



**Data Mining**

**NOVA-IMS**  
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**Clustering**

## Agenda

- Cluster analysis
- Variables to use
- Similarity criterion
- Clustering algorithms
  - A Priori Grouping
  - RFM Analysis

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# Cluster Analysis

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## Clustering

- **Cluster Analysis**
  - Cluster Analysis is a **basic conceptual activity of human beings**;
  - A **fundamental process**, common to many sciences, essential to the development of scientific theories;
  - The possibility of **reducing the infinite complexity of real** to sets of objects or similar phenomena, is one of the most powerful tools in the service of mankind.

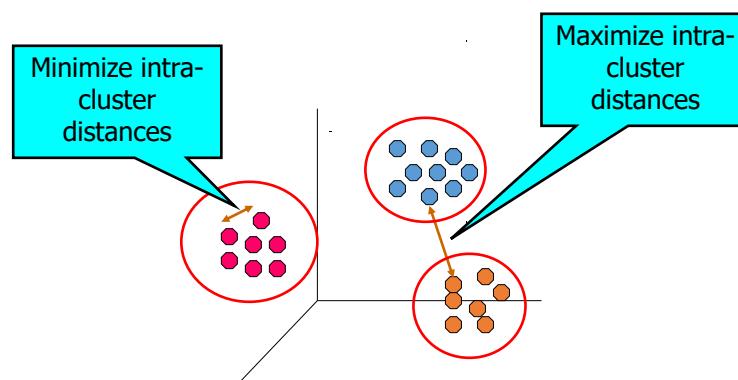
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- **Cluster Analysis**

- Cluster analysis is a generic name for a variety of methods that are used to **group entities**;
- Objective: **To form groups of objects that are similar to each other**;
- From a data collection about a group of entities, seeks to organize them **in homogeneous groups**, assessing a "frame" of similarities/differences between units.

- **Cluster Analysis**



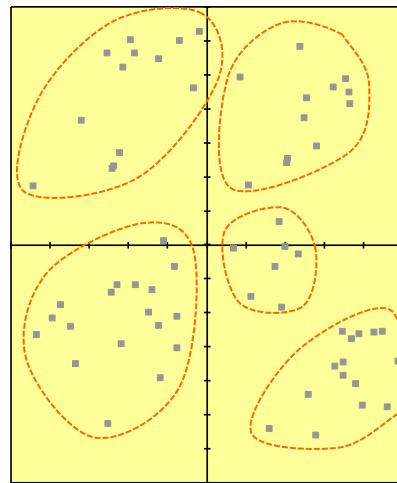
- **Cluster Analysis**

- Classification:
  - Starts out with a **pre-classified training set**, that is, the method has a set of data which contains not only the variables to use in classification but also the class to which each of the records belongs;
  - Attempts to develop a model capable of predicting how a new record will be classified.

- **Cluster Analysis**

- Clustering:
  - There is **no pre-classified data**;
  - We search for groups of records (clusters) that are similar to one another;
  - Underlying is the expectation that similar customers in terms of the variables used will behave in similar ways.

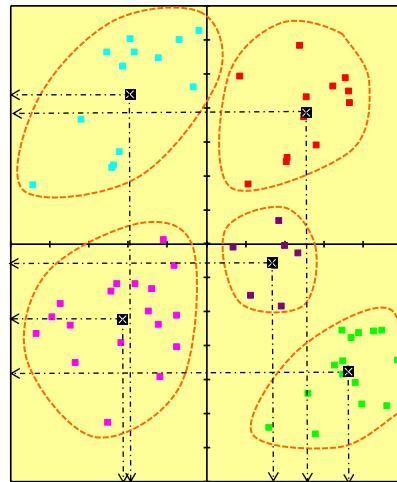
## Clustering



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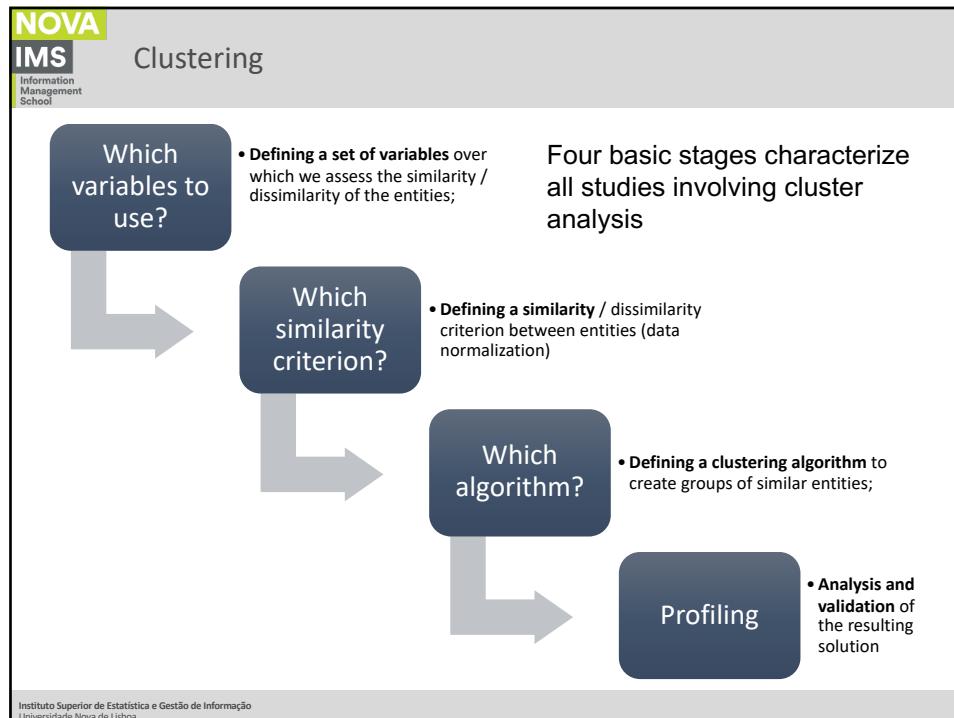
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## Clustering

