



**Data Mining**

**Semi-supervised classification**

**Classification trees**

24/11/2021

**NOVA-IMS**

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1



**AGENDA**

- Cluster analysis
  - Semi-supervised classification
    - Classification Trees

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## Classification Trees

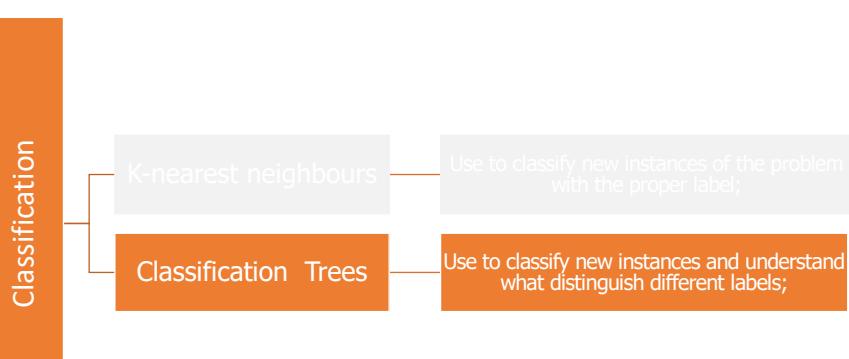
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### Going from clustering into classification



Classification

- K-nearest neighbours
- Classification Trees
  - Use to classify new instances of the problem with the proper label;
  - Use to classify new instances and understand what distinguish different labels;

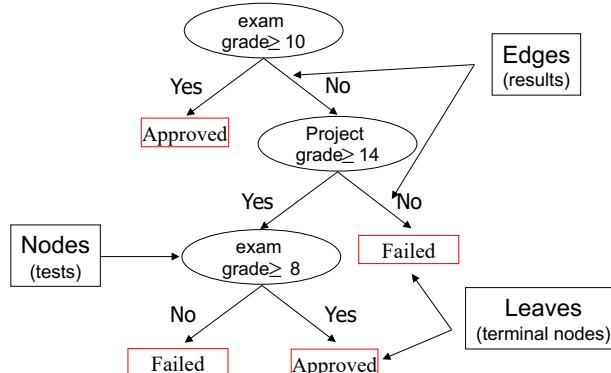
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4

- **Classification Trees:**

- Classification tree are typically considered to be classification and regression tools
- One of its most important advantages relates with the simplicity of the interpretation of its results
- Thus, the end result of a classification tree can easily be expressed in English or SQL.

- A classification tree is a decisional algorithm
- It can be seen as a way of storing knowledge
- The objective is to discriminate between Class
- Obtain leaves as pure as possible
- If possible each leave should represent only individuals from a specific class



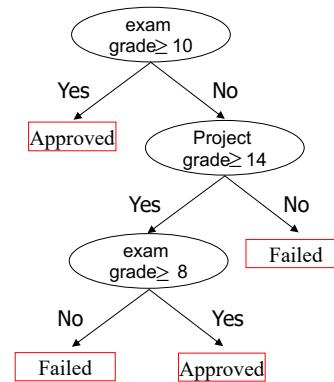
## Classification Trees

$$aprovado \Leftrightarrow (exame \geq 10)$$

$$aprovado \Leftrightarrow (exame < 10) \wedge (projeto \geq 14) \wedge (exame \geq 8)$$

$$reprovado \Leftrightarrow (exame < 10) \wedge (projeto < 14)$$

$$reprovado \Leftrightarrow (exame < 10) \wedge (projeto > 14) \wedge (exame < 8)$$



## Classification Trees

- **Classification Trees (strengths):**

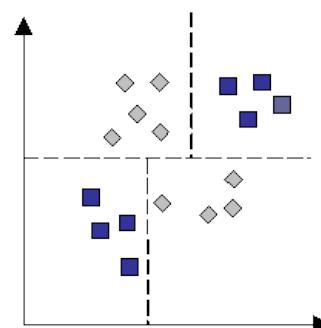
- Interpretation
  - We can easily understand the reasons behind a specific classification decision
- May use different types of data
  - Interval, ordinal, nominal, etc.
- Insensitive to scale factors
  - Variables measured in different scales may be used without any type of normalization

- **Classification Trees (strengths):**

- Automatically defines the most relevant variables
  - These are the variables used at the top of the tree
- Can be adapted to a regression
  - Each leave becomes a linear model

- **Classification Trees (weaknesses):**

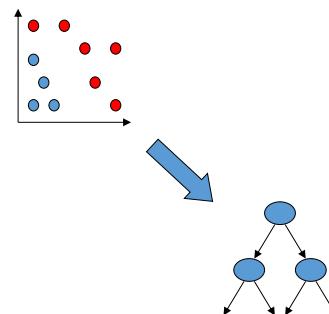
- Boundaries are linear and perpendicular to the variables axys
- Sensitive to small perturbations in the data



From Gahegan and West  
[http://divcom.otago.ac.nz/SIRC/GeoComp98/61/gc\\_61.htm](http://divcom.otago.ac.nz/SIRC/GeoComp98/61/gc_61.htm)

- **Classification Trees:**

- Build (induce) a tree from data
- Problems:
  - What to do?
  - Which variable to use?
  - What partition to use?
  - Which node to split?
  - How many edges per node?
  - When to stop?



