Business Cycle : TA Session 9

Zero Lower Bound and Forward Guidance

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1 Objective of This Session

The goal of this session is to demonstrate two of the three main predictions of the standard New Keynesian models with a zero lower bound :

Result 1 *When the zero lower bound is binding, the fall in consumption can be very large.*

Result 2 *When the zero lower bound is binding, a commitment by the central bank to raise future inflation ("Odyssean forward guidance") is a powerful tool to raise current consumption.*

You will have to demonstrate the third one for your next Take Home Exam

Result 3 *When the zero lower bound is binding, the government spending multiplier can be very large*

2 Take the standard New Keynesian Model :

Consumption Euler equation

$$c_t = \mathbb{E}_t \left[-\frac{1}{\gamma}(\varepsilon^d_{t+1} - \varepsilon^d_t + r_t - \pi_{t+1}) + c_{t+1} \right]$$

New Keynesian Phillips curve

$$\pi_t = \kappa c_t + \beta\, \mathbb{E}_t(\pi_{t+1})$$

Monetary policy rule

$$r_t = \max\{-\ln(R), \phi_\pi \pi_t\}$$

3 Zero lower bound

Step 1 : Guess

We guess that there exists an equilibrium with the following properties: (i) consumption, inflation, and the nominal interest rate are constant until the preference shock reverts permanently back to zero, and (ii) the economy is in the non-stochastic steady state thereafter.

You can verify that $c_t = \pi_t = r_t = 0$ in every period $t \geqslant T$ (by showing that it satisfy the monetary policy rule, the consumption Euler equation and the New Keynesian Phillips curve).

Furthermore, we guess whether or not the zero lower bound is binding.

First, remember that the preference shock keeps its current value with probability μ and reverts permanently back to zero with probability $1-\mu$. Moreover, consumption, inflation, and the nominal interest rate are constant (but depend on the size of the aggregate shock) from period zero until preference shocks revert permanently back to zero (in period T) . Then, in periods $0 \leqslant t \leqslant T-1$

$$\begin{aligned} \mathbb{E}_t\, \varepsilon^d_{t+1} &= \mu \times \varepsilon^d_t + (1-\mu) \times 0 \\ &= \mu \varepsilon^d_t \end{aligned}$$

And,

$$\begin{aligned} \mathbb{E}_t\, c_{t+1} &= \mu c_t + (1-\mu) \times 0 = \mu c_t \\ \mathbb{E}_t\, \pi_{t+1} &= \mu \pi_t + (1-\mu) \times 0 = \mu \pi_t \end{aligned}$$

Then you can rewrite the Euler equation as the following :

$$c_t = \mathbb{E}_t \left[-\frac{1}{\gamma}((\mu-1)\varepsilon^d_t + r_t - \mu\pi_t) + \mu c_t \right] \tag{1}$$

And the New Keynesian Phillips curve becomes :

$$\begin{aligned} \pi_t &= \kappa c_t + \beta\mu\pi_t \\ \iff \pi_t &= \frac{\kappa c_t}{1-\beta\mu} \end{aligned}$$

Case 1 : the zero lower bound binding

Guess that the zero lower bound is binding. Then,

$$r_t = -\ln(R)$$

Step 2 : Verify

Step 2.1 : Substitute in the guess

Substituting $r_t = -\ln(R)$ and the New Keynesian Phillips curve into the Euler equation yields :

$$c_t = -\frac{1}{\gamma}\left((\mu-1)\varepsilon^d_t - \ln(R) - \mu(\frac{\kappa c_t}{1-\beta\mu}) \right) + \mu c_t$$

Solving for consumption yields :

$$c_t(1-\mu-\frac{\mu\kappa}{\gamma(1-\beta\mu)}) = -\frac{1}{\gamma}\left((\mu-1)\varepsilon^d_t - \ln(R)\right)$$