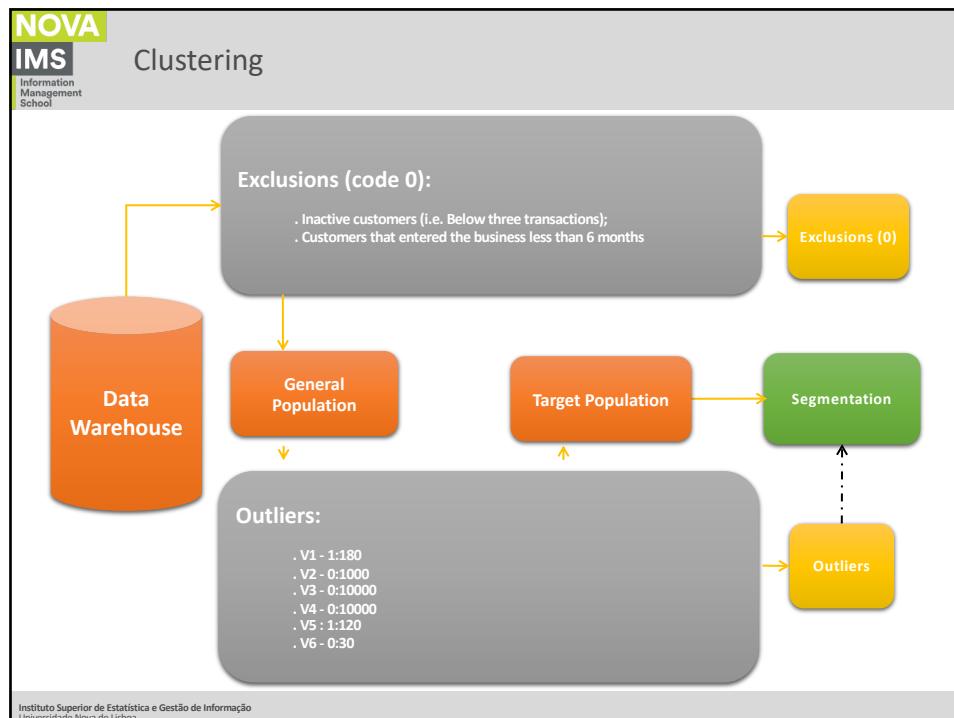


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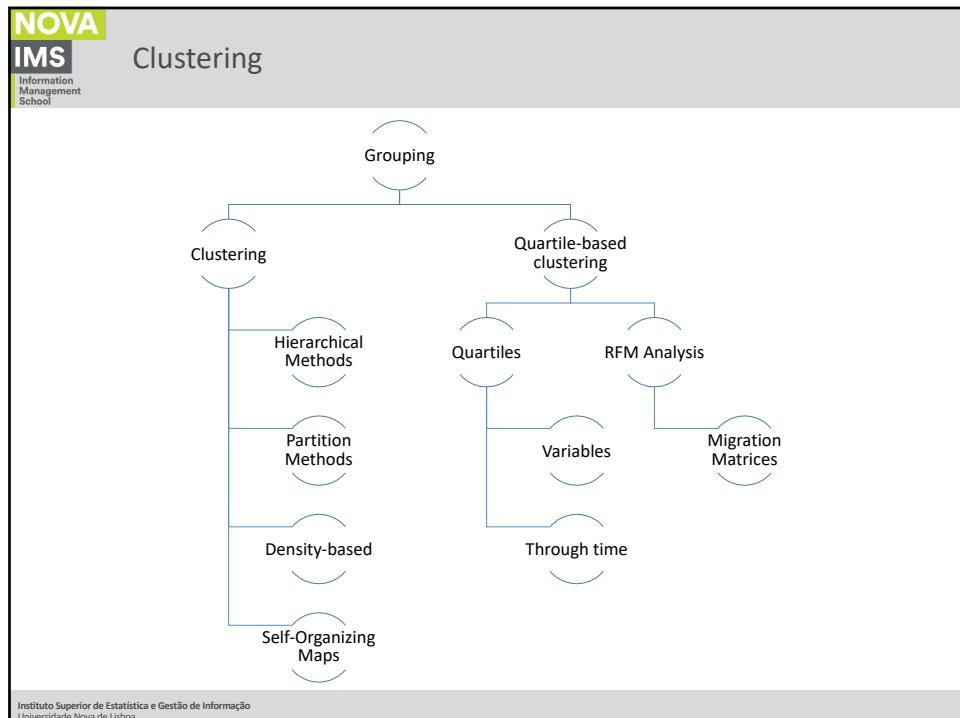
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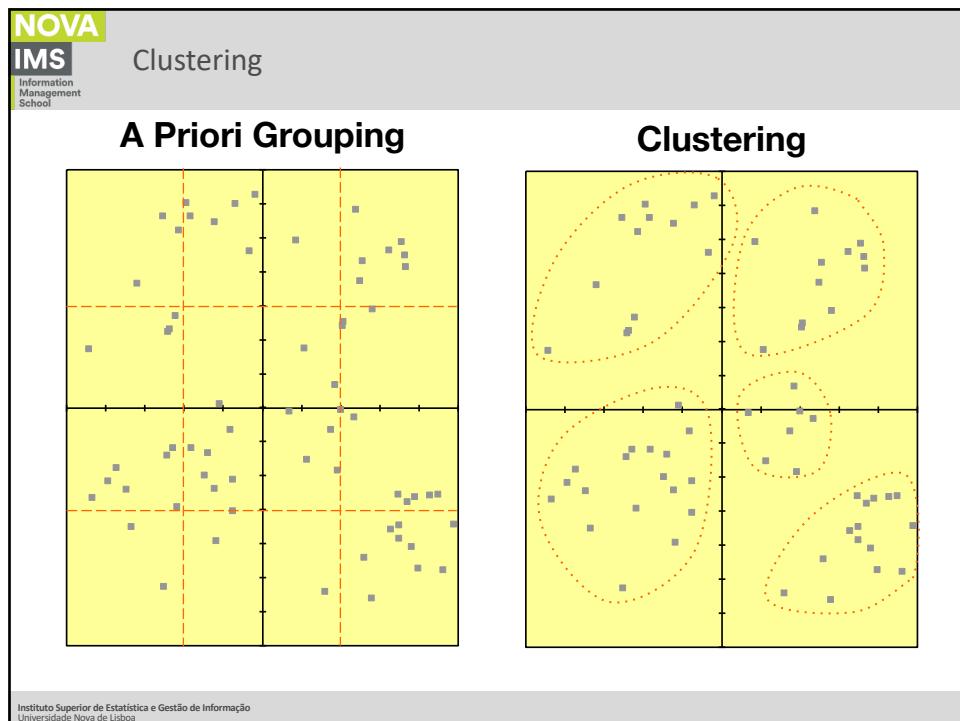
