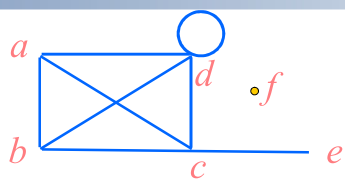
CHAPTER 10 Graphs

1. Basic Terminology

Undirected Graphs $G=(V, E)$



- ***Vertex, edge***
- **Two vertices, *u* and *v* in an undirected graph *G* are called *adjacent* (or *neighbors*) in *G*, if {*u*, *v*} is an edge of *G*.**
- **An edge *e* connecting *u* and *v* is called *incident with vertices u and v*, or is said to connect *u* and *v*.**
- **The vertices *u* and *v* are called *endpoints* of edge {*u*, *v*}.**
- ***loop***
- **The *degree of a vertex* in an undirected graph is the number of edges incident with it, except that a loop at a vertex contributes twice to the degree of that vertex**

Notation: deg(*v*)

- **If deg(*v*) = 0, *v* is called *isolated*.**
- **If deg(*v*) = 1, *v* is called *pendant*.**

【Theorem 1】 The Handshaking Theorem

Let $G = (V, E)$ be an undirected graph with e edges. Then

$$\sum_{v \in V} \deg(v) = 2e$$

The sum of the degrees over all the vertices is twice the number of edges.

Proof:

Each edge contributes twice to the degree count of all vertices.

Note:

This applies even if multiple edges and loops are present.

Questions:

If a graph has 5 vertices, can each vertex have degree 3? 4?

- The sum is 3•5 = 15 which is an odd number.

  Not possible.

- The sum is 20 = 2 | E | and 20/2 = 10.

  Possible.

【 Theorem 2】 An undirected graph has an even number of vertices of odd degree.

Proof:

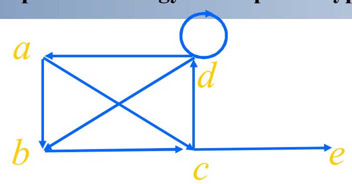
Let V_1, V_2 **be the set of vertices of even degree and the set of vertices of odd degree, respectively.**

$$\sum_{v \in V_1} d(v) + \sum_{v \in V_2} d(v) = 2m$$

Question:

Is it possible to have a graph with 3 vertices each of which has degree 3?

Directed Graphs $G=(V, E)$



Let (u, v) **be an edge in** G**. Then** u **is an** ***initial vertex*** **and is** ***adjacent to*** v **and** v **is a** ***terminal vertex*** **and is** ***adjacent from*** u**.**

The ***in degree*** **of a vertex** v**, denoted** $\deg^-(v)$ **is the number of edges which terminate at** v**.**

Similarly, the ***out degree*** **of** v**, denoted** $\deg^+(v)$**, is the number of edges which initiate at** v**.**

underlying undirected graph

【Theorem 3】 Let $G = (V, E)$ be a graph with direct edges. Then

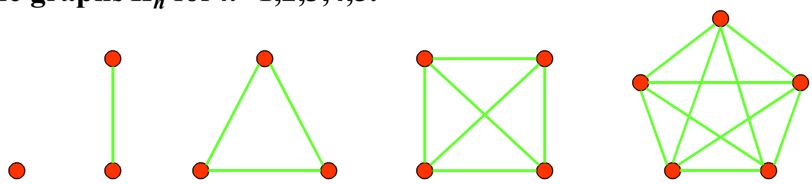
$$\sum_{v \in V} d^+(v) = \sum_{v \in V} d^-(v) = |E|$$

2. Some Special Simple Graphs

(1) Complete Graphs - K_n: the simple graph with

- n vertices
- exactly one edge between every pair of distinct vertices.

The graphs K_n for n=1,2,3,4,5.

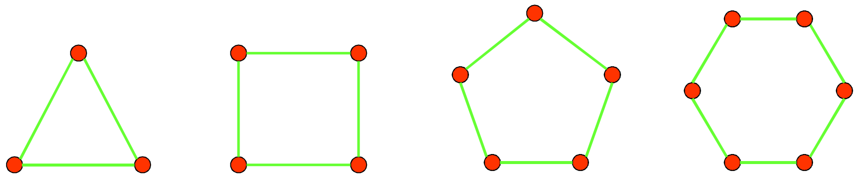


K_1 K_2 K_3 K_4 K_5

Question:

The number of edges in K_n ?

