Chapter 1: Introduction to internal magnetic field inversion using altitude-cognizant gradient Vector Slepian functions

The Slepian Working Group

October 4, 2017

1 Testing the software

After installing the software and starting Octave or Matlab, and initializing (by running `initialize_octave` or `initialize`), try out the demo function `demos_slepian_hotel`. For example. run

```
demos_slepian_hotel(1)
```

This will take a while to calculate. If it takes too long you can abort by pressing “Ctrl-c”. Let’s have a look why it takes so long. Open the function `demos_slepian_hotel.m` by double-clicking on it in Octave/Matlab.

You will first see a couple of lines with `%` signs in the beginning. This is the help component of the program. Any later line with a `%` in the beginning is simply a comment that does not do anything but can be very helpful for anyone who reads the program.

You see that a few lines down there is a comment that says “Maximum spherical-harmonic degree:”. Underneath this comment is the line “Lmax=20”. Here, Lmax defines the maximum spherical-harmonic degree. I will give a more detailed explanation about spherical harmonics at a later point but basically: The maximum spherical-harmonic degree defines the resolution of features in our analysis. The larger it is, the more details we see, but also the longer the computations take and the more sensitive we are to noise in the data!

Set Lmax to a lower value. Maybe 10. Save, and then run again

```
    demos_slepian_hotel(1)
```

This should, after other things, finally plot a sphere with some blurry colors on it. You can rotate the sphere by clicking on “R” in Octave, or the rotate button in Matlab and then rotate with your mouse.

If you now run the example again

```
demos_slepian_hotel(1)
```

you will see that the calculations are much faster. This is because in-between steps were stored.

Congrats, you calculated and plotted your first "altitude-cognizant gradient vector Slepian function". Play around with the other examples. You might need to set the maximum degree to a lower value to make calculations faster, or to a higher value if you want a more realistic and challenging example.

Also, in `demos_slepian_hotel.m` on line 30 you see "% Region", below that `%dom=[30.5 7.3];` and below that `%clon=18;` and `ccola=90+33`. Remove the `%` on each of these lines and instead type a `%` before `dom='africa'` and `clon=[]; ccola=[];`.

With this you commented (made invisible to the program) the line setting Africa as domain and instead set a spherical ring between opening angles 7.3° and 30.5° centered at longitude 18° and latitude 33° (which is given as colatitude 123°).

You can try other named regions such as `'namerica'` for North America, or `'samerica'`, or `'eurasia'`. Or you can choose a polar caps for example of opening angle 10 degrees by setting "dom=10" or a polar ring by giving it two numbers like "dom=[20 10];". It is also possible to run calculations for spherical cap centered somewhere else than the north pole. See the help comments for `glmalphaup.m`, `gradvecglmalphaup.m`, etc. You can even write a wrapper function for a list of lon/lat locations of the outline of your generic region. Generally, spherical caps and rings are much faster to calculate and require much less memory than irregular regions. This is particularly important for very large spherical-harmonic degrees (200 and beyond).

`demos_slepian_hotel(3)` and `demos_slepian_hotel(4)` are inversions for artificial data with a noise level you can choose. These are the key examples for estimating crustal fields from satellite altitude. In principle you can take these examples (3 for radial component data only and 4 for three component vectorial data), put in your data, select your region, satellite altitude, planet radius, maximum spherical harmonic degree, and number of Slepian functions you want to use, and run it to calculate your crustal magnetic field model.

`demos_slepian_hotel(5)` finally puts everything together and more. In this function we have:

- An internal field (for example a magnetic field from a planet's crust or core),
- an external field (for example solar wind)

and we want to find the field within an spherical ring centered at a chosen location and we know that the internal field only has power within the spherical harmonic degrees of a chosen *bandwidth*, for example spherical-harmonic degrees 10 to 40.