Then,

$$\Longrightarrow c_t(1-\mu-\frac{\mu\kappa}{\gamma(1-\beta\mu)}) = -\frac{1}{\gamma}\left((\mu-1)\varepsilon_t^d - \ln(R)\right)$$
$$\Longrightarrow c_t(\frac{\gamma(1-\beta\mu)-\mu\gamma(1-\beta\mu)-\mu\kappa}{\gamma(1-\beta\mu)}) = -\frac{1}{\gamma}\left((\mu-1)\varepsilon_t^d - \ln(R)\right)$$
$$\Longrightarrow c_t = \frac{\gamma(1-\beta\mu)}{\gamma(1-\beta\mu)-\mu\gamma(1-\beta\mu)-\mu\kappa} \times -\frac{1}{\gamma}\left((\mu-1)\varepsilon_t^d - \ln(R)\right)$$
$$\Longrightarrow c_t = -\frac{(1-\beta\mu)}{\gamma(1-\beta\mu)-\mu\gamma(1-\beta\mu)-\mu\kappa} \times \left((\mu-1)\varepsilon_t^d - \ln(R)\right)$$
$$\Longrightarrow c_t = -\frac{(\mu-1)\varepsilon_t^d - \ln(R)}{\gamma-\mu\gamma-\frac{\mu\kappa}{(1-\beta\mu)}}$$
$$\Longrightarrow c_t = -\frac{\frac{\varepsilon_t^d}{\gamma}-\frac{\ln(R)}{\gamma(\mu-1)}}{\frac{1}{\mu-1}-\frac{\mu}{\mu-1}-\frac{\mu\kappa}{(1-\beta\mu)\gamma(\mu-1)}}$$

$$\Longrightarrow c_t = -\frac{\frac{\varepsilon_t^d}{\gamma}-\frac{\ln(R)}{\gamma(\mu-1)}}{-1-\frac{\mu\kappa}{(1-\beta\mu)\gamma(\mu-1)}}$$
$$\Longrightarrow c_t = \frac{\frac{-\varepsilon_t^d}{\gamma}-\frac{\ln(R)}{\gamma(1-\mu)}}{-1+\frac{\mu\kappa}{(1-\beta\mu)\gamma(1-\mu)}}$$
$$\Longrightarrow c_t = \frac{\frac{\varepsilon_t^d}{\gamma}+\frac{\ln(R)}{\gamma(1-\mu)}}{1-\frac{\mu\kappa}{(1-\beta\mu)\gamma(1-\mu)}} \tag{2}$$

Case 2 : the zero lower bound is not binding

Guess that the zero lower bound is not binding. Then,

$$r_t = \phi_\pi \pi_t$$

Step 2 : Verify

Step 2.1 : Substitute in the guess

Substituting $r_t = \phi_\pi\pi_t$ and the New Keynesian Phillips curve into the Euler equation yields :

$$c_t = -\frac{1}{\gamma}\left((\mu-1)\varepsilon_t^d + \phi_\pi\frac{\kappa c_t}{1-\beta\mu} - \mu\frac{\kappa c_t}{1-\beta\mu}\right) + \mu c_t$$
$$c_t = -\frac{1}{\gamma}\left((\mu-1)\varepsilon_t^d + \frac{\kappa c_t}{1-\beta\mu}(\phi_\pi-\mu)\right) + \mu c_t$$

Solving for consumption yields :

$$\Longrightarrow c_t(1-\mu+\frac{\kappa(\phi_\pi-\mu)}{\gamma(1-\beta\mu)}) = -\frac{1}{\gamma}\left((\mu-1)\varepsilon_t^d +\right)$$
$$\Longrightarrow c_t(\frac{\gamma(1-\beta\mu)-\mu\gamma(1-\beta\mu)+\kappa(\phi_\pi-\mu)}{\gamma(1-\beta\mu)}) = -\frac{1}{\gamma}\left((\mu-1)\varepsilon_t^d\right)$$
$$\Longrightarrow c_t = -\frac{(\mu-1)\varepsilon_t^d}{\gamma}\times\frac{\gamma(1-\beta\mu)}{\gamma(1-\beta\mu)-\mu\gamma(1-\beta\mu)+\kappa(\phi_\pi-\mu)}$$
$$\Longrightarrow c_t = -\frac{(\mu-1)\varepsilon_t^d(1-\beta\mu)}{\gamma(1-\beta\mu)-\mu\gamma(1-\beta\mu)+\kappa(\phi_\pi-\mu)}$$
$$\Longrightarrow c_t = -\frac{\frac{\varepsilon_t^d}{\gamma}}{\frac{1}{(\mu-1)}-\frac{\mu}{(\mu-1)}+\frac{\kappa(\phi_\pi-\mu)}{\gamma(\mu-1)(1-\beta\mu)}}$$
$$\Longrightarrow c_t = \frac{\frac{\varepsilon_t^d}{\gamma}}{1+\frac{\kappa(\phi_\pi-\mu)}{\gamma(1-\mu)(1-\beta\mu)}}$$