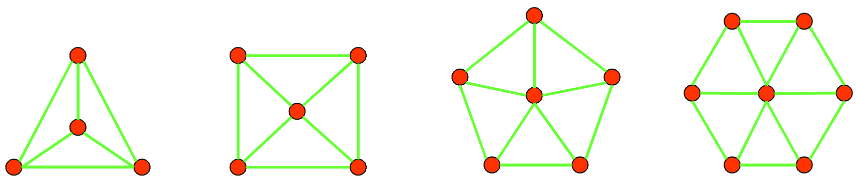
(2) Cycles C_n (n>2)



C_3 C_4 C_5 C_6

(3) Wheels W_n (n>2)

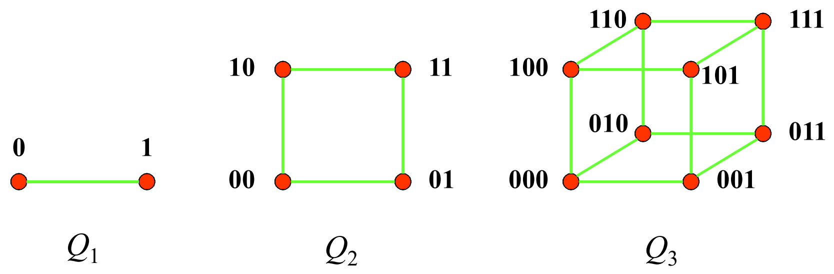
Add one additional vertex to the cycle C_n and add an edge from each vertex in C_n to the new vertex to produce W_n.



W_3 W_4 W_5 W_6

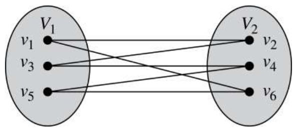
(4) *n-Cubes* Q_n (n>0)

Q_n is the graph with 2^n vertices representing bit strings of length n. An edge exists between two vertices that differ in exactly one bit position.



3. Bipartite Graphs

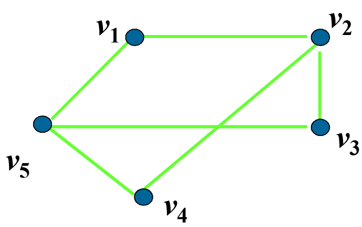
A simple graph G is *bipartite* if V can be partitioned into two disjoint subsets V_1 and V_2 such that every edge connects a vertex in V_1 and a vertex in V_2.



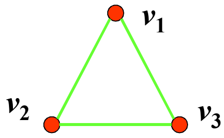
Note:

There are no edges which connect vertices in V_1 or in V_2.

For example,

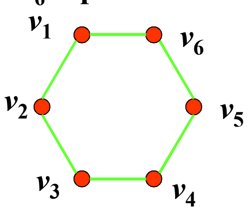


〖Example 1〗 Is C_3 bipartite?

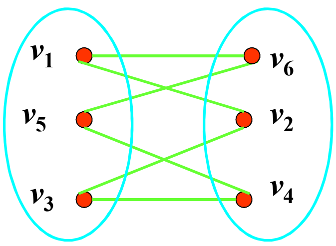


No.

〖Example 2〗 Is C_6 bipartite?



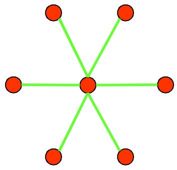
Yes. Because we can display C_6 like this:



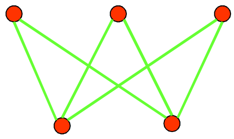
The *complete bipartite graph* is the simple graph that has its vertex set partitioned into two subsets V_1 and V_2 with m and n vertices, respectively, and *every vertex* in V_1 is connected to *every vertex* in V_2, denoted by $K_{m,n}$, where $m = | V_1 |$ and $n = | V_2 |$.

For example,

(1) A Star network is a $K_{1,n}$



(2) $K_{3,2}$



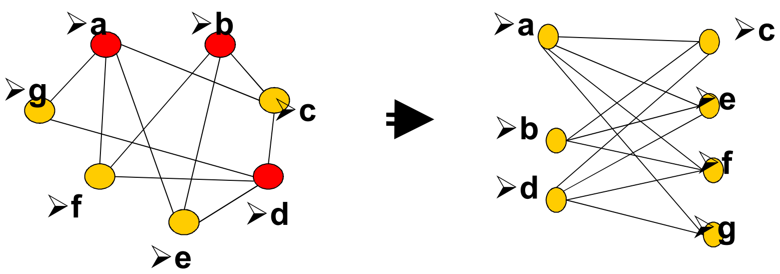
【Theorem 4】 A simple graph is bipartite if and only if it is possible to assign one of two different colors to each vertex of the graph so that no two adjacent vertices are assigned the same color.

