Scalar Spherical Harmonics and Slepian Functions

Contents

1 Plot a single spherical harmonic function

We will demonstrate plotting a spherical-harmonic on a sphere, in a standard Matlab plot, on a Mollweide projection, and on random points of a sphere.

First, designate a spherical-harmonic to be plotted:

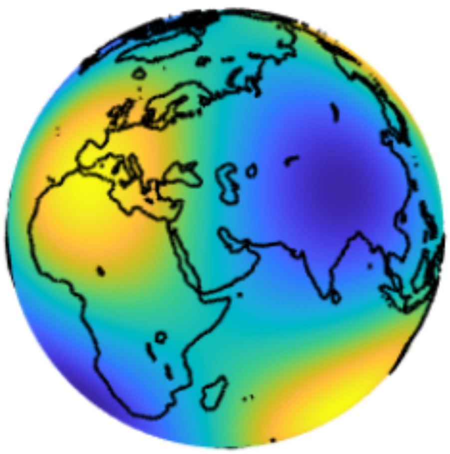
For example,

`l` = 3; `m` = -2;

0 `l` fixes the degree and `m` fixes the order.

1.1 Plot on sphere

```
l = 3; m = -2;
lon = 0:0.5:360;
lat = -90:0.5:90;
Y = ylm(l, m, pi/180*(90-lat), pi/180*lon);
figure;
plotplm(Y, pi/180*lon, pi/180*lat,2)
```



1. Create a grid on the sphere

```
lon = 0:0.5:360; lat = -90:0.5:90;
```

This creates a coordinate point every half-degree.

2. Calculate the values of the function for coordinate points on the sphere

```
Y = ylm(l, m, pi/180*(90-lat), pi/180*lon);
```

The function `slepian_alpha/ylm.m` evaluates the spherical harmonic function of degree `l` and order `m` at every point `pi/180*(90-lat)`, `pi/180*lon` on the grid. We name the vector of the spherical-harmonic values `Y`.

Note that `90-lat` is needed to convert latitude to colatitude and `pi/180` is needed to convert degrees to radians.

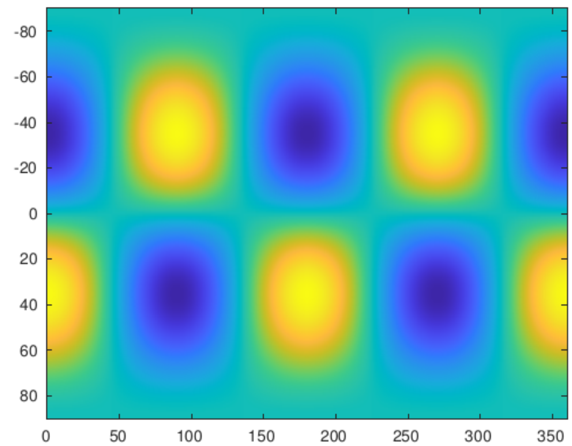
3. Plot

```
figure; plotplm(Y, pi/180*lon, pi/180*lat,2)
```

The function `slepian_alpha/plotplm.m` is here used to plot the vector `Y` using the grid specified by `lon` and `lat` in step 1. The input `2` dictates that the graph be on a sphere.

1.2 Plot in standard Matlab plot

```
l = 3; m = -2;
lon = 0:0.5:360;
lat = -90:0.5:90;
Y = ylm(l, m, pi/180*(90-lat), pi/180*lon);
imagesc(lon, lat, Y)
```

