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# Solomonoff - kompilator transduktorów skończenie stanowych

Solomonoff - Finite state tranducer compiler

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# Oświadczenie

Ja, niżej podpisana **Bohdan Bondar, Marcin Jałoński, Aleksander Mendoza-Drosik**, studentka Wydziału Matematyki i Informatyki Uniwersytetu im. Adama Mickiewicza w Poznaniu oświadczam, że przedkładaną pracę dyplomową pt. *Solomonoff - kompilator transduktorów skończenie stanowych* napisałam samodzielnie. Oznacza to, że przy pisaniu pracy, poza niezbędnymi konsultacjami, nie korzystałam z pomocy innych osób, a w szczególności nie zlecałam opracowania rozprawy lub jej części innym osobom, ani nie odpisywałam tej rozprawy lub jej części od innych osób. Oświadczam również, że egzemplarz pracy dyplomowej w wersji drukowanej jest całkowicie zgodny z egzemplarzem pracy dyplomowej w wersji elektronicznej. Jednocześnie przyjmuję do wiadomości, że przypisanie sobie, w pracy dyplomowej, autorstwa istotnego fragmentu lub innych elementów cudzego utworu lub ustalenia naukowego stanowi podstawę stwierdzenia nieważności postępowania w sprawie nadania tytułu zawodowego.

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# Streszczenie

**Słowa kluczowe:** klasa

# Abstract

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

**Keywords:** klasa

Tu możesz umieścić swoją dedykację.

# Spis treści

Rozdział 1

# Introduction

This project focuses on research in the field of automata theory and inductive inference. The main product of our work is the "Solomonoff" regular expression compiler for finite state transducers. Plenty of research has gone into development of the theory behind this system. As a result the transducers contain several features not know before.

The most innovative achievements is the lexicographic arctic semiring of weights, specialized adaptation of Glushkov's construction for subsequential transducers and the most significant flagship feature - built-in support for inductive inference and machine learning of transducers. Thanks to the cooperation with LearnLib and Dortmund University, Solomonoff supports learning algorithms such as RPNI and several of it's derivatives. Solomonoff contributes its own more specialized for transducers inference. We implemented OSTIA for efficient learning of deterministic transducers. For nondeterministic ones we developed our own OFTIA algorithm, that was not known before.

All those features together make Solomonoff a unique library that stands out from all the alternatives. We support most of the features of UNIX regexes, including look-aheads and look-behinds, which is unusual for automata-based regex engine. The key that allows Solomonoff for doing that is the possibility of emulating look-aheads with careful placement of transducer outputs. As a result Solomonoff can compete and do much more than existing projects such as RE2 developed by Google, BRICKS automata developed at Aarhus University or even to certain extent with Pearl/Java based regular expression engines. Another, much stronger competitor for Solomonoff is the OpenFST project developed by Google. Their Thrax grammars are capable of doing most of the things that Solomonoff can and they also support probabilistic automata. OpenFST is much older and more established in the scientific community. They support a lot more features that were developed by scientists, by the course of many years. Solomonoff cannot compete with this level of sophistication, but perhaps, what might seem like a limitation, is in fact our strongest advantage. Solomonoff

focuses on functional transducers and enforces this property at compilation time. Any arising nondeterministic output is automatically rejected as an error. This allows Solomonoff to perform a lot more optimisations, the automata are smaller and their behaviour is more predictable. Moreover, lexicographic weights allow for precise disambiguation of nondeterministic paths whenever necessary and their most important advantage is that lexicographic semiring is not commutative and hence it does not "propagate" throughout entire automaton. This only increases Solomonoff's robustness and allows for (exponentially) smaller automata, without sacrificing predictability of the regular expression. On th contrary, probabilistic weights in Thrax, make the whole system, more heavyweight, unpredictable and difficult to maintain. The results are especially palpable when comparing our benchmarks. Solomonoff was written in Java and Thrax uses C++, but despite this our compiler is several magnitudes more efficient. We performed efficiency tests on a large corpus of linguistic data (dictionary with 6000 records). Solomonoff compilation times were around 2 seconds, whereas Thrax took 19 minutes. Solomonoffs automata were also much smaller, as thanks to Glushkov's construction, there is a 1:1 relationship between size of regular expression and size of transducer. As a result Thrax's transducer takes of 6336K of RAM, whereas Solomonoff only takes 738K. Execution time for such large corpora was about 10 miliseconds in Solomonoff (which is roughly the same performance as using Java's HashMap), while Thrax took about 250 miliseconds. One might argue that, such great differences are achievable, only because Thrax supports a lot more features. OpenFST uses epsilon transitions, while Solomonoff does not implement them and instead all automata are always and directly produced in epsilon-free form. OpenFST performs operations such as sorting of edges, determinization, epsilon-removal and minimization. Solomonoff always has all of its edges sorted and it doesn't need a special routine for it (which makes for additional performance gains), it has no epsilons to remove and it does not need determinization procedure, because it can pseudo-minimise nondeterministic transducers, using heuristics inspired by Kameda-Weiner's NFA minimization. One could, half-jokingly, summarize that the difference between Thrax and Solomonoff is like that between "Android and Apple". Thrax wants to support "all of the features at all cost", whereas Solomonoff carefully chooses the right features to support. We believe this approach will be our strongest asset, that will make our compiler a serious alternative to the older and more established OpenFST. An additional strength that favours Solomonoff over OpenFST is that we support inductive inference out-of-the-box, require no programming (regular expressions are the primary user interface and Java API is minimal and optional) and we provide automatic

conversion from Thrax to Solomonoff, so that existing codebases can be easily migrated.

