Chapter 4: Vector Spherical Harmonics

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1 Representation 2 of Vector Fields

The second representation of vector fields include E_{lm}, F_{lm}, and C_{lm}.

E_{lm}: vector components from the gradient of a potential field from a planet.
F_{lm}: vector components from the gradient of a potential field from outside the satellite radius (space).
C_{lm}: same as in representation 1.

1.1 Example One:

Let's calculate the spherical harmonics first for E_{lm}. Let's set some parameters:

```
L = 2;
theta = 0:0.1:pi;
phi = 0:0.1:2*pi;
```

We can now calculate them by running:

```
[E,theta,phi] = elm(L,theta,phi);
```

This provides us with `E`, which contains the three components of the E_{lm} vector spherical harmonics for degrees $l = 0$ to $l = L$. `E{1}` contains $(L+1)^2$ pixel maps for the radial component. Each row of E1 is a pixel map represented as a vector of numbers such that we when we reshape............**SKYPE

`E{2}` contains $(L+1)^2$ pixel maps for the colatitudinal components and `E{3}` contains $(L+1)^2$ pixel maps for the longitudinal components. From the `help` function, we can see that `elm` function provides the pixel map values in the *addmout* format.

Let's plot the radial component of E_{lm} with `l=2` and `m=-1`. With the *addmout* format, we will have this as the sixth row, which is denoted as `E{1}(6,:)`. Our first step would be to label the radial, colatitudinal, and longitudinal components. The best way to do this would be to also create an index such that we can change it easily:

```
index = 6;
E2m1_rad = E{1}(index,:);
E2m1_theta = E{2}(index,:);
E2m1_phi = E{3}(index,:);
```

Now we can reshape this into a pixel matrix by running:

```
P = reshape(E2m1_rad,length(theta),length(phi));
```

Now we can plot `E2m1_rad` on the Mallweide projection******:

```
plotplm(P,phi,pi/2-theta,1)
kelicol(1)
```

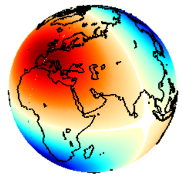
We can also plot the colatitudinal component with the same (l,m). We will need to reshape this as above and plot:

```
U = reshape(E2m1_theta,length(theta),length(phi));
plotplm(U,phi,pi/2-theta,1)
```

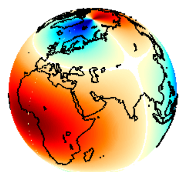
Plot the longitudinal component by running:

```
V = reshape(E2m1_phi,length(theta),length(phi));
plotplm(V,phi,pi/2-theta,1)
```

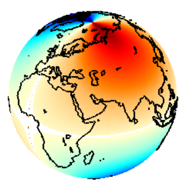
Compare your plots to figure 1.



(a) Radial component



(b) Colatitudinal component



(c) Longitudinal component

Figure 1: Vector spherical harmonic with E_{2-1} plot of the radial component in (a), colatitudinal component in (b), and the longitudinal component in (c).

We can also plot each component on one figure. To do this, we will need to use a function called `quiver`, which can be used as the `help` function describes. Open a new figure and run the following:

```
plotplm(P,phi,pi/2-theta,4);
kelicol(1)
caxis([-1,1]*max(abs(caxis)))
hold on
quiver(phi*180/pi,90-theta*180/pi,U,V,'k','LineWidth',1)
hold off
```

This will produce figure ......

We can plot another (l,m), say E_{20}, which would be the the seventh row in E_{lm}. Let's look at the radial component:

```
index = 7;
E20_rad = reshape(E{1}(index,:),length(theta),length(phi));
plotplm(E20_rad,phi,pi/2-theta,1)
```

Compare your plot to figure 2.

Figure 2: Vector spherical harmonic with E_{20} plot of the radial component.

1.2 Linear Combinations of Vector Spherical Harmonics

Exercise: Let's try a linear combination, choose: $2E_{00} - 1.5E_{10} + 3E_{11}$. We can plot this in a couple different ways, either by first reshaping each of the E_{lm} values then make the linear combination or make the linear combination first then reshape it. Try both and compare the results. For the sake of computation, let's try the second way:

```
lincomb = 2*E{1}(1,:)-1.5*E{1}(3,:)+3*E{1}(4,:);
linr = reshape(lincomb,length(theta),length(phi));
```

Now we can plot:

```
plotplm(linr,[],[],2)
```

Compare your plot to figure 3.

Currently working on this part.

Figure 3: Linear combination of vector spherical harmonics, $2E_{00} - 1.5E_{10} + 3E_{11}$, with the radial component.

We can also create the same linear combination by putting the values into *addmout* format in the following way:

```
Elmcosi = [l,m,cosine expansion coefficient,sine expansion coefficient]
```

The cosine and sine expansion coefficients are determined by whether m is positive, negative, or zero. As described in Plattner and Simons (2017) in equation (2), we know that if m is negative or zero, the coefficient is the cosine portion and if m is positive, the coefficient is the sine portion. So for our linear combination above, we can create a matrix:

```
Elmcosi = [0,0,2,0 ; 1,0,-1.5,0 ; 1,1,0,3]
```

Currently working on this part.

We can also change all of the E_{lm} coefficients into lmcosi format by running:

```
elmcosi_rad = coef2lmcosi(E{1}(:,1),1);
```

We can then create a gradient spherical harmonic vector field by running:

```
elm_rad = elm2xyz(elmcosi_rad,1);
```

We can do this with each of the components:

```
elmcosi_theta = coef2lmcosi(E{2}(:,1),1);
elm_theta = elm2xyz(elmcosi_theta,1);
elmcosi_phi = coef2lmcosi(E{3}(:,1),1);
elm_phi = elm2xyz(elmcosi_phi,1);
```

Now we can plot these.

If the vector field is represented as a linear combination of elm and flm, then we will need to evaluate elm and flm separately then sum them.

Currently working on this part.

This tutorial is currently under construction. Please check back later for more by keeping your software updated.