When should we tax firms? Optimal corporate taxation with firm heterogeneity

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Abstract

Corporate fiscal policy over the business cycle is carried out in very different ways over time and across countries. Moreover, little is known about how it should be conducted. This paper studies the design of optimal fiscal policy in a heterogeneous firm environment, when the economy is hit by aggregate shocks. It provides tools to understand when and how heterogeneous firms should be taxed or subsidized over cycles. To tackle this issue, I first solve a tractable model which delivers a simple distribution of firms. In this framework, I provide an analytical characterization of the corporate tax rate over the business cycle. Then, using a fully fledged heterogeneous firm model and cutting-edge computational method, I solve for the optimal path of the tax rate in this environment. My main result is that, in both exercises, the variation of the optimal tax rate depends on the expected persistence of the aggregate shock. This is due to the presence of financial constraints that prevent the allocation of capital from being optimal. I show that the magnitude of this problem varies over the business cycle depending on the persistence of the aggregate shock. When the shock is very persistent, this problem decreases and the optimal tax rate is pro-cyclical. On the contrary, when the shock is not persistent, this problem increases and the optimal tax rate is counter-cyclical.

Keywords: Heterogeneous firms, optimal policy, fiscal policy, investment.
JEL Classification : E44, E22, E32, E62 .

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1 Introduction

Fiscal policy is an important tool to smooth the business cycle. Moreover, if "firms are, for the most part, absent from the modern theory of optimal taxation" (Kopczuk and Slemrod (2006)), they remit the quasi-totality of taxes in an economy[1] (firms remit 93% of the taxes in the US (Milanez (2017))). Even considering only their tax liabilities, firms are important contributors to total tax revenue[2]. Therefore, firms' contribution to total tax revenue contrasts greatly with their consideration in the optimal fiscal policy literature. In addition, if consumption has been at the core of business cycle research, investment is another important dimension of aggregate fluctuations. Indeed, it accounts for a large part of GDP and it is very volatile. Thus, understanding how to use corporate taxes to influence aggregate investment over the business cycle seems crucial. The behavior of tax revenue over the business cycle is well identified. In contrast, little is known about the path of corporate tax rates (except few papers reviewed below). Empirically, the cyclicality of corporate tax rates changes a lot over time and across countries[3]. Understanding the rationale behind those seemingly conflicting policy decisions is thus challenging. It raises the question of the optimal way to set corporate tax over the business cycle. Should corporate tax vary over the business cycle and, if yes, how?

The literature on heterogeneous firm provides a solid foundation to study this question. This literature has first investigated the distribution of capital across firms to quantify the misallocation of capital (for instance Buera and Shin (2013) or Hsieh and Klenow (2009)). Then, the introduction of aggregate shocks has allowed to identify time variations in this misallocation (among others Khan and Thomas (2013)), opening the possibility to study optimal time-varying policies in such environments. Some papers have studied optimal policies in such framework (González et al. (2021) study optimal monetary policy with heterogeneous firms in continuous time), but, to the best of my knowledge, I am the first one to study optimal fiscal policy in such setup. Specifically, I

[1]Besides paying their own taxes, firms play an important role in the tax remittance process. For instance, they remit employee social security contributions, VAT or they withhold taxes on labour income.

[2]In the US, the firms legal tax liability represents 28.9% of the total tax revenue (in 2014) Milanez (2017).

[3]Vegh and Vuletin (2015) study in depth the volatility of corporate income tax rates on 62 countries over the period 1960–2020

study optimal corporate tax rate dynamics in this environment.

More precisely, I follow the Khan and Thomas (2013) literature and I assume the existence of constraints that prevent firms from investing at their optimal level. In this paper, firms need to rely on their retained earnings to finance their investment. Uninsurable and idiosyncratic productivity risk generates a realistic firm productivity distribution. In such general equilibrium environments, there are many direct and indirect effects of corporate taxes or subsidies. For this reason, I first solve a tractable model to obtain an analytical characterization of the optimal corporate tax policy over the business cycle. To do that, I build a model that generates a simple structure of heterogeneity and a finite distribution of firms at equilibrium. I am able to provide an analytical expression for the dynamics of the corporate tax rate after aggregate total-factor productivity (TFP thereafter) shocks[4].

My main result is that the path of the optimal tax rate over the business cycle depends on the expected persistence of the aggregate shock. Indeed, aggregate shocks affect the current profit and the future optimal capital stock of a firm. Then, when some firms are constrained by their earnings, an aggregate shock has a direct effect on their level of investment. Whether it is optimal to decrease or increase corporate income taxes rate after such a shock depends on whether the variation in firm constrained investment is larger than the variation in the optimal level of investment. Hence, it depends on the persistence of the shock. After a negative TFP shock, constrained firms face more severe constraints, what lower their level of investment. But the optimal level of investment also decreases following this shock. I show that for TFP shocks with a low persistence, the negative effect on constraints is more important than the reduction in the desired investment level. As a consequence, corporate tax should decrease to stimulate investment of constrained firms. When shocks are persistent, the second effect dominates and corporate tax should increase.

[4]Although the model could solve with various types of shocks, such as demando shock and preference shocks, the paper focuses on TFP shocks as they are the main shocks studied in the literature. It could be interesting to solve for optimal policies after simultaneous shocks, reproducing the Covid crisis. I let that for future work

To check the relevance of this new result, I verify if my results hold in a quantitatively relevant environment. I assume an incomplete insurance markets for idiosyncratic productivity risks and I solve a model with a full distribution of firms over different levels of capital and productivity. This type of model is hard to solve because the distribution of capital is a state variable and is an infinite-dimensional object. I use a projection and perturbation method derived from Reiter (2009) to solve this model. Then, I simulate this model and determine the tax rate that maximizes the aggregate welfare in the economy, taking into account the transitions after aggregate shocks.

Related literature This paper is related to different strands of the literature. My paper closest literature is that which studies optimal stabilization policies in heterogeneous firm models. A first part of this literature studies optimal monetary policy in such frameworks, for instance Andrés and Burriel (2018) design optimal monetary policy when there is heterogeneity of total factor productivity and strategic price interactions between firms, Adam and Weber (2019) show that taking into account heterogeneous firms and systematic firm-level productivity trends change predictions for the optimal inflation rate. González et al. (2021) show that central bank should implement monetary expansion after a TFP shock to relax constraints on firms. Another part of this literature, studies optimal fiscal policy in heterogeneous firm agent models without aggregate shock. Cerda and Saravia (2013): studies steady-state optimal (Ramsey) taxation with heterogeneous firms and multiple sectors. Hall and Laincz (2020) determines optimal fiscal policy when the planner can tax or subsidy R&D of observably heterogeneous firms in a duopoly model of R&D competition. Dávila and Hébert (2020) study the optimal design of corporate taxes when firms face financial frictions. Itskhoki and Moll (2019) study optimal dynamic Ramsey policies in a standard growth model with financial frictions so they can investigate if governments should interfere with markets in emerging countries.

To the best of my knowledge, González et al. (2021) is the only paper that studies optimal monetary policy in heterogeneous firm model with aggregate shock and my paper is the first one that studies optimal fiscal policy in this framework.

My paper uses different methods developed in the literature on optimal policies in heterogeneous household models. Aiyagari (1995) initiated this literature studying a

Ramsey allocation and the optimal capital tax in a heterogeneous household model. Aiyagari and McGrattan (1998) or Krueger and Ludwig (2015) derive optimal policies by maximizing the aggregate steady-state welfare. Açıkgöz (2015) further developed in Açıkgöz (2018), uses an explicit Lagrangian approach to derive the planner's first-order conditions at the steady state and relies on a numerical procedure to approximate the value of Lagrange multipliers. Dyrda and Pedroni (2018) and Chang et al. (2018) compute optimal policies without considering the planner's first-order conditions, and instead directly maximizing the intertemporal welfare over all possible paths for the planner's instruments. Nuño and Moll (2018) consider a continuous-time framework in which they use the techniques of Ahn et al. (2017) to simplify the derivation of the planner's first-order conditions. Bhandari et al. (2020) derive optimal Ramsey policy in a general environment with incomplete insurance markets and aggregate shocks. Le Grand and Ragot (forthcoming) solve for optimal Ramsey policies in heterogeneous household models with aggregate shocks using truncation theory of idiosyncratic histories.

This paper is also related to a literature that studies the link between taxes and firms investment on a positive perspective. From Hall and Jorgenson (1967) this question is key to understand how to design fiscal policy (see Abel (1990) for a review of the begining of this theoretical literature). Empirical papers followed, trying to measure this effect, for instance Summers et al. (1981), Auerbach and Hassett (1992), Cummins et al. (1994), Goolsbee (1998), Chirinko et al. (1999), Desai and Goolsbee (2004), House and Shapiro (2008), Yagan (2015), Edgerton (2010), Zwick and Mahon (2017). Public finance literature also studies the effect of tax reform on investment. Korinek and Stiglitz (2009) study the effects of the Jobs and Growth Tax Relief Reconciliation Act in a partial equilibrium framework where firms can be are heterogeneous in their financing. Gourio and Miao (2010) study a dynamic general equilibrium model version of the previous article in which there is a continuum of firms subject to idiosyncratic productivity shocks. After them, Anagnostopoulos et al. (2012) investigate the effect of the same reform in a model where households are heterogeneous. Finke et al. (2010) use microsimulation to assess the effect of the German 2008 corporate tax reform on heterogeneous firms.

The literature which studies the link between firm heterogeneity and financial frictions to understand aggregate fluctuations is also important to my work. Part of this literature models frictions in firms external financing, for instance the seminal works of Cooley and Quadrini (2001) and Cooley and Quadrini (2006) or Arellano et al. (2012), Khan and Thomas (2013), Buera and Moll (2015), Khan et al. (2017); Gomes (2001), Gilchrist et al. (2017), Crouzet and Mehrotra (2020) or Begenau and Salomao (2015). Another part of this literature provides empirical evidences of such mechanisms, among others, Zwick and Mahon (2017), Ottonello and Winberry (2018), Cloyne et al. (2018), Jeenas (2020). A large corporate finance literature studies the importance of firms liquidity constraints to understand firm investment behavior. For instance, and among others, the work of Fazzari et al. (1988) and Kaplan and Zingales (1997).

Another strand of the literature models firm dynamics within heterogeneous firm models to understand aggregate fluctuations and the effect of macroeconomics policies. Canonical examples are Hopenhayn (1992) and Hopenhayn and Rogerson (1993). Erosa and González (2019) for instance show the importance of firms life cycle for understanding how taxation has an effect on investment. Other examples are Bartelsman et al. (2013), Clementi and Palazzo (2016) or Sedlacek and Sterk (2019).

Finally, the literature on open-economy macroeconomics and international trade theory has initiated the study of heterogeneous firms. This literature has emerged with the increasing availability of micro-level data and grew, trying to explain patterns revealed by those new databases. Popularized by Melitz (2003) and developed in multiple papers as Ghironi and Melitz (2005) and Bilbiie et al. (2012) this field studies the importance of taking into account firms heterogeneity to understand business cycles and optimal policy over them. For instance, in such a framework with heterogeneous firms, Becker and Fuest (2011) study optimal taxation when firms are internationally mobile. Demidova and Rodríguez-Clare (2013) analyse the effect of tariffs and subsidies on aggregate productivity and welfare, Chor (2009) studies the effects of a production subsidy, Davies and Eckel (2010) study tariffs and taxes when there is an informal sector in the economy. Other examples are Pflüger and Suedekum (2013), Krautheim and Schmidt-Eisenlohr (2016), Dharmapala et al. (2011), Bauer et al. (2014).

The rest of the paper is organized as follows. In Section 2 I motivate empirically this paper. In Section 3 I present the environment of the tractable model and I derive optimal Ramsey policies in this framework. In Section 4 I present the environment of the quantitative model, the algorithm used to solve it and the path of the optimal tax rate in this environment. Section 5 concludes.

2 Empirical Motivation

There exist different ways to tax firms. In this paper, I consider corporate income tax[5] as the main fiscal instrument. Indeed, corporate income taxes account for the majority of firms' tax liabilities, they are widely used over the world and they apply to the majority of firms in an economy. Corporate income taxes are not the only tax on firms, but even taken alone, they represent an important amount of the total tax revenue. Figure 1 represents CIT as proportion of total tax revenue in different industrial countries[6] in 2019[7]. It shows that, even if it is a great source of revenue, its importance varies across countries.

[5]Other taxes are, among others, social contribution tax or specific local taxes
[6]I plot this Figure using a larger sample of countries in Appendix A.
[7]In 2018 for Austria and Greece

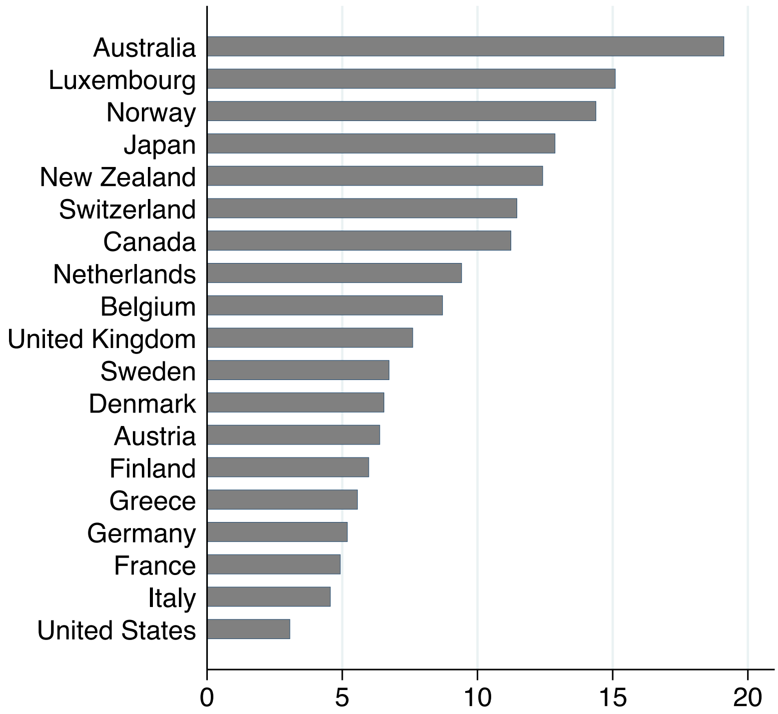


Figure 1: CIT as Proportion of Total Tax Revenue in 2019 (Source: OECD, Revenue Statistics)

Corporate income taxes depends on a statutory tax rate τ_{cit} that is applied to the corporate tax base Y = Revenues - Expenses. Different tax credits can be deduced from this initial level of tax afterward. Corporate income tax (CIT), is defined as $CIT = \tau_{cit}Y - ITC - RTC$ with ITC is the Investment tax credit (equivalent to accelerated depreciation) and RTC is the Research tax credit (based on R&D spending). Thus, CIT can be negative.

The heterogeneity in the percentage of the corporate income tax revenue over total tax revenue shown in Figure 1 reflects, in part, heterogenity in statutory tax rates across countries. Using the novel dataset build by Vegh and Vuletin (2015), I represent in Figure 2 the average and the standard deviation of the statutory income corporate tax rate in a sample of industrial countries[8] over the period 1960-2020. This graph shows that this

[8] I plot this Figure using a larger sample of countries in Appendix A.

tax rate is heterogeneous across countries. It ranges from an average over the period of 7.4% in Switzerland to 49.8% in Germany. Moreover, using the standard deviation of the tax rate over the entire period, this graph shows also that corporate income tax rates have changed a lot within each country between 1960 and 2018.