The characteristic feature of Solomonoff, is that its development focuses on the compiler and regular expressions instead of library API. Everything can be done without writing any Java code. It also allows the developers for much greater flexibility, because the internal implementation can be drastically changed at any time, without breaking existing regular expressions. There is very little public API that needs to be maintained. As a result backwards-compatibility is rarely an issue.

The primary philosophy used in implementing this library is the top-down approach. Features are added conservatively in a well thought-through manner. No features will be added ad-hoc. Everything is meant to fit well together and follow some greater design strategy. For comparison, consider the difference between OpenFst and Solomonoff.

— OpenFst has Matcher that was meant to compactify ranges. In Solomonoff all transitions are ranged and follow the theory of (S,k)-automata. They are well integrated with regular expressions and Glushkov's construction. They allow for more efficient squaring and subset construction. Instead of being an ad-hoc feature, they are well integrated everywhere.
— OpenFst had no built-in support for regular expression and it was added only later in form of Thrax grammars, that aren't much more than another API for calling library functions. In Solomonoff the regular expressions are the primary and only interface. Instead of having separate procedures for union, concatenation and Kleene closure, there is only one procedure that takes arbitrary regular expression and compiles it in batches. This way everything works much faster, doesn't lead to introduction of any $\epsilon$-transitions and allows for more optimisation strategies by AST manipulations. This leads to significant differences in performance.

While, most of other automata libraries (RE2/BRICKS) were not designed for large codebases and have no build system, Solomonoff comes with it's very own build system out-of-the-box. Thrax is the only alternative that does have a "build system" but it's very primitive and relies on generating Makefiles. Solomonoff has a well integrated tool that assembles large projects, detects cyclic dependencies, allows for parallel compilation and performs additional code optimisations. It also ships with syntax highlighting and tools that assist code refactoring. Our project strives to make automata as easy to use and accessible to mass audiences as possible. For this reason we developed a website with online playground where visitors could test Solomonoff and experiment without

any setup required. The backend technology we used is Spring Boot, because it allowed for convenient integration with existing API in Java.

While at the beginning our attempts focused on implementing the library is C, switching to Java turned out to be a major advantage. While, it's true that C allows for more manual optimisations, having a garbage collector proved to be a strong asset. The compilation relies on building automata in form of directed graphs. Each vertex itself is a separate Java object. The automaton is always kept trim because all the unreachable vertices are lost and free to be garbage collected at any time. If we tried to implement such data structure in C we would need to reinvent a simple (and most likely, less efficient) garbage collector ourselves. Manipulation of linked lists, linked graphs and other recursive data structures is not as convenient in C as it is in Java. Another advantage that Java gave us over C is that the plenty of existing infrastructure was already implemented in Java. Hence we became more compatible with Samsung's systems where Solomonoff could be deployed. We could also easily integrate our compiler with LearnLib.

For environments where Java is not an option, Solomonoff will allow for generating automata in form of C header files that can be easily included and deployed in production code. It will not allow for manipulation of automata from C level, but our compiler was never intended to be used programmatically. There is, in fact, not much benefit from doing so. If user needs to create their custom automata manually, it's much easier to generate them in AT&T format and then have them read by the compiler. Later they can by manipulated with regular expressions like any other automaton. Generating AT&T has also additional benefits, as such custom script for generating AT&T can be attached to Solomonoff build system as an external routine and then compilation can be optimised more efficiently. An example scenario would be having two independent routines that the build system could decide to run in parallel and then it could cache the results for subsequent rebuilds. For comparison, if anybody decided to use OpenFST manually from from C++, they would need to first learn the API (which is less user friendly than learning AT&T format) and then they would also need to implement caching and parallelization themselves. Moreover, the users are also tied to C++, whereas, AT&T format can be generated in any language. This shows, why API for programmatic manipulation offers little to no benefit, while imposes much more limitations. It should also be mentioned that Solomonoff allows user for implementing their own Java functions that could then be incorporated into the regular expressions as "native" calls.

To guarantee the better utility of Solomonoff, it has to provide ability of interactive evaluation in order to enable efficient and convenient work with the compiler. It can be also vastly useful while learning. The mentioned feature is

implemented with a REPL. The biggest challenge here is adjusting the entire stack of compilation processes to work interactively with a sensible manner. The other condition of usefulness is a fast build automation. To make our system meet the need it has to support both parallel compilation and caching of previously compiled units. Also the ability to resolve dependencies may be be important in some more complex projects. It this case the main challenge is to find satisfying solution of ordering in a parallel compilation. This problem can be broken down to a topological sort in a direct acyclic graph. To make the use of Solomonoff even more practical it provides an export option for transducers in a way that allows to include them into a C code during a recompilation phase.