Proof:

(1) Suppose that $G=(V, E)$ is a bipartite simple graph. Then $V=V_1\cup V_2$,where V_1,V_2 are disjoint sets and every edge in E connects a vertex in V_1 and a vertex in V_2 .

(2) Suppose that it is possible to assign colors to the vertices of the graph using just two colors so that no two adjacent vertices are assigned the same color.

【 Theorem 4】 **A simple graph is bipartite if and only if it is possible to assign one of two different colors to each vertex of the graph so that no two adjacent vertices are assigned the same color.**



Regular graph

A simply graph is called *regular* if every vertex of this graph has the same degree.

A *regular graph* is called *n*-regular if every vertex in this graph has degree *n*.

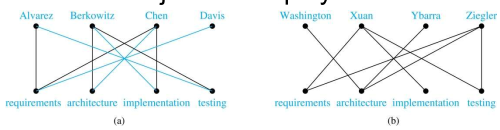
For example,

(1) k_n **is a (*n*-1)-regular.**

(2) For which values of *m* and *n* is $K_{m,n}$ regular?

Bipartite Graphs and Matchings

- Bipartite graphs are used to model applications that involve matching the elements of one set to elements in another, for example:
- *Job assignments* - vertices represent the jobs and the employees, edges link employees with those jobs they have been trained to do. A common goal is to match jobs to employees so that the most jobs are done.



- *Marriage* - vertices represent the men and the women and edges link a a man and a woman if they are an acceptable spouse. We may wish to find the largest number of possible marriages.

See the text for more about matchings in bipartite graphs.

Bipartite Graphs and Matchings

- A **matching** M in a simple graph $G = (V, E)$ is a subset of the set E of edges of the graph such that no two edges are incident with the same vertex.
- A vertex that is the endpoint of an edge of a matching M is said to be **matched** in M
- A **maximum matching** is a matching with the largest number of edges
- We say that a matching M in a bipartite graph $G = (V, E)$ with bipartition ($V1$, $V2$) is a **complete matching from** $V1$ **to** $V2$ if every vertex in $V1$ is the endpoint of an edge in the matching

Theorem 5 (Hall 1935)

- **HALL'S MARRIAGE THEOREM The bipartite graph *G = (V ,E) with bipartition*** *(V1, V2) has a complete matching from V1 to V2 if and only if $|N(A)| \geq |A|$ for all $A \subseteq V1$.*
- ($\Rightarrow$) ***Proof: We first prove the only if part of the theorem***
  - $a \in V1$ has at least one neighbor, i.e., $|N(\{a\})| \geq |\{a\}| = 1$.
    - For any $a \in V1$, there is an edge ab in M for some $b \in V2$
  - Neighbors of a and a' in M can be distinct if $a \neq a'$.
    - We can choose the other ends of a, a' in M as the neighbors.
  - For any A, at least $|A|$ distinct neighbors can be found.
    - That is, $|N(A)| \geq |A|$ for all $A \subseteq V1$.

Theorem 5 (Hall 1935) (2/6)

- ($\Leftarrow$) To prove the *if part of the theorem, the more difficult part, we need to show that if $|N(A)| \geq |A|$ for all $A \subseteq V_1$, then there is a complete matching M from V_1 to V_2. We will* use strong induction on $|V_1|$ *to prove this.*
- *Basis step: If $|V_1| = 1$, then V_1 contains a single vertex v_0. Because $|N(\{v_0\})| \geq |\{v_0\}| = 1$,*
- there is at least one edge connecting v_0 *and a vertex $w_0 \in V_2$. Any such edge forms a complete* matching from V_1 to V_2.

Theorem 5 (Hall 1935) (3/6)

- *Inductive step: We first state the inductive hypothesis.*
- *Inductive hypothesis: Let k be a positive integer. If $G = (V, E)$ is a bipartite graph with bipartition $(V1, V2)$, and $|V1| = j \leq k$, then there is a complete matching M from $V1$ to $V2$ whenever* the condition that $|N(A)| \geq |A|$ *for all* $A \subseteq V1$ *is met.*
- Now suppose that $H = (W, F)$ *is a bipartite graph with bipartition* $(W1, W2)$ *and* $|W1| = k + 1$. *We will prove that the inductive holds using a proof by cases, using two case.*
- *Case (i)* applies when for all integers j *with* $1 \leq j \leq k$, *the vertices in every subset of* j *elements from* $W1$ *are* adjacent to at least $j + 1$ *elements of* $W2$. *Case (ii) applies when for some* j *with* $1 \leq j \leq k$ *there* is a subset $W1$ of j *vertices such that there are exactly* j *neighbors of these vertices in* $W2$.
- *Because* either Case *(i) or Case (ii) holds, we need only consider these cases to complete the inductive step.*