- **Classification Trees:**

- ID3, C4.5 e C5 [Quinlan 86,93]
  - Iterative Dichotomizer 3
- CART
  - Classification and regression trees [Breiman 84]
- CHAID [Hartigan 75]
  - Used in SPSS and SAS...
- Muitas (mesmo muitas) outras variantes...
  - In SAS you can choose different parameters to build your tree.

## Worked-Example

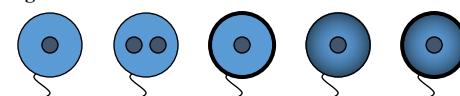
### General Idea

Langley, P: 1996, Elements of Machine Learning, Morgan and Kaufmann Publishers.

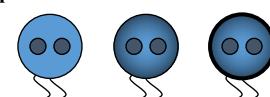
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13

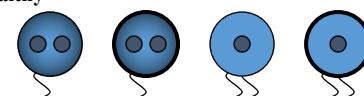
Lethargia



Burpoma



Healthy



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### Classification Trees

**Table**

# Nucleus	# Tails	Color	Membrane	<i>Class</i>
1	1	Light	Thin	<i>Lethargia</i>
2	1	Light	Thin	<i>Lethargia</i>
1	1	Light	Thick	<i>Lethargia</i>
1	1	Dark	Thin	<i>Lethargia</i>
1	1	Dark	Thick	<i>Lethargia</i>
2	2	Light	Thin	<i>Burpoma</i>
2	2	Dark	Thin	<i>Burpoma</i>
2	2	Dark	Thick	<i>Burpoma</i>
2	1	Dark	Thin	<i>Healthy</i>
2	1	Dark	Thick	<i>Healthy</i>
1	2	Light	Thin	<i>Healthy</i>
1	2	Light	Thick	<i>Healthy</i>

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### Classification Trees

- **Classification Trees:**
  - Measure to discriminate the attribute

$$f(A) = \frac{1}{n} \sum_{i=1}^{|A|} C_i$$

- $n$  is the total number of examples and  $C_i$  the number of examples correctly classified based on the most frequent class.
- This is a measure of “dominance” or “purity”

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### Classification Trees

Table		# Nucleus	# Tails	Color	Membrane	Class
# Nucleus	1	1	Light	Thin	<b>Lethargia</b>	
Lethargia	4	1	Light	Thin	<b>Lethargia</b>	
Burpoma	0	3	Dark	Thin	<b>Lethargia</b>	
Healthy	2	2	Dark	Thin	<b>Lethargia</b>	
		1	1	Light	Thin	<b>Lethargia</b>
		2	2	Light	Thin	<b>Burpoma</b>
		2	2	Dark	Thin	<b>Burpoma</b>
		2	1	Dark	Thin	<b>Healthy</b>
		2	1	Dark	Thick	<b>Healthy</b>
		1	2	Light	Thin	<b>Healthy</b>
		1	2	Dark	Thick	<b>Healthy</b>

Discrimination:  
 $(4 + 3) / 12 = 0.58$

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### Classification Trees

Table		# Nucleus	# Tails	Color	Membrane	Class
# Tails	1	1	Light	Thin	<b>Lethargia</b>	
Lethargia	5	0	Light	Thin	<b>Lethargia</b>	
Burpoma	0	3	Light	Thin	<b>Lethargia</b>	
Healthy	2	2	Dark	Thin	<b>Lethargia</b>	
		1	1	Light	Thin	<b>Lethargia</b>
		2	2	Light	Thin	<b>Burpoma</b>
		2	2	Dark	Thin	<b>Burpoma</b>
		2	1	Dark	Thin	<b>Healthy</b>
		2	1	Dark	Thick	<b>Healthy</b>
		1	2	Light	Thin	<b>Healthy</b>
		1	2	Light	Thick	<b>Healthy</b>

Discrimination:  
 $(5 + 3) / 12 = 0.67$

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Classification Trees							
Table			# Nucleus	# Tails	Color	Membrane	Class
Color	Light	Dark	1	1	Light	Thin	Lethargia
Lethargia	3	2	2	1	Light	Thin	Lethargia
Burpoma	1	2	1	1	Dark	Thin	Lethargia
Healthy	2	2	1	1	Dark	Thick	Lethargia
			2	2	Light	Thin	Burpoma
			2	2	Dark	Thin	Burpoma
			2	1	Dark	Thick	Healthy
			2	1	Dark	Thin	Healthy
			1	2	Light	Thin	Healthy
			1	2	Light	Thick	Healthy