Rozdział 2

# Transducers

# Abstract

Glushkov's construction has many interesting properties and they become even more evident when applied to transducers. This article strives to show the vast range of possible extensions and optimisations for this algorithm. Special flavour of regular expressions is introduced, which can be efficiently converted to $\epsilon$-free functional subsequential weighted finite state transducers. Produced automata are very compact, as they contain only one state for each symbol (from input alphabet) of original expression and only one transition for each range of symbols, no matter how large. Such compactified ranges of transitions allow for efficient binary search lookup during automaton evaluation. All the methods and algorithms presented here were used to implement open-source compiler of regular expressions for multitape transducers.

are not many open source solutions available for working with transducers. The most significant and widely used library is OpenFst. Their approach is based on theory of weighted automata[**?**][**?**][**?**]. Here we propose an alternative approach founded on lexicographic transducers [**?**] and Glushkov's algorithm [**?**].

Let $W$ be some set of weight symbols. The free monoid $W^*$ will be out set of weight strings. We assume there is some lexicographic order defined as

$$b_1 w_1 > b_2 w_2 \iff w_1 > w_2 \text{ or } (w_1 = w_2 \text{ and } b_1 > b_2)$$

where $w_1, w_1 \in W$ and $b_1, b_2 \in W^*$. The order is defined only on strings of equal lengths. Let $\Sigma$ be the input alphabet, $\Sigma^*$ is the monoid of input strings and $D$ is the monoid of output strings. Lexicographic transducer is defined as tuple $(Q, I, W, \Sigma, D, \delta, \tau)$ where $Q$ is some finite set of states, $I$ is the set of initial states, $\tau$ is a state output (partial) function $Q \to D \times W$ and lastly $\delta$ represents transitions of the form $\delta \subset Q \times W \times \Sigma \times D \times Q$.

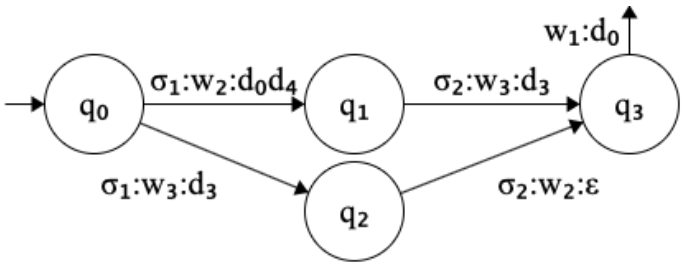
Thanks to $\tau$, such machines are subsequential [**?**][**?**][**?**][**?**]. As an example consider the simple transducer from figure 2.1. The states $q_0$, $q_1$ and $q_2$ have no output, which can be denoted with $\tau(q_0) = \emptyset$. The only set which does have

output is $q_3$. Every time automaton finishes reading input string and reaches $q_3$, it will append $d_0$ to its output and then accept. For instance, on input $\sigma_1\sigma_2$ it will first read $\sigma_1$, produce output $d_0d_4$ and go to state $q_1$, then read $\sigma_2$ and append output $d_3$, go to state $q_3$, finally reaching end of input, appending $d_0$ and accepting. The total output would be $d_0d_4d_3d_0$. Note that the automaton is nondeterministic, as it could take alternative route passing through $q_2$ and producing $d_3d_0$. In such scenarios weights are used to disambiguate output. The first route produces weight string $w_2w_3w_1$, while the second produces $w_3w_2w_1$. According to our definition of lexicographic order we have $w_2w_3w_1 > w_3w_2w_1$ (assuming that $w_3 > w_2$). Throughout this article we will consider smaller weights to be "better". Hence the automaton should choose $d_3d_0$ as the definitive output for input $\sigma_1\sigma_2$. There might be situations in which two different routes have the exact same (equally highest) weight while also producing different outputs. In such cases, the automaton is ambiguous and produces multiple outputs for one input.

## 2.1. Expressive power

There are some remarks to be made about lexicographic transducers. They recognize relations on languages, unlike "plain" finite state automata (FSA) which recognize languages. If $M$ is some transducer, then we denote its recognized relation with $\mathcal{L}(M)$. Those relations are subsets of $\Sigma^* \times D$. The set of strings $\Sigma^*$ accepted by $M$ must be a regular language (indeed, if we erased output labels, we would as a result obtain FSA). The weights are erasable [**?**] in the sense that, give any lexicographic transducer we can always build an equivalent automaton without weighted transitions. If we didn't have $\tau$, the only output possible to be expressed for empty input would be an empty string as well. With $\tau$ we can express pairs like $(\epsilon, d) \in \mathcal{L}$ where $d \neq \epsilon$.