Theorem 5 (Hall 1935) (4/6)

- *Case (i): Suppose that for all integers* j *with* $1 \leq j \leq k$, *the vertices in every subset of* j *elements* from $W1$ *are adjacent to at least* $j + 1$ *elements of* $W2$. *Then, we select a vertex* $v \in W1$ *and an* element $w \in N(\{v\})$, *which must exist by our assumption that* $|N(\{v\})| \geq |\{v\}| = 1$. *We delete* v *and* w *and all edges incident to them from* H. *This produces a bipartite graph* H with bipartition $(W1 - \{v\}, W2 - \{w\})$. *Because* $|W1 - \{v\}| = k$, *the inductive hypothesis tells us* there is a complete matching from $W1 - \{v\}$ *to* $W2 - \{w\}$. *Adding the edge from* v *to* w *to this*complete matching produces a complete matching from $W1$ *to* $W2$.

Theorem 5 (Hall 1935) (5/6)

- *Case (ii): Suppose that for some* j *with* $1 \le j \le k$*, there is a subset* $W'1$ of j *vertices such that* there are exactly j *neighbors of these vertices in* $W2$*. Let* $W'2$ be the set of these neighbors. Then, by the inductive hypothesis there is a complete matching from $W'1$ to $W'2$. Remove these $2j$ vertices from $W1$ *and* $W2$ *and all incident edges to produce a bipartite graph* K *with bipartition*$(W1 - W'1, W2 - W'2)$.

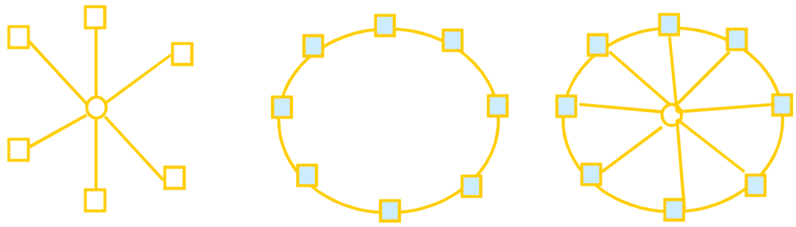
Theorem 5 (Hall 1935) (6/6)

- We will show that the graph K *satisfies the condition* $|N(A)| \ge |A|$ *for all subsets* A *of* $W1 - W'1$. If not, there would be a subset of t *vertices of* $W1 - W'1$ where $1 \le t \le k + 1 - j$ such that the vertices in this subset have fewer than t *vertices of* $W2 - W'2$ as neighbors. Then, the set of $j + t$ *vertices of* $W1$ *consisting of these* t *vertices together with the* j *vertices we* removed from $W1$ *has fewer than* $j + t$ *neighbors in* $W2$*, contradicting the hypothesis that* $|N(A)| \ge |A|$ *for all* $A \subseteq W1$.

4. Some applications of special types of graphs

〖Example 3〗 Local Area Networks.

1. Star topology
2. Ring topology
3. Hybrid topology

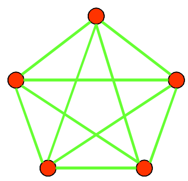


5. New Graphs from Old

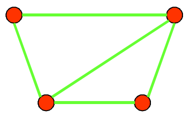
【Definition】 $G=(V, E)$, $H=(W, F)$

- H is a ***subgraph*** of G if $W \subseteq V, F \subseteq E$.
- subgraph H is a ***proper subgraph*** of G *if* $H \neq G$.
- H is a ***spanning***(生成） ***subgraph*** of G if $W = V, F \subseteq E$.

For example,



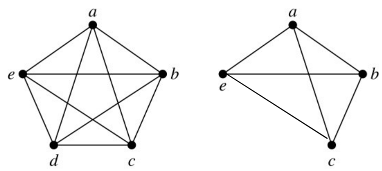
K_5



subgraph of K_5

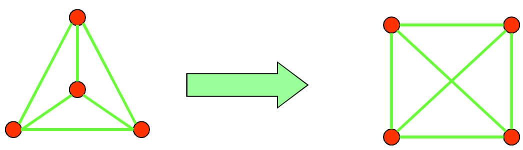
【Definition】 Let $G = (V, E)$ be a simple graph. The *subgraph induced*（诱导） by a subset W of the vertex set V is the graph (W,F), where the edge set F contains an edge in E if and only if both endpoints are in W.