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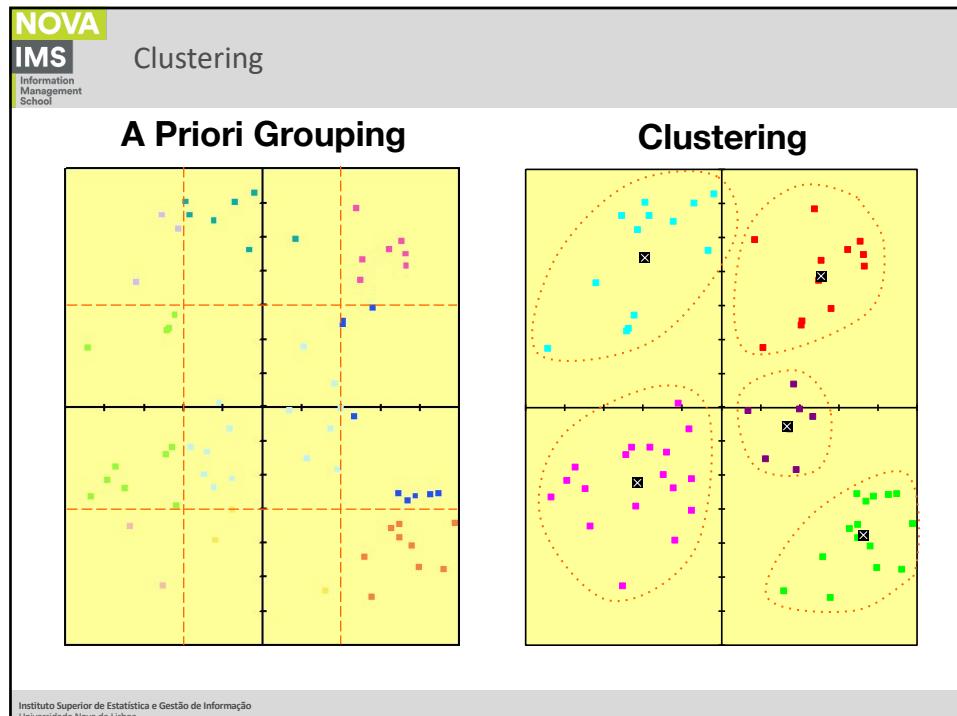
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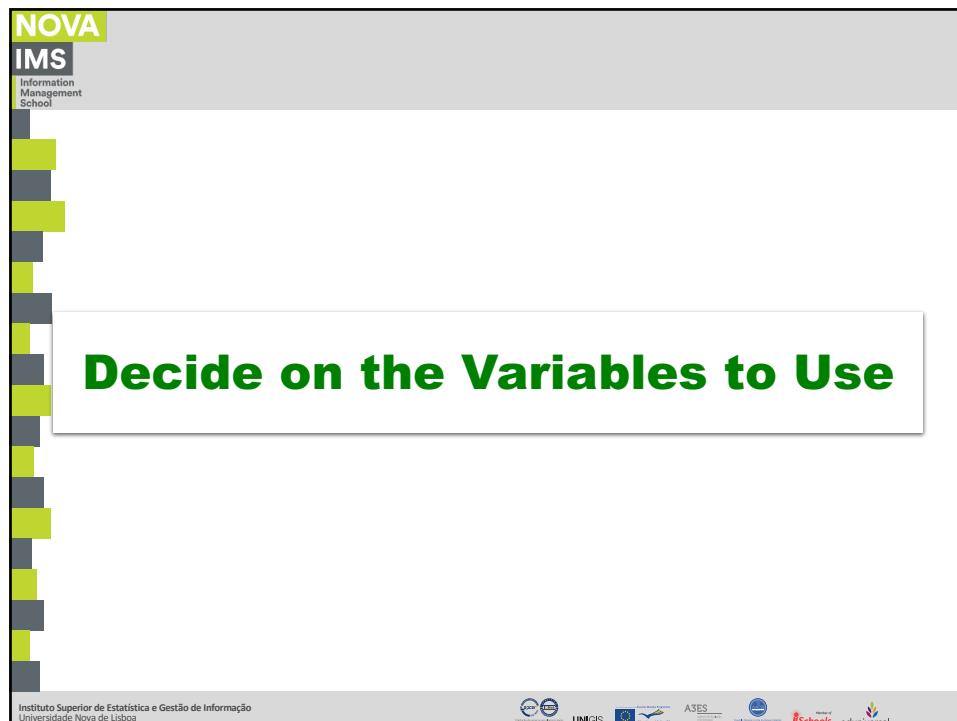
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