The transducers can return at most finitely many outputs for any given input (see *infinite superposition*[**?**]). If we allowed for $\epsilon$-transitions (transitions that have $\epsilon$ as input label) we could build $\epsilon$-cycles and produce infinitely many outputs. However, automata that do so are not very interesting from practical point of view. Therefore we shall focus only on functional transducers, that is those which produce at most one output. If automaton does not have any $\epsilon$-cycles and is functional, then it's possible to erase all $\epsilon$-transitions (note that it would not be possible without $\tau$, because $\epsilon$-transitions allow for producing output given empty input). Therefore $\epsilon$-transitions don't increase power of functional transducers.




**Rysunek 2.1.** Example of lexicographic transducer. State $q_0$ is initial. State $q_3$ in accepting, in the sense that $\tau(q_3) = (w_1, d_0)$. The remaining states have state output $\emptyset$.

We say that transducer has **conflicting states** $q_1$ and $q_2$ if it's possible to reach both of them simultaneously (there are two possible routes with the same inputs and weights) given some input $\sigma$ and there is some another state $q_3$ to which both of those states can transition over the same input symbol $\sigma_i$. Alternatively, there might be no third state $q_3$, but instead both $q_1$ and $q_2$ have non-empty $\tau$ output (so $\tau$ can in a sense be treated like $q_3$). We say that transitions $(q_1, \sigma_i, w, d, q_3)$ and $(q_2, \sigma_i, w', d, q_3)$ are **weight-conflicting** if they have equal weights $w = w'$. For instance in figure 2.1 the states $q_1$ and $q_2$ are indeed conflicting because they both transition to $q_3$ over $\sigma_2$ but their transitions are not weight conflicting. It can be shown that transducers without weight-conflicting transitions are functional. Moreover, if a transducer is functional but contains weight-conflicting transitions, then the weights can be reassigned in such a way that eliminates all conflicts [**?**]. The only requirement is that there are enough symbols in $W$ (for instance, if $W$ had only one symbol, then all transitions of conflicting states would always be weight-conflicting). If there are at least as many weight symbols as there are states $|W| = |Q|$, then every functional transducer on $|W|$ states can be built without weight-conflicting transitions. For convenience we can assume that $W = \mathbb{N}$, but in practice all algorithms presented here will work with bounded $W$. Hence transducers without weight-conflicting transitions are equivalent in power to functional transducers. This is important because by searching for weight-conflicting transitions we can efficiently test whether transducer is functional or not.

## 2.2. Ranged automata

Often when implementing automata the algorithm behind $\delta$ function needs to efficiently find the right transition for a given $\sigma$ symbol. It's beneficial to optimise UNIX-style ranges like `[0-9]` or `[a-z]` as they arise often in practical settings. Even the `.` wildcard can be treated as one large range spanning entire $\Sigma$. If the alphabet is large (like ASCII or UNICODE), then checking every one of them in a loop is not feasible. A significant improvement can be made by only checking two inequalities like $\sigma_1 \ x \ \sigma_{10}$, instead of large number of equalities. The current paper presents a way in which simplified model of $(\mathcal{S}, k)$-automata**[?][?]**, can be used to obtain major improvements. In particular we consider only automata that don't have any registers apart from constant values, that is $k = 0$. Therefore we provide a more specialized definition of "ranged automata".

Let $\Sigma$ be the (not necessarily finite) alphabet of automaton. Let $\chi$ be the set of subsets of $\Sigma$ that we will call **ranges** of $\Sigma$. Let $\overline{\chi}$ be the closure of $\chi$ under countable union and complementation (so it forms a sigma algebra). For instance, imagine that there is total order on $\Sigma$ and $\chi$ is the set of all intervals in $\Sigma$. Now we want to build an automaton whose transitions are not labelled with symbols from $\Sigma$, but rather with ranges from $\chi$. Union $\chi_0 \cup \chi_1$ of two elements from $\chi$ "semantically" corresponds to putting two edges, $(q, \chi_0, q') \in \delta$ (for a moment forget about outputs and weights) and $(q, \chi_1, q') \in \delta$. There is no limitation on the size of $\delta$. It might be countably infinite, hence it's natural that $\overline{\chi}$ should be closed under countable union. Therefore, $\chi$ is the set of allowed transition labels and $\overline{\chi}$ is the set of all possible "semantic" transitions. We could say that $\overline{\chi}$ is discrete if it contains every subset of $\Sigma$. An example of discrete $\overline{\chi}$ would be finite set $\Sigma$ with all UNIX-style ranges `[`$\sigma$`-`$\sigma'$`]` included in $\chi$.

Another example would be set $\Sigma = \mathbb{R}$ with $\chi$ consisting of all ranges, whose ends are computable real numbers (real number $x$ is computable if the predicate $q < x$ is decidable for all rational numbers $q$). If we also restricted $\delta$ to be a finite set, then we could build effective automata that work with real numbers of arbitrary precision.

## 2.3. Regular expressions

Here we describe a flavour of regular expressions specifically extended to interplay with lexicographic transducers and ranged automata.

