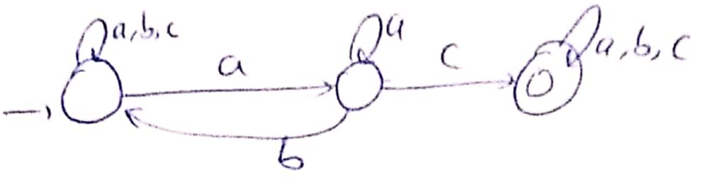
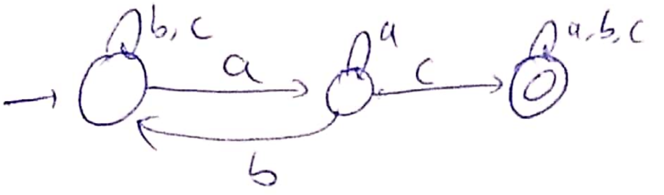
Ex 1 - 24 pts

a) $(a \cup b \cup c)^* ac (a \cup b \cup c)^*$

b)

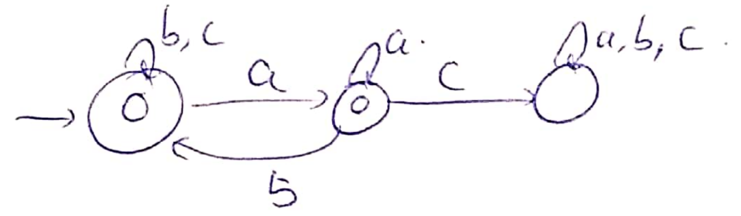


NFA



DFA

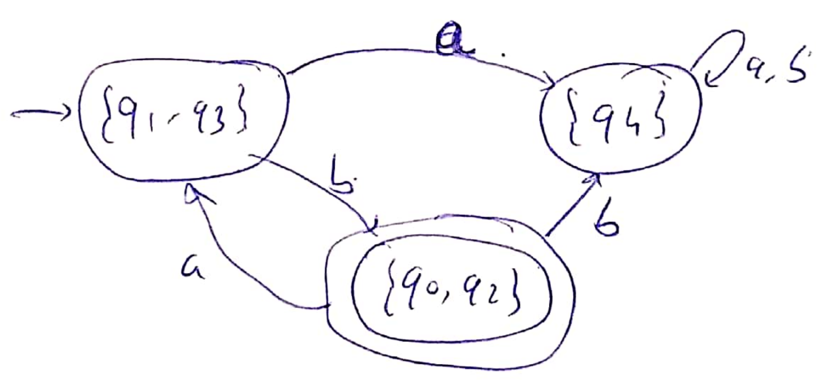
c) we cannot convert the NFA into complement of the language
we switch the stats in a DFA ~~not~~ NFA

=> 

d) is the DFA given in part c.

Ex 2 - 16 pts

0 - equivalence $\{q_1, q_3, q_4\}$ $\{q_0, q_2\}$
1 - equivalence $\{q_1, q_3\}$ $\{q_4\}$ $\{q_0, q_2\}$

=> 

Ex 3 - 20 pts

$L = \{a^n b^m c^k \mid k = m + n\}$

To prove we should verify that $L(G) \subseteq L$ & also $L \subseteq L(G)$

$S \overset{*}{\Rightarrow} a^n S c^n \to a^n B c^n \overset{*}{\Rightarrow} a^n b^m B c^m c^n \to a^n b^m c^{m+n}$.

since all words in $L(G)$ follows the same pattern, so $L(G) \subseteq L$.

Let $w \in L \Rightarrow w = a^n b^m c^{n+m}$ for $n, m \geq 0$

the derivation $S \xRightarrow{*} a^n S c^n \to a^n B c^n \xRightarrow{*} a^n b^m B c^m c^n \to a^n b^m c^{n+m}$

clearly produces w for any n & m $\Rightarrow L \subseteq L(G)$

Ex 4: 20 pts

1) $L = \{a^i b^{2i} c^j \mid i, j \geq 0\}$ is not regular

Let $w = a^p b^{2p} c^p \in L$.

$|w| \geq p$

for any decomposition of w into xyz. we will have the following

suppose L regular. $w = a^n a^m a^l b^{2p} c^p$ / $n+m+l = p$

$x = a^n$ / $n \geq 0$

$y = a^m$ / $m > 0$

$z = a^l b^{2p} c^p$ / $l \geq 0$

$xy^iz = a^n (a^m)^i a^l b^{2p} c^p$

$= a^{n+m \times i+l} b^{2p} c^p$

$\Rightarrow n + m \times i + l = p$

if $i = 2 \Rightarrow n + 2m + l = p$ & we have $m + n + l = p$

$\Rightarrow m = 0$ which is not correct since m should be $> 0 \Rightarrow$ not regular

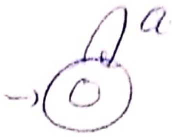
2) $S \to Sc \mid A$

$A \to aAbb \mid \varepsilon$.

Ex 5 20 pts

1 - False.

Let $L = a^*$ which is regular



Let $L' = a^p$ where p is prime

L' is not regular & $L' \subset L$.

so a non-regular language is a subset of a regular language.

2 - True

Let $L_1 = a$. (regular)

Let $L_2 = a^*$ (regular)

Let $L_3 = a^p$

$L_1 L_2 L_3 = a a^* a^p = a^k \mid k \geq 2 \Rightarrow$ regular