Example: Here we show K_5 and the subgraph induced by $W = \{a,b,c,e\}$.



〖Example 4〗 **How many subgraphs with at least one vertex does W_3 have?**

Solution:



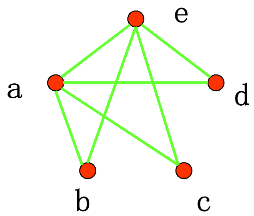
$$C(4,1)+C(4,2)\times 2+C(4,3)\times 2^3+C(4,4)\times 2^6$$

The union of G_1 and G_2

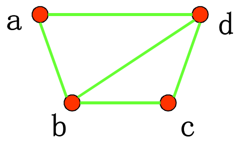
The *union* of two simple graphs $G_1 = (V_1 , E_1)$ and $G_2 = (V_2 , E_2)$ is the simple graph with vertex set $V = V_1 \cup V_2$ and edge set $E = E_1 \cup E_2$.

Notation: $G_1 \cup G_2$

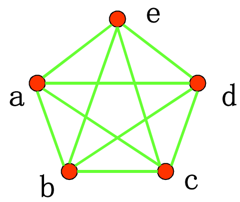
For example,



G_1



G_2



$G_1 \cup G_2 = K_5$

Homework:

第8版 Sec. 10.2 5, 24, 25, 44(b, f, h), 55, 62

CHAPTER 10 Graphs

1. Representing Graphs

Methods for representing g

- **Graphs**
- **Adjacency lists**

  -- lists that specify
  to each vertex
- **Adjacency matrice**
- **Incidence matrices**

An adjacency list for a directed graph	
Initial vertex	terminal vertices
a	b,c,d,e
b	b,d
c	a,c,e
d	
e	b,c,d

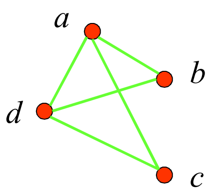
2. Adjacency Matrices

A simple graph $G = (V, E)$ with n vertices ($v_1, v_2, \cdots, v_n$) can be represented by its adjacency matrix, A, with respect to this listing of the vertices, where

$a_{ij} = 1$ if $\{v_i, v_j\}$ is an edge of G,
$a_{ij} = 0$ otherwise.

Note: An adjacency matrix of a graph is based on the ordering chosen for the vertices.

〖Example 1〗 What is the adjacency matrix A_G for the following graph G based on the order of vertices a, b, c, d ?



Solution:

$$A_G = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Note: Adjacency matrices of undirected graphs are always symmetric.

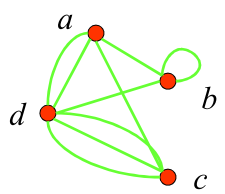
◆ The adjacency matrix of a multigraph or pseudograph

For the representation of graphs with multiple edges, we can no longer use zero-one matrices.

Instead, we use matrices of nonnegative integers.

The (i, j)th entry of such a matrix equals the number of edges that are associated to $\{v_i, v_j\}$.

〖Example 2〗 What is the adjacency matrix A_G for the following graph G based on the order of vertices a, b, c, d ?



Note: For undirected multigraph or pseudograph, adjacency matrices are symmetric.

Solution:

$$A_G = \begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 3 \\ 2 & 1 & 3 & 0 \end{bmatrix}$$

◆ **The adjacency matrix of a directed graph**

Let $G = (V, E)$ be a directed graph with $|V| = n$. Suppose that the vertices of G are listed in an arbitrary order as v_1, v_2, …, v_n.

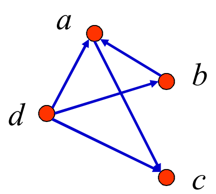
The adjacency matrix A (or A_G) of G, with respect to this listing of the vertices, is the $n \times n$ zero-one matrix with 1 as its (i, j)th entry when there is an edge from v_i to v_j, and 0 otherwise.

In other words, for an adjacency matrix $A = [a_{ij}]$,

$a_{ij} = 1$ if (v_i, v_j) is an edge of G,

$a_{ij} = 0$ otherwise.

〖Example 3〗 What is the adjacency matrix A_G for the following graph G based on the order of vertices a, b, c, d ?

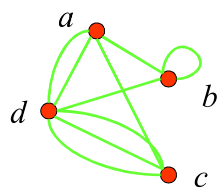


Solution:

$$A_G = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Question:

1. **What is the sum of the entries in a row of the adjacency matrix for an undirected graph?**



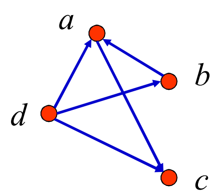
$$A_G = \begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 3 \\ 2 & 1 & 3 & 0 \end{bmatrix}$$

Question:

1. **What is the sum of the entries in a row of the adjacency matrix for an undirected graph?**

   The number of edges incident to the vertex *i*, which is the same as degree of *i* minus the number of loops at *i*.