Transducers with input $\Sigma^*$ and output $\Gamma^*$ can be seen as FSA working with single input $\Sigma^* \times \Gamma^*$. Therefore we can treat every pair of symbols $(\sigma, \gamma)$ as an

atomic formula of regular expressions for transducers. We can use concatenation $(\sigma, \gamma_0)(\epsilon, \gamma_1)$ to represent $(\sigma, \gamma_0\gamma_1)$. It's possible to create ambiguous transducers with unions like $(\epsilon, \gamma_0) + (\epsilon, \gamma_1)$. To make notation easier, we will treat every $\sigma$ as $(\sigma, \epsilon)$ and every $\gamma$ as $(\epsilon, \gamma)$. Then instead of writing lengthy $(\sigma, \epsilon)(\epsilon, \gamma)$ we could introduce shortened notation $\sigma : \gamma$. Because we would like to avoid ambiguous transducers we can put restriction that the right side of : should always be a string of $\Gamma^*$ and writing entire formulas (like $\sigma : \gamma_1 + \gamma_2^*$) is not allowed. This restriction will later simplify Glushkov's algorithm.

We define $\mathcal{A}^\Sigma$ to be the set of atomic characters. For instance we could choose $\mathcal{A}^\Sigma = \Sigma \cup \{\epsilon\}$ for FSA/transducers and $\mathcal{A}^\Sigma = \chi$ for ranged automata.

We call $RE^{\Sigma:D}$ the set of all regular expression formulas with underlying set of atomic characters $\mathcal{A}^\Sigma$ and allowed output strings $D$. It's possible that $D$ might be a singleton monoid $\{\epsilon\}$ but it should not be empty, because then no element would belong to $\Sigma^* \times D$. By inductive definition, if $\phi$ and $\psi$ are $RE^{\Sigma:D}$ formulas and $d \in D$, then union $\phi + \psi$, concatenation $\phi \cdot \psi$, Kleene closure $\phi^*$ and output concatenation $\phi : d$ are $RE^{\Sigma:D}$ formulas as well. Define $V^{\Sigma:D} : RE^{\Sigma:D} \to \Sigma^* \times D$ to be the valuation function:
$V^{\Sigma:D}(\phi + \psi) = V^{\Sigma:D}(\phi) \cup V^{\Sigma:D}(\psi)$
$V^{\Sigma:D}(\phi \cdot \psi) = V^{\Sigma:D}(\phi) \cdot V^{\Sigma:D}(\psi)$
$V^{\Sigma:D}(\phi^*) = (\epsilon, \epsilon) + V^{\Sigma:D}(\phi) + V^{\Sigma:D}(\phi)^2 + ...$
$V^{\Sigma:D}(\phi : d) = V^{\Sigma:D}(\phi) \cdot (\epsilon, d)$
$V^{\Sigma:D}(a) = a$ where $a \in \mathcal{A}^{\Sigma:D}$
Some notable properties are:
$x : y_0 + x : y_1 = x : (y_0 + y_1)$
$x : \epsilon + x : y + x : y^2 ... = x : y^*$
$(x : y_0)(\epsilon : y_1) = x : (y_0 y_1)$
$x_0 : (y_0 y') + x_1 : (y_1 y') = (x_0 : y_0 + x_1 : y1) \cdot (\epsilon : y')$
$x_0 : (y' y_0) + x_1 : (y' y_1) = (\epsilon : y') \cdot (x_0 : y_0 + x_1 : y1)$
Therefore we can see that expressive power with and without : is the same.

It's also possible to extend regular expressions with weights. Let $RE_W^{\Sigma:D}$ be a superset of $RE^{\Sigma:D}$ and $W$ be the set of weight symbols. If $\phi \in RE_W^{\Sigma \to D}$ and $w_0, w_1 \in W$ then $w_0\phi$ and $\phi w_1$ are in $RE_W^{\Sigma \to D}$. This allows for inserting weight at any place. For instance, the automaton from figure 2.1 could be expressed using

$$((\sigma_1 : d_0 d_4) w_2 (\sigma_2 : d_3) w_3 + (\sigma_1 : d_3) w_3 \sigma_2 w_2) : d_0$$

The definition of $V^{\Sigma:D}(\phi w)$ depends largely on $W$ but associativity $(\phi w_1) w_2 = \phi(w_1 + w_2)$ should be preserved, given that $W$ is a multiplicative monoid. This also implies that $w_1 \epsilon w_2 = w_1 w_2$, which is semantically equivalent to the addition $w_1 + w_2$.

We showed that regular expressions for transducers can be expressed using pairs of symbols $(\sigma, \gamma)$. There is an alternative approach. We can encode both input and output string by interleaving their symbols like $\sigma_1\gamma_1\sigma_2\gamma_2$. Such regular expressions "recognize" relations rather than "generate" them. This approach has one significant problem. We have to keep track of the order. For instance, this $(\sigma_1\gamma_1\sigma_2 + \sigma_3)\gamma_4$ is a valid interleaved expression but this is not $(\sigma_1\gamma_1 + \sigma_3)\gamma_4$.

