Business Cycle Facts

TA Session 1

Alaïs Martin-Baillon

SciencesPo

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Objective of this first TA session

At the end of this session you should be able to :

Part I :

- Download series from the fred database, either manually or using Stata
- Plot a serie and customize a graph using Excel and Stata
- Detrend a serie using the Hodrick Prescott Filter and understand the role of its key parameter
- Understand the notions of procyclical and countercyclical behavior of a serie
- Use the Baxter-King filter and understand its main differences with the HP filter

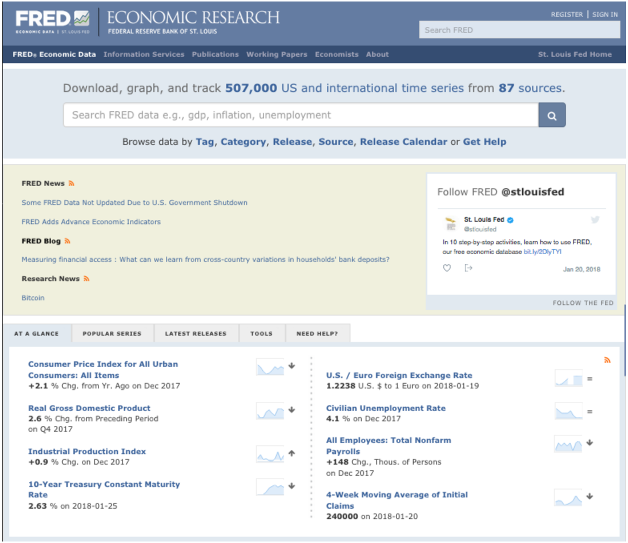
Part II :

- Understand the two ways to state the "centralized problem"

Part I

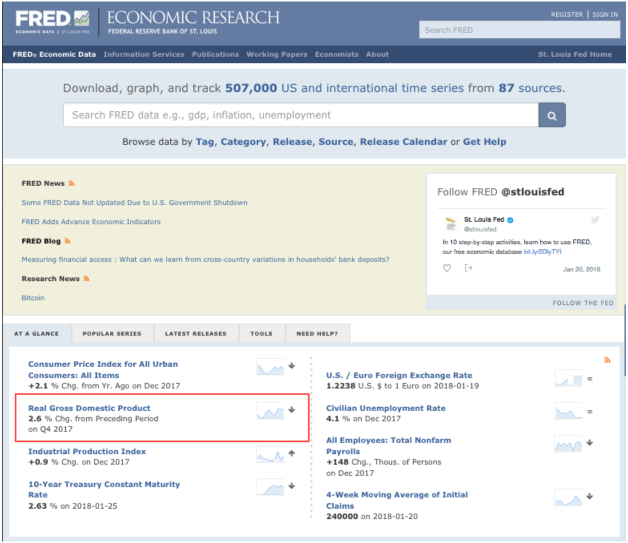
The Fred Database : `https://fred.stlouisfed.org/`

Objective : be able to look for specific data and to download them



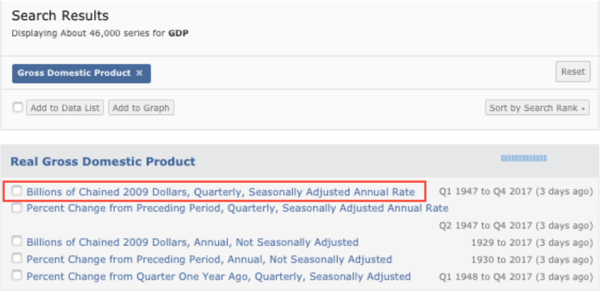
The Fred Database

Looking for the real GDP serie



The Fred Database

Or just Search for "GDP" and select : Billions of Chained 2009 Dollars, Quarterly, Seasonally Adjusted Annual Rate