   For a directed graph?



$$A_G = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Question:

1. **What is the sum of the entries in a row of the adjacency matrix for an undirected graph?**

   The number of edges incident to the vertex *i*, which is the same as degree of *i* minus the number of loops at *i*.

   For a directed graph?

   $\deg^+(v_i)$

2. **What is the sum of the entries in a column of the adjacency matrix for an undirected graph?**

   The number of edges incident to the vertex *i*, which is the same as degree of *i* minus the number of loops at *i*.

   For a directed graph?

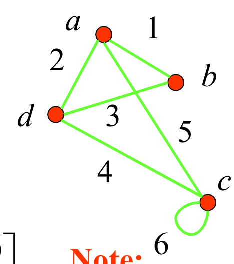
   $\deg^-(v_i)$

3. Incidence matrices

Let $G = (V, E)$ be an undirected graph. Suppose that $v_1, v_2, \cdots, v_n$ are the vertices and $e_1, e_2, \cdots, e_m$ are the edges of G. Then the *incidence matrix* with respect to this ordering of V and E is $n \times m$ matrix $\mathrm{M} = [m_{ij}]_{n\times m}$, where

$$m_{ij} = \begin{cases} 1 & \text{when edge } e_j \text{ is incident with } v_i \\ 0 & \text{otherwise} \end{cases}$$

〖Example 4〗 What is the incidence matrix M for the following graph G based on the order of vertices a, b, c, d and edges 1, 2, 3, 4, 5, 6?



Solution:

$$M = \begin{array}{c} a \\ b \\ c \\ d \end{array} \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Note:

Incidence matrices of undirected graphs contain two 1s per column for edges connecting two distinct vertices and one 1 per column for loops.

4. Isomorphism of Graphs

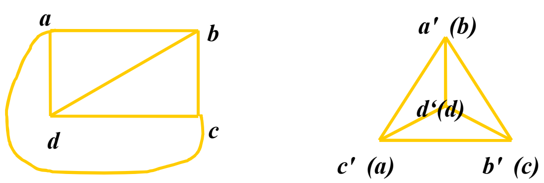
Graphs with the same structure are said to be *isomorphic*.

Formally, two simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are isomorphic if there is a 1-1 and onto function f from V_1 to V_2 such that for all a and b in V_1, a and b are adjacent in G_1 iff $f(a)$ and $f(b)$ are adjacent in G_2.

Such a function f is called an *isomorphism*.

In other words, when two simple graphs are isomorphic, there is a one-to-one correspondence between vertices of the two graphs that preserves the adjacency relationship.

For example,



Question:

How to determine whether two simple graphs are isomorphic?

It is usually difficult since there are *n*! possible 1-1 correspondence between the two vertex sets with *n* vertices. However, some properties (called *invariants*) in the graphs may be used *to show that they are not isomorphic*.

invariants

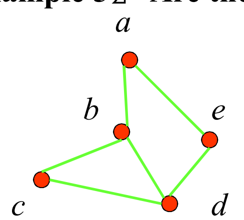
-- things that G_1 and G_2 must have in common to be isomorphic.

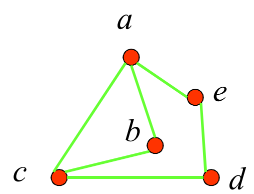
Important *invariants in isomorphic graphs*:

- **the number of vertices**
- **the number of edges**
- **the degrees of corresponding vertices**
- **if one is bipartite, the other must be**
- **if one is complete, the other must be**
- **if one is a wheel, the other must be**

etc.

〖Example 5〗 Are the following two graphs isomorphic?





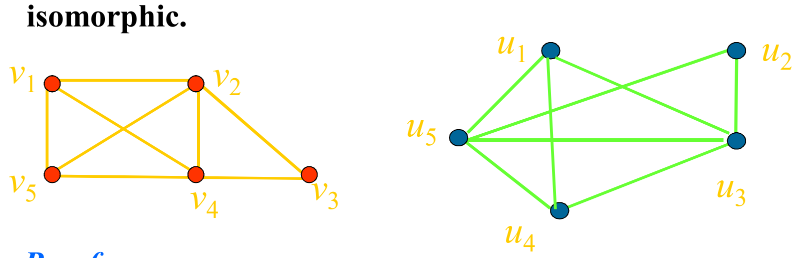
Solution:

They are isomorphic, because they can be arranged to look identical.