In period zero, the economy is hit by a negative preference shock, $\varepsilon_t^d < 0$ what generate an increase in desire to save. Central bank lowers nominal interest rate as much as possible but hit the zero lower bound at some point. Saving must be zero in equilibrium, so a force has to be pulling savings down, this force is a reduction in income. This reduction in income implies a reduction in consumption, what yields marginal costs to fall. Then prices fall today and in the future and households expect deflation in the futur. Then, because nominal interest rate is constant, the real interest rate increases and consumption fall even more, etc.

The zero lower bound on the nominal interest rate is binding if the preference shocks is sufficiently negative ie if $\varepsilon_t^d < \bar{\varepsilon}$

$$-\ln(R) = \phi_\pi\pi_t$$
$$\Longleftrightarrow\ -\ln(R) = \phi_\pi\frac{\kappa c_t}{1-\beta\mu}$$
$$\Longleftrightarrow\ -\ln(R) = \phi_\pi\kappa\frac{\frac{\varepsilon_t^d}{\gamma}}{1+\frac{\kappa(\phi_\pi-\mu)}{\gamma(1-\mu)(1-\beta\mu)}}\frac{1}{1-\beta\mu}$$
$$\Longleftrightarrow\ -\ln(R)\times(1-\beta\mu)\times(1+\frac{\kappa(\phi_\pi-\mu))}{\gamma(1-\mu)(1-\beta\mu)}\gamma\times\frac{1}{\phi_\pi\kappa} = \varepsilon_t^d$$
$$\Longleftrightarrow\ -\frac{1+\frac{\frac{1}{\gamma}\kappa(\phi_\pi-\mu)}{(1-\mu)(1-\beta\mu)}}{\frac{1}{\gamma}\frac{\phi_\pi}{(1-\mu)}\frac{\kappa}{(1-\beta\mu)}}\frac{1}{1-\mu}\ln(R) = \varepsilon_t^d$$

Then, the zero lower bound is binding if and only if :

$$\varepsilon_t^d < \bar{\varepsilon} = -\frac{1+\frac{\frac{1}{\gamma}\kappa(\phi_\pi-\mu)}{(1-\mu)(1-\beta\mu)}}{\frac{1}{\gamma}\frac{\phi_\pi}{(1-\mu)}\frac{\kappa}{(1-\beta\mu)}}\frac{1}{1-\mu}\ln(R)$$

You can see that a positive denominator on the right-hand side of equation 2 with the condition 3 implies a negative numerator on the right-hand side of equation. To study more in details the response of consumption you can decompose (1) as following :

$$c_t = \mathbb{E}_t \left[-\frac{1}{\gamma}((\mu - 1)\varepsilon_t^d + r_t - \mu\pi_t) + \mu c_t \right]$$

$$c_t = \frac{\bar{\varepsilon}_t^d}{\gamma} + \frac{r_t}{\gamma(1-\mu)} + \frac{\mu\pi_t}{\gamma(1-\mu)}$$

Consumption equals the sum of three terms: the first term is the direct effect of the preference shock on consumption, the second term is the effect of the nominal interest rate on consumption, and the third term is the effect of expected inflation on consumption. Substituting in the equilibrium nominal interest rate when the zero lower bound is binding and equilibrium inflation when the zero lower bound is binding yields :

$$c_t = \frac{\bar{\varepsilon}_t^d}{\gamma} + \frac{1}{\gamma(1-\mu)}\ln(R) + \frac{\mu}{\gamma(1-\mu)}\frac{\kappa}{1-\beta\mu}\frac{\frac{1}{\gamma}\bar{\varepsilon}_t^d + \frac{\frac{1}{\gamma}}{1-\mu}\ln(R)}{1 - \frac{\frac{1}{\gamma}\mu}{1-\mu}\frac{\kappa}{1-\beta\mu}}$$

The first term is negative. The second term is positive because the monetary authority can lower the nominal rate to some extent before the zero lower bound becomes binding, which actually increases consumption. The third term is negative and re ects the effect of the aggregate shock on aggregate consumption coming from movements in inflation expectations. The reason why the fall in consumption can be arbitrarily large for a given size of the shock is the third term. The model of this subsection predicts that the fall in consumption can be arbitrarily large for a given size of the shock, because the fall in in ation expectations can be arbitrarily large for a given size of the shock. The amplification of the shock comes from movements in household in ation expectations. If household in ation expectations did not move in response to the shock, consumption would be given by the sum of the first two terms.