In order to decide whether an interleaved regular expression is valid, we should annotate every symbol with its respective alphabet (like $(\sigma_1^{\Sigma}\gamma_1^{\Gamma}\sigma_2^{\Sigma} + \sigma_3^{\Sigma})\gamma_4^{\Gamma}$). Then we rewrite the expression, treating alphabets themselves as the new symbols (for instance $(\Sigma\Gamma\Sigma + \Sigma)\Gamma$). If the language recognized by such expression is a subset of $(\Sigma\Gamma)^*$, then the interleaved expression valid.

This leads us to introduce *interleaved alphabets*. We should notice that $(\Sigma\Gamma)^*$ is in fact a local language. What it means is that in order to define interleaved alphabet we need 3 sets - set of initial alphabets $U$, set of allowed 2-factors of $V$ and set of final alphabets $W$. Moreover all the elements of $U$ must be pairwise disjoint alphabets. Similarly for $V$ if $(\Sigma_1, \Sigma_2) \in V$ and $(\Sigma_1, \Sigma_3) \in V$ then $\Sigma_2$ and $\Sigma_3$ must be disjoint. (For instance, in case of $(\Sigma\Gamma)^*$ we have $U = \{\Sigma\}$, $V = \{(\Sigma, \Gamma)\}$ and $W = \Gamma$).

With interleaved alphabets we can encode much more complex "multitape automata". In fact it has certain resemblance to recursive algebraic data structures built from products (like $\{(\Sigma, \Gamma)\}$ in $V$) and coproducts (like $\{(\Sigma, \Gamma_1), (\Sigma, \Gamma_2)\} \in V$) .

It's possible to use interleaved alphabets together with $RE_W^{\Sigma;D}$ to express multitape inputs and mutitape outputs.

## 2.4. Extended Glushkov's construction

The core result of this paper is Glushkov's algorithm capable of producing very compact, $\epsilon$-free, weighted, ranged, functional, multitape transducers and automatically check if any regular expression is valid, when given specification of interleaved alphabets.

Let $\phi$ be some $RE_W^{\Sigma;D}$ formula. We will call $\Sigma$ the *universal alphabet*. We also admit several subaphabets $\Sigma_1, \Sigma_2, ...$ all of which are subsets of $\Sigma$. Each $\Sigma_i$ admits their own set of atomic characters $\mathcal{A}^{\Sigma_i}$ and we require that $\mathcal{A}^{\Sigma_i} \subset \mathcal{A}^{\Sigma}$. Let $U_\Sigma, V_\Sigma, W_\Sigma$ be the interleaved alphabet consisting of all the subalphabets. For example $\Sigma$ could be the set of all 64-bit integers and then $V_\Sigma$ could contain its subsets like ASCII, UNICODE or binary alphabet $\{0, 1\}$ (possibly with offsets to ensure disjointness). In cases when $D = \Gamma^*$, we can similarly define $U_\Gamma, V_\Gamma, W_\Gamma$,

but there might be cases where $D$ is more a exotic set (like real numbers) and interleaved alphabet's don't make much sense. Moreover, we require $W$ to be a semiring. For instance, lexicographic weights have concatenation as multiplicative operation and *min* is used for addition.

First step of Glushkov's algorithm is to create a new alphabet $\Omega$ in which every atomic character (including duplicates but excluding $\epsilon$) in $\phi$ is treated as a new individual character. As a result we should obtain new rewritten formula $\psi \in RE_W^{\Omega \to D}$ along with mapping $\alpha : \Omega \to \mathcal{A}^\Sigma$. This mapping will remember the original atomic character, before it was rewritten to unique symbol in $\Omega$. For example

$$\phi = (\epsilon : x_0)x_0(x_0 : x_1x_3)x_3w_0 + (x_1x_2)^*w_1$$

will be rewritten as

$$\psi = (\epsilon : x_0)\omega_1(\omega_2 : x_1x_3)\omega_3w_0 + (\omega_4\omega_5)^*w_1$$

with $\alpha = \{(\omega_1, x_0), (\omega_2, x_0), (\omega_3, x_3), (\omega_4, x_1), (\omega_5, x_2)\}$.

Every element $x$ of $\mathcal{A}^\Sigma$ may also be member of several subalphabets. For simplicity we can assume that all expressions are annotated and we know exactly which subalphabet a given $x$ belongs to. In practice, we would try to infer the annotation automatically and ask user to manually annotate symbols only when necessary.

Next step is to define function $\Lambda : RE_W^{\Omega \to D} \rightharpoonup (D \times W)$. It returns the output produced for empty word $\epsilon$ (if any) and weight associated with it. (We use symbol $\rightharpoonup$ to highlight the fact that $\Lambda$ is a partial function and may fail for ambiguous transducers.) For instance in the previous example empty word can be matched and the returned output and weight is $(\epsilon, w_1)$. Because both $D$ and $W$ are monoids, we can treat $D \times W$ like a monoid defined as $(y_0, w_0) \cdot (y_1, w_1) = (y_0y_1, w_0 + w_1)$. We also admit $\emptyset$ as multiplicative zero, which means that $(y_0, w_0) \cdot \emptyset = \emptyset$. We denote $W$'s neutral element as 0. This facilitates recursive definition:
$\Lambda(\psi_0 + \psi_1) = \Lambda(\psi_0) \cup \Lambda(\psi_1)$ if at least one of the sides is $\emptyset$, otherwise error
$\Lambda(\psi_0\psi_1) = \Lambda(\psi_0) \cdot \Lambda(\psi_1)$
$\Lambda(\psi_0 : y) = \Lambda(\psi_0) \cdot (y, 0)$
$\Lambda(\psi_0 w) = \Lambda(\psi_0) \cdot (\epsilon, w)$
$\Lambda(w\psi_0) = \Lambda(\psi_0) \cdot (\epsilon, w)$
$\Lambda(\psi_0^*) = (\epsilon, 0)$ if $(\epsilon, w) = \Lambda(\psi_0)$ or $\emptyset = \Lambda(\psi_0)$, otherwise error
$\Lambda(\epsilon) = (\epsilon, 0)$
$\Lambda(\omega) = \epsilon$ where $\omega \in \Omega$

