

Chapter 2: Introduction to Slepian Functions

The Slepian Working Group

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After having worked through Chapter 1: Introduction to Spherical Harmonics, we will now have a look at what spherical Slepian functions are. The goal of this chapter is not to give a solid foundation of the mathematical intricacies. This can be obtained from the many research articles in the literature. Rather, we will look at some basic ideas and plot the first few Slepian functions.

1 Basic ideas

When plotting spherical harmonics in chapter 1, it became clear that all of these spherical harmonics are functions that cover the entire sphere. This is of course an advantage if we try to describe functions (or maps), that cover the entire planet, such as gravity fields, magnetic fields, etc. We saw in chapter 1 that we can generate pretty much any spatial pattern we like by simply summing up different spherical harmonics each multiplied by a different factor called “coefficient”.

But what can we do if we only have information within a specific region? Sometimes it’s fine to just describe that information as point values. But in some cases, as for example when we measure gravity or magnetic data at satellite altitude, and we want to calculate, what the corresponding magnetic or gravity field looks like on the planet’s surface (assuming there are no gravity or magnetic sources between the planet and the satellite, or that they are negligible), we need to describe these fields in spherical harmonics.

This is where the Slepian functions come in. Slepian functions are linear combinations of spherical harmonics which means that they are constructed by multiplying each spherical harmonic function with a factor (called Slepian coefficient) and then add them all up. The name Slepian function stems from the first author of a research article, that described the original idea for 1-dimensional functions.

There are different types of Slepian functions that are constructed in different ways but the basic idea is always to solve an optimization problem, in which we try to balance the number of spherical-harmonic functions we use (the maximum spherical-harmonic degree) in the construction of the Slepian functions, and how much they are concentrated within the region that we are interested in.

[This tutorial is currently under construction. Please check back later for more by keeping your software updated.](#)