4 Forward Guidance

Define **forward guidance** as a direct statement by the central bank about the future path of its policy tools. These statements can take different forms. Campbell et al. (2012) distinguish between **Odyssean forward guidance**, which publicly commits the central bank to a future action, and **Delphic forward guidance**, which merely forecasts macroeconomic performance and likely monetary policy actions. Eggertsson and Woodford (2003) demonstrate that in a standard New Keynesian model a commitment by the central bank to create inflation in the future is a powerful way of stimulating the economy when the zero lower bound is binding.

Suppose that in period zero the central bank publicly commits itself to a future path of the nominal interest rate, the central bank announces that it will set the nominal interest rate in periods $t \geqslant T$ so as so as to achieve a long-run inflation target of $\bar{\pi} > 0$ in order to raise inflation expectations in periods $t \leqslant T$. Suppose moreover that the zero lower bound is binding.

Without forward guidance, the consumption is given by equation (2), with forward guidance, consumption is given by the following :

First, you can rewrite the consumption Euler equation, knowing that the ZLB is binding as following.

$$c_t = \mathbb{E}_t \left[-\frac{1}{\gamma}((\mu - 1)\varepsilon_0^d - \ln(R) - \pi_{t+1}) + c_{t+1} \right]$$

Moreover, agents know that consumption and inflation are constant for period $t < T$ and equal to $\bar{\pi}$ and $\bar{c}$ in period $t \leqslant T$. Last thing, remember that preference shocks revert permanently back to zero in period T and it happends with probability $(1-\mu)$.

Then $\mathbb{E}_t\, c_{t+1} = \mu c_t + (1-\mu)\bar{c}$ and $\mathbb{E}_t\, \pi_{t+1} = \mu\pi_t + (1-\mu)\bar{\pi}$. Substituing,

$$c_t = -\frac{1}{\gamma}\left[(\mu-1)\varepsilon_0^d - \ln(R) - \mu\pi_t - (1-\mu)\bar{\pi}\right] + \mu c_t + (1-\mu)\bar{c}$$

The new Keynesian Phillips curve becomes :

$$\begin{aligned}
&\pi_t = \kappa c_t + \beta(\mu\pi_t + (1-\mu)\bar{\pi}) \\
&\implies \pi_t(1-\beta\mu) = \kappa c_t + \beta(1-\mu)\bar{\pi} \\
&\implies \pi_t = \frac{\kappa c_t + \beta(1-\mu)\bar{\pi}}{(1-\beta\mu)}
\end{aligned}$$

Then,

$$c_t = -\frac{1}{\gamma}\left[(\mu-1)\varepsilon_0^d - \ln(R) - \left(\mu\frac{\kappa c_t + \beta(1-\mu)\bar{\pi}}{(1-\beta\mu)} + (1-\mu)\bar{\pi}\right)\right] + \mu c_t + (1-\mu)\bar{c}$$

Solving for consumption :