Next step is to define $B : RE_W^{\Omega \to D} \to (\Omega \rightharpoonup D \times W)$ which for a given formula $\psi$ returns set of $\Omega$ characters that can be found as the first in any

string of $V^{\Omega\to D}(\psi)$ and to each such character we associate output produced "before" reaching it. For instance, in the previous example of $\psi$ there are two characters that can be found at the beginning: $\omega_1$ and $\omega_4$. Additionally, there is $\epsilon$ which prints output $x_0$ before reaching $\omega_1$. Therefore $(\omega_1, (x_0, 0))$ and $(\omega_3, (\epsilon, 0))$ are the result of $B(\psi)$. For better readability, we admit operation of multiplication $\cdot : (\Omega \rightharpoonup D \times W) \times (D \times W) \to (\Omega \rightharpoonup D \times W)$ that performs monoid multiplication on all $D \times W$ elements returned by $\Omega \rightharpoonup D \times W$.
$B(\psi_0 + \psi_1) = B(\psi_0) \cup B(\psi_1)$
$B(\psi_0\psi_1) = B(\psi_0) \cup \Lambda(\psi_0) \cdot B(\psi_1)$
$B(\psi_0 w) = B(\psi_0)$
$B(w\psi_0) = (\epsilon, w) \cdot B(\psi_0)$
$B(\psi_0^*) = B(\psi_0)$
$B(\psi_0 : d) = B(\psi_0)$
$B(\epsilon) = \emptyset$
$B(\omega) = \{(\omega, (\epsilon, 0))\}$
It's worth noting that $B(\psi_0) \cup B(\psi_1)$ always yields function (instead of relation) because every $\Omega$ character appears in $\psi$ only once and it cannot be both in $\psi_0$ and $\psi_1$.

Next step is to define $E : RE_W^{\Omega\to D} \to (\Omega \rightharpoonup D \times W)$, which is very similar to $B$, except that $E$ collects characters found at the end of strings. In our example it would be $(\omega_3, (\epsilon, w_0))$ and $(\omega_5, (\epsilon, w_1))$. Recursive definition is as follows:
$E(\psi_0 + \psi_1) = E(\psi_0) \cup E(\psi_1)$
$E(\psi_0\psi_1) = E(\psi_0) \cdot \Lambda(\psi_1) \cup B(\psi_1)$
$E(\psi_0 w) = E(\psi_0) \cdot (\epsilon, w)$
$E(w\psi_0) = E(\psi_0)$
$E(\psi_0^*) = E(\psi_0)$
$E(\psi_0 : d) = E(\psi_0) \cdot (d, 0)$
$E(\epsilon) = \emptyset$
$E(\omega) = \{(\omega, (\epsilon, 0))\}$

Next step is to use $B$ and $E$ to determine all two-character substrings that can be encountered in $V^{\Omega\to D}(\psi)$. Given two functions $b, e : \Omega \rightharpoonup D \times W$ we define product $b \times e : \Omega \times \Omega \rightharpoonup D \times W$ such that for any $(\omega_0, (y_0, w_0)) \in b$ and $(\omega_1, (y_1, w_1)) \in c$ there is $((\omega_0, \omega_1), (y_0 y_1, w_0 + w_1)) \in b \times e$. Then define $L : RE_W^{\Omega\to D} \to (\Omega \times \Omega \rightharpoonup D \times W)$ as:
$L(\psi_0 + \psi_1) = L(\psi_0) \cup L(\psi_1)$
$L(\psi_0\psi_1) = L(\psi_0) \cup L(\psi_1) \cup E(\psi_0) \times B(\psi_1)$
$L(\psi_0 w) = L(\psi_0)$
$L(w\psi_0) = L(\psi_0)$
$L(\psi_0^*) = L(\psi_0) \cup E(\psi_0) \times B(\psi_0)$

$L(\psi_0 : d) = L(\psi_0)$
$L(\epsilon) = \emptyset$
$L(\omega) = \emptyset$
One should notice that all the partial functions produced by $B$, $E$ and $L$ have finite domains, therefore they are effective objects from computational point of view.

