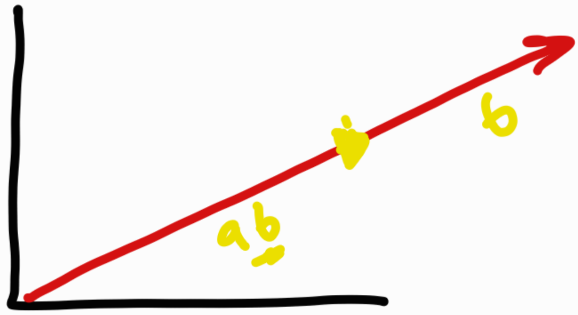
Vector projection

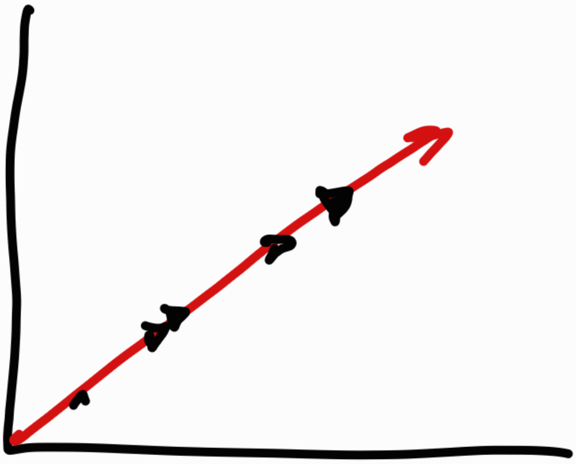
ML → feature + feature → efficiency X ✓

performance ✓ X



ML





Matrix

1- Square Matrix $[\ ]_{2\times2}$ $[\ ]_{3\times3}$

2- Rectangular $r \neq c$

$[\ ]_{2\times3}$

$[\ ]_{3\times2}$

3- Diagonal :

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

4- Identity matrix!

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

5- Scalar matrix

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

6- Zero matrix

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

7- Lower Triangular $\begin{bmatrix} 5 & 0 & 0 \\ 6 & 7 & 0 \\ 8 & 9 & 9 \end{bmatrix}$

8- upper

$$\begin{bmatrix} 2 & 3 & 4 \\ 0 & 5 & 6 \\ 0 & 0 & 7 \end{bmatrix}$$

9- Symmetric:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & 6 & 9 \end{bmatrix}$$

---

* Basic operation:

1- M - M Addition:

$$\begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 5 & -3 \end{bmatrix}$$

2- Negative

$$-\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -3 & -4 \end{bmatrix}$$

3- Subtracting:

$$\begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix} - \begin{bmatrix} -4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} -1 & 8 \\ 3 & 15 \end{bmatrix}$$

4- Scalar Multiplication:

$$2 * \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$$

5- Division of matrix within Scalar:

$$3 \div \begin{bmatrix} 2 & 3 \\ 4 & 3 \end{bmatrix} \rightarrow \frac{1}{3} * \begin{bmatrix} 2 & 3 \\ 4 & 3 \end{bmatrix}$$

* Matrix Multiplication:

$$A_{r \times c} \; B_{r \times c} = Z_{r \times c}$$

equal

$$A_{2 \times 3} \; B_{3 \times 5} = Z_{2 \times 5}$$

$$A_{2 \times 3} * B_{3 \times 2} = Z_{2 \times 2}$$

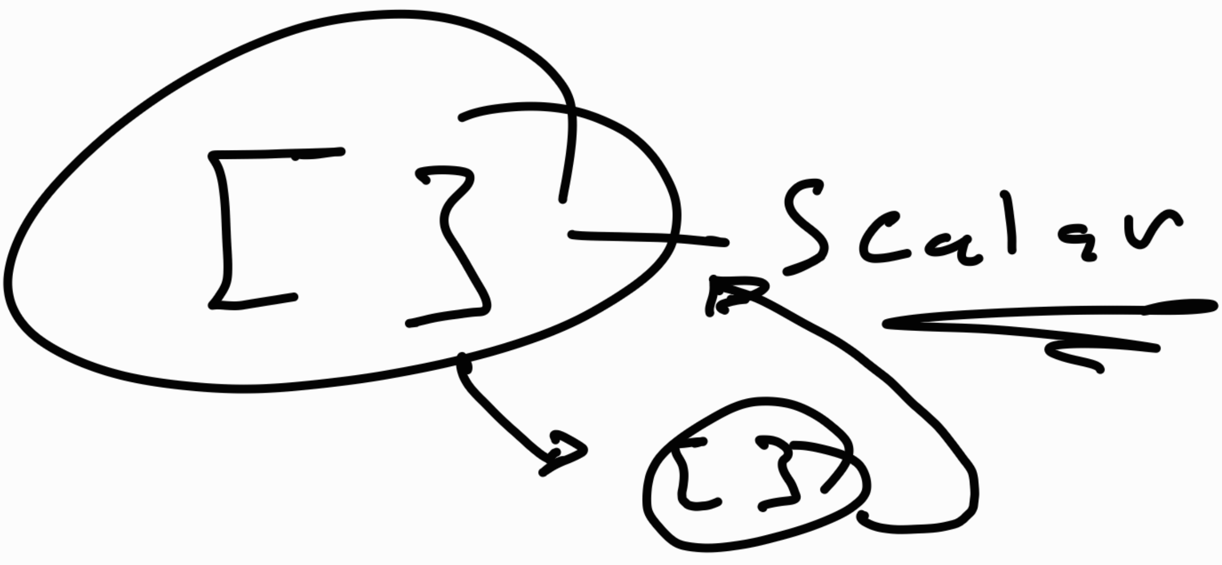
Ex!

$$A_{2 \times 3} * B_{3 \times 2} = Z_{2 \times 2}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{array}{cc} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \end{array}$$

$$(1 * 7) + (2 * 9) + (3 * 11) = 58$$

* Determinate Matrix



$$A_{2\times 2} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad 2\times 2$$

$$|A| = a * d - b * c$$

Ex!

$$F_{2\times 2} = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \quad |F| = 2\times 4 - 3\times 1$$

$$= 8 - 3 = 5$$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & I \end{bmatrix} \qquad \begin{bmatrix} d & f \\ g & I \end{bmatrix}$$

3×3

$$a(eI - f \cdot H) - b(d \cdot I - f \cdot g)$$

$$+ \; c(d \cdot h - e \cdot g)$$

$$= [\;]$$

* Identity Matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} & \end{bmatrix}$$

$$\underline{A} \cdot \underline{I} = \underline{A}$$

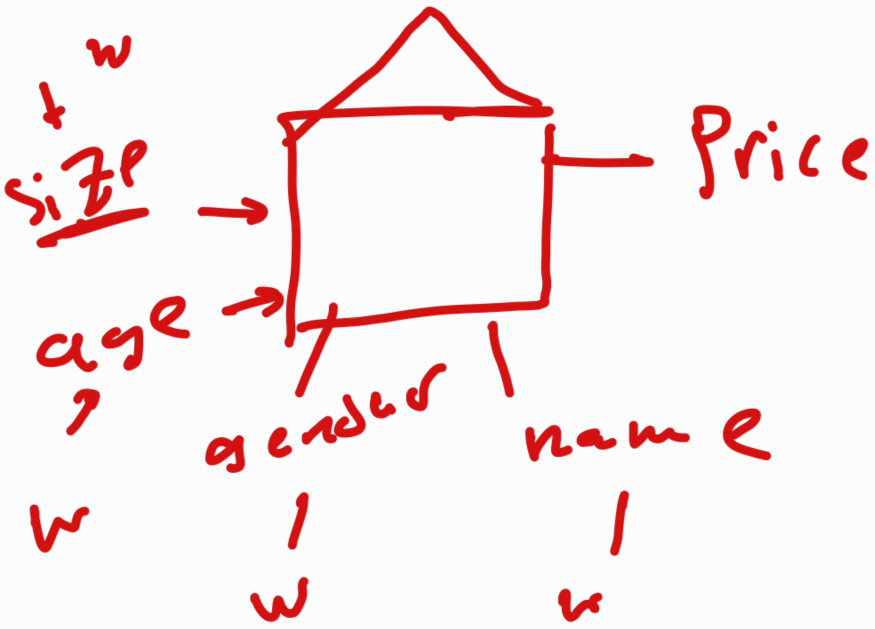
$$A_{3\times 3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