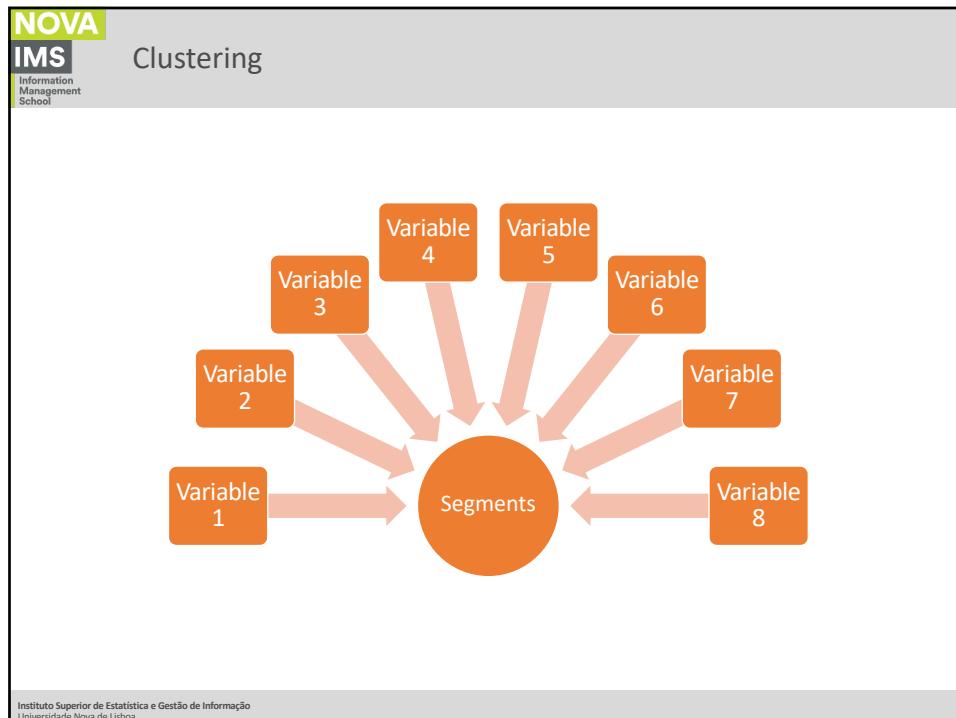
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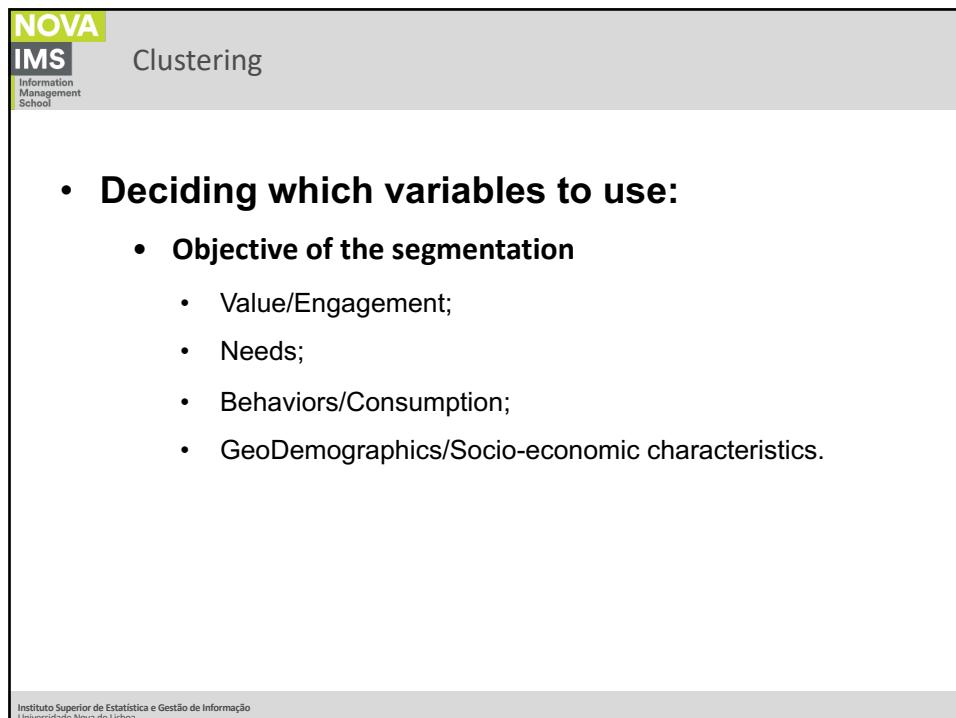
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- **Deciding which variables to use:**