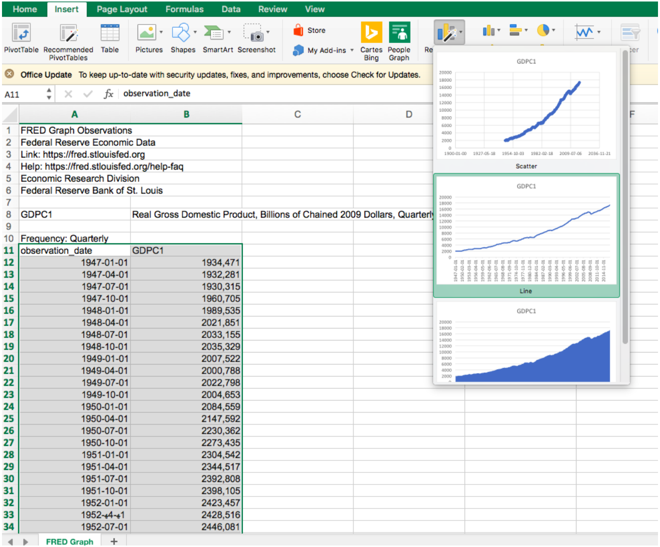


The Fred Database

Then download the serie using Excel

You can change the frequency, unit, axis, time window, etc. using "Edit Graph"

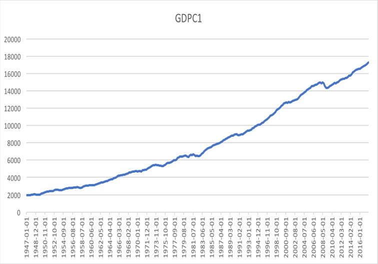
On the Art of Plotting Data

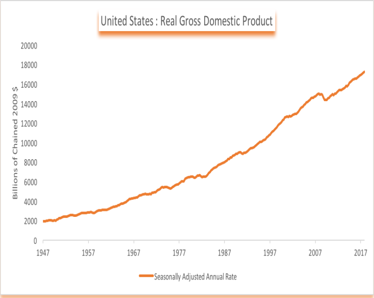


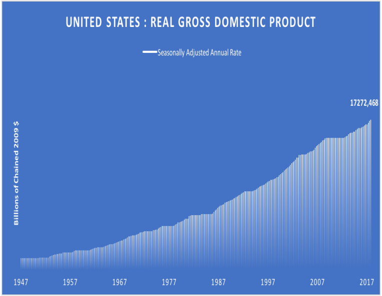
Select "Insert" and choose the kind of graph that you want to plot

On the Art of Plotting Data

And customise it : add axis label (unit), title, and whatever you like







The Hodrick Prescott Filter

Excel Plugin : `http://www.web-reg.de/hp_addin.html`

"Download" and follow the instruction "Extract the zip-file and declare HPFilter.xla as an add-in (Tools - Adds-In Manager - Browse").

LN =HP(C12:C295;1600)

	A	B	C	D	E	F
1	FRED Graph Observations					
2	Federal Reserve Economic Data					
3	Link: https://fred.stlouisfed.org					
4	Help: https://fred.stlouisfed.org/help-faq					
5	Economic Research Division					
6	Federal Reserve Bank of St. Louis					
7						
8	GDPC1	Real Gross Domestic Product, Billions of Chained 2009 Dollars, Quarterly, Seasonally Adjusted Annual Rate				
9						
10	Frequency: Quarterly					
11	observation_date	Seasonally Adjusted Annual Rate				
12	1947-01-01	1934,471	12,17275937	=HP(C12:C295;1600)		
13	1947-04-01	1932,281	12,17162664			
14	1947-07-01	1930,315	12,17060867			
15	1947-10-01	1960,705	12,18622957			
16	1948-01-01	1989,535	12,20082641			
17	1948-04-01	2021,851	12,21693889			
18	1948-07-01	2033,155	12,22251424			
19	1948-10-01	2035,329	12,22358294			
20	1949-01-01	2007,522	12,20982659			
21	1949-04-01	2000,788	12,20646657			
22	1949-07-01	2022,798	12,21740717			
23	1949-10-01	2004,653	12,20839644			
24	1950-01-01	2084,559	12,24748279			
25	1950-04-01	2147,592	12,27727268			
26	1950-07-01	2230,362	12,31508937			
27	1950-10-01	2273,435	12,33421737			
28	1951-01-01	2304,542	12,34780742			
29	1951-04-01	2344,517	12,36500488			
30	1951-07-01	2392,808	12,38539304			
31	1951-10-01	2398,105	12,38760431			
32	1952-01-01	2423,457	12,3981205			
33	1952-04-01	2428,516	12,40020584			
34	1952-07-01	2446,081	12,40741262			
35	1952-10-01	2526,359	12,4397046			
36	1953-01-01	2573,372	12,45814257			
37	1953-04-01	2593,456	12,46591681			
38	1953-07-01	2578,922	12,46029695			