* Inverse Matrix:

$$A \longrightarrow A^{-1}$$

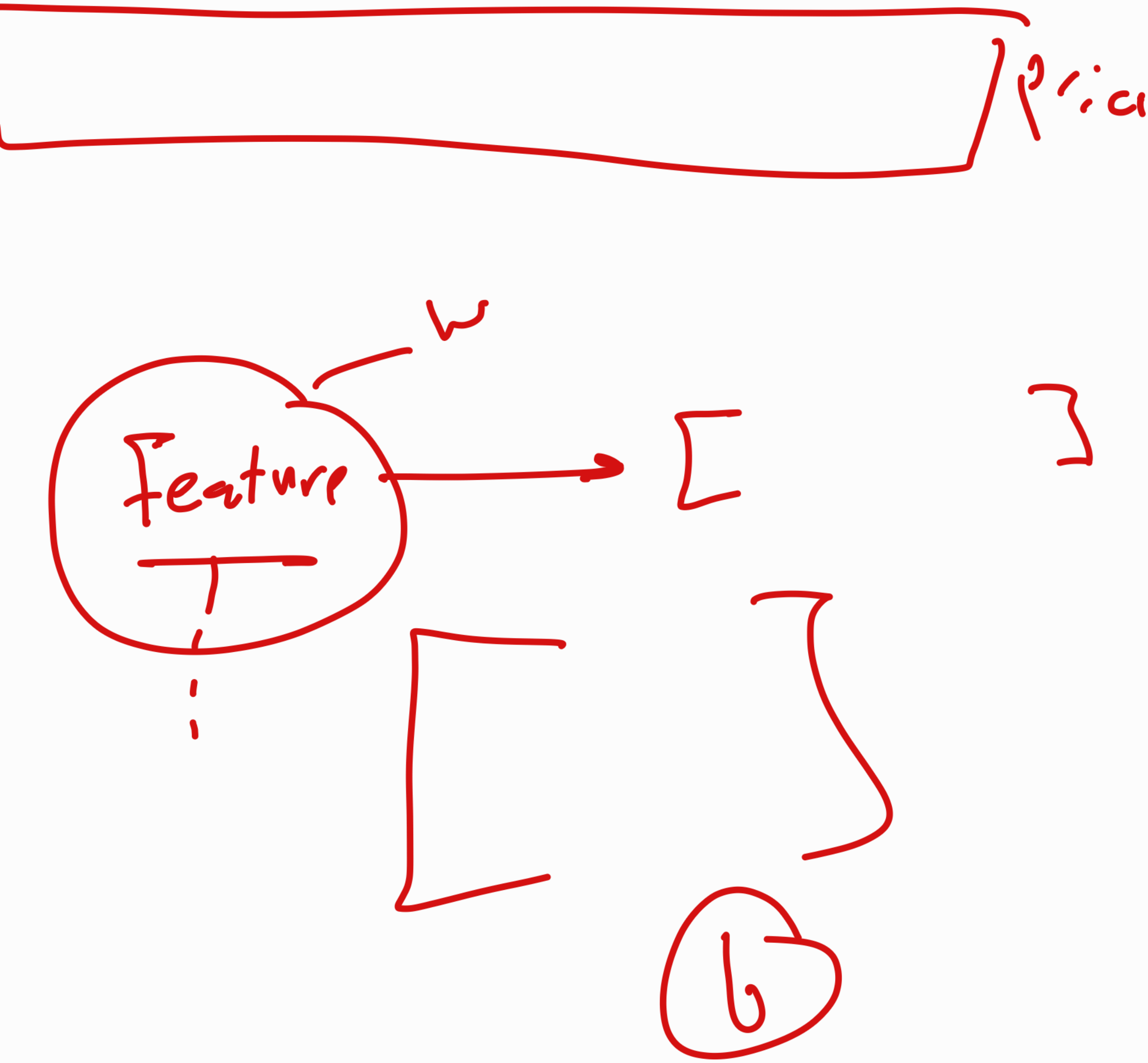
$$3x + 4y = z$$

$$3x = -4y + z$$



$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \end{bmatrix}^T \begin{bmatrix} x & x & x & x & x \\ x & x & x & x & x \\ x & x & x & x & x \\ x & x & x & x & x \end{bmatrix}$$