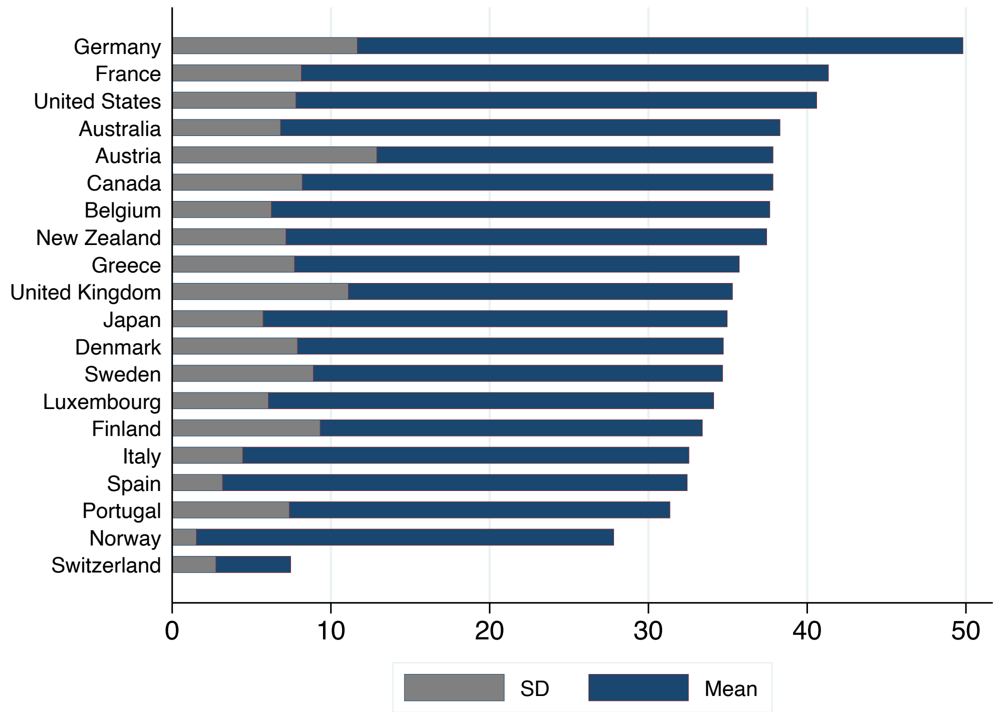


Figure 2: Corporate Income Tax Rate by Country, 1960-2018 (Source: Vegh and Vuletin (2015))

It is now well known that, because of the volatility of the tax base, corporate income tax revenue is very volatile over the business. However, little is known about the behavior of the tax rate over the business cycle. Vegh and Vuletin (2015) have shown in a pioneering paper that the cyclicality of corporate income tax rates differ greatly between countries. Using their new dataset, I illustrate in Figure 3 this heterogeneity. I represent the correlation between real GDP percentage changes and tax rate percentage changes[9]

[9]Following Vegh and Vuletin (2015) I use percentage change in tax rates in my analysis. The cyclical component of the tax rate is usually used when studying tax revenue but the infrequent change in the corporate tax rate motivates this choice.

in industrial countries[10]

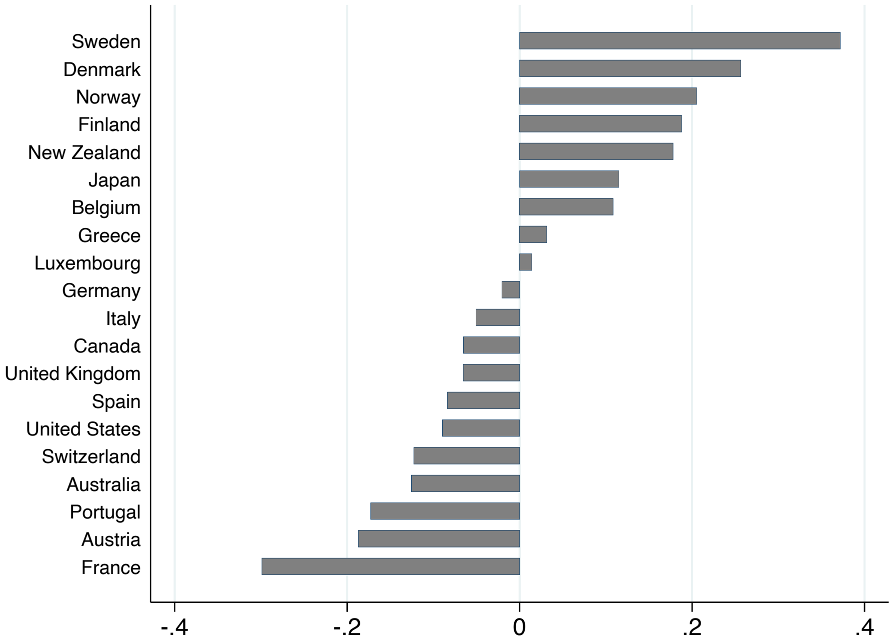


Figure 3: Correlation between CIT rate growth and real GDP growth (Source: Vegh and Vuletin (2015))

We see in Figure 3 that the cyclicality of the tax rate changes a lot across countries. Over 1980-2018, corporate tax policy is counter-cyclical, in average, in countries where the correlation is positive and pro-cyclical, in average, in countries where the correlation is negative. But, even within countries, those correlations are far from perfect. I present in Figure 4 the comovement of the percentage change of the real GDP (taken as a deviation from the overall means of the percentage change over the period) and the percentage change of the tax rate over a subsample of four countries[11]. We see that, in each country, there are episodes when the government decides to implement counter-cyclical, pro-cyclical or acyclical policy.

[10] I reproduce the same graph over the entire dataset in Appendix A.

[11] The graph for the other industrial countries can be found in appendix

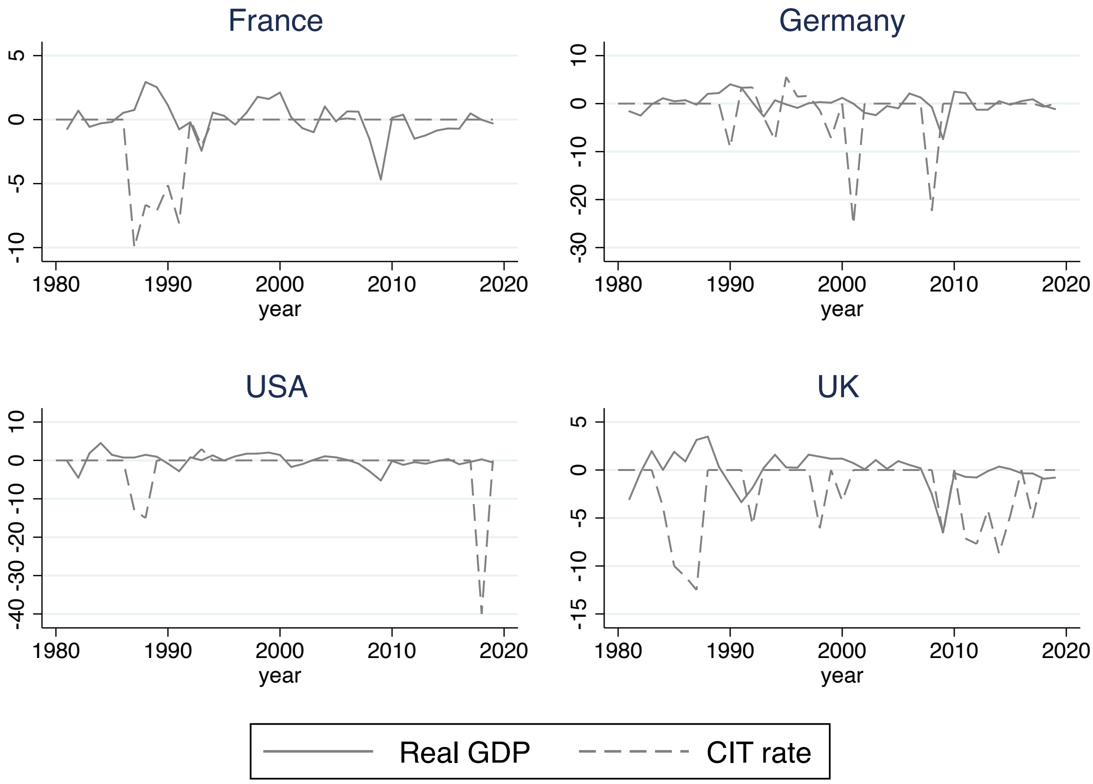


Figure 4: Percentage changes of Real GDP and CIT rate over 1980-2018 (Source: Vegh and Vuletin (2015), OECD)

Therefore, it seems that tax rates changes depends on multiple factors. (Vegh and Vuletin (2015)) explore, for instance, the determinant of tax rates cyclicality across countries focusing on the role of institutional quality and financial integration. To take into account that corporate income tax can be negative, I use the effective tax rate as the main fiscal instrument in this paper. This tax rate accounts for the deductions and credits mentioned above and . It is defined as $ETR = \frac{r^g - r^n}{r^g}$ with r^g and r^n the rate of return gross and net of taxes.

I investigate, in the rest of this paper, what should be the economic determinants of corporate income tax rate cyclicality within countries. Could we rationalize the change in corporate income tax rate that we observe in the data?When should a government increases this tax rate over a boom and when should he increases this same tax rate over a recession?To to so, I derive the optimal corporate income tax rate over the business cycle in a heterogeneous firm model. First, I build a tractable model to obtain an analytical characterization of the optimal tax policy over the business cycle. To obtain such results, I make different assumptions. For instance, I assume full depreciation of capital, no labor in the production function, and that households have a risk-neutral pricing kernel. More importantly, I assume that firm's productivity only depends on their level of capital and on aggregate shocks. Second, I relax those different assumptions and I indroduce idiosyncratic productivity shocks. I show that my analytical results hold in this quantitatively relevant framework. After deriving these policies, I show - in the last section of the paper - using the same dataset as in this first section, that they are consistent with the data.

3 The simple Model

The simple model is a heterogeneous-firm model, where I remove idiosyncratic productivity risk. Some firms exit the economy at each period and some firms enter the economy at each period. Entrants have a low capital stock. Moreover, firms are constrained on their level of investment by their retained earnings. Thus, firm heterogeneity comes from the life-cycle profile of investment. I show that it is enough to capture the relevant trade-offs.

Discussion of the main assumptions. The two important elements of this model are that firms face financing constraints and that new firms have a low level of capital. This structure of heterogeneity is consistent with micro data. Indeed, firm life-cycle is a key dimension of firms heterogeneity. For instance, it is crucial to understand the dynamics of aggregate productivity. Among others, Haltiwanger et al. (2013) show that new firms are numerous, small and that they grow more rapidly than the others. Hsieh and Klenow (2009) shows that the life-cycle growth of firms is important to understand the difference of aggregate productivity across countries. Moreover, there exist numerous empirical evidences showing that firms face important financing costs and that those frictions bind more severely for young firms. For instance, Hennessy and Whited (2007) show that firms face important equity issuance costs and that this problem is more stringent for young firms. Cloyne et al. (2018) show using micro data that young firms are more affected by financial frictions and that those frictions shape their investment decisions.

Model Setup Time is discrete and indexed by $t = 0, 1, ...$ The economy is populated by a continuum of firms of measure 1 distributed on an interval $\mathcal{J}$ following a non-atomic measure ℓ: $J(\ell) = 1$. The economy is also populated by a representative household and a government.

3.1 Preferences

Household In each period, there is a unique good in the economy. The representative household derives utility from private consumption C according to a period utility function denoted $U(C)$. Endogenous labor and labor disutility are introduced in the general model. The household has standard intertemporal preferences, she has a constant discount factor $0 < \beta < 1$. She is expected utility maximizers and ranks consumption stream, denoted by $(C_t)_{t\geqslant 0}$ using the intertemporal utility criterion $\sum_{t=0}^{\infty} \beta^t U(C_t)$ where $u : \mathbb{R}+ \to \mathbb{R}$ is twice continuously derivable, increasing, and concave, with $U'(0) = \infty$.

Firms Firms maximize their value, that is the expected discounted value of dividends D_t returned to their shareholders at each period. In this economy, firms are own by the representative households. Then, I assume that the firm's pricing kernel is independent of firms type and is equal to the household pricing kernel. I denote it $\frac{M_t}{M_0}$.

3.2 Risks

The only aggregate risk in this economy is an aggregate productivity risk. Here, firms don't face idiosyncratic productivity risk, I will introduce such risks in the quantitative part of this paper.

Aggregate risk. The aggregate risk affects the technology level in the economy. At a given date t, the risk is denoted by $(z_t)_{t\geqslant 0}$. We assume that it follows an $AR(1)$ process $z_t = \rho z_{t-1} + u_t$ with ρ_t the persistence parameter and the shock u_t being a white noise with a normal distribution $\mathcal{N}(0, \sigma^2)$. The aggregate productivity, denoted $(Z_t)_{t \geqslant 0}$ is assumed to relate to z_t through the following functional form: $Z_t = Z_0 e^{z_t}$.

Exit risk. At the beginning of each period, each firm $j \in \mathcal{J}$ faces a constant probability $0 < \theta < 1$ to exit the economy. Then, θ firms exit the economy at each period t. To keep the number of firms constant in this economy, θ firms also enter the economy at each period.

3.3 Production

The only good in this economy is produced by heterogeneous firms. A firm j is endowed with a production technology that transforms, at date t, capital $k_{t,j}$ into $y_{t,j}$ output units of the single good. The production function is a Cobb-Douglas function with parameter $\alpha < 1$ featuring decreasing returns-to-scale. Capital must be installed one period before production and the total productivity factor Z_t is stochastic. To further simplify results, capital depreciation is constant and equal to 1.

When a firm exit the economy, its capital is destroyed. Newly created firms are endowed with a level of capital k_0.

3.4 Government

A benevolent government can levy corporate income tax at rate τ_t[12] to finance a transfer T_t to the household. She can also subsidize corporate income levying lump-sum tax on households. Therefore, this tax rate τ_t can be seen as the effective tax rate paid by a firm. Then, fiscal policy is characterized by two instruments $(\tau_t, T_t)_{t \geq 0}$. Those instruments will be optimally chosen. The lump-sum transfer to the household is equal to the sum of the tax over each firm.

$$T_t = \int_0^1 \tau_t y_{j,t} d_j$$

[12]As discussed in the first section, it is the most important and the most widespread of the fiscal instrument that apply to firms. Here, I use a stylized version of the corporate income tax and I do not allow for depreciation allowance deduction in the tax base.

3.5 Agents program

Consider a firm $j \in \mathcal{J}$. At time t, it can invest in capital that will be used at the next period and / or it can pay dividends to its shareholders. Firms cannot extract equity from their shareholders and their dividends are thus prevented from being non-positive [13].

At time 0 the firm j chooses the investment plan $(k^j_{t+1})_{t\geqslant 0}$ and dividend plan $(D^j_t)_{t\geqslant 0}$ that maximize the expected discounted value of dividends paid to its shareholders. It has a probability $(1-\theta)^t$ not to have exited the economy at period $t \geqslant 0$. Formally, the program of the firm j can be written, for a given initial shock z_0 as:

$$\max_{(k_{j,t+1})_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} (\beta(1-\theta))^t \frac{M_t}{M_0} \left((1-\tau_t) Z_t k_{j,t}^{\alpha} - k_{j,t+1}\right) \right] \tag{1}$$

$$\text{S.t} \quad D_{j,t} = (1-\tau_t) Z_t k_{j,t}^{\alpha} - k_{j,t+1} \geq 0 \tag{2}$$

$$k_{j,t+1} \geq 0 \tag{3}$$

Where $\mathbb{E}_0$ is an expectation operator over the aggregate productivity risk.

I denote by $\beta^t \nu_{j,t}$ the Lagrange multiplier on the constraint on the positivity of the firm j dividends, then:

$$\beta^t \nu_{j,t} \left((1-\tau_t) Z_t k_{j,t}^{\alpha} - k_{j,t+1}\right) = 0$$

The Lagrange multiplier is null when the firm is not constrained by its financing constraint.

Decisions Among Constrained Firms Consider a firm j that is constrained at time t, then $\beta^t \nu_{j,t} > 0$ and:

$$(1-\tau_t) Z_t (k^j_t)^{\alpha} - k^j_{t+1} = 0$$

Finally,

[13] I show in appendix that my results hold when firms can extract a finite amount of equity $\bar{D}$ from their shareholders and their dividends are thus prevented from being too negative.

$$k_{t+1}^j = (1-\tau_t) Z_t (k_t^j)^\alpha \quad (4)$$

Those firms would like to invest more than what is allowed by their level of retained earnings and the amount of equity they can extract from their shareholder. It means that they would like to extract more equity from their shareholders, which is prohibited in this model. Therefore, they invest as much as they can.