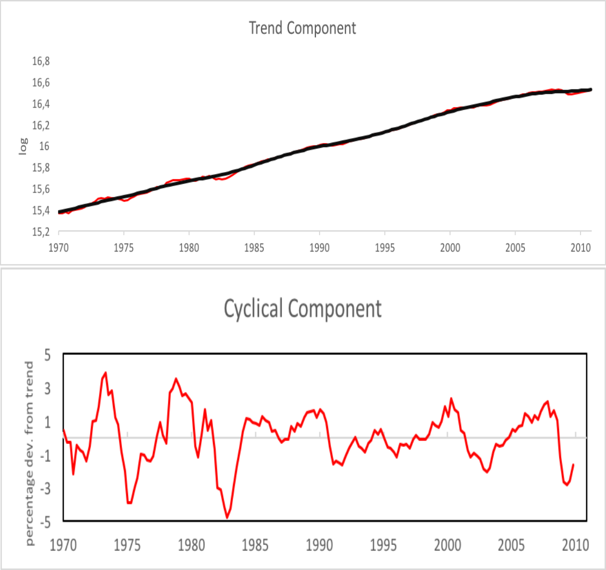
FRED Graph

HP(timeseries as Range,lambda as Double).

Lambda : 100 for yearly data, 1600 for quarterly data and 14400 for monthly data.

The Hodrick Prescott Filter

Plot them as your Prof.



Do it with Stata

Install Freduse

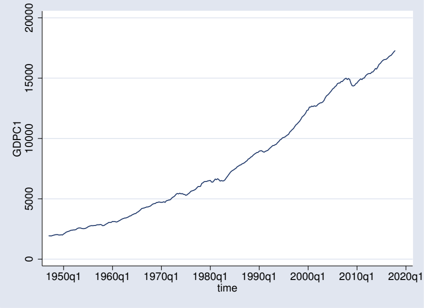
```
clear

findit freduse

/*----------------------
To Do:
         1.select "st010"
         2."click here to install"
         3. close the window
-----------------------*/
```