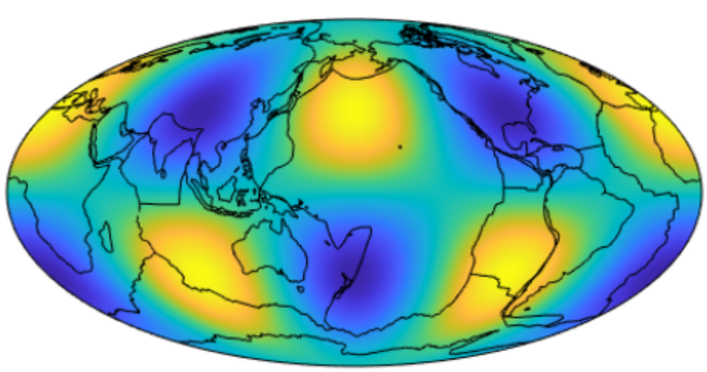


Do steps 1 and 2, and then run

```
imagesc(lon, lat, Y)
```

1.3 Plot on Mollweide projection

```
l = 3; m = -2;
lon = 0:0.5:360;
lat = -90:0.5:90;
Y = ylm(l, m, pi/180*(90-lat), pi/180*lon);
figure;
plotplm(Y, pi/180*lon, pi/180*lat,1)
```



Do steps 1 and 2, and then run

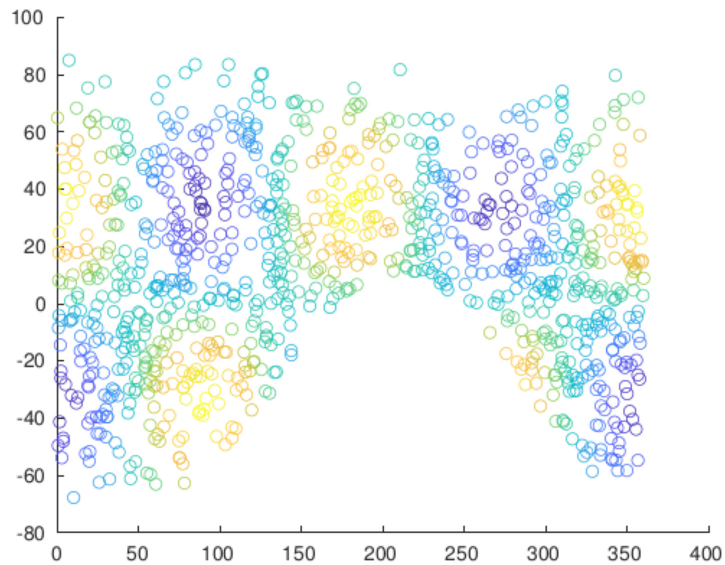
```
figure;

plotplm(Y, pi/180*lon, pi/180*lat,1)
```

The input `1` dictates that the graph be on the Mollweide projection.

1.4 Plot for random points on a sphere

```
l = 3; m = -2;
TH = 120; lon0 = 30; cola0 = 40; N=1000;
[lon, lat] = randpatch(N,TH,lon0,cola0);
Y = ylm(l, m, pi/180*(90-lat), pi/180*lon,[],[],[],1);
scatter(lon, lat, [], Y)
```



1. Generate a subset of the sphere consisting of random points

In particular, we will create `N` randomly-generated coordinate points within a spherical cap of opening angle `TH` and centered at longitude `lon0` and colatitude `cola0`

For example,

```
TH = 120; lon0 = 30; cola0 = 40; N=1000;

[lon, lat] = randpatch(N,TH,lon0,cola0);
```

The function `slepian_alpha/randpatch.m` creates the set of random points within the spherical cap of the specified values. We name those coordinate points `[lon,lat]`.

2. Calculate the values of the spherical harmonic at those points

```
Y = ylm(l, m, pi/180*(90-lat), pi/180*lon,[],[],[],1);
```

`ylm.m` takes the arguments `l`, `m`, `pi/180*(90-lat)`, `pi/180*lon` as before. Run `help ylm` for information on all eight arguments.