- The type of problem determines the variables to choose;
- If the purpose is to group objects, the choice of variables with discrimination ability is crucial;
- The quality of any cluster analysis is, first of all, conditioned by the variables used.

- **Deciding which variables to use:**

- The choice of variables should replicate a theoretical context, a reasoning;
- This process is carried out based on a set of variables that we know to be good discriminators for the problem at hand;
- First of all, the quality of the cluster analysis reflects the discrimination ability of the variables we decided to use in our study.



## Similarity criterion

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### Clustering

- **Similarity criterion:**
  - The analysis of similarity relations has been dominated by metrics based on Euclidean Spaces;
  - Objects as points in a multidimensional space, in a way that the observed dissimilarities between the objects correspond to distances between the respective points;
  - Thus, the use of clustering methods most times means the use of similarity ratios that respect these metrics:

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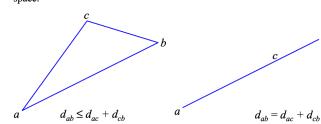
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- **Similarity criterion:**

- In mathematics, a true measure of distance, called a *metric*, obeys three properties. These metric axioms are as follows, where  $d_{ab}$  denotes the distance between objects  $a$  and  $b$ :

1.  $d_{ab} = d_{ba}$  (*measure is symmetric*)
2.  $d_{ab} \geq 0$  and  $= 0$  if and only if  $a = b$  (*distances are always positive except when the objects are identical*)
3.  $d_{ab} \leq d_{ac} + d_{cb}$  (*triangle inequality*)

*Exhibit 5.1* Illustration of the triangle inequality for distances in Euclidean space.



- **Similarity criterion:**

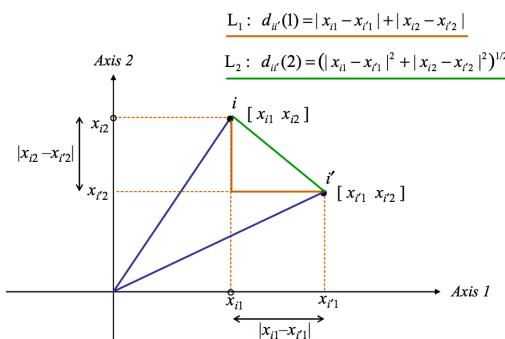
- Euclidian distance: the distance between two elements  $(i,j)$  is the square root of the sum of the squares of the differences between  $i$  and  $j$  values for all variables ( $v=1, 2, \dots, p$ ):

$$d_{ij} = \sqrt{\sum_{v=1}^p (X_{iv} - X_{jv})^2} \text{ euclidean also known as } L_2$$

$$d_{ij} = \sum_{v=1}^p |X_{iv} - X_{jv}| \text{ City Block or } L_1$$

- **Similarity criterion:**

- Euclidian distance: the distance between two elements  $(i,j)$  is the square root of the sum of the squares of the differences between  $i$  and  $j$  values for all variables ( $v=1, 2, \dots, p$ ):



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- **Similarity criterion:**

- Minkowski distance: is defined from the absolute distance, and can be considered as a generalization of both the Euclidean distance and the Manhattan distance. It coincides with Euclidean distance when  $r=2$  and with Manhattan distance when  $r=1$ :

$$d_{ij} = \left( \sum_{v=1}^p |X_{iv} - X_{jv}|^r \right)^{1/r}$$

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- **Similarity criterion:**

- If a weight is assigned to each variable, according to their importance for the analysis, the weighted Euclidean distance takes the following form:

$$d_{ij} = \sqrt{\sum_{v=1}^p w_v (X_{iv} - X_{jv})^2}$$

- **Similarity criterion:**

- Pearson correlation coefficient: its function is to measure the degree of linear correlation between two elements, for a number of variables:

### Correlation

$$\rho(x_i, x_{i'}) = \frac{\sum_j (x_{ij} - \bar{x}_i)(x_{i'j} - \bar{x}_{i'})}{\sqrt{\sum_j (x_{ij} - \bar{x}_i)^2 \sum_j (x_{i'j} - \bar{x}_{i'})^2}}$$

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## Choose the Algorithm

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### Clustering

```

graph TD
    Grouping --> Clustering
    Grouping --> QuartileBasedClustering
    Clustering --> HierarchicalMethods
    Clustering --> PartitionMethods
    Clustering --> DensityBased
    Clustering --> SOM
    QuartileBasedClustering --> Quartiles
    QuartileBasedClustering --> RFMAnalysis
    Quartiles --> Variables
    Quartiles --> ThroughTime
    RFMAnalysis --> MigrationMatrices
  
```

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# A Priori Grouping

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## Clustering

```

graph TD
    Grouping --> Clustering
    Clustering --> HierarchicalMethods
    Clustering --> PartitionMethods
    Clustering --> DensityBased
    Clustering --> SOM
    HierarchicalMethods --- QuartileBased
    PartitionMethods --- RFMAnalysis
    DensityBased --- Variables
    DensityBased --- ThroughTime
    RFMAnalysis --- MigrationMatrices
    
```

The diagram illustrates the classification of clustering methods. It starts with a general category 'Grouping' at the top, which branches down to 'Clustering'. 'Clustering' further divides into four main categories: 'Hierarchical Methods', 'Partition Methods', 'Density-based', and 'Self-Organizing Maps'. The 'Hierarchical Methods' category is highlighted with a light blue background. The 'Partition Methods' category is shown in a light grey box. The 'Density-based' and 'Self-Organizing Maps' categories are also shown in light grey boxes. The 'Hierarchical Methods' category leads to 'Quartile-based clustering', which then branches into 'Quartiles' and 'RFM Analysis'. 'Quartiles' further branches into 'Variables' and 'Through time'. 'RFM Analysis' branches into 'Migration Matrices'.

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## Quartile-based clusters

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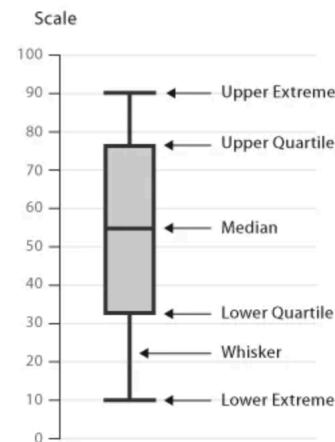
### Clustering

- **Quartile-based clusters**
  - A percentile is a measure used in statistics indicating the value below which a given percentage of observations in a group of observations fall.
  - For example, the 20th percentile is the value (or score) below which 20 percent of the observations may be found.
  - The 25th percentile is also known as the first quartile (Q1), the 50th percentile as the median or second quartile (Q2), and the 75th percentile as the third quartile (Q3).
  - In general, percentiles and quartiles are specific types of quantiles.

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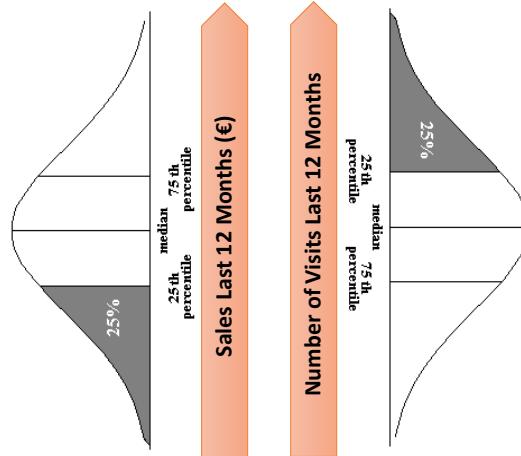
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- Quartile-based clusters



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## Clustering

Sales last 12M\_VL\_X\_Visits Last 12M\_NR

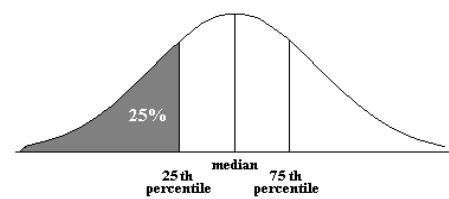
Tabela de Contingência		TICKETS_ULT_12M_NR				Totais
		<Q1	[Q1,Q2[	[Q2,Q3[	>=Q3	
VENDAS_ULT_12M_VL	<Q1				Buy very small amounts very frequently	
	[Q1,Q2[					
	[Q2,Q3[					
	>=Q3	A lot of money few times				
Totais						

