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Solve for real numbers:

$$\sin^2 x \cdot \cos^2 t + \sin^2 y \cdot \cos^2 x + \sin^2 z \cdot \cos^2 y + \sin^2 t \cdot \cos^2 z = 2$$

Proposed by Daniel Sitaru-Romania

Solution 1 by Adrian Popa-Romania, Solution 2 by Fayssal Abdelli-Bejaia-Algerie

Solution 1 by Adrian Popa-Romania

$$\sin^2 x \cdot (1 - \sin^2 t) + \sin^2 y \cdot \cos^2 x + \sin^2 z \cdot \cos^2 y + \sin^2 t \cdot \cos^2 z = 2 \Leftrightarrow$$

$$\sin^2 x \cdot \cos^2 t + (1 - \cos^2 y) \cdot \cos^2 x + \sin^2 z \cdot (1 - \sin^2 y) + (1 - \cos^2 t) \cdot \cos^2 z = 2$$

$$\Leftrightarrow \sin^2 x \cdot \sin^2 t + \cos^2 y \cdot \cos^2 x + \sin^2 z \cdot \sin^2 y + \cos^2 t \cdot \cos^2 z$$

$$\Leftrightarrow \sin x \cdot \sin t = 0, \cos y \cdot \cos x = 0, \sin z \cdot \sin y = 0, \cos t \cdot \mathrm{cox} z = 0$$

I) $\quad \sin x = \cos y = \sin z = \cos t = 0 \Leftrightarrow$

$$(x, y, z, t) = \left(m\pi, n\pi + \frac{\pi}{2}, p\pi, q\pi + \frac{\pi}{2}\right), m, n, p, q \in \mathbb{Z}$$

ii) $\quad \sin t = \cos z = \sin y = \cos x = 0$

$$(x,y,z,t)=\left(m'\pi+\frac{\pi}{2},n'\pi,p'\pi+\frac{\pi}{2},q'\pi\right),m',n',p',q'\in\mathbb{Z}$$

Solution 2 by Fayssal Abdelli-Bejaia-Algerie

$$sin^2x\cdot(1-sin^2t)+sin^2y\cdot cos^2x+sin^2z\cdot cos^2y+sin^2t\cdot cos^2z=2\Leftrightarrow$$

$$sin^2x\cdot cos^2t+(1-cos^2y)\cdot cos^2x+sin^2z\cdot(1-sin^2y)+(1-cos^2t)\cdot cos^2z=2$$

$$\Leftrightarrow sin^2x\cdot sin^2t+cos^2y\cdot cos^2x+sin^2z\cdot sin^2y+cos^2t\cdot cos^2z$$

$$\Leftrightarrow sinx\cdot sint=0,cosy\cdot cosx=0,sinz\cdot siny=0,cost\cdot coxz=0$$

II) $\quad sinx=cosy=sinz=cost=0\Leftrightarrow$

$$(x,y,z,t)=\left(m\pi,n\pi+\frac{\pi}{2},p\pi,q\pi+\frac{\pi}{2}\right),m,n,p,q\in\mathbb{Z}$$

ii) $\quad sint=cosz=siny=cosx=0$

$$(x,y,z,t)=\left(m'\pi+\frac{\pi}{2},n'\pi,p'\pi+\frac{\pi}{2},q'\pi\right),m',n',p',q'\in\mathbb{Z}$$

Note by editor:
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