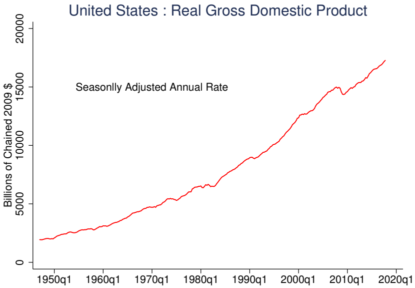
Do it with Stata

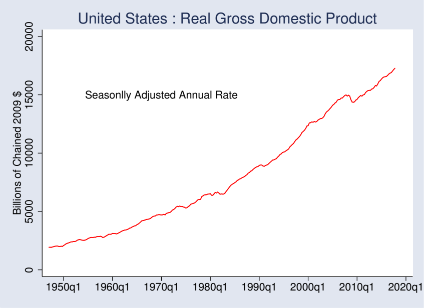
Load the data and plot the serie

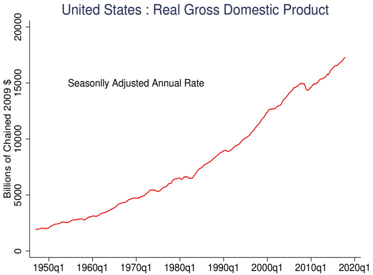
```
freduse GDPC1

generate time = q(1947q1) + _n -1
format time %tq
tsset time

 twoway tsline GDPC1

 twoway tsline GDPC1, title(United States : Real Gross
     Domestic Product) ytitle(Billions of Chained
    2009 $) xtitle("") lcolor(red) ttext( 15000 1970
    q1 "Seasonlly␣Adjusted␣Annual␣Rate" ) ylab(,
    nogrid)
```

Do it with Stata







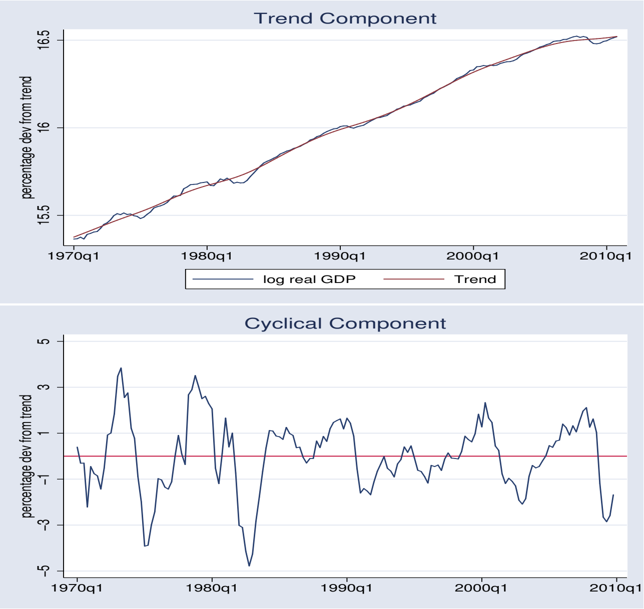


Do it with Stata

Detrend the serie and plot

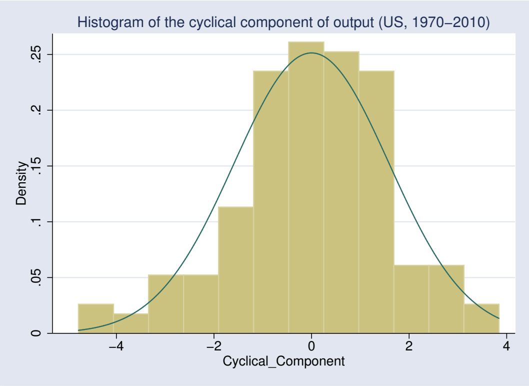
```
gen ln_gdp = ln(GDPC1*1000) if time >= tq(1970q1) &
    time < tq(2010q1)

tsfilter hp gdp_cycle =ln_gdp, trend(gdp_trend) smooth
    (1600)

gen Cyclical_Component = gdp_cycle  * 100
label var ln_gdp "log␣real␣GDP"
label var gdp_trend "Trend"

twoway tsline Cyclical_Component if time >= tq(1970q1)
     & time < tq(2010q1), title(Cyclical Component)
    ytitle(percentage dev from trend) xtitle("")
    ylabel(-5(2)5) yline(0)

twoway tsline ln_gdp gdp_trend if time >= tq(1970q1) &
     time < tq(2010q1), title(Trend Component) ytitle(
    percentage dev from trend) xtitle("")
```