Decisions Among Unconstrained Firms Consider a firm j that is not constrained by its level of earnings at time t. Then $\beta^t \nu_{j,t} = 0$ and this firm solves the following program:

$$\max_{(k_{j,t+1})_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} (\beta(1-\theta))^t \frac{M_t}{M_0} \left((1-\tau_t) Z_t k_{j,t}^\alpha - k_{j,t+1} \right) \right]$$

The first-order condition of this program is :

$$M_t = \beta(1-\theta) \, \mathbb{E}_t \left[M_{t+1} \alpha Z_{t+1} (1-\tau_{t+1}) \, (k_{t+1}^j)^{\alpha-1} \right]$$

This firm invests until the household marginal utility of consumption is equal to the expected marginal return of the capital invested. The optimal level of investment is thus given by:

$$k_{j,t+1} = \left(\beta(1-\theta) \, \mathbb{E}_t [\frac{M_{t+1}}{M_t} \alpha Z_{t+1} (1-\tau_{t+1})] \right)^{\frac{1}{1-\alpha}} \quad (5)$$

Therefore, the investment of a firm j is given by the expression (4) or by the expression (5) depending on whether the investment constraint is binding or not.

3.6 Equilibrium

This model generates a reduced-heterogeneity equilibrium as firm just face an investment constraint and no idiosyncratic shock[14]. The relevant equilibrium is characterised as the following. At equilibrium, new firms face a binding constraint. They would like to invest

[14]Reduced heterogeneity equilibrium have been used to identify effects in the heterogeneous household literature for instance in Ravn and Sterk (2017), Challe and Ragot (2016), Ragot (2014), Bilbiie and Ragot (2020) (see Ragot (2017) for a survey of the different method used to solve such model in heterogenous household models).

more than what is allowed by their level of earnings and the amount of equity they can extract from their shareholder. If those firms have not exit the economy after N period (this number is not time-varying), this constraint does not bind anymore. Indeed, at a given period of time, firms have accumulated a sufficient level of capital to ensure that their earnings no longer constrain their level of investment. Therefore, they are able to invest at their optimal level. Thus, at each period t, the economy is characterized by a simple and constant distribution of firms. The existence of this equilibrium can be derived using a simple guess-and-verify structure.

Firm Distribution Here, I choose to work with the simplest possible distribution to understand the mechanisms of this model. For this, I consider that firms are constrained only for one period. It means that I assume $N = 1$. Firms that enter the economy at period t face a binding constraint over this period. At the following period $(t + 1)$, if these firms still exist, this constraint no longer binds[15].

At period t, θ firms denoted firms of type 0 enter the economy and face a constraint on their level of investment. In addition, $\theta(1 - \theta)$ firms denoted firms of type 1 have been existing for exactly one period. Those firms are not constrained on their level of investment but produce using a non-optimal level of capital inherited from the previous period. Finally, $1 - \theta - \theta(1 - \theta) = (1 - \theta)^2$ firms, denoted firms of type 2 have been existing for more than one period. Those firms are not constrained on their level of investment, and they produce using an optimal level of capital.

$$\begin{aligned}
n_0 &= \theta \\
n_1 &= \theta\left(1-\theta\right) \\
n_2 &= 1-\theta\left(1-\theta\right)-\theta=\left(1-\theta\right)^2
\end{aligned}$$

Where n_i, $i \in [0, 1, 2]$ is the number of firms of type $0, 1, 2$.

Budget Constraints The household consumes using only the dividends paid by type 1 and type 2 firms minus the amount of equity extracted by type 0, the lump-sum transfer

[15] Obviously, for some parameter values, equilibria with higher N can be constructed. I choose the simplest distribution on purpose, to obtain transparent analytical results.

made by the government. Thus, at time t, the budget constraint of the household can be expressed as:

$$\begin{aligned} C_t &= n_1 D_{1,t} + n_2 D_{2,t} + \tau_t \left[n_0 Z_t k_0^\alpha + n_1 Z_t k_{1,t}^\alpha + n_2 Z_t k_{2,t}^\alpha \right] \\ &= n_1 \left(\left(1-\tau_t\right) Z_t k_{1,t}^\alpha - k_{2,t+1} \right) + n_2 \left(\left(1-\tau_t\right) Z_t k_{2,t}^\alpha - k_{2,t+1} \right) \\ &\quad + \tau_t \left[n_0 Z_t k_0^\alpha + n_1 Z_t k_{1,t}^\alpha + n_2 Z_t k_{2,t}^\alpha \right] \end{aligned}$$

Before solving the Ramsey-Problem, I study the previously described problem without taxes.

Existence of This Equilibrium I show that there exists parameters such that an equilibrium of this problem exists. An equilibrium of this problem exists if and only if I can find k_0 such that $\forall t$:

1. New firms are always constrained, i.e:
   - $Z_t k_0^\alpha - k_{2,t} < 0$
2. Firms that have been existing for one period and more are never constrained, i.e:
   - $Z_t k_{1,t}^\alpha - k_{2,t} \geq 0$
   - $Z_t k_{2,t}^\alpha - k_{2,t} \geq 0$

I show in appendix B that, with $0 < \alpha \leqslant 1$, $0 < \theta < 1$ and $0 < \beta < 1$, an equilibrium of this problem exists if and only if:

$$[\alpha\beta(1-\theta)]^{\frac{1}{\alpha^2(1-\alpha)}} \leqslant k_0 < [\alpha\beta(1-\theta)]^{\frac{1}{\alpha(1-\alpha)}}$$

3.7 Misallocation of capital over the business cycle

In this model, misallocation of capital stems from the presence of a financing constraint. Because there are no idiosyncratic shocks, all firm should invest the same amount of capital and marginal returns of capital should be homogeneous across firms. Without this constraint, the low endowment of capital of young firms would not be relevant for understanding their investment behavior. The relevant measure to understand misallocation here is therefore the misallocation of firms' investment. Indeed, misallocation of firms' investment translates one period later into misallocation of firms' capital, what is revealed by the presence of heterogeneous marginal returns of capital across firms.

To quantify the misallocation of capital it is therefore important to measure the departure of aggregate investment from form its optimal level. I define $\frac{\hat{I}_t}{I_t}$ as the measure of misallocation of investment at time t. It is the ratio of the optimal level of aggregate investment at time t over the aggregate level of investment that prevails when firms face binding financing constraints.

$$\frac{\hat{I}_t}{I_t} = \frac{[(1-\theta)\alpha\beta E_t(Z_{t+1})]^{\frac{1}{1-\alpha}}}{\theta\left(Z_t k_0^\alpha\right) + (1-\theta)\left[\alpha\beta(1-\theta)E_t(Z_{t+1})\right]^{\frac{1}{1-\alpha}}}$$

The optimal level of investment $\hat{I}_t$ is given by $[(1-\theta)\alpha\beta E_t(Z_{t+1})]^{\frac{1}{1-\alpha}}$ (because there is a measure 1 of firm and they all invest at the optimal level). The level of aggregate investment when θ firms face financing constraint is given by $\theta\left(Z_t k_0^\alpha\right) + (1-\theta)\left[\alpha\beta(1-\theta)E_t(Z_{t+1})\right]^{\frac{1}{1-\alpha}}$.

An aggregate shock affects both side of this ratio. First, an aggregate shock affects the optimal level of investment. Indeed, an aggregate shock has an effect on the expected futur aggregate shock trought its persistence. Therefore, it modifies the expected marginal return of capital that depends on the expected futur aggregate shock. Finally, because the optimal level of investment depends on the expected marginal return of capital, an aggregate shock modify the investment behavior of firms. We can see below that the optimal level of investment is pro-cyclical, after a positive aggregate shock it increases and it decreases after a negative aggregate shock.

$$\frac{\partial \hat{I}_t}{\partial u_t} = \frac{\rho}{1-\alpha}\left[\alpha\beta(1-\theta)E_t(Z_{t+1})\right]^{\frac{1}{1-\alpha}} > 0$$

Second, the aggregate level of investment in an economy where some firms face financing constraint is also pro-cyclical. Following a positive aggregate shock, firms' profit increases, it relaxes their financing constraint and it increase the amount of good they can use to invest.

$$\frac{\partial I_t}{\partial u_t} = \theta Z_t k_0^\alpha + (1-\theta)\frac{\rho}{1-\alpha}\left[\alpha\beta(1-\theta)E_t(Z_{t+1})\right]^{\frac{1}{1-\alpha}} > 0$$

But, those two levels of investment do not evolve to the same extent after an aggregate shock and the evolution of the measure of misallocation after a shock reflects this pattern:

$$\frac{\partial(\hat{I}_t/I_t)}{\partial u_t} = \frac{[\alpha\beta(1-\theta)E_t(Z_{t+1})]^{\frac{1}{1-\alpha}}\,\theta\left(Z_t k_0^\alpha\right)\left(\frac{\rho}{1-\alpha}-1\right)}{\left[\theta\left(Z_t k_0^\alpha\right)+(1-\theta)\left[\alpha\beta(1-\theta)E_t(Z_{t+1})\right]^{\frac{1}{1-\alpha}}\right]^2}$$

This ratio can increase or decrease after an aggregate shock depending on the value of $\frac{\rho}{1-\alpha}$. It follows that the measure of misallocation increases or decreases after a positive aggregate shock depending on the persistence of the aggregate shock for a given value of the capital share.

$$\begin{aligned}\frac{\partial(\hat{I}_t/I_t)}{\partial u_t} > 0 &\iff \left(\frac{\rho}{1-\alpha}-1\right) > 0\\ &\iff \rho > 1-\alpha\end{aligned}$$

This result can be explained as follows: an aggregate shock as a direct effect on constrained firm investment, this effect does not change depending on the characteristic of the shock. In contrast, the variation in firms' optimal level of investment in response to this shock is proportional to $\left(\frac{\rho}{1-\alpha}\right)$. Then, depending on the value of this ratio, the firm optimal level of investment is more or less reponsive to an aggregate shock. If the increase in the firm optimal level of investment is low compared to the increase in the firm constrained instment, the misallocation of investment decreases after a shock, on the contrary, if the increase in the firm optimal level of investment is high compared to the increase in the firm constrained investment, the misallocation of investment increases after a shock.

In this model, there is a room for fiscal policy and for redistribution between agents because of this friction. The existence of financing constraint preventing firm to invest at their optimal level implies that a planner can increase the aggregate welfare by acting on this friction. I have also shown here that the magnitude of this friction changes over the business cycle, which justifies studying the optimal fiscal policy over such cycles.

3.8 Ramsey Program

The goal of this paper is to determine the optimal fiscal policy that generates the equilibrium-maximizing aggregate welfare. Aggregate welfare is defined using an explicit criterion. Fiscal policy is a path of two instruments $(\tau_t, T_t)_{t\geq 0}$, tax rate and transfer. The role of the government is to choose the competitive equilibrium maximizing the aggregate welfare and satisfying its budget constraint.

Aggregate Welfare The aggregate welfare is the intertemporal discounted sum of the utility of the representative household. Formally, the aggregate welfare criterion is written as:

$$W = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t)$$

The Ramsey Program I determine the optimal fiscal policy solving a Ramsey problem. In the quantitative model, I follow another strategy to compute the optimal dynamics of corporate tax rate, relying on simulations of the model.

Formally, the Ramsey Program can be expressed as:

$$\max_{(k_{2,t+1},\tau_t\}_{t\geqslant 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t\right) \tag{6}$$

$$\text{S.t} \quad k_{1,t+1} = (1-\tau_t) Z_t k_0^{\alpha} \tag{7}$$

$$M_t = \beta(1-\theta)\mathbb{E}_t\left[\alpha M_{t+1}\left(1-\tau_{t+1}\right) Z_{t+1} k_{2,t+1}^{\alpha-1}\right] \tag{8}$$

$$\begin{aligned} C_t = {} & n_1\left((1-\tau_t)Z_t k_{1,t}^{\alpha} - k_{2,t+1}\right) + n_2\left((1-\tau_t)Z_t k_{2,t}^{\alpha} - k_{2,t+1}\right) \\ & + \tau_t\left(n_0 Z_t k_0^{\alpha} + n_1 Z_t k_{1,t}^{\alpha} + n_2 Z_t k_{2,t}^{\alpha}\right) \end{aligned} \tag{9}$$

And subject to different other constraints such that the definition of the household pricing kernel, the positivity of capital and consumption choices, and initial conditions. The constraints in this Ramsey program include the investment constraint on type 0 firm (7), unconstrained firms Euler equations (8) and the household budget constraint (9. I simplify the formulation of this problem, using the factorization of the Lagrangian method developed by Marcet and Marimon (2019) and applied to incomplete-market environments. I denote by $\beta^t \lambda^t$ the Lagrange multiplier on the unconstrained firms investment Euler equation. I factorize the Euler equation into the objective of the planner. The objective of the Ramsey program can be rewritten as:

$$\begin{aligned} J &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t\right) - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \left[M_t - \beta(1-\theta)\alpha\, \mathbb{E}_t \left[M_{t+1}\left(1-\tau_{t+1}\right) Z_{t+1} k_{2,t+1}^{\alpha-1}\right]\right] \\ &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t\right) - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_t M_t + \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \lambda_{t-1} \left[(1-\theta)\alpha M_t \left[\left(1-\tau_t\right) Z_t k_{2,t}^{\alpha-1}\right]\right] \end{aligned}$$

And, using $\lambda_{-1} = 0$

$$J = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t\right) - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t M_t \left(\lambda_t - \lambda_{t-1}(1-\theta)\alpha\left(1-\tau_t\right) Z_t k_{2,t}^{\alpha-1}\right)$$

Finally, the Ramsey-problem can be express as:

$$\begin{aligned} &\max_{\{k_{2,t+1},\tau_t\}_{t\geqslant 0}} J = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t\right) - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t M_t \left(\lambda_t - \lambda_{t-1}(1-\theta)\alpha\left(1-\tau_t\right) Z_t k_{2,t}^{\alpha-1}\right) \\ &\text{S.t} \quad k_{1,t+1} = (1-\tau_t) Z_t k_0^{\alpha} \\ &\qquad C_t = n_1\left((1-\tau_t) Z_t k_{1,t}^{\alpha} - k_{2,t+1}\right) + n_2\left((1-\tau_t) Z_t k_{2,t}^{\alpha} - k_{2,t+1}\right) \\ &\qquad\qquad + \tau_t\left(n_0 Z_t k_0^{\alpha} + n_1 Z_t k_{1,t}^{\alpha} + n_2 Z_t k_{2,t}^{\alpha}\right) \end{aligned}$$

In the rest of this paper, I assume, for the sake of simplicity, that the household has a linear utility function.

3.8.1 The benchmark: the constrained efficient equilibrium

To understand the impact of frictions on my results, I compare an economy with frictions to a frictionless economy. I define the frictionless economy as an economy where no firms are constrained on their level of investment. The government can choose, in each period, firm's investment and corporate taxes and transfers so as to optimize the aggregate welfare. Formally, the frictionless economy allocation is the solution of the following program:

$$\max_{\{k_{t+1},\tau_t\}_{t\geqslant 0}} J = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t) - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\lambda_t - \lambda_{t-1}(1-\theta)\alpha(1-\tau_t) Z_t k_t^{\alpha-1}\right) \quad (10)$$

$$\text{S.t} \qquad C_t = \theta\left((1-\tau_t)Z_t k_0^\alpha - k_{t+1}\right) + (1-\theta)\left((1-\tau_t)Z_t k_t^\alpha - k_{t+1}\right) \quad (11)$$
$$+ \tau_t\left(\theta Z_t k_0^\alpha + (1-\theta) Z_t k_t^\alpha\right)$$

No firm is constrained in its level of investment. It implies that there are only two types of firm in this frictionless economy. There are θ firms of type 0. Those firms enter the economy at time t and they are characterised by a low level of capital k_0. Because they are not constrained or their level of investment, they invest to the optimal level. It means that, at the following period, they will produce using an optimal level of capital and invest to the optimal level. There are $(1-\theta)$ of such firms, which have been existing for one period or more. The objective of the planner (10) includes the factorization of firms' Euler equations. The consumption of the household is now the sum of dividends paid by all firms and of taxes levied on those 2 types of firms.