3. Plot

If necessary, use the Matlab command

```
clf;
```

To clear existing figures, and then run the Matlab command

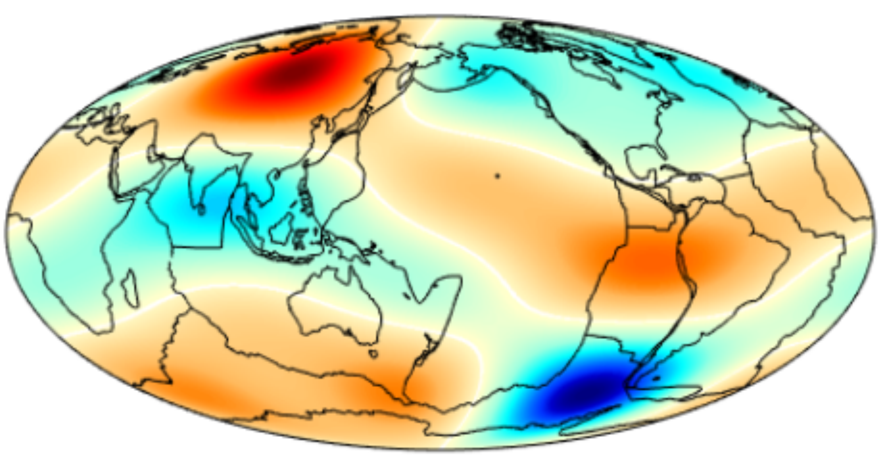
```
scatter(lon, lat, [], Y)
```

To create a scatter plot of circles having locations `[lon, lat]`. Here, `[]` indicates the default value for circle size and the vector of spherical-harmonic values `Y` is used to determine circle color.

Please see `Ch_01` in the `.edu` folder for more detailed information.

2 Plot a linear combination of spherical harmonics

```
lon = 0:0.5:360;
lat = -90:0.5:90;
Y1=ylm(3,1,pi/180*(90-lat),pi/180*lon);
Y2=ylm(1,-1,pi/180*(90-lat),pi/180*lon);
Y3=ylm(5,-2,pi/180*(90-lat),pi/180*lon);
Y4=4*Y1-0.2*Y2+2*Y3;
plotplm(Y4, pi/180*lon, pi/180*lat,1);
kelicol(1)
```



This task is a simple variation on the first.

Let us define three spherical harmonics:

```
lon = 0:0.5:360;

lat = -90:0.5:90;

Y1=ylm(3,1,pi/180*(90-lat),pi/180*lon);

Y2=ylm(1,-1,pi/180*(90-lat),pi/180*lon);

Y3=ylm(5,-2,pi/180*(90-lat),pi/180*lon);
```

Next, create a vector which is a linear combination of these three. For example,

```
Y4=4*Y1-0.2*Y2+2*Y3;
```

To plot the function, use `plotplm.m`. For example,

```
plotplm(Y4, pi/180*lon, pi/180*lat,1)
```

If you're interested in another color scheme, try out

```
kelicol(1)
```

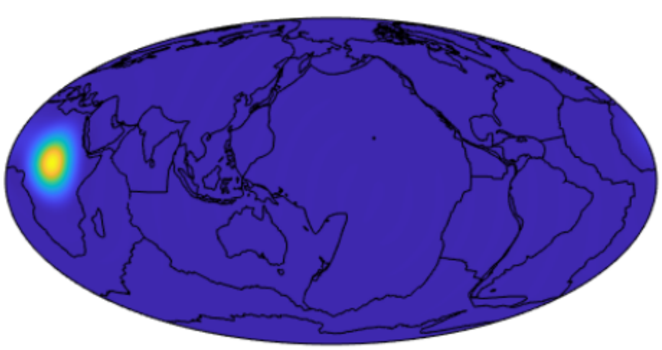
Please see `Ch_01` in the `.edu` folder for more detailed information.

3 Create and plot scalar Slepian functions

3.1 Named region and polar cap

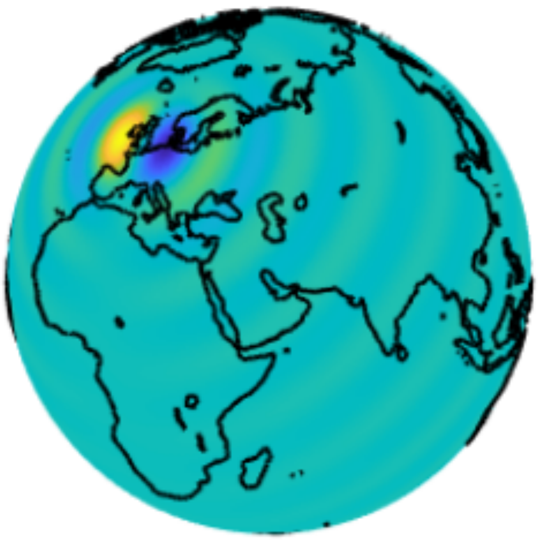
Named region example 1:

```
[G] = glmalpha('africa',20,[],0);
lmcs = coef2lmcosi(G(:,1),1);
data=plm2xyz(lmcs,0.5);
plotplm(data, [], [], 1, 0.5)
```