You can see this if in the right graph you move vertex *b* to the left of the edge {*a*, *c*}. Then the isomorphism *f* from the left to the right graph is:

$f(a) = e, \ f(b) = a,$
$f(c) = b, \ f(d) = c, \ f(e) = d.$

〖Example 6〗 Show that the following two graphs are isomorphic.



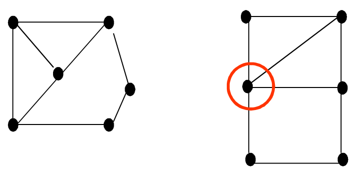
Proof:

- Try to find an 1-1 and onto function *f*
- Show that *f* isomorphism (preserves adjacency relation)

  - The adjacency matrix of a graph *G* is the same as the adjacency matrix of another graph *H*, when rows and columns are labeled to correspond to the images under *f* of the vertices in *G* that are the labels of these rows and columns in the M_G

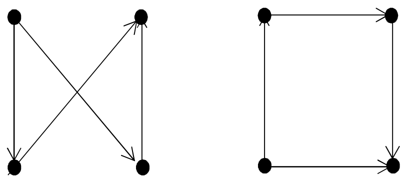
〖Example 7〗 Determine whether the given pair of graphs is isomorphic?

(1)



(2)

(3)



Homework:

第8版 Sec. 10.3 **8, 15, 17, 38-41**

CHAPTER 10 Graphs

1. Paths

A ***path of length n*** in a **simple graph** is a sequence of vertices $v_0, v_1, \ldots, v_n$ such that $\{v_0, v_1\}$, $\{v_1, v_2\}$, ..., $\{v_{n-1}, v_n\}$ are n edges in the graph.

The path is a ***circuit*** if it begins and ends at the same vertex (length greater than 0).

A path is ***simple*** if it does not contain the same edge more than once.

Note:

1. There is nothing to prevent traversing an edge back and forth to produce arbitrarily long paths. This is usually not interesting which is why we define a simple path.
2. The notation of a path: vertex sequence
3. A path of length zero consists of a single vertex.

Path in directed graph

A *path of length n* in a directed graph is a sequence of vertices $v_0, v_1, \ldots v_n$ such that (v_0, v_1) , (v_1, v_2) , ..., (v_{n-1}, v_n) are n directed edges in the graph.

Circuit or cycle : the path begins and ends with the same vertex.

Simple path: the path does not contain the same edge more than once.

Paths represent useful information in many graph models.

〖Example 1〗 Path in Acquaintanceship Graphs

In an acquaintanceship graph there is a path between two people if there is a chain of people linking these people, where two people adjacent in the chain know one other.

Many social scientists have conjectured that almost every pair of people in the world are linked by a small chain of people, perhaps containing just five or fewer people.

The play *Six Degrees of Separation* by John Guare is based on this notion.

2. Counting paths between vertices

The number of paths between two vertices in a graph can be determined using its adjacency matrix.

【 Theorem 2】 The *number of different paths of length r* from v_i to v_j is equal to the (i, j)th entry of A^r, where A is the adjacency matrix representing the graph consisting of vertices $v_1, v_2, \ldots v_n$.

Note: This is the standard power of A, not the Boolean product.

Proof:

Let $A = [a_{ij}]_{n\times n}$

(1) A

$a_{ij} = 1$

$a_{ij} = 0$

(3)

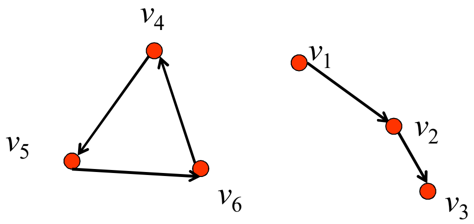
$$A^r = (c_{ij})_{n\times n}$$

$$A^{r+1} = A^r \cdot A = (d_{ij})_{n\times n}$$

$$d_{ij} = c_{i1}a_{1j} + c_{i2}a_{2j} + \cdots + c_{in}a_{nj} = \sum_{k=1}^{n} c_{ik}a_{kj}$$

from v_i to v_j

〖Example 2〗



(1) How many paths of length 2 are there from v_5 to v_4?

a_{54} in A^2: 1

(2) The number of paths not exceeding 6 are there from v_4 to v_5?

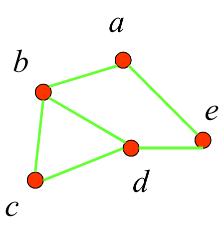
a_{45} in $A+A^2+A^3+A^4+A^5+A^6$: 2

(3) The number of circuits starting at vertex v_5 whose length is not exceeding 6?