The resolution of this Ramsey program in Appendix B shows that a solution to this program is characterized by the following condition:

$$1 = \beta(1-\theta)\alpha k_{t+1}^{\alpha-1}\, \mathbb{E}_t(Z_{t+1})$$

This expression determines the allocation in this economy. The equation means that firms invest until the household marginal utility of consumption (the left-hand side of the equation) is equal to the expected marginal product of capital. The marginal utility of consumption is equal to 1 because the utility function of the household is linear.

In the rest of this section, I derive the first-order conditions of the planner in economies with frictions. I determine the optimal path of the tax rate in such frameworks. I consider two different cases. In the first one, the planner knows the type of each firm and she is able to design taxes that depend on those types. Thus, the planner has as many instruments as there are types of firms. In the second one, the planner uses a unique tax rate, either because she can't observe firms' type or because it is too costly to do so. In the data, within a country, the statutory corporate income tax rate is globally unique across firms. But there exists few exceptions that shows that governments are able - and sometimes willing - to discriminate across firms' type. For instance, in France, under given conditions, small firms have a reduced CIT rate[16]. Thus, governments know firm types and could design tax rates that depend on such characteristics.

3.8.2 Case 1: Heterogeneous tax rates

I first consider the simple and intuitive case. The planner knows firms' type and she is able to design taxes that depend on such type. Then, the planner has a set of three tax rates $\{\tau_t^0, \tau_t^1, \tau_t^2\}_{t=0}^{\infty}$.

In this case, the Ramsey-problem can be express as:

$$\max_{\{k_{2,t+1},\tau_t^0,\tau_t^1,\tau_t^2\}_{t\geqslant 0}} J = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t\right) - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t M_t \left(\lambda_t - \lambda_{t-1}(1-\theta)\alpha\left(1-\tau_t^2\right) Z_t k_{2,t}^{\alpha-1}\right)$$

$$\begin{aligned}
\text{S.t} \quad & k_{1,t+1} = (1-\tau_t^0) Z_t k_0^{\alpha} \\
& C_t = n_1\left((1-\tau_t^1) Z_t k_{1,t}^{\alpha} - k_{2,t+1}\right) + n_2\left((1-\tau_t^2) Z_t k_{2,t}^{\alpha} - k_{2,t+1}\right) \\
& \qquad + \left(n_0 \tau_t^0 Z_t k_0^{\alpha} + n_1 \tau_t^1 Z_t k_{1,t}^{\alpha} + n_2 \tau_t^2 Z_t k_{2,t}^{\alpha}\right)
\end{aligned}$$

The resolution of this Ramsey program in Appendix B shows a set of results that can be gathered in the following propositions:

Proposition 1 :

(i) At steady-state, the planner does not tax firms. She does not tax nor subsidize unconstrained firm and she subsidizes constrained firms.

(ii) When the planner designs taxes that depend on firm types, the frictionless allocation can be recovered.

(iii) This allocation can be implemented using time varying taxes on type 0 firms. Those times varying taxes depend on the expected persistence of the aggregate productivity shock.

[16] In 2021, a CIT of 15% for firms realising a turnover of max 10 million euros (and only on their first 38 120e of taxable profits) and of 26.5% for firms realising a turnover of max 250 million euros. The other ones face a CIT rate of 27.5%.

Proofs are in Appendix B.

Discussion of the Proposition 1.3 As shown in Appendix B, the optimal path of the tax rate on type 0 firm can be express, for $t \geq 0$, as:

$$\tau_t^0 = 1 - \overbrace{\frac{\mathbb{E}_t\left[Z_{t+1}\right]^{\frac{1}{1-\alpha}}}{Z_t}}^{\text{(i) Dynamics}} \overbrace{\frac{\left[\beta(1-\theta)\alpha\right]^{\frac{1}{1-\alpha}}}{k_0^{\alpha}}}^{\text{(ii) Steady-State}}$$

The tax rate on type 0 firm depends on:

(ii) the ratio of the steady-state value of the optimal level of investment over the steady-state value of the constrained level of investment. The greater the difference between those two values, the more the planner subsidies type 0 firms at steady-state. This ratio depends on the different parameters of the model, k_0 the capital of new firms, α that governs firms return to scale, β the household discount factor and θ that determines the probability to exit the economy at each period. Therefore, the steady-state value of the tax rate is governed by the magnitude of the main friction of this model. If the two investment values are equal, the planner has no reason to tax or subsidy firms at steady-state. Therefore, firm subsidies at steady-state can be motivated by the existence of financing constraints.

(i) the dynamics of the tax rate. The dynamics of the tax rate after an aggregate productivity shock is given by the ratio of a function of the expected future value of the aggregate productivity and the current value of the aggregate productivity.

As discussed in the previous section, after a positive productivity shock, the current value of the aggregate productivity increases. The constrained level of investment depends on the current aggregate level of productivity. Indeed, aggregate productivity has an effect on firms' current earnings which relax (or tighten following a negative shock) the financing constraints of constrained firms and, finally, which increases their level of investment. The current value of the aggregate productivity does not affect the unconstrained firm level of investment that depends only on the expected future value of the aggregate productivity. But, an aggregate productivity shock has an effect on the expected future value of the aggregate productivity through the persistence of the shock over time.

If a shock is not persistent at all, its unique effect is through the current level of aggregate productivity. A positive shock increases the constrained level of investment and reduces the difference between the constrained and the unconstrained level of investment. Thus, it reduces the magnitude of the friction of this model and the incentive of the planner to subsidy those firms. Therefore, the planner increases the tax rate on type 0 firm following a positive productivity shock. As we can see in the expression, if the shock is not persistence, $\mathbb{E}_t(Z_{t+1}) = 1$. In this case, the dynamics of the tax rate depends only on $1 - \frac{1}{Z_t}$. Then, when Z_t increases, the tax rate also increases.

If the shock is persistent, the aggregate productivity shock increases the unconstrained firm level of investment through the persistence of its effect over time. In this case, the level of investment of unconstrained firm also increases after a positive productivity shock. The planner increases the tax rate if the increase in the constrained firm level of investment is still greater than the increase in the unconstrained firm level of investment. In this case, the friction in this model is still reduced after a shock and so is the incentive of the planner to subsidy type 0 firms. In contrast, if the increase in the constrained firm level of investment is lower than the increase in the unconstrained firm level of investment, the magnitude of the friction increases and so is the incentive of the planner to subsidy type 0 firm. In this situation, the planner increases further the subsidy rate on type 0 firm.

The evolution of this ratio following a shock depends therefore on how the current aggregate shock translates into a future expected aggregate shock, and how this future expected aggregate shock modify the unconstrained firm level of investment. Then, it depends on the persistence of the aggregate shock and on the parameters α that governs the marginal value of capital.

Threshold I show in Appendix B that I can determine conditions on those two parameters that determine the cyclicality of the tax rate. The tax rate on type 0 firms decreases after a positive producivity shock i.e., it is pro-cyclical with respect to its steady-state value if and only if:

$$Z_t\left((1-\alpha)-\rho\right)-\frac{\sigma^2}{2}\leq 0$$

The tax rate on type 0 firms increases after a positive producivity shock, i.e. it is counter-cyclical with respect to its steady-state value otherwise.

Around small shocks, $Z_t = 1$ and the tax rate is pro-cyclical if and only if:

$$(1-\alpha)\leq\rho$$

The tax rate is countercyclical otherwise.

Therefore, the optimal tax rate follows the path of the misallocation of investment in this model. When the problem of misallocation decreases after an aggregate shock, the planner increases the tax rate, on the contrary, when this problem increases, the planner should decrease this tax rate.

I derived here an analytical characterization of the optimal corporate tax rate after an aggregate productivity shock. It shows in a transparent way that the variation in the tax rates should depend on the relative effect of an aggregate shock on heterogeneous firms. The heterogeneous firm framework here is crucial to observe such mechanism. Because the key of the optimal response of the planner is not given by the variation in the level of investment of constrained or unconstrained firms after a shock but by the variation of their relative level of investment, it is crucial to model firms that differ in the severity of their constraints to design optimal fiscal policy. Then, the design of optimal fiscal policy over the business cycle depends on the response to aggregate shock of the entire firm distribution.

I illustrate this effect on the figure 5 below. I calibrate the previously described model as following:

Parameter	Description	Value
β	discount factor	0.99
α	capital share	0.3
θ	exit risk	0.5
k_0	new firms' capital	such that the eq. exists

The period is a quarter, the discount factor is $\beta = 0.99$, the production function of all firms $j \in \mathcal{J}$ in each period t is Cobb-Douglas: $y_{j,t} = Z_t(k_{j,t})^\alpha$, the capital share is $\alpha = 30\%$. k_0 is set such that the equilibrium of this model exists. θ is the probability for a firm to exit the economy at each period, it is set to 0.5. ρ is the persistence of the aggregate shock. I represent τ_t^0 over time in deviation with respect to its steady-state value, after an aggregate productivity shock[17].

[17] I checked that the steady-state of the economy is well defined.

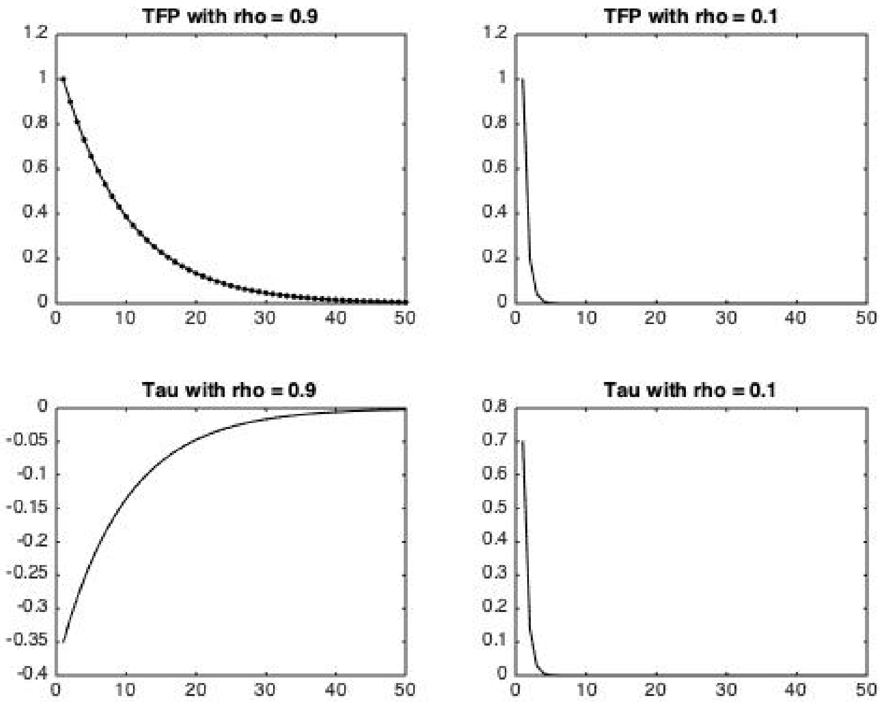


Figure 5: Illustration of the Proposition 1.3

Figure (5) sums-up my main finding and illustrates my analytical characterization of the optimal path of the tax rate over the business cycle. We can see on the left panel of this Figure, that, when the shock is very persistent, the planner increases the tax rate on type 0 firm after a positive productivity shock. This is so because the optimal level of investment of the constrained firm increases more than the level of investment of the unconstrained firm. Thus, it is due to the fact that the friction of this model is reduced by the different firms' reaction to the shock. On the contrary, we can see on the right panel of this graph, that, when the shock is not persistent, the planner decreases the tax rate on type 0 firm after a positive productivity shock. Indeed, the optimal level of investment of the unconstrained firm increases more than the level of investment of the constrained firm. This is so because, in this case, the friction of this model is worsened by the shock.

3.8.3 Case 2: Unique tax rate

Now, I consider the case where the planner uses a unique tax rate to tax heterogeneous firms. It can be because she does not observe the different type of firms or because it is too costly for the government to discriminate across firms. Then, the planner is left with one set of fiscal instruments: $(\tau_t)_{t\geq 0}$. As discussed earlier in this paper, the reality is a mix of those two extreme cases. The corporate tax rate is mainly flat across firms, but the planner discriminate over some firms characteristics (mostly over the very small or the very young firms).

In this case, the Ramsey-problem can be express as:

$$\max_{\{k_{2,t+1},\tau_t\}_{t\geq 0}} J = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t\right) - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t M_t \left(\lambda_t - \lambda_{t-1}(1-\theta)\alpha\left(1-\tau_t\right) Z_t k_{2,t}^{\alpha-1}\right)$$

$$\begin{aligned}
\text{S.t} \quad & k_{1,t+1} = (1-\tau_t) Z_t k_0^{\alpha} \\
& C_t = n_1\left((1-\tau_t) Z_t k_{1,t}^{\alpha} - k_{2,t+1}\right) + n_2\left((1-\tau_t) Z_t k_{2,t}^{\alpha} - k_{2,t+1}\right) \\
& \qquad + \tau_t\left(n_0 Z_t k_0^{\alpha} + n_1 Z_t k_{1,t}^{\alpha} + n_2 Z_t k_{2,t}^{\alpha}\right)
\end{aligned}$$

This program is the same as in the previous Section, except that now, the same tax rate applies to all three types of firms. Now, when the planner taxes or subsidizes type 0 firms, she needs to take into account that she also distorts types 1 and 2 firms' optimal decisions.

The resolution of this Ramsey program shows that a solution to this Ramsey program is characterized by the following conditions:

(i) The first order condition on τ_t can be written as:

$$\overbrace{n_0 Z_t k_0^{\alpha}}^{\text{(i)}} - \overbrace{\lambda_{t-1}(1-\theta)\alpha Z_t k_{2,t}^{\alpha-1}}^{\text{(ii)}} = \overbrace{Z_t k_0^{\alpha} \beta n_1 \alpha E_t Z_{t+1} k_{1,t+1}^{\alpha-1}}^{\text{(iii)}}$$

The planner increases the unique subsidy rates until the cost of increasing this rate (left-hand side of the equation) equalizes the benefit of increasing the subsidy rate (right-hand side of the equation). The trade-off faced by the planner appears cleary in this equation. When the planner increases the subsidy rate: (i) she increases the lump-sum taxes on the household by the amount of the additional subsidy given to the type 0 firm, i.e. proportional to the subsidy base of the type 0 firm. Because type 1 and type 2 firms reverse as dividends to the household excess earning over their optimal level of investment, additional dividends compensate exactly the lump-sum taxes needed to subsidy those two types of firms. Moreover, as the planner increases the subsidy rate, she distorts the firms' Euler equation (ii). Indeed, because the optimal level of investment at time $t-1$ depends on the tax rate at time t, changing the tax rate at time t affects the optimal investment of the previous period. Then λ_{t-1} is the cost (because it is negative) of increasing the optimal level of investment at time $t-1$. It is scaled by the marginal return of capital at time t. So, when the planner has a unique tax rate, she has to take into account the fact that, if she wants to subsidy type 0 firm to make them grow, it also distorts the optimal choice of capital of unconstrained firms.

The second condition is given by the first order condition on $k_{2,t+1}$:

$$\overbrace{(n_1+n_2)}^{(i)} = \overbrace{\beta n_2 \alpha E_t Z_{t+1} k_{2,t+1}^{\alpha-1}}^{(ii)} - \overbrace{\beta \lambda_t (1-\theta)\alpha(1-\alpha) E_t \left[(1-\tau_{t+1}) Z_{t+1} k_{t+1,2}^{\alpha-2}\right]}^{(iii)}$$

The left-hand side of this equation determines the cost of increasing the optimal level of capital and the right-hand side of this equation is the benefit of increasing the optimal level of capital. The cost of increasing the optimal level of capital (i) is equal to the decrease in type 2 and type 1 firms' dividends at period t. The benefit of increasing the optimal level of capital is (ii) proportional to the additional amount of production of type 2 firms at the next period due to the higher level of investment at time t. Moreover, it decreases the marginal cost of distorting further the unconstrained firms' Euler equations.

Then, the optimal path of the unique tax rate depends on the benefit of increasing the subsidy rate on type 0 firm to make them grow, taking into account the cost of the subsidy for the planner. It also depends on the fact that when the unique subsidy rate increases, it increases the level of investment of unconstrained firm. The planner needs to take into account this distortion when setting the path of the tax rate of the cycle.