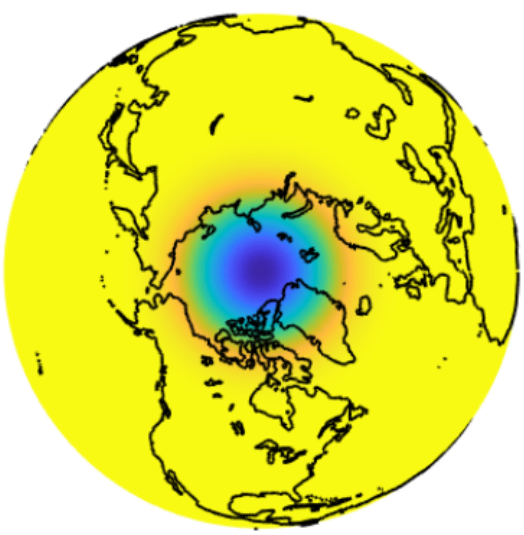
Named region example 2:

```
[G] = glmalpha('england',20,[],0);
lmcs = coef2lmcosi(G(:,3),1);
data=plm2xyz(lmcs,0.5);
plotplm(data, [], [], 2, 0.5)
```



Polar cap example:

```
[G] = glmalpha(40,20,1,0)
lmcs = coef2lmcosi(G(:,1),1);
data=plm2xyz(lmcs,0.5);
plotplm(data, [], [], 2, 0.5)
view(2)
```



1. Generate the coefficients of the function

We will use the function `glmalpha.m`, which will essentially compute for us the best spatially-concentrated Slepian functions given two constraints. Those constraints are the first input, `TH`, and the second, `L`. `TH` can be either a named region or the opening angle in degrees of a polar cap. `L` is the bandwidth.

You may choose among the named regions 'england', 'eurasia', 'namerica', 'australia', 'greenland', 'africa', 'samerica', 'amazon', 'orinoco', 'antarctica', 'contshelves', and 'alloceans'.

For example,

```
[G] = glmalpha('africa',20,[],0);
```

2. Plot

`[G]` in both cases is a matrix whose nth column holds the coefficients of the nth-best spatially-concentrated Slepian function. In order to plot the nth function we need to convert the nth column of `[G]` into the form recognized by the function `plotplm.m`, the so-called "lmcosi" format.

```
lmcs = coef2lmcosi(G(:,1),1);
```

Here we have chosen to use the first column `G(:,1)`. The second input `1` is necessary when the coefficients are calculated using `glmalpha`.

Now we input the (lmcosi-formatted) matrix `lmcs` and a resolution into the function `plm2xyz`. We choose the resolution to be 0.5 and name the ouput "data":

```
data=plm2xyz(lmcs,0.5);
```

To plot, run

```
plotplm(data, [], [], 1, 0.5)
```

The input `1` specifies Mollweide projection and the input `0.5` is just the resolution again.

The same sequence is used to plot a Slepian function on a polar cap, except for a change in the inputs to `glmalpha.m` Specifically, we will now let `TH` denote an opening angle in degrees.

For example, let `TH = 40`.

```
[G] = glmalpha(40,20,2,0)
```

The command

```
view(2)
```

may be used to rotate the spherical figure so that the North pole is faced toward the viewer.

We suggest reading the `help` section for the relevant functions and/or Chapter 2 Section 1 in the folder .edu for a detailed discussion.

3.2 Rotated polar cap

The general method of calculuating coefficients for a rotated polar cap is the same as above, but involves slightly different commands.