The last step is to use results of $L, B, E, \Lambda$ and $\alpha$ to produce automaton $(Q, q_\epsilon, W, \Sigma, D, \delta, \tau)$ with
$\delta : Q \times \Sigma \to (Q \rightharpoonup D \times W)$
$\tau : Q \rightharpoonup D \times W$
$Q = \{q_\omega : \omega \in \Omega\} \cup \{q_\epsilon\}$
$\tau = E(\psi)$
$(q_{\omega_0}, \alpha(\omega_1), q_{\omega_1}, d, w) \in \delta$ for every $(\omega_0, \omega_1, d, w) \in L(\psi)$
$(q_\epsilon, \alpha(\omega), q_\omega, d, w) \in \delta$ for every $(\omega, d, w) \in B(\psi)$

This concludes the Glushkov's construction. Now it's possible to use specification $U_\Sigma$, $V_\Sigma$ and $W_\Sigma$ of interleaved alphabet to check if regular expression was correct. We can treat alphabets $\Sigma_1, \Sigma_2, ...$ as colours and then colour each state with it's respective alphabet. If transition leads from state of colour $\Sigma_i$ to $\Sigma_j$ then we check that the pair $(\Sigma_i, \Sigma_j)$ is indeed present in $V_\Sigma$. Similarly we check colours of initial and accepting states.

## 2.5. Optimisations

The above construction can detect some obvious cases of ambiguous transducers, but it doesn't give us complete guarantee. We can check in quadratic time[**?**] for weight conflicting transitions to be sure. If there are none, then transducer must be functional. If we find at least one, it doesn't immediately imply that the transducer is ambiguous. In such cases we can warn the user and demand additional weight annotations in the regular expression.

When $\mathcal{A}^\Sigma$ consists of all possible ranges $\chi$, then the obtained $\delta$ is of the form $Q \times W \times \chi \times D \times Q$. While, theoretically equivalent to $Q \times W \times \Sigma \times D \times Q$, in practice it allows for more efficient implementations. For instance given two ranges `[1-50]` and `[20-80]`, we do not need to check equality for all 80 numbers. The only points worth checking are $1, 50, 20, 80$. Let's arrange them in some sorted array. Then given any number $x$, we can use binary search to find which of those points is closest to $x$ and then lookup the full list of intervals that $x$ is a member of. This approach works even for real numbers. More precise algorithm can be given a follows. Let $(x_0, y_0), (x_1, y_1), ...(x_n, y_n)$ be closed ranges.

Build an array P sorted in ascending order that contains all $y_i$ and also for every $x_i$ contains the largest element of $\Sigma$ smaller than $x_i$ (or more generally the supremum). Build a second array R that to every $i^{th}$ element of R assign list of ranges containing $i^{th}$ element of P. Then in order to find ranges containing any $x$, run binary search on P that returns index of the largest element smaller or equal to $x$. Then lookup the list of ranges in R.

In Glushkov's construction epsilons are not rewritten to $\Omega$, which means that there are also no $\epsilon$-transitions. Hence we can use dynamic programming to efficiently evaluate automaton for any input string $x \in \Sigma^*$. The algorithm is as follows. Create two dimensional array $c_{i,j}$ of size $|Q| \times (|x| + 1)$ where $i$-th column represents all nondeterministically reached states after reading first $i - 1$ symbols. Each cell should hold information about the previously used transition. This also tells us the weight, output and source state of transition. For instance cell $c_{i,j} = k$ should encode transition coming from state $k$ to state $i$, after reading $j - 1^{th}$ symbol. If state $q_i \in Q$ does not belong to $j^{th}$ superposition, then $c_{i,j} = \emptyset$. The first column is initialized with arbitrary value at $c_{i,1} = -1$ for $i$ referring to initial state $q_\epsilon$ and set to $c_{i,1} = \emptyset$ for all other $i$. Then algorithm progresses building next column from previous one. After filling out the entire array. The last column should be checked for any accepting states according to $\tau$. There might be many of them but the one with largest weight should be chosen. If we checked that the automaton has no weight-conflicting transitions, then there should always be only one maximal weight. Finally we can backtrack, to find out which path "won". This will determine what outputs need to be concatenated together to obtain path's output. This algorithm is quadratic $O(|Q|, |x|)$, but in practice each iteration itself is very efficient, especially when combined with binary search described in previous paragraph. By observing that states of automata are often sparsely connected, additional optimisation can be made by representing the two dimensional array with list of indices, as it's often done for sparse matrices.

## 2.6. Conclusion

Interleaved alphabets could find numerous applications with many possible extensions. In the context of natural language processing, they could be used to annotate human sentences with linguistic meta-information like parts of speech. Then transducers could built to take advantage of those tag. Using grammatical inference methods, one could also train such transducers to as POS taggers.

This approach cannot fully replace OpenFST, because it lacks their flexibility.

The goal of OpenFST is to provide general and extensible implementation of many different transducer's, whereas the approach presented in this paper sacrifices extensibility for highly integrated design and optimal efficiency. For instance, Glushkov's algorithm could never support such operations as inverses, projections, reverses or composition.

Rozdział 3

# Build system

Rozdział 4

# Web technologies