Do it with Stata



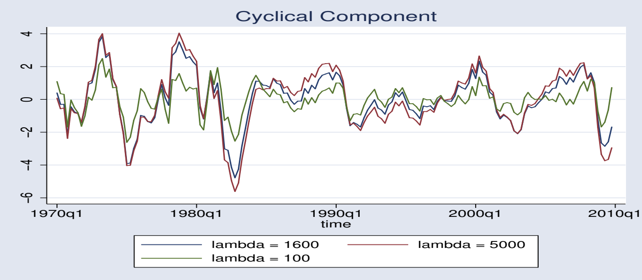
Do it with Stata

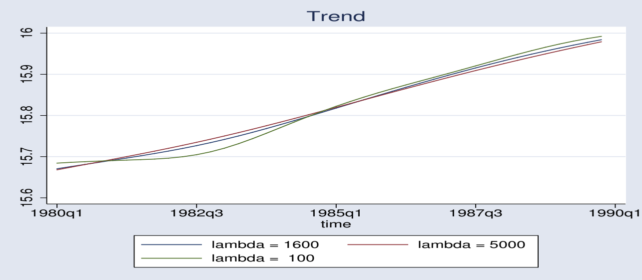
```
hist Cyclical_Component, normal title("Histogram␣of␣
    the␣cyclical␣component␣of␣output␣(US,␣1970-2010)",
     size(medium))

egen sd = sd(Cyclical_Component)
mean = mean(Cyclical_Component)
```



The Hodrick Prescott Filter

The dependence on lambda





The Hodrick Prescott Filter

Why ?

The Hodrick Prescott Filter
Why ?

$$\min_{\tau}\left(\sum_{t=1}^{T}(y_t-\tau_t)^2+\boxed{\lambda}\sum_{t=2}^{T-1}\left[(\tau_{t+1}-\tau_t)-(\tau_t-\tau_{t-1})\right]^2\right)$$

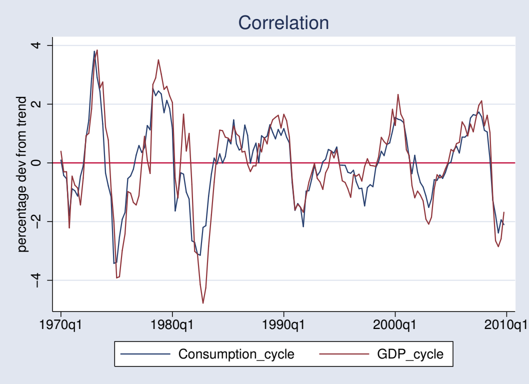
The second term is a multiple λ of the sum of the squares of the trend component's second differences. This second term penalizes variations in the growth rate of the trend component. The larger the value of λ , the higher is the penalty.
With y_t the serie, $\tau thetrendcomponent$,

Correlation between variables

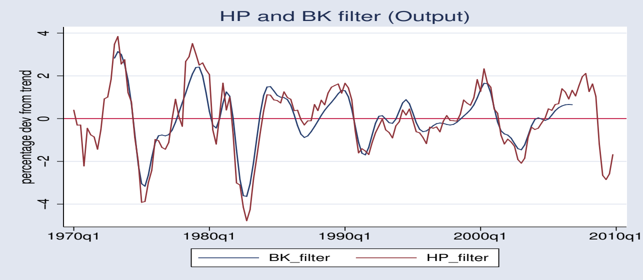
Real Personal Consumption Expenditures

Cross correlation with output

L.Consumption	Consumption	F.Consumption
0.8882	0.8778	0.7229



The Baxter-King filter



Part II

The centralized problem

The common notation

Let's copy and past the problem of your lecture :
The planner chooses $\{C_{t+i}, S_{t+i}, K_{t+i+1}\}_{i=0}^{\infty}$ to maximize :

$$\mathbb{E}_t \left[\sum_{i=0}^{\infty} \beta^i U(C_{t+i}) \right]$$

subject to

$$\begin{aligned} C_{t+i} + S_{t+i} &\leqslant Z_{t+i} F(K_{t+i}, 1) \\ K_{t+i+1} &= (1-\delta) K_{t+i} + S_{t+i} \\ K_{t+i+1} \geqslant 0 \ ; \ K_t &> 0 \ \textit{given} \end{aligned}$$

The centralized problem

A problem of notation

This notation is widely use in macroeconomics but is not really precise and can lead you to misunderstand what is really going on with this optimization program.