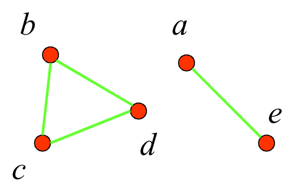
3. Connectedness in undirected graphs

An undirected graph is called *connected* if there is a path between every pair of distinct vertices of the graph.

〖Example 3〗 Are the following graphs connected?



Yes.



No.

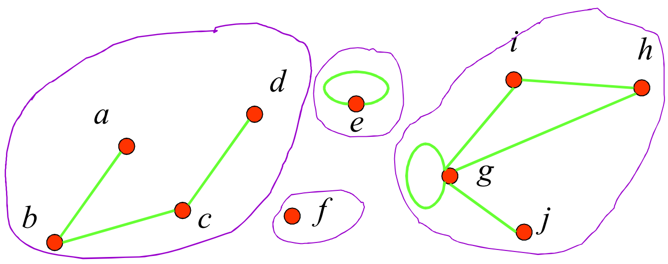
【 Theorem 1】 There is a simple path between every pair of distinct vertices of a connected undirected graph.

Proof:

Because the graph is connected there is a path between *u* and *v*. Throw out all redundant circuits to make the path simple.

The maximally connected subgraphs of *G* are called the *connected components* or just the *components*.

For example,

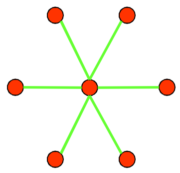


A vertex is a *cut vertex* (or *articulation point*), if removing it and all edges incident with it results in more connected components than in the original graph.

Similarly if removal of an edge creates more components the edge is called a *cut edge* or *bridge*.

For example,

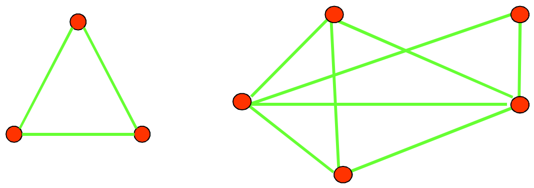
（1）



In the star network the center vertex is a cut vertex.

All edges are cut edges.

（2）



There are no cut edges or vertices in the graph *G* above.

Removal of any vertex or edge does not create additional components.

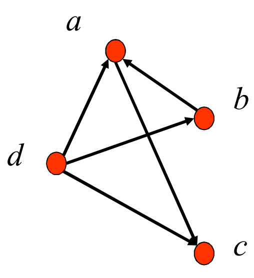
4. Connectedness in directed graphs

A directed graph is *strongly connected* if there is a path from *a* to *b* and from *b* to *a* for all vertices *a* and *b* in the graph.

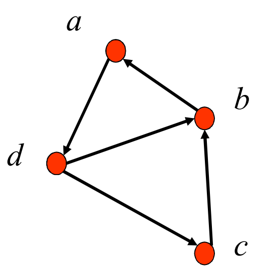
The graph is *weakly connected* if the underlying undirected graph is connected.

Note: By the definition, any strongly connected directed graph is also weakly connected.

For example,



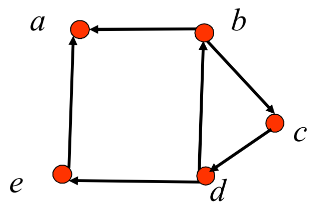
Weakly connected



Strongly connected

For directed graph, the maximal strongly connected subgraphs are called the ***strongly connected components*** or just the ***strong components.***

For example,



Problems:

1. How to determine whether a given directed graph is strongly connected or weakly connected ?
2. How to find the strongly connected components in a directed graph ?

   Kosararu

   Tarjan

5. Paths and Isomorphism

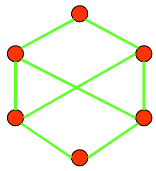
Idea:

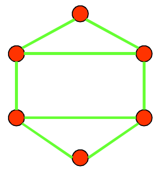
(1) Some other invariants

- ■ The number and size of connected components
- ■ Path
- ✓ Two graphs are isomorphic only if they have simple circuits of the same length.
- ✓ Two graphs are isomorphic only if they contain paths that go through vertices so that the corresponding vertices in the two graphs have the same degree.

(2) We can also use paths to find mapping that are potential isomorphism.

〖Example 4〗 Are these two graphs isomorphic?





Solution:

No.

Because the right graph contains circuits of length 3, while the left graph does not.

Homework:

Sec. 10.4 27(e), 28, 29, 62