$$\cdot \begin{bmatrix} w_1 & w_2 & w_3 \end{bmatrix} + b =$$


price
w
Feature
b

$A_{2\times 2} =$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} \rightarrow \frac{1}{|A|} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \frac{1}{a \cdot d - b \cdot c} \cdot \begin{bmatrix} a & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}^{-1}$$

Ex:

$$A = \begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix}^{-1} = \frac{1}{4 \cdot 6 - 7 \cdot 2} \cdot \begin{bmatrix} 6 & -7 \\ -2 & 4 \end{bmatrix}$$

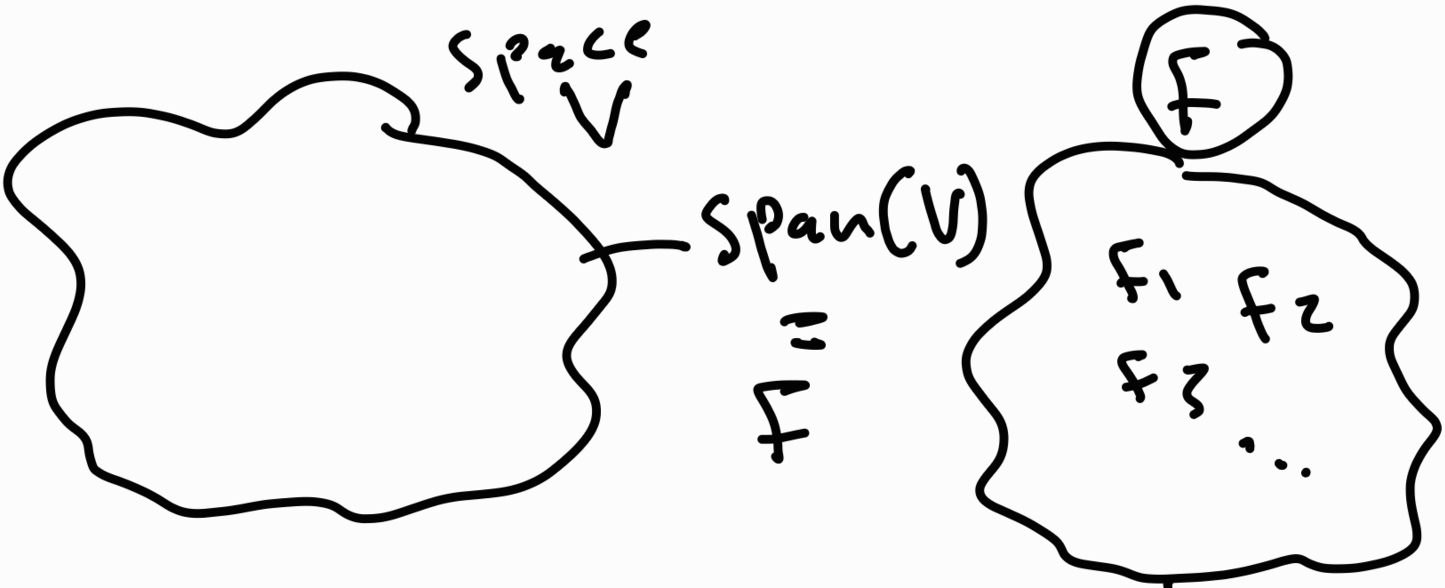
$$\cdot\, A^{-1} = \begin{bmatrix} 0.6 & -0.7 \\ -0.2 & 0.4 \end{bmatrix}$$