This notation can lead you to think that the planner chooses at time t every C, S and K from now to infinity when what it does it to choose a *plan* for those variables conditional on what happens in the economy (and so here on the history of z).

Let's restate it in a different way !

The centralized problem

A problem of notation

Define the history z^t of the random variable Z by :

$$z^t \equiv [Z_t, Z_{t-1}, ..., Z_0]$$

And define Π^{t+i} as the probability of the history $t + i$

The centralized problem

Let's restate the centralized problem

With these notations :
The planner chooses $\{C(z^{t+i}), S(z^{t+i}), K'(z^{t+i})\}_{i=0}^{\infty}$ to maximize :

$$\sum_{i=0}^{\infty} \sum_{z^{t+i}} \beta^i \Pi^{t+i} U\left(C(z^{t+i})\right)$$

subject to

$$\begin{aligned} C(z^{t+i}) + S(z^{t+i}) \leqslant Z(z^{t+i}) F(K'(z^{t+i-1}), 1) \\ K'(z^{t+i}) = (1-\delta) K'(z^{t+i-1}) + S(z^{t+i}) \\ K'(z^{t+i}) \geqslant 0 \ ; \ K(z^t) > 0 \ \textit{given} \end{aligned}$$

The first order conditions

- As in the lecture, we combine the two constraints :

$$K'(z^{t+i}) = (1-\delta)K'(z^{t+i-1}) + Z(z^{t+i})F(K'(z^{t+i-1}),1) - C(z^{t+i})$$

There is one constraint for each period and for each state of the world possible

- And write the Lagrangian :

$$\begin{aligned}\mathcal{L} = &\sum_{i=0}^{\infty}\sum_{z^{t+i}}\beta^i \Pi^{t+i} U\left(C(z^{t+i})\right) \\ &+ \sum_{i=0}^{\infty}\sum_{z^{t+i}}\beta^i \Pi^{t+i}\lambda(z^{t+i})[(1-\delta)K'(z^{t+i-1}) \\ &+ Z(z^{t+i})F(K'(z^{t+i-1}),1) - C(z^{t+i}) - K'(z^{t+i})]\end{aligned}$$

The first order conditions

- Take the derivative with respect to $C(z^{t+i})$:

$$\frac{\partial \mathcal{L}}{C(z^{t+i})} = \beta^i \Pi^{t+i} U'\left(C(z^{t+i})\right) - \beta^i \Pi^{t+i} \lambda(z^{t+i}) = 0$$
$$\iff U'\left(C(z^{t+i})\right) = \lambda(z^{t+i})$$

- Take the derivative with respect to $K'(z^{t+i})$ $(\star)$:

$$\frac{\partial \mathcal{L}}{K'(z^{t+i})} = -\beta^i \Pi^{t+i} \lambda(z^{t+i}) + \sum_{z^{t+i+1}} \beta^{i+1} \Pi^{t+i+1} \lambda(z^{t+i+1}) R(z^{t+i+1}) = 0$$
$$\iff \beta\, \mathbb{E}(\lambda(z^{t+i+1}) R(z^{t+i+1}) |\, z^{t+i}) = \lambda(z^{t+i})$$

Remember : $R(z^{t+i}) = (1-\delta) + Z(z^{t+i}) F_1(K'(z^{t+i-1}), 1)$

$(\star)$There is a sum because $K'(z^{t+i})$ appears in all histories z^{t+i+1} that start with the same history z^{t+i}

The first order conditions

Take the two first order conditions :

$$\begin{aligned} \beta\,\mathbb{E}(\lambda(z^{t+i+1})R(z^{t+i+1})|\ z^{t+i}) &= \lambda(z^{t+i}) \\ U'\left(C(z^{t+i})\right) &= \lambda(z^{t+i}) \end{aligned}$$

Combine them to get the **Euler equation**

$$U'\left(C(z^{t+i})\right) = \beta\,\mathbb{E}(U'\left(C(z^{t+i+1})\right) R(z^{t+i+1})|\ z^{t+i})$$