I cannot derive an analytical characterization of the path of the tax rate using those two equations. Therefore, I rely on numerical simulations to determine the path of the tax rate after an aggregate shock. I use the same calibration as presented in the heterogenous tax rates case[18]. As before, I simulate the optimal path of the tax rate following an aggregate shock, depending on its persistence. On Figure 6, I plot τ_t over time, in deviation with respect to its steady-state value, after a positive aggregate productivity shock.

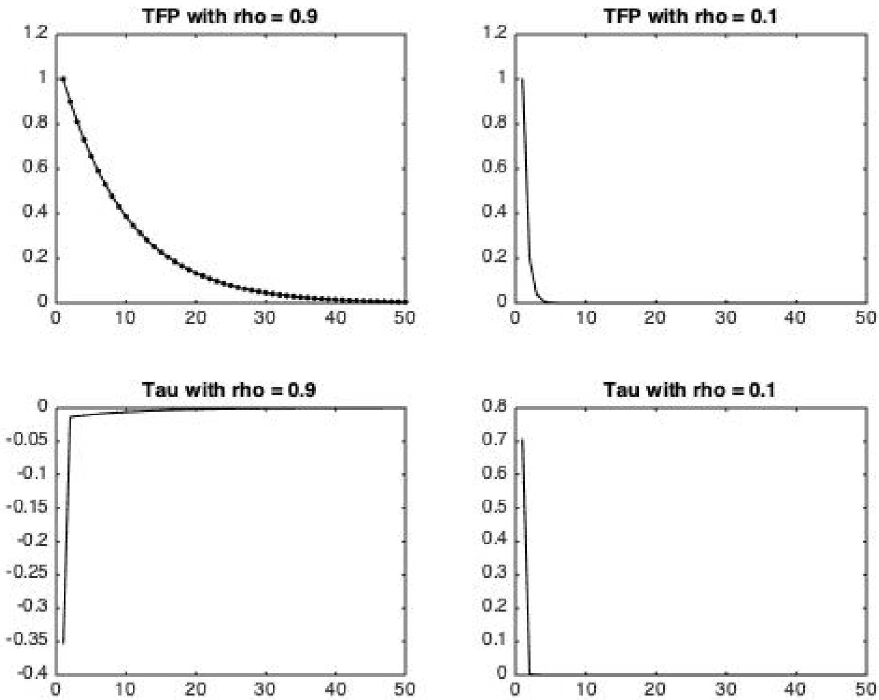


Figure 6: Optimal path of the tax rate τ_t depending on the value of ρ

We can see on Figure 6 that the main result derived in the heterogeneous tax rates case holds. Indeed, the optimal path of the tax rate following a shock still depends on the persistence of the aggregate shock. When the shock is not persistent, the difference between the two cases is small because the investment of the unconstrained firm barely react to an aggregate shock. Moreover, in both cases, the change in the tax rate following an aggregate shock is not persistent. Then, the distortion in the optimal level of investment implied by the change in the unique tax rate is limited. The main difference with

[18] I checked that the steady-state is well defined.

the previous case in that, when the shock is very persistent, the optimal decrease in the tax rate is very short lasting. Indeed, when the shock is very persistent, the optimal level of investment of the unconstrained firm reacts greatly to current aggregate productivity shock. Then, the planner has to take into account the cost of increasing the optimal level of capital of unconstrained firm. The, she changes only briefly the value of the tax rate, trying to minimize those additional distortions.

The importance of taking into account firms heterogeneity when designing optimal fiscal policy is still clear here. Firm heterogeneity still implies that the cyclicality of the tax rate has to change according to the relative variation in the different type of firm level of investment. What changes in this case is that a variation in the tax rate has an impact on the different level of investment. Then, the planner has to take into account that, when trying to correct the friction on type 0 firms, she also distorts the optimal level of investment of unconstrained firms.

In this section, I derived the path of the optimal corporate tax rate when the planner has access to a set of tax rates that depends on the type of firms or when she has a unique tax rate. I show that in both cases, the dynamics of the tax rate depends on the persistence of the aggregate shock. In the first one, the planner can reproduce the frictionless allocation. In the second one, she has to take into account that subsidizing constrained firm distorts the optimal level of investment of unconstrained firms. In both cases, firm heterogeneity is a key to the design of optimal fiscal policy.

4 Quantitative assessment

Now, I solve for the optimal fiscal policy over the business cycle in a quantitatively relevant environment. In the first part of this paper, I have made different assumptions to make my model tractable and to derive analytical and intuitive results. In this section, I relax those assumptions, keeping the main ingredients of this model. Now, capital depreciates at rate $\delta < 1$ and I introduce endogenous labor in the production function and in the household utility function. Most importantly, I introduce idiosyncratic productivity shocks, which generate a time-varying joint distribution of capital and productivity.

Deriving the optimal path of the tax rate in this framework allows to test the robustness of my analytical results. I check here - in a quantitatively relevant heterogenous firm framework - if the optimal fiscal policy should depend on the relative behavior of the different types of firm after an aggregate shock.

4.1 Model

This model builds on Khan et al. (2017). It is a quantitative generalization of the model presented in the Section 2 of this paper. In this framework, because of idiosyncratic productivity shocks, firms have persistent differences in their total factor productivity. Moreover, investment is funded by retained earnings only. It implies that firms with low levels of capital have low levels of production and can fund only low levels of investment. All firms draw productivity from the same distributions. It results in a misallocation of capital, reducing in return aggregate capital and GDP.

As in the previous Section, there are three types of agents in this model. A continuum of heterogeneous firms, a continuum of representative households and a government. Time is discrete and infinite. Each period, there is a fix mass 1 of heterogeneous firms distributed on an interval $\mathcal{J}$.

Preferences Now, in each period, the economy has two goods: a consumption good - as before - and labor. Households are expected utility maximizers and they rank streams of consumption $(C_t)_{t\geq 0}$ and labor $(N_t)_{t\geq 0}$ using the intertemporal utility criterion $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$ where the utility function satisfies the usual conditions. As before, in each period, firms maximize the expected discounted value of dividends D_t returned to their shareholders - the representative household.

Risks Firms are subject to three types of shocks in this economy. As in the first part of this paper, they face two aggregate risks, an aggregate productivity risk and an exit risk. The aggregate productivity risk is denoted $(z_t)_{t\geqslant 0}$ at time t and I assume that it follows an $AR(1)$ process $z_t = \rho_z z_{t-1} + u_t^z$ with ρ_z the persistence parameter and the shock u_t^z being a white noise with a normal distribution $\mathcal{N}(0, \sigma_z^2)$. The aggregate productivity, denoted $(Z_t)_{t\geqslant 0}$ is assumed to relate to z_t through the following functional form: $Z_t = Z_0 e^{z_t}$. Moreover, at the beginning of each period, firm $j \in \mathcal{J}$ faces a constant probability $\theta < 1$ to exit the economy.

Now, firms also face an idiosyncratic productivity risk. I assume that the idiosyncratic productivity shock follows an $AR(1)$ process such that the idiosyncratic level of productivity of a firm j at time t is given by $\varepsilon_{j,t} = \rho_\varepsilon \varepsilon_{j,t-1} + u^\varepsilon_{j,t}$ with ρ^ε the persistence of the idiosyncratic shock u^ε_t and this shock u^ε_t being a white noise with a normal distribution $\mathcal{N}(0, \sigma^2_\varepsilon)$ and $\varepsilon \in E = \{\varepsilon_1, ..., \varepsilon_n\}$.

Production As before, firms produce using capital. Here, I also introduce labor in the production function. Moreover, in addition to the aggregate productivity shock, an idiosyncratic productivity shock enters their production function. Each firm j produces the same good $y_{j,t}$ using a predetermined stock of capital $k_{j,t}$ and labor $n_{j,t}$ with the following production function, for $t \geq 0$:

$$y_{j,t} = Z_t \varepsilon_{j,t} k^\alpha_{jt} n^\nu_{j,t}$$

Where $\alpha > 0$ is the capital share, $\nu > 0$ is the labor share and $\alpha + \nu < 1$. $\varepsilon_{j,t}$ is the firm j idiosyncratic level of productivity and Z_t is the economy-wide level of productivity.

Firms Heterogeneity At the beginning of each period, a firm is identified by its predetermined stock of capital k and its current idiosyncratic productivity level ε. The aggregate state of this economy is characterised by the distribution of firms (k, ε), by the aggregate shock Z and by the tax rate τ. I summarize the distribution of firms over (k, ε) using the probability measure μ defined on the Borel algebra generated by the open subsets of the product space $K \times E$. The distribution of the firms evolve over time according to a mapping Γ such that $\mu' = \Gamma(\mu, Z)$. The evolution of the firm distribution depends on two factors. An endogenous evolution that depends on firm's decision and the exogenous process of firms entry and exit from the economy.

Indeed, as before, in each period, there is an exogenous mass of firms that enter the economy in order to keep the mass of firms constant and equal to one. Those new firms are endowed with a fix level of capital and they draw their idiosyncratic level of productivity from the time-invariant distribution.

Government As before, in each period the government levies a distorting corporate income tax (or subsidy) τ on firms to finance a lump-sum transfer (or tax) T redistributed to the household. Because empirically the corporate income tax is mainly flat across firms, I study a case where the fiscal policy is characterized by two time varying instruments $(\tau_t, T_t)_{t\geqslant 0}$[19]. Thus, the planner does not discriminate across firms types. But, now, the aggregate tax T depends on the distribution of firms over their idiosyncratic level of productivity and over their level of capital. Therefore, the budget constraint of the government is, written in a recursive form:

$$T(Z,\mu,\tau) = \int \tau(Z,\mu)\,[y(Z,\mu,\tau,k,\varepsilon)]\,d(\mu(k\times\varepsilon))$$

Household The household consumes using aggregate firm dividends, aggregate transfer and its wage. Therefore, the budget constraint of the household is, written in a recursive form:

$$C(Z,\mu,\tau) = w(Z,\mu,\tau)N + D(Z,\mu,\tau) + T(Z,\mu,\tau)$$

With $D(Z,\mu,\tau)$ the aggregate level of dividends, $T(Z,\mu,\tau)$ a lump sum tax or subsidy, $w(Z,\mu,\tau)$ the hourly wage and N the household endogenous level of labor supply.

The utility function of the household is:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log(C_t) - \chi \frac{N_t^{1+\frac{1}{\phi}}}{1+\frac{1}{\phi}} \right)$$

With $\beta \in (0,1)$ a constant discount factor, $\phi > 0$ the Frisch elasticity of labor supply and $\chi > 0$ a parameter which scales labor disutility. The representative household owns

[19] Here, I also use a stylized version of the corporate income tax. I still don't allow for depreciation allowance in the tax base. Costs of production are not deduced either from the tax base. I show in Appendix C different characterizations of this tax.

all firms in the economy.

Timing At the beginning of each period, firms produce. To do that, at time t, firm j draw a idiosyncratic level of productivity $\varepsilon_{j,t}$ and hires labor $n_{j,t}$ from a competitive labor market at wage w_t. After production, the exit shock is realized and firms learn if they have to exit the economy or if they can continue to the following period. If they are allowed to continue, given the aggregate state of the economy, (Z, μ, τ) and their beginning of period individual state, they choose their level of investment and the amount of dividend to redistribute to their shareholder. As before, firms cannot extract equity from their shareholders, what bounds their level of dividends to 0.

Discussion of the main assumption In this paper friction is generated by the fact that firms cannot access external finance. I study here an extreme case in which firms need to finance their investment using only retained earnings. In a future extension, I will study a more realistic setup, allowing for a costly access to external finance.

I state here the firm dynamic optimization problem, written in a recursive form. At the beginning of the period, before firms know if they will be allowed to continue into the next period, their value function is given by:

$$\begin{aligned} V_0(k, \varepsilon; Z, \mu, \tau) = \quad & \theta \max_n [(1 - \tau(Z, \mu)) Z \varepsilon k^{\alpha} n(k, \varepsilon; Z, \mu, \tau)^{\nu} - w(Z, \mu, \tau) n(k, \varepsilon; Z, \mu, \tau) \\ & \qquad + (1 - \delta) k] \\ & + (1 - \theta) V(k, \varepsilon; Z, \mu, \tau) \end{aligned}$$

After the realization of the exit shock, firms know if they have to exit the economy or not. If they have to exit the economy, they simply choose their level of labor demand as to maximize the dividend that they pay to their shareholder during the period. If firms are allowed to continue, they have to solve for the optimal level of investment and dividend. Firms take as given the evolution of the firm distribution and they solve the following optimization problem. Firms are constrained by the fact that they cannot extract equity from their shareholders:

$$
\begin{aligned}
V(k,\varepsilon;Z,\mu,\tau) = \max_{k'} \quad & (1-\tau(Z,\mu))\left(Z\varepsilon k^{\alpha} n(k,\varepsilon;Z,\mu,\tau)^{\nu}\right) - w(Z,\mu,\tau)n(k,\varepsilon;Z,\mu,\tau) \\
& + (1-\delta)k - k' \\
& + \mathbb{E}[\lambda'(\theta(1-\tau'(Z',\mu'))\left(Z'\varepsilon' k'^{\alpha} n(k',\varepsilon';Z',\mu',\tau')^{\nu}\right) - w(Z',\mu',\tau')n(k',\varepsilon';Z',\mu',\tau') \\
& + (1-\theta)V'(k',\varepsilon';Z',\mu',\tau'))] \\
\text{S.t} \qquad & (1-\tau(Z,\mu))\left(Z\varepsilon k^{\alpha} n(k,\varepsilon;Z,\mu,\tau)^{\nu}\right) - w(Z,\mu,\tau)n(k,\varepsilon;Z,\mu,\tau) + (1-\delta)k - k' \geqslant 0 \\
& \mu' = \Gamma(\mu, Z)
\end{aligned}
$$

With λ the firms' pricing kernel, because firms are own by the households they have the household pricing kernel: $\lambda' = \beta \frac{U(C')}{U(C)}$

All firm with the same level of idiosyncratic productivity and the same level of capital choose the same level of labor demand $n(k,\varepsilon;Z,\mu,\tau)$ and the same level of production $y(k,\varepsilon;Z,\mu,\tau)$.

Firm labor demand of (k,ε) firms is given by:

$$
n(k',\varepsilon';Z',\mu',\tau') = \left(\frac{\nu(1-\tau'(Z',\mu'))Z'\varepsilon' k'^{\alpha}}{w(Z',\mu',\tau')}\right)^{\frac{1}{1-\nu}}
$$

And their level of production is given by:

$$
y(k,\varepsilon;Z,\mu,\tau) = Z\varepsilon k^{\alpha}\left(\frac{\nu(1-\tau'(Z',\mu'))Z'\varepsilon' k'^{\alpha}}{w(Z',\mu',\tau')}\right)^{\frac{\nu}{1-\nu}}
$$

Solving this optimization problem, firms determine their optimal choice of capital accumulation. If their level of retained earnings is sufficient to finance this level of investment, they invest to this level and redistribute as dividends to their shareholders the residual earnings.

The optimal capital accumulation rule is given by:

$$
\begin{aligned}
k'^{\star}(\varepsilon) = argmax_{k'} \quad & - k' \\
& + \beta\,\mathbb{E}[(1-\tau(Z',\mu'))Z'\varepsilon' k'^{\alpha} n(k',\varepsilon';Z',\mu',\tau')^{\nu} - w(Z',\mu',\tau')n(k',\varepsilon';Z',\mu',\tau') \\
& + (1-\delta)k']
\end{aligned}
$$

Thus, they are not constrained by their level of retained earnings if:

$$(1-\tau(Z,\mu))\left(Z\varepsilon k^{\alpha}n(k,\varepsilon,Z,\mu,\tau)^{\nu}\right)-w(Z,\mu,\tau)n(k,\varepsilon,Z,\mu,\tau)+(1-\delta)k-k'^{\star}(\varepsilon)\geq 0$$

In such case, firms invest to their optimal level of investment $k'^{\star}(\varepsilon)$ and they set their level of dividends to $d(k,\varepsilon;Z,\mu,\tau)$ such that :

$$\begin{aligned}&d(k,\varepsilon;Z,\mu,\tau)=\\&\quad(1-\tau(Z,\mu))\left(Z\varepsilon k^{\alpha}n(k,\varepsilon,Z,\mu,\tau)^{\nu}\right)-w(Z,\mu,\tau)n(k,\varepsilon,Z,\mu,\tau)+(1-\delta)k-k'^{\star}(\varepsilon)\end{aligned}$$

Thus, all firms that share the same level of idiosyncratic productivity ε and with a level of capital k sufficient to finance the optimal capital policy choose the same level of investment. Moreover, all firms with the same (k,ε) and that are not constrained on their level of investment pay the same level of dividends to the household.

