Zewail City of Science and Technology
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Math 404 - fall 2019

Report 3

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1| Approach

To implement interior point method along with Mehrotra , we need to implement three following algorithm from Numerical Optimization by Jorge Nocedal, Steve Wright chapter 14 . I am still working on code and you will find updates of it on [this google drive link]

2| First Algorithm: Primal-Dual Path-Following

Framework 14.1 (Primal-Dual Path-Following).

Given (x^0, λ^0, s^0) with $(x^0, s^0) > 0$;

for $k = 0, 1, 2, \ldots$

Choose $\sigma_k \in [0, 1]$ and solve

$$\begin{bmatrix} 0 & A^T & I \\ A & 0 & 0 \\ S^k & 0 & X^k \end{bmatrix} \begin{bmatrix} \Delta x^k \\ \Delta \lambda^k \\ \Delta s^k \end{bmatrix} = \begin{bmatrix} -r_c^k \\ -r_b^k \\ -X^k S^k e + \sigma_k \mu_k e \end{bmatrix},$$

where $\mu_k = (x^k)^T s^k / n$;

Set

$$(x^{k+1}, \lambda^{k+1}, s^{k+1}) = (x^k, \lambda^k, s^k) + \alpha_k (\Delta x^k, \Delta \lambda^k, \Delta s^k),$$

choosing α_k so that $(x^{k+1}, s^{k+1}) > 0$.

end (for).

2| Second Algorithm: Long-Step Path-Following

CENTRAL PATH NEIGHBORHOODS AND PATH-FOLLOWING METHODS

Algorithm 14.2 (Long-Step Path-Following).

Given γ, $\sigma_{\min}$, $\sigma_{\max}$ with $\gamma \in (0, 1)$, $0 < \sigma_{\min} \leq \sigma_{\max} < 1$,
and $(x^0, \lambda^0, s^0) \in \mathcal{N}_{-\infty}(\gamma)$;

for $k = 0, 1, 2, \ldots$

Choose $\sigma_k \in [\sigma_{\min}, \sigma_{\max}]$;

Solve (14.10) to obtain $(\Delta x^k, \Delta\lambda^k, \Delta s^k)$;

Choose α_k as the largest value of α in $[0, 1]$ such that

$$(x^k(\alpha), \lambda^k(\alpha), s^k(\alpha)) \in \mathcal{N}_{-\infty}(\gamma);$$

Set $(x^{k+1}, \lambda^{k+1}, s^{k+1}) = (x^k(\alpha_k), \lambda^k(\alpha_k), s^k(\alpha_k))$;

end (for).

3| Third Algorithm: Predictor –Corrector (Mehrotra)

Algorithm 14.3 (Predictor-Corrector Algorithm (Mehrotra [207])).

Calculate (x^0, λ^0, s^0) as described above;

for $k = 0, 1, 2, \ldots$

Set $(x, \lambda, s) = (x^k, \lambda^k, s^k)$ and solve (14.30) for $(\Delta x^{\text{aff}}, \Delta \lambda^{\text{aff}}, \Delta s^{\text{aff}})$;

Calculate $\alpha_{\text{aff}}^{\text{pri}}, \alpha_{\text{aff}}^{\text{dual}}$, and μ_{aff} as in (14.32) and (14.33);

Set centering parameter to $\sigma = (\mu_{\text{aff}}/\mu)^3$;

Solve (14.35) for $(\Delta x, \Delta \lambda, \Delta s)$;

Calculate α_k^{pri} and α_k^{dual} from (14.38);

Set

$$x^{k+1} = x^k + \alpha_k^{\text{pri}} \Delta x,$$
$$(\lambda^{k+1}, s^{k+1}) = (\lambda^k, s^k) + \alpha_k^{\text{dual}} (\Delta \lambda, \Delta s);$$

end (for).

Figure 1 Fibonacci testing