$$\begin{aligned}
c_t &= -\frac{1}{\gamma}\left[(\mu-1)\varepsilon_0^d - \ln(R) - \left(\mu\frac{\kappa c_t + \beta(1-\mu)\bar{\pi}}{(1-\beta\mu)} + (1-\mu)\bar{\pi}\right)\right] + \mu c_t + (1-\mu)\bar{c} \\
c_t(1 - \frac{1}{\gamma}\frac{\mu\kappa}{(1-\beta\mu)} - \mu) &= -\frac{1}{\gamma}\left[(\mu-1)\varepsilon_0^d - \ln(R) - (\frac{\mu\beta(1-\mu)\bar{\pi}}{(1-\beta\mu)} + (1-\mu)\bar{\pi})\right] + (1-\mu)\bar{c} \\
c_t\frac{(1-\beta\mu)\gamma - \mu\kappa - \mu(1-\beta\mu)\gamma}{(1-\beta\mu)\gamma} &= -\frac{1}{\gamma}\left[(\mu-1)\varepsilon_0^d - \ln(R) - (\frac{\mu\beta(1-\mu)\bar{\pi}}{(1-\beta\mu)} + (1-\mu)\bar{\pi})\right] + (1-\mu)\bar{c} \\
c_t &= \left[-\frac{1}{\gamma}((\mu-1)\varepsilon_0^d - \ln(R) - (\frac{\mu\beta(1-\mu)\bar{\pi}}{(1-\beta\mu)} + (1-\mu)\bar{\pi})) + (1-\mu)\bar{c}\right]\frac{(1-\beta\mu)\gamma}{(1-\beta\mu)\gamma - \mu\kappa - \mu(1-\beta\mu)\gamma} \\
c_t &= \left[\frac{1}{\gamma}\left(\varepsilon_0^d + \frac{\ln(R)}{1-\mu} + \bar{\pi}(1 + \frac{\mu\beta}{(1-\beta\mu)})\right) + \bar{c}\right]\frac{(1-\mu)}{1-\mu-\frac{\kappa}{(1-\beta\mu)\gamma}} \\
c_t &= \left[\frac{1}{\gamma}(\varepsilon_0^d + \frac{\ln(R)}{1-\mu} + \bar{\pi}(1 + \frac{\mu\beta}{(1-\beta\mu)}) + \bar{c}\right]\frac{1}{1-\frac{\mu\kappa}{(1-\beta\mu)\gamma(1-\mu)}}
\end{aligned}$$

This policy increases aggregate consumption if and only if :

$$\begin{aligned}
&c_t^{fg} > c_t \\
&\iff \left[\frac{1}{\gamma}(\varepsilon_t^d + \frac{\ln(R)}{1-\mu} + \bar{\pi}(1 + \frac{\mu\beta}{(1-\beta\mu)}) + \bar{c}\right]\frac{1}{1-\frac{\mu\kappa}{(1-\beta\mu)\gamma(1-\mu)}} > \frac{\frac{\varepsilon_t^d}{\gamma} + \frac{\ln(R)}{\gamma(1-\mu)}}{1-\frac{\mu\kappa}{(1-\beta\mu)\gamma(1-\mu)}} \\
&\iff \left[\frac{1}{\gamma}(\varepsilon_t^d + \frac{\ln(R)}{1-\mu} + \bar{\pi}(1 + \frac{\beta}{(1-\beta\mu)}) + \bar{c}\right] > \frac{\varepsilon_t^d}{\gamma} + \frac{\ln(R)}{\gamma(1-\mu)} \\
&\iff \frac{1}{\gamma}(\varepsilon_t^d + \frac{\ln(R)}{1-\mu} + \bar{\pi}(1 + \frac{\beta}{(1-\beta\mu)})) + \bar{c} > \frac{\varepsilon_t^d}{\gamma} + \frac{\ln(R)}{\gamma(1-\mu)} \\
&\iff \bar{c} + \bar{\pi}\frac{1}{(1+\beta\mu)\gamma} > 0
\end{aligned}$$

You know that

$$
\begin{aligned}
& \bar{\pi} = \kappa\bar{c} + \beta\bar{\pi} \\
\iff & \bar{\pi} = \frac{\kappa\bar{c}}{1-\beta} \\
\iff & \bar{c} = \bar{\pi}\frac{(1-\beta)}{\kappa}
\end{aligned}
$$

Then,

$$
\begin{aligned}
\iff & \bar{c} + \bar{\pi}\frac{1}{(1+\beta\mu)\gamma} > 0 \\
\iff & \bar{\pi}\frac{(1-\beta)}{\kappa} + \bar{\pi}\frac{1}{(1+\beta\mu)\gamma} > 0 \\
\iff & \bar{\pi}\left[\frac{(1-\beta)(1+\beta\mu)\gamma+\kappa}{(1+\beta\mu)\gamma\kappa}\right] > 0 \\
\iff & \left[\frac{(1-\beta)(1+\beta\mu)\gamma+\kappa}{(1+\beta\mu)\gamma\kappa}\right] > 0
\end{aligned}
$$