On the contrary, if firms level of capital is not sufficient to finance their optimal level of investment, they invest their entire retained earnings and set their level of dividend equal to zero. Thus, if

$$(1-\tau(Z,\mu))\left(Z\varepsilon k^{\alpha}n(k,\varepsilon,Z,\mu,\tau)^{\nu}\right)-w(Z,\mu,\tau)n(k,\varepsilon,Z,\mu,\tau)+(1-\delta)k-k'^{\star}(\varepsilon)<0$$

Firms set their level of constraint investment to their retained earnings:

$$k'^{c}(k,\varepsilon,\tau)=(1-\tau(Z,\mu))Z\varepsilon k^{\alpha}n(k,\varepsilon;Z,\mu)^{\nu}-w(Z,\mu)n(k,\varepsilon,Z;\mu,\tau)+(1-\delta)k$$

And,

$$d(k,\varepsilon;Z,\mu,\tau)=0$$

4.2 Equilibrium Definition

Firms distribution For a given policy instrument τ, a recursive competitive equilibrium for this model is a collection of individual functions ($v(k,\varepsilon;Z,\mu,\tau)$, $k^{\star}(k,\varepsilon;Z,\mu,\tau)$, $k^{c}(k,\varepsilon;Z,\mu,\tau)$, $n(k,\varepsilon;Z,\mu,\tau)$, $d(k,\varepsilon;Z,\mu,\tau)$, $\lambda(Z,\mu,\tau)$, $N(Z,\mu,\tau)^{s}$), of aggregate quantities ($K(Z,\mu,\tau)$, $N(Z,\mu,\tau)^{d}$, $Y(Z,\mu,\tau)$, $D(Z,\mu,\tau)$, of price processes $w(Z,\mu,\tau)$ and of fiscal policy $(\tau(Z,\mu),T(Z,\mu,\tau))$, a law of motion of the distribution of firms $\Gamma(\mu,Z)$ such that such, for an initial capital and productivity distribution:

1. taking $\lambda(Z,\mu,\tau), w(Z,\mu,\tau), (Z', \mu'(Z'), \tau'(Z'))$ as given, the functions $v(k,\varepsilon;Z,\mu,\tau)$, $k^\star(k,\varepsilon;Z,\mu,\tau)$, $k^c(k,\varepsilon;z,\mu,\tau), n(k,\varepsilon;Z,\mu,\tau), d(k,\varepsilon,Z,\mu,\tau)$ solve the firm optimization program

2. The firs-order conditions of the planner are fulfilled: $\lambda(Z,\mu,\tau) = C(Z,\mu,\tau)^{-\gamma}$ with $C(Z,\mu,\tau) = \int Z\varepsilon(1-\tau)k^\alpha n(k,\varepsilon;Z,\mu)^\nu + (1-\delta)k - (1-\theta)(1_\xi k^o(\varepsilon;z,\mu,\tau) - (1-1_\xi)k^c(k,\varepsilon,z,\mu,\tau))\mu(\varepsilon,k)d\varepsilon dk$

3. given $w(z,\mu,\tau)$, the functions $N^d(Z,\mu,\tau), C(Z,\mu,\tau)$ solve the household optimization program

4. $w(Z,\mu,\tau)$ satisfies $\int n(k,\varepsilon;Z,\mu,\tau)d\varepsilon d\mu(k\times\varepsilon) = (\frac{w(Z,\mu,\tau)\lambda(Z,\mu,\tau)}{\chi})^\phi$

5. labor, and good markets clear at all dates

6. the government budget is balanced at all dates

7. the law of motion $\Gamma(\mu,Z)$ is consistent with individual firm decisions

To be more precise above the firm distribution law of motion, the distribution of firms is the sum of the firms that have not exit the economy at the previous period and of firms that enter the economy at the beginning of the period. The law of motion of firm distribution is then given by: $\forall(k',\varepsilon')$

$$\begin{aligned}
&\mu'(k',\varepsilon';Z,\mu,\tau) = \\
&\quad (1-\theta)\int \left((\mathbb{1}\{k^o(k,\varepsilon)=k'\} + \mathbb{1}\{k^c(k,\varepsilon)=k'\}) \times p(\varepsilon'=\varepsilon'|\varepsilon=\varepsilon)\right) d\varepsilon d\mu(k\times\varepsilon) \\
&\quad + \theta\int (\{k^o(k_0,\varepsilon_0)=k'\} + \{k^c(k_0,\varepsilon_0)=k'\})
\end{aligned}$$

With $\mathbb{1}_\xi$ an indicator function equal to 1 if the firm is not constrained on its level of investment and 0 if the firm is constrained, ε_0 is drawn from the ergodic distribution of the idiosyncratic productivity and $\mathbb{1}_\xi$ and $\mathbb{1}(A)$ denote an indicator function equal to 1 if A is true and 0 otherwise.

4.3 Determination of the optimal tax policy

To determine the optimal tax policy I follow two steps. First, I solve this heterogeneous agent model with aggregate shocks. Then, I compute the time varying tax rate that maximizes the aggregate welfare given by:

$$W = \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

4.3.1 Dynamics of the model

Solving heterogeneous agent models with aggregate shocks is computationally challenging. Indeed, in such model the vector of state variables, which contained the distribution of the agents is infinite-dimensional. Then, the policy function depends on an infinite object. The vector of state variables here is $\Omega = (Z, \mu, \tau)$ with μ that is of infinite dimension.

I use the Reiter (2009)'s projection and perturbation method together with the Young (2010)'s method to simulate a cross-section to solve for the recursive competitive equilibrium [20]. This method consists in three steps. The first one is to discretize the model. The second is to solve for the non-stochastic steady-state of this model with idiosyncratic uncertainty but without aggregate uncertainty. The last one is to linearize around this non-stochastic steady-state and to solve for the dynamics using a usual rational expectation solver. Details of the algorithm used to solve this model can be found in Appendix C.

4.3.2 Maximization of the Aggregate Welfare

I solve for the path of the tax rate over the business cycle that maximizes the aggregate welfare W relying on numerical simulations. The aggregate welfare criterion is the discounted intertemporal sum of the household utility:

[20] It consists in using a histogram to approximate the law of motion of firm distribution, and to avoid relying on Monte-Carlo simulations as in the Krussel and Smith (1998) algorithm. Winberry (2018) approximates the distribution with a flexible parametric family. The Reiter (2009)'s algorithm is well suited for my problem but it limits the model to be low-dimensional. When extending this model, a possibility would be to use the Winberry (2018)'s algorithm to be able to reduce the dimensionality of the problem to a finite set of endogenous parameters.

$$W = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\log(C_t) - \chi \frac{N_t^{1+\phi}}{1+\phi})$$

My goal is to determine how the corporate income tax rate should vary over the business cycle. I want to determine if the cyclicality of the tax rate should change depending on the heterogenous variation of different firms' investment over the business cycle. Put differently, I want to test if the results I found analytically in a simple model hold in this quantitative environment.

Thus, I need to determine how the corporate income tax rate should vary with the aggregate productivity shock. It means that I need to determine the parameter η that maximizes the aggregate welfare such that:

$$\tau_t = \eta(Z_t - 1)$$

η can be defined as the cyclicality of the tax rate.

4.3.3 Numerical Simulations

To determine the optimal η, I proceed as the following. I fix η and I simulate the model over 10,000 periods around a steady-state where the tax rate is null i.e. around $\tau^\star = T^\star = 0$.

Then, at each period I compute the utility function of the household and I approximate the aggregate welfare with the finite discounted sum of the household utility:

$$\hat{W} = \mathbb{E}_0 \sum_{t=0}^{10{,}000} \beta^t U(C_t, N_t)$$

I simulate the model M times and I take the average of the aggregate welfare $\bar{\hat{W}}$ over those M simulations. Then, I solve for the η that maximizes $\bar{\hat{W}}$ and that satisfies the constraint of the model at each period[21]. M is such that the estimations converge.

[21] For instance, I prevent the planner from choosing a path of the tax rate that implies a negative level of consumption or a negative level of capital at some period.

Discussion about the choice of the steady-state value of the tax rate Here, I set the average tax rate to 0, i.e. $\tau^\star = 0$. This analysis can be run using different steady-state level of the tax rate. The average level of the tax rate depends on different elements such as the government's funding needs or on the existence of different taxes that I don't model here. Thus, I focus on the study of the dynamics of the tax rate over the business cycle.

4.3.4 Calibration

Preferences and Production I set the model period to one quarter. I follow standard calibration of this type of model. I set the discount factor to 0.99. I set the labor share to 0.595% and the capital share to 21% to have a total returns to scale of 80%. The depreciation rate of capital is set quarterly to 0.025 to match the average aggregate investment rate of nonresidential fixed investment.

Parameter	Description	Value
α	capital share	0.21
ν	labor share	0.595
β	discount factor	0.99
ϕ	frisch elasticity of labor supply	1
N^s	aggregate labor supply	$\frac{1}{3}$

Shocks The first two parameters ρ^ε and σ^ε determine the process of the idiosyncratic shocks. I use $\rho^\varepsilon = 0.8$ and $\sigma^\varepsilon = 0.15$ following Jeenas (2020) using an estimation based on the data of Compustat firms. The following two ρ^z and σ^z determine the process of the aggregate shocks. I set $\sigma^z = 0.31\%$ to have a deviation of the aggregate productivity shock Z_t equal to 1% at the quarterly frequency (as in Den Haan and Rendahl (2010)). I use different values of ρ^z over my simulations.

Parameter	Description	Value
ρ^{ε}	Persistence of idiosyncratic productivity shock	0.8
σ^{ε}	SD of innovations to idiosyncratic productivity	0.15
ρ^{z}	Persistence of aggregate productivity shock	depends
σ^{z}	SD of innovations to aggregate productivity	0.0031

Life-cycle parameters The remaining parameters govern the firm's life-cycle. k_0 determines the initial capital stock of new entrants and θ the probability of leaving the economy at each period.

Parameter	Description	Value
θ	Mean exit rate	8.8%
k_0	Size of new firms (relative to mean)	28%

I follow Xiao (2018) and I set the exit rate of the firm in the economy to 8.8% and the capital of firm that enter the economy such that it is about 28% of the capital of all firms to match micro data.

I run the different simulations of this model using three idiosyncratic productivity levels and 100 points in the capital grid.

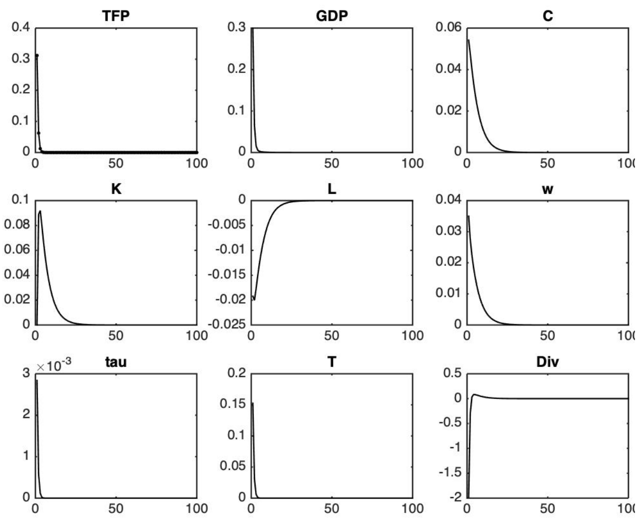
4.3.5 Results

Finally, using the previously described calibration, I solve for the dynamics of this model. Then, using the algorithm described earlier, I solve for the optimal η that maximizes the aggregate welfare in this economy. I solve for this parameter as a function of the persistence of the aggregate shock. I show that the optimal path of the tax rate still depends on the persistence of the aggregate shock. Then, the analytical results derived in Section 2 holds in this quantitative environment. It means that heterogeneity matters in the design of the optimal fiscal policy over the business cycle. Indeed, as we saw earlier, the difference in the path of the optimal tax can be explained by the relative heterogenous response of firm's investment to the aggregate shock and by the fact that those heterogenous responses change with the persistence of the aggregate shock.

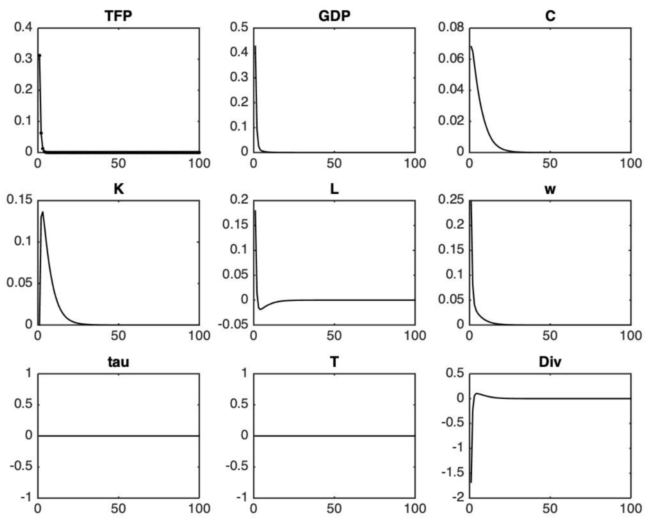
	ρ^z	η
High persistence of the aggregate shock	0.95	-1.75
Low persistence of the aggregate shock	0.2	1.1

Table 1: Optimal η depending of ρ^z

I present here the dynamics of the economy after an aggregate shock depending on the persistence of the aggregate shock and on wheter η is at its optimal value or not. I report the IRFs using the values for the persistence ρ^z of the aggregate shock and η presented on Table 1. Each panel of Figure 7 and 8 presents the proportional change for the specified variable in percentage points. The only exceptions are for the tax rate and the aggregate tax, for which I present the absolute variation.



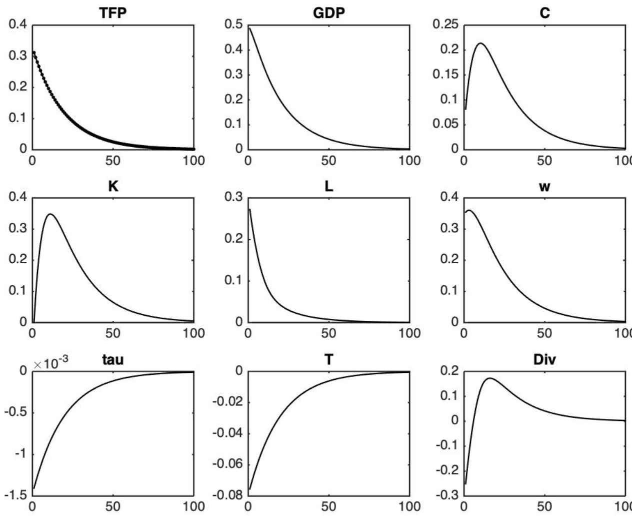
(a) $\eta = 1.1$



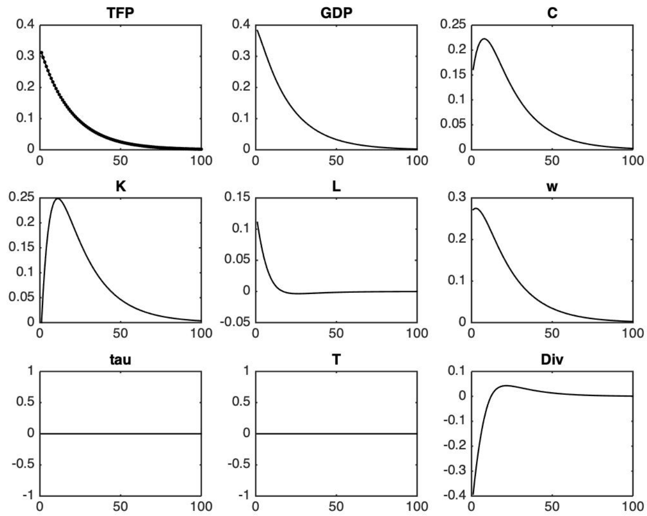
(b) $\eta = 0$

Figure 7: IRFs with $\rho = 0.2$

The response of the economy is quite different depending on the path of the corporate income tax rate. When the path of the tax rate is set optimally and the shock is not persistent, the planner increases the tax rate following a shock. The aggregate level of capital and GDP decreases, but the aggregate consumption is sustained by the large increase in the lump sum transfer to the household. Because the shock is not long-lasting, the planner is incentivized to tax firms heavily on the impact and to transfer those funds to the household (because the unconstrained firms are almost not affected either by the current aggregate shock nor by the current level of the corporate income tax).