I

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Extra

* Basis Vector



Two vector

$\begin{bmatrix}0\\1\end{bmatrix}$ $\begin{bmatrix}1\\0\end{bmatrix}$

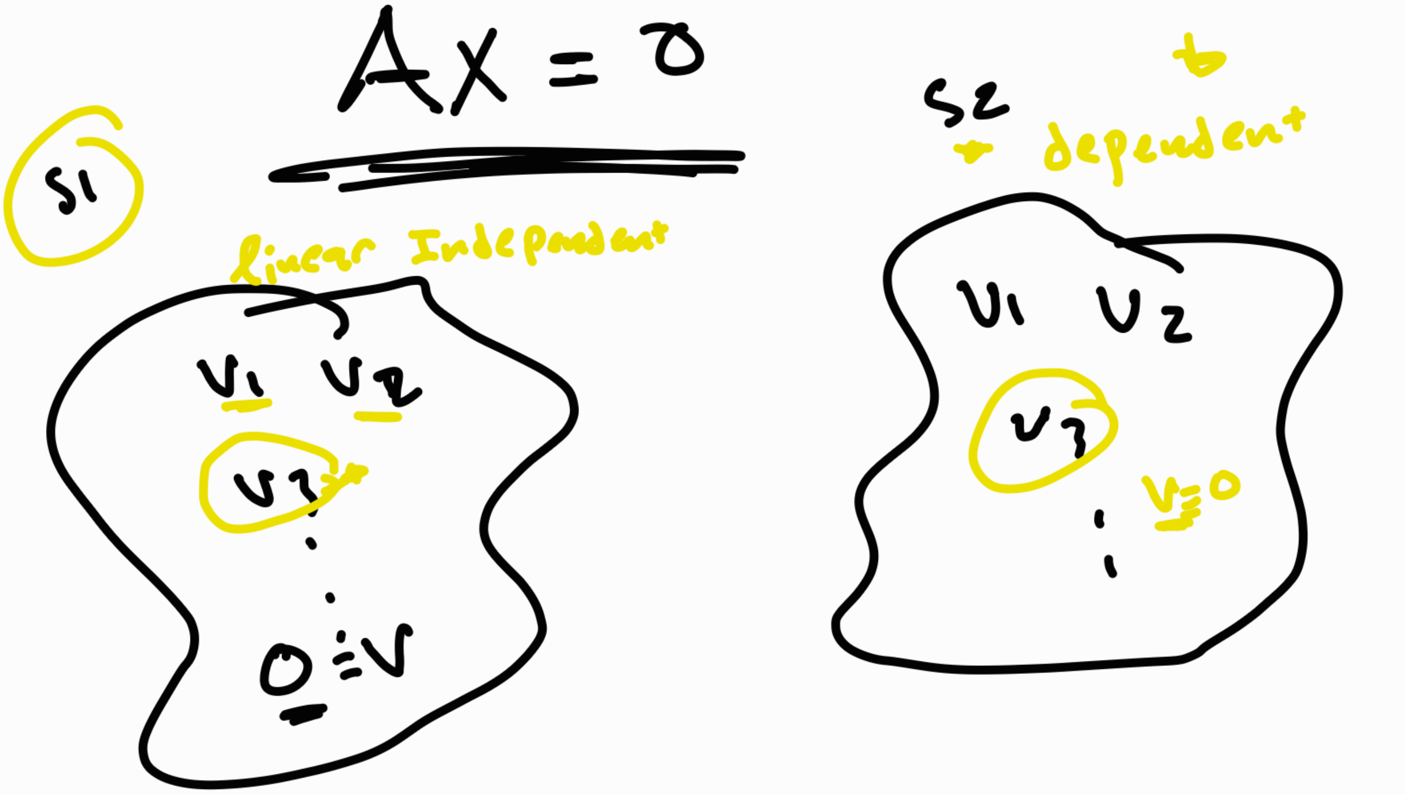
$$1\begin{bmatrix}1\\0\end{bmatrix}+3\begin{bmatrix}0\\1\end{bmatrix}=V\begin{bmatrix}1\\3\end{bmatrix}$$

linear → Independenc

Homogeneous system:

Equation = 0

$$Ax = 0$$



Span(V, V) →

└→ □ R³ → dependent