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Find:

$$\Omega = \lim_{n\to\infty} \frac{\sqrt{(1+n!)^{n!}}}{n\cdot(n!)!}$$

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Solution by Ahmed Yackoube Chach-Lille-France

$$\Omega_n = \frac{\sqrt{(1+n!)^{n!}}}{n\cdot(n!)!} = \frac{\sqrt{(n!)^{n!}}\sqrt{\left(1+\frac{1}{n!}\right)^{n!}}}{n\sqrt{2\pi n!}\left(\frac{n!}{e}\right)^{n!}} = \frac{\sqrt{\left(1+\frac{1}{n!}\right)^{n!}}\sqrt{n!^{n!}}\,e^{n!}}{n\sqrt{2\pi}\sqrt{n!^{n!+1}}} =$$

$$= \frac{\sqrt{\left(1+\frac{1}{n!}\right)^{n!}}}{n\sqrt{2\pi}}e^{n!-\frac{n!+1}{2}log(n!)}$$

$$\lim_{n\to\infty}\sqrt{\left(1+\frac{1}{n!}\right)^{n!}} = \sqrt{e}$$

$$\lim_{n\to\infty} e^{n!-\frac{n!+1}{2}log(n!)} = e^{-\infty}$$

Therefore,

$$\Omega = \lim_{n\to\infty} \frac{\sqrt{(1+n!)^{n!}}}{n\cdot(n!)!} = \lim_{n\to\infty} \frac{1}{n}\sqrt{\frac{e}{2\pi}}e^{-\infty} = 0$$

Note by editor:
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