(a) $\eta = -1.75$



(b) $\eta = 0$

Figure 8: IRFs with $\rho = 0.95$

When the path of the tax rate is set optimally and the shock is persistent, the planner decreases the tax rate following a shock. The planner subsidies firms further and the aggregate capital stock increases significantly. The lump sum tax on the household also increases, but is compensated by the increase in the aggregate level of dividends. In both cases, it is not costly for the planner to transfer funds to the household. In future work, I will introduce fixed cost to this transfer, such as administrative cost to see how it changes those results.

Then, we can see on Figure 8 that the response of the economy to an aggregate shock changes a lot depending on the optimal path of the corporate income tax. If, in both cases, the value of η is the one that maximizes the aggregate welfare, the means to achieve this maximization differ greatly depending on the persistence of the aggregate shock. When the shock is not persistent, the planner triggers a decrease in the aggregate level of capital, in the aggregate level of production and in the aggregate level of dividends and transfers lump sum the product of the tax to the household after a positive aggregate productivity shock. When the shock is persistent, the planner triggers an increase in the aggregate level of production, in the aggregate level of dividends and she taxes lump-sum the household to finance those subsidies.

I show that those results are confirmed by the analyze of the second-order moments presented on Table 9.

ρ		0.95		0.2	
η		Optimal	Exogenous	Optimal	Exogenous
		-4.3878	0.0000	4.0816	0.0000
K	Mean	1.4634	1.4650	1.4654	1.4653
	Std	0.0931	0.0163	0.0007	0.0026
Y	Mean	0.5369	0.5374	0.5376	0.5375
	Std	0.0845	0.0184	0.0077	0.0055
C	Mean	0.5652	0.5653	0.5654	0.5654
	Std	0.0272	0.0146	0.0009	0.0015
L	Mean	0.3329	0.3332	0.3333	0.3332
	Std	0.0872	0.0040	0.0174	0.0032
τ	Mean	0.0008	0.0000	-0.0001	0.0000
	Std	0.0608	0.0000	0.0176	0.0000
Correlations					
$corr(\tau, K)$		-0.8658	-	-0.0171	-
$corr(\tau, Y)$		-0.9998	-	-0.9979	-
$corr(K, Y)$		0.8603	0.8615	0.0479	0.1211
$corr(Y, Y_{-1})$		0.9545	0.9532	0.2594	0.2175
$corr(K, K_{-1})$		0.9955	0.9954	0.9725	0.8940

Figure 9: Moments of the simulated models

I present four economies characterized by the two values of the persistence of the aggregate shock used in the rest of the analysis and depending on if η is chosen optimally or not. Those moments are computed over the simulation of the model. I report the unconditional first and second-order moments for the key variables of this model. For each variable, I report the steady-state value (labeled "Mean") and the normalized standard deviation, equal to the standard deviation divided by the mean (labeled "Std") of the variable, except for the tax rate and the aggregate tax for which I report the standard deviation. The second part of the table reports correlations. This Table 9 presents similar evidences than the analysis of the IFRs. The aggregate level of capital and consumption are less volatile when the tax is optimal and when the aggregate shock is not persistent. In contrast, they are more volatile when the aggregate shock is persistent. In both cases, GDP and aggregate labor are more volatile when the tax is set optimally.

Finally, the tradeoff faced by the planner and identified in the first part of this paper is still present here. When the aggregate shock is persistent, the planner chooses a counter-cyclical tax rate as to maximises the welfare in the economy. When the aggregate shock is not persistent, she chooses a pro-cyclical tax rate as to maximises the welfare in the economy.

5 Conclusion

In this paper, I derive the optimal path of the corporate tax over the business cycle in a heterogenous firm model. This paper studies how and when a government should tax firms. I show, both analytically in a tractable model and numerically in a quantitative model that the optimal path of this tax depends on the persistence of the aggregate shock. Indeed, in this model, financial frictions prevent firm investment to be optimal. As a consequence, capital is not optimally allocated across firms and the severity of this problem depends on aggregate shocks. More precisely, this problem improves or get worse over the business cycle depending on the persistence of aggregate shocks. Then, depending on the persistence of aggregate shock, the optimal path of the corporate tax is pro-cyclical or counter-cyclical. In a future work, I will develop this project in two directions. First, I will introduce costly access to external finance and sticky prices in this model. It will allow me to study the interaction of monetary policy and fiscal policy. It will also allow me to determine which type of policy is more efficient to stimulate aggregate investment. Second, I will introduce distortive taxation on the household side and study the relative benefit of taxing or subsidizing households and firms over the business cycle.

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A Appendix

A.1 Empirical Data

A.1.1 CIT as Proportion of Total Tax Revenue in 2019

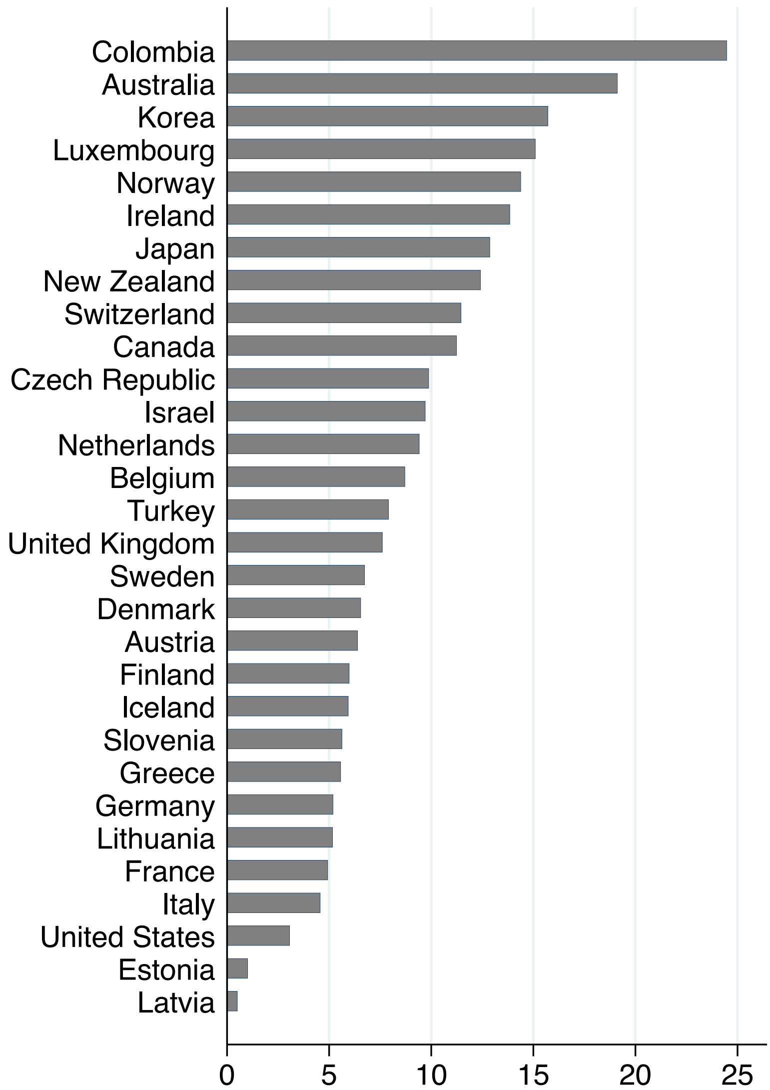

Figure 10: CIT as Proportion of Total Tax Revenue in 2019 (Source: OECD, Revenue Statistics)

A.1.2 Corporate Income Tax Rate by Country over 1960-2018

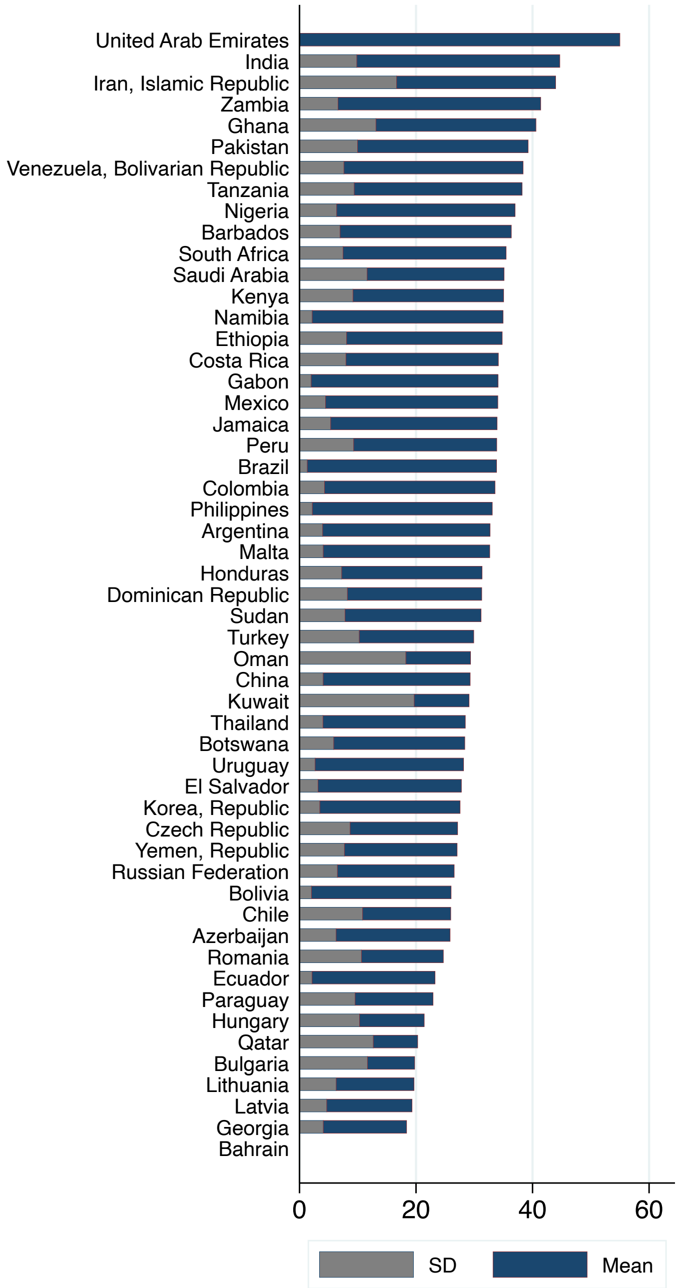

Figure 11: Corporate Income Tax Rate by Country, 1960-2018 (Source: Vegh and Vuletin (2015))

A.1.3 Cyclicality of Corporate Income Tax Rate by Country over 1980-2018

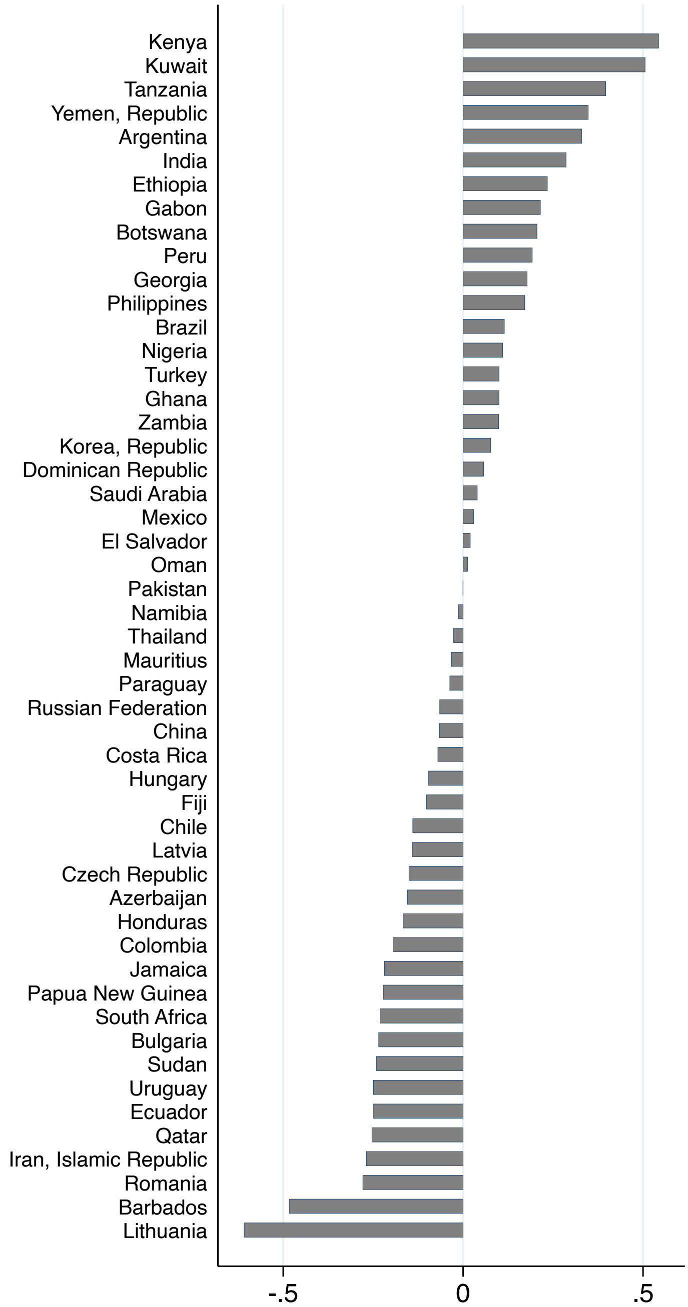

Figure 12: Correlation between CIT rate growth and real GDP growth (Source: Vegh and Vuletin (2015))

A.1.4 Comovement between CIT percentage change and real GDP percentage change over 1980-2018

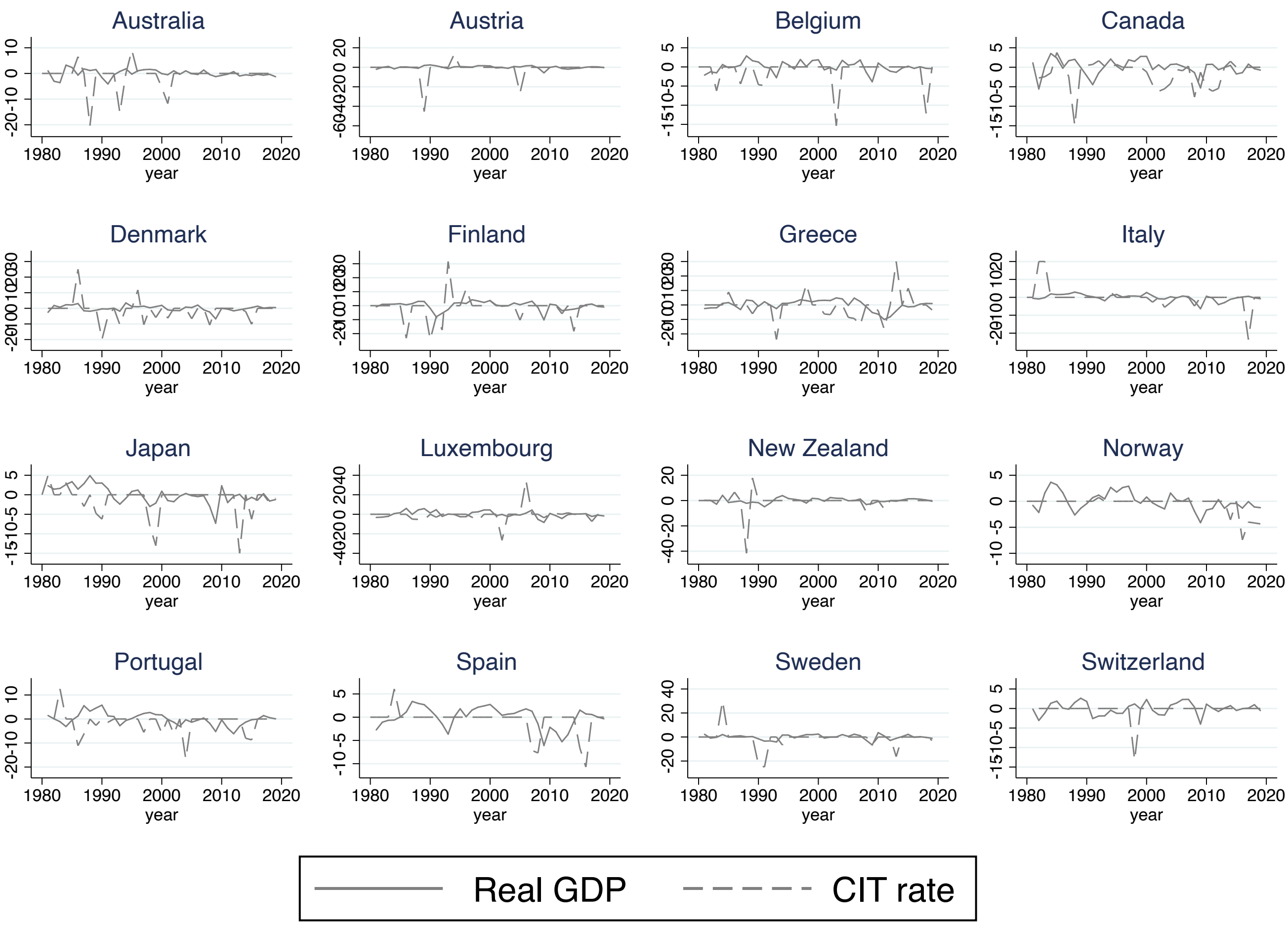

Figure 13: Percentage changes of Real GDP and CIT rate over 1980-2018 (Source: Vegh and Vuletin (2015), OECD)