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## Clustering

Migration Matrix



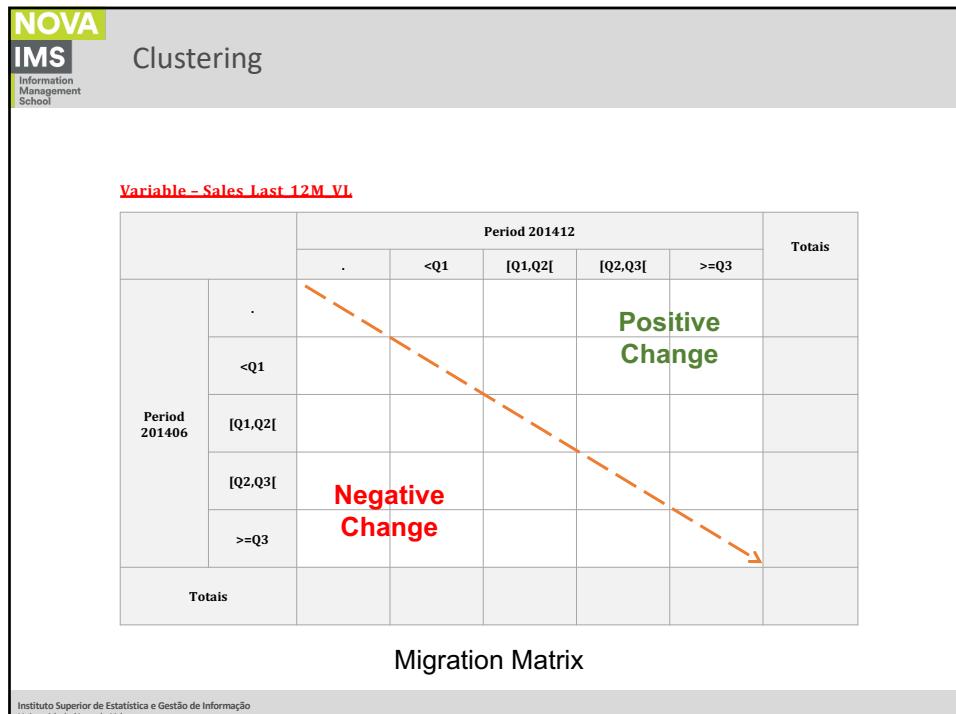
12/2012 – 12/2013

06/2013 – 06/2014

Evolution Sales Last 12 Months (12.2012 – 06.2014)

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### Clustering

**Tabela de Contingência**

		Period 06.2013/06.2014					Totais
		. 0	<Q1 a b	[Q1,Q2[ f g h	[Q2,Q3[ k l m n o	>=Q3 p q r s t	
Periodo 12.2012/ 12.2013	.	30603	17050	8815	6427	62895 13.5%	
	<Q1	28734	55411	13600	2772	178	100695 21.6%
	[Q1,Q2[	14834	15506	52421	16838	1097	100696 21.6%
	[Q2,Q3[	6540	3450	19346	59756	11608	100700 21.6%
	>=Q3	3765	760	1835	14733	79604	100697 21.6%
	<b>Totais</b>		53873 11.6%	105730 22.7%	104252 22.4%	102914 22.1%	98914 21.2%

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**RFM Analysis**

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## Clustering

- **RFM**
  - Based on the following principles:
    - Customers who have purchased more recently are more likely to purchase again;
    - Customers who have made more purchases are more likely to purchase again;
    - Customers who have made larger purchases are more likely to purchase again.

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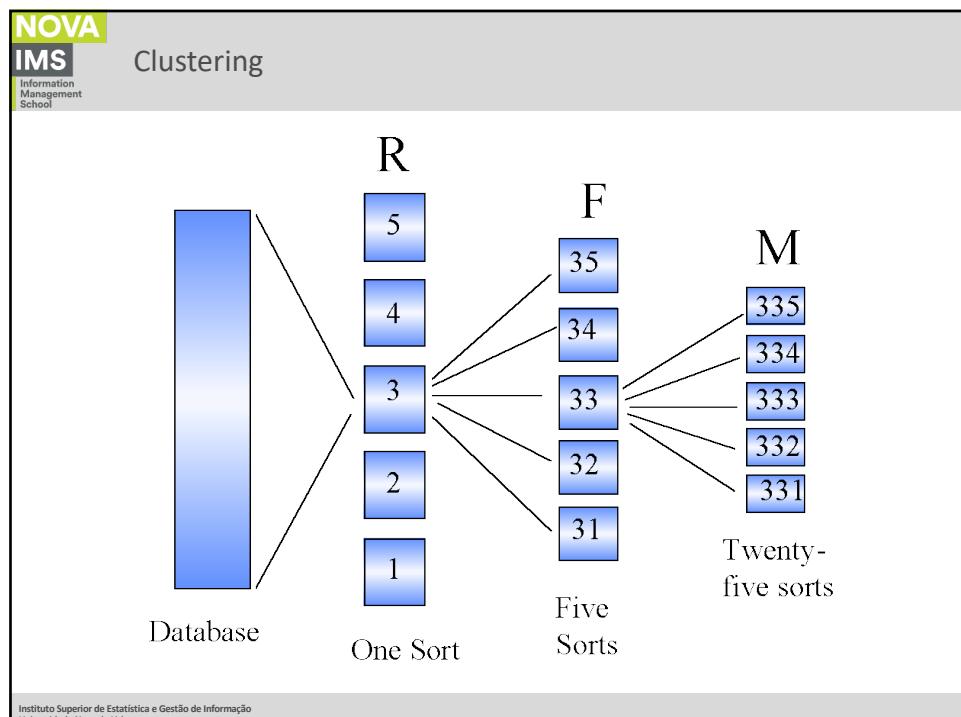
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- **RFM**