A.2 Tractable Model

A.2.1 Existence:

Condition 1. I know that $k_2 = [\alpha\beta(1-\theta)]^{\frac{1}{1-\alpha}}$, then, :

$$k_0^\alpha - k_2 < 0 \iff k_0 < [\alpha\beta(1-\theta)]^{\frac{1}{\alpha(1-\alpha)}}$$

Condition 2. I know that $k_1 = k_0^\alpha$, then,

$$k_1^\alpha - k_2 \geqslant 0 \iff k_0 \geq [\alpha\beta(1-\theta)]^{\frac{1}{\alpha^2(1-\alpha)}}$$

Condition 2.

$$k_2^\alpha - k_2 \geq 0 \iff \frac{1}{\alpha\beta(1-\theta)} \geq 1$$

Always true with the value of the different parameters.

A.2.2 Heterogeneous tax rates: Proposition 1

Equilibrium: At equilibrium:

$$\begin{aligned}
&\tau_t^1 = \tau_t^2 = 0 \\
&\tau_t^0 = 1 - \frac{E_t\left[Z_{t+1}\right]^{\frac{1}{1-\alpha}}}{Z_t}\left[\beta(1-\theta)\alpha\right]^{\frac{1}{1-\alpha}} \times \frac{1}{k_0^\alpha} \\
&k_0 > 0 \\
&k_{1,t+1} = E_t\left[Z_t\right]^{\frac{1}{1-\alpha}}\left[\beta(1-\theta)\alpha\right]^{\frac{1}{1-\alpha}} \\
&k_{2,t+1} = E_t\left[Z_{t+1}\right]^{\frac{1}{1-\alpha}}\left[\beta(1-\theta)\alpha\right]^{\frac{1}{1-\alpha}}
\end{aligned}$$

(i) Taxes at steady-state:

$$\begin{aligned}
&\tau^1 = \tau^2 = 0 \\
&\tau_t^0 = 1 - \left[\beta(1-\theta)\alpha\right]^{\frac{1}{1-\alpha}} \times \frac{1}{k_0^\alpha}
\end{aligned}$$

Then,

$$\tau = 1 - [\beta(1-\theta)\alpha]^{\frac{1}{1-\alpha}} \times \frac{1}{k_0^\alpha} \geq 0 \iff k_0 \geq [\beta(1-\theta)\alpha]^{\frac{1}{\alpha(1-\alpha)}}$$

And, to insure that $k_0^\alpha < k_2$, $k_0 < [\beta(1-\theta)\alpha]^{\frac{1}{\alpha(1-\alpha)}}$, then, $\tau < 0$.

(ii): implementation of the frictionless allocation with time varying tax rates

I can substitute the two remaining constraints into the objective of the planner.

$$\max_{(k_{2,t+1},\tau_t^0,\tau_t^1,\tau_t^2)} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t) - E_0 \sum_{t=0}^{\infty} \beta^t M_t \left[\lambda_t - \lambda_{t-1}(1-\theta)\alpha \left[(1-\tau_t^2) Z_t k_{t,2}^{\alpha-1}\right]\right]$$

$$\text{S.t} \qquad C_t = n_1 \left[(Z_t((1-\tau_{t-1}^0)Z_{t-1}k_{t-1,0}^\alpha)^\alpha - k_{2,t+1}\right] + n_2 \left[Z_t k_{2,t}^\alpha - k_{2,t+1}\right] + \left[n_0 \tau_t^0 Z_t k_0^\alpha\right]$$

- τ_t^1 is indeterminate. The planner is indifferent between taxing or subsidizing type 1 firm and I set $\tau_t^1 = 0$

The first-oder condition on τ_t^2 is given by the following condition:

$$\lambda_{t-1}(1-\theta)\alpha \left[k_{t,2}^{\alpha-1}\right] = 0$$

I know that $0 < \theta < 1$ and $0 < \alpha < 1$ and $k_{t,2} > 0$ then, $\lambda_{t-1} = 0$.

The first-oder condition on k_{t+1}^2 is given by the following condition:

$$-\beta\lambda_t(1-\theta)\alpha(\alpha-1)E_t\left[(1-\tau_{t+1}^2)Z_{t+1}k_{t+1,2}^{\alpha-2}\right] + \beta\alpha n_2 E_t Z_{t+1} k_{2,t+1}^{\alpha-1} = (n_1+n_2)$$

Then, with $\lambda_t = 0$

$$\beta\alpha n_2 E_t Z_{t+1} k_{2,t+1}^{\alpha-1} = (n_1+n_2)$$

I.e

$$\beta\alpha(1-\theta)E_t Z_{t+1} k_{2,t+1}^{\alpha-1} = 1$$

The optimal path of the unconstrained investement is given by the expected return on capital, the planner doesn't distort the firms' Euler equations.

The first-oder condition on τ_t^0 is given by the following condition:

$$n_0 Z_t k_0^\alpha = \beta n_1 \alpha ((1-\tau_t^0))^{\alpha-1} E_t Z_{t+1} \left(Z_t k_0^\alpha\right)^\alpha$$

Then,

$$1-\tau_t^0 = \frac{E_t\left[Z_{t+1}\right]^{\frac{1}{1-\alpha}}}{Z_t} \frac{\left[\beta(1-\theta)\alpha\right]^{\frac{1}{1-\alpha}}}{k_0^\alpha}$$

We can see clearly in this expression that the path of the optimal tax rate depends on the ratio $\frac{E_t[Z_{t+1}]^{\frac{1}{1-\alpha}}}{Z_t}$ and then, on the persistence of the expected aggregate shock.

Let's prove that this time varying tax rate can reproduce the frictionless allocation.

We know that:

$$k_{1,t+1} = Z_t(1-\tau_t^0)k_0^\alpha$$

Then,

$$\begin{aligned} k_{1,t+1} &= Z_t k_0^\alpha \frac{E_t\left[Z_{t+1}\right]^{\frac{1}{1-\alpha}}}{Z_t} \frac{\left[\beta(1-\theta)\alpha\right]^{\frac{1}{1-\alpha}}}{k_0^\alpha} \\ &= \left[\beta(1-\theta)\alpha E_t(Z_{t+1})\right]^{\frac{1}{1-\alpha}} \end{aligned}$$

The type 0 firm investment is then equal to the frictionless level of investment. The planner reproduces the frictionless allocation using time varying corporate income tax on type 0 firm.

And,

$$k_{1,t+1} = k_{2,t+1} = \left[\beta(1-\theta)\alpha E_t(Z_{t+1})\right]^{\frac{1}{1-\alpha}}$$

The frictionless level of investment.

(iii) Threshold: I know that $Z_t = Z_0 e^{z_t}$ and that $z_t = \rho z_{t-1} + u_t$ with ρ_t the persistence parameter and the shock u_t being a white noise with a normal distribution $\mathcal{N}(0, \sigma^2)$.

Then, with $Z_0 = 1$, I can determine the threshold such that the tax rate is pro-cyclical (with respect to the steady-state value of the tax). This threshold depends on ρ the persistence of the shock and on σ^2, the variance of the shock.

$$
\begin{aligned}
\tau_t^0 \leqslant \tau^0 \iff 1 - \frac{E_t\left[Z_{t+1}\right]^{\frac{1}{1-\alpha}}}{Z_t}\frac{[\beta(1-\theta)\alpha]^{\frac{1}{1-\alpha}}}{k_0^\alpha} &\leqslant 1 - \frac{[\beta(1-\theta)\alpha]^{\frac{1}{1-\alpha}}}{k_0^\alpha} \\
\iff \frac{E_t\left(Z_0 e^{z_{t+1}}\right)^{\frac{1}{1-\alpha}}}{Z_0 e^{z_t}} &\geq 1 \\
\iff \frac{e^{\rho z_t + \frac{1}{2}\sigma^2}}{e^{(1-\alpha)z_t}} &\geq 1 \\
\iff z_t\left(\frac{\rho}{(1-\alpha)} - 1\right) + \frac{1}{2(1-\alpha)}\sigma^2 &\geq 0
\end{aligned}
$$

A.2.3 Equity Constraint

Consider now that firms can extract a finite amount of equity $\bar{D}$ from their shareholders. The program of the firm j can be rewritten as:

$$
\max_{(k_{j,t+1})_{t=0}^{\infty}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}(\beta(1-\theta))^t \frac{M_t}{M_0}\left((1-\tau_t)Z_t k_{j,t}^\alpha - k_{j,t+1}\right)\right] \tag{12}
$$

$$
\text{S.t} \quad D_{j,t} = (1-\tau_t)Z_t k_{j,t}^\alpha - k_{j,t+1} \geq -(1-\tau_t)\bar{D} \tag{13}
$$

$$
k_{j,t+1} \geq 0 \tag{14}
$$

The investment of a firm that is not constrained is thus given by:

$$
(1-\tau_t)\left(Z_t(k_t^j)^\alpha + \bar{D}\right) = k_{t+1}^j
$$

At time t, the budget constraint of the household can be expressed as:

$$
\begin{aligned}
C_t &= n_1 D_{1,t} + n_2 D_{2,t} - n_0\bar{D} + \tau_t\left[n_0 Z_t k_0^\alpha + n_1 Z_t k_{1,t}^\alpha + n_2 Z_t k_{2,t}^\alpha\right] \\
&= n_1\left((1-\tau_t)Z_t k_{1,t}^\alpha - k_{2,t+1}\right) + n_2\left((1-\tau_t)Z_t k_{2,t}^\alpha - k_{2,t+1}\right) - n_0\bar{D} \\
&\quad + \tau_t\left[n_0(Z_t k_0^\alpha + \bar{D}) + n_1 Z_t k_{1,t}^\alpha + n_2 Z_t k_{2,t}^\alpha\right]
\end{aligned}
$$

Here, the optimal tax rate on firm 0 is given by:

$$
\frac{\left[(1-\theta)\beta E_t Z_{t+1}\alpha\right]^{\frac{1}{1-\alpha}}}{\left((Z_t k_0^\alpha + \bar{D})\right)} = (1-\tau_t^0)
$$

A.3 Quantitative Model

A.3.1 Algorithm

Step 1: Discretization of the model Objective of the governemnt: maximize opitaml policy one that maximizes the objective of the govenrment when the allocation is given by the competitive equilibrium.

I discretize every equilibrium condition to have a dynamic such that:

$$F(X', X, \eta', z') = 0$$

I discretize the policy rules (k^o, k^c) using the vector $\boldsymbol{K^o}$. The policy rules have to satisfy the Euler equations for each k, ε or to satisfy the constraint. I discretize the labor demand nd using the vector $\boldsymbol{n^d}$ I discretize the distribution of capital μ using an histogram $\boldsymbol{S}$ as in Young (2010).

Moreover, $\boldsymbol{S'} = \Pi \boldsymbol{S}$ with Π the transition matrix implied by the decision rules.

I also have 8 aggregate variables: the aggregate capital, the aggregate labor, the level of aggregate productivity, the tax rates, the aggregate level of tax, the aggregate level of dividend, the firm pricing kernel, wage. Then, $\boldsymbol{X} = (\boldsymbol{K^o}, \boldsymbol{S}, \boldsymbol{n^d}, K, Z, \tau, T, D, \lambda, w, N^d)$

η is the vector of expectation error. Once written in this form, we know from how to solve this system.

The approximate equilibrium conditions are written as a system of $3 * n_k n_\varepsilon + 8$ equations.

Expectation Error Expectation errors come from the aggregate productivity shock. I rewrite the expectation operator $\mathbb{E}$ as $\mathbb{E}_{\boldsymbol{S_{-1}}, Z, \tau}(x) = x + \eta$. As many expectation operator as Euler equations.

Computable recursive competitive equilibrium I can then replace the aggregate state (Z, μ, τ) with an approximate one $(Z, \boldsymbol{S}, \tau)$ and compute the recursive competitive equilibrium.

Step 2: Now, I solve for a steady-state of the discretized model such that

$$f(X^{\star}, X'^{\star}, 0, 0) = 0$$

I use the algorithm developed in **?** to solve this system. It consists in solving a root-finding problem in the wage $w^{\star}$. Ie., I find the wage that equalized labor supply and labor demand.

I set $\eta^{\star} = 0$ and $z^{\star} = 0$. I guess, a level of wage $w^{\star}$.

Using this wage, I compute the firm labor demand. To do that:

1. I solve for the firms value function by value function iteration.
2. Using the policy rule I determine the stationary distribution $s\star$ iterating on the law of motion.
3. I determine the level of labor demand for each level of capital and idiosyncratic productivity and I aggregate the labor demand using the stationry firm distribution over capital and idiosyncratic productivity.
4. I determine the aggregate variables $T^{\star}, D^{\star}, Y^{\star}, I^{\star}$ using the stationary distribution. It allows to derive $C^{\star} = Y^{\star} - I^{\star} + D^{\star} + T^{\star}$ what gives $\lambda = (C^{\star})^{-\gamma}$.
5. Using those objects, I derive the labor supply using the household first order condition on labor together with the wage guess. $N^{s\star} = (\frac{w^{\star}\lambda^{\star}}{\chi})^{\phi}$.
6. I solve for the wage $wage^{\star}$ that clears the labor market using a root-finding algorithm.

Step 3: Linearization $F(X, X', \eta, z)$ is numerically differentiated around the non stochastic steady-state $X = X' = X^{\star}$:

$$
\begin{aligned}
F_1 &= (\frac{\partial F}{\partial X'})_{|X=X^{\star}} \\
F_2 &= (\frac{\partial F}{\partial X})_{|X=X^{\star}} \\
F_3 &= (\frac{\partial F}{\partial \eta'})_{|X=X^{\star}} \\
F_4 &= (\frac{\partial F}{\partial z'})_{|X=X^{\star}}
\end{aligned}
$$

Then,

$$F_1(X' - X^{\star}) + F_2(X - X^{\star}) + F_3\eta' + F_4 z' = 0$$

Then, I can write this system into Sims form (Sims (2001)):

$$\Gamma_0 y' = \Gamma_1 y + C + \psi z' + \varphi\eta'$$

With $y' = X' - X^{\star}$, $\Gamma_0 = -F_1$, $\Gamma_1 = -F_2$, $C = 0$, $\psi = F_3$, $\varphi = F_4$.

The outcome of the Sims (2001) method are the matrix A and B such that:

$$y' = Ay + Bz'$$

The linearization and the perturbation steps are automatized in Dynare (Adjemian et al. (2011)). It computes the partial derivatives of this system and solve this system.