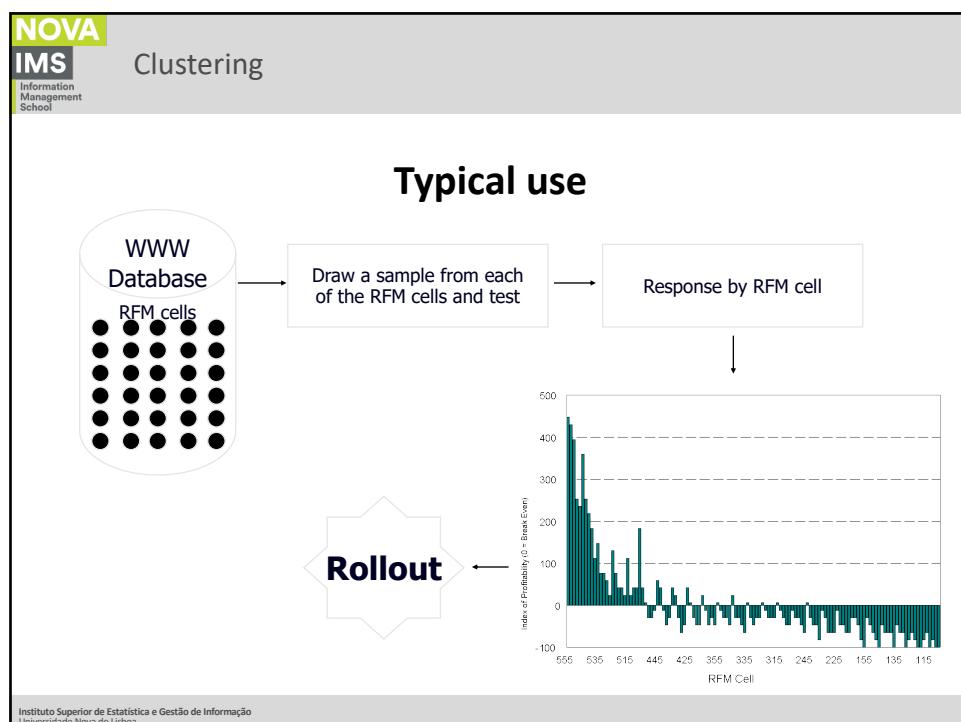
- Has been in active use in Direct Marketing for more than 40 years;
- It can be used only for customer files that contain purchase history;
- There are two methods:
  - Exact Quintiles;
  - Hard coding;

- **RFM**

- How to do it (Exact Quintiles)?
  - We sort the database according to recency and divide into 5 quintiles (5 equal segments);
  - Do the same for the variables frequency and monetary;
  - Result: 125 cells of equal size (5\*5\*5).



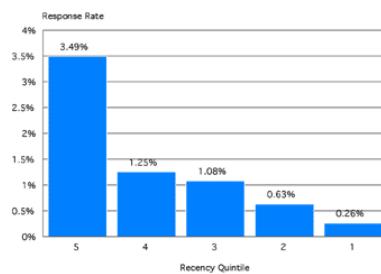
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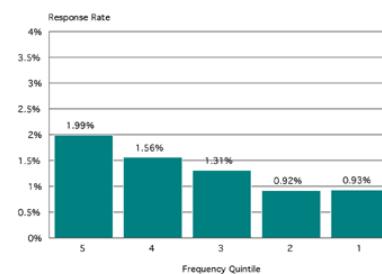
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## Clustering

Response By Recency Quintile



Response By Frequency Quintile



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## Clustering

## Migration Matrix

Segment in YY/YY/YYYY	Segment 1 XX/XX/XXXX							Total YYYY
	44	45	51	52	53	54	55	
44	.	41914	209	4362	1862	253	22	48622
45	34200	<b>58714</b>	7875	14961	8968	1652	128	126498
51	505	9089	<b>7823</b>	4895	5420	30	.	27762
52	9109	7044	7151	<b>83963</b>	11103	8820	208	127398
53	3572	5758	4211	5578	<b>29736</b>	3691	9	52555
54	382	124	93	6507	2190	<b>36300</b>	4128	49724
55	69	10	22	156	62	4089	<b>14446</b>	18854
<b>Total XXXX</b>	47837	122653	27384	120422	59341	54835	18941	

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- **RFM**

- Hard coding
  - Categories are divided by exact values (0-3 months; 4-6 months; 7-9 months; etc.);
  - More expensive in terms of programming, categories tend to change over time;
  - Very different quantities from cell to cell.

- **RFM**

- Its popularity comes from its simplicity, low cost and capacity to classify customers based on their behavior;
- Opportunity to carry out tests in small, representative groups of each cell;
- A more sophisticated modeling is almost always better, but is it worth it? Not always.



Questions?

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