Discrimination:  
 $(3 + 2) / 12 = 0.41$

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Classification Trees							
Table			# Nucleus	# Tails	Color	Membrane	Class
Membrane	Thin	Thick	1	1	Light	Thin	Lethargia
Lethargia	3	2	2	1	Light	Thin	Lethargia
Burpoma	2	1	1	1	Light	Thick	Lethargia
Healthy	3	1	1	1	Dark	Thin	Burpoma
			2	2	Light	Thin	Burpoma
			2	2	Dark	Thin	Burpoma
			2	1	Dark	Thick	Healthy
			2	1	Dark	Thin	Healthy
			1	2	Light	Thin	Healthy
			1	2	Light	Thick	Healthy

Discrimination:  
 $(3 + 2) / 12 = 0.41$

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### Classification Trees

**Choice: # Tails**

# Nucleus	1	2
Lethargia	4	1
Burpoma	0	3
Healthy	2	2

0.58

Color	Light	Dark
Lethargia	3	2
Burpoma	1	2
Healthy	2	2

0.41

# Tails	1	2
Lethargia	5	0
Burpoma	0	3
Healthy	2	2

0.67

Membrane	Thin	Thick
Lethargia	3	2
Burpoma	2	1
Healthy	3	1

0.41

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### Classification Trees

**Initial Partition**

```

graph TD
    Tails((Tails)) -- one --> TableOne
    Tails -- two --> TableTwo
    
```

# Nucleus	Color	Membrane	Class
1	Light	Thin	Lethargia
2	Light	Thin	Lethargia
1	Light	Thick	Lethargia
1	Dark	Thin	Lethargia
1	Dark	Thick	Lethargia
2	Dark	Thin	Healthy
2	Dark	Thick	Healthy

# Nucleus	Color	Membrane	Class
2	Light	Thin	Burpoma
2	Dark	Thin	Burpoma
2	Dark	Thick	Burpoma
1	Light	Thin	Healthy
1	Light	Thick	Healthy

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### Classification Trees

**Second Partition**

```

graph TD
    Tails((Tails)) -- one --> Nucleus((Nucleus))
    Tails -- two --> Data2[Data]
    Nucleus -- one --> Data1[Data]
    Nucleus -- two --> Data3[Data]
  
```

# Nucleus	Color	Membrane	Class
2	Light	Thin	Burpoma
2	Dark	Thin	Burpoma
2	Dark	Thick	Burpoma
1	Light	Thin	Healthy
1	Light	Thick	Healthy

Color	Membrane	Class
Light	Thin	Lethargia
Light	Thick	Lethargia
Dark	Thin	Lethargia
Dark	Thick	Lethargia

Color	Membrane	Class
Light	Thin	Lethargia
Dark	Thin	Healthy
Dark	Thick	Healthy

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### Classification Trees

**Tree (cont.2)**

```

graph TD
    Tails((Tails)) -- one --> Nucleus((Nucleus))
    Tails -- two --> Data2[Data]
    Nucleus -- one --> Lethargia4[Lethargia (4)]
    Nucleus -- two --> Data3[Data]
  
```

# Nucleus	Color	Membrane	Class
2	Light	Thin	Burpoma
2	Dark	Thin	Burpoma
2	Dark	Thick	Burpoma
1	Light	Thin	Healthy
1	Light	Thick	Healthy

Color	Membrane	Class
Light	Thin	Lethargia
Dark	Thin	Healthy
Dark	Thick	Healthy

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### Classification Trees

**Tree (cont.3)**

# Nucleus	Color	Membrane	Class
2	Light	Thin	Burpoma
2	Dark	Thin	Burpoma
2	Dark	Thick	Burpoma
1	Light	Thin	Healthy
1	Light	Thick	Healthy

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### Classification Trees

**Final Tree**

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### The description of the three Classss

$$(tails = 1) \wedge (nucleous = 1) \\ \vee \\ (tails = 1) \wedge (nucleous = 2) \wedge (color = light) \rightarrow Lethargic$$

$$(tails = 2) \wedge (nucleous = 1) \\ \vee \\ (tails = 1) \wedge (nucleous = 2) \wedge (color = dark) \rightarrow Healthy$$

$$(tails = 2) \wedge (nucleous = 2) \rightarrow Burpoma$$

- **Classification Trees:**
  - In each level it divides the set into alternative partitions.
    - Using a measure of quality selects the best partition.
  - The process is repeated for each element of the partition.
  - Stops when a given criteria is reached

- **Classification Trees:**

- It assumes the existence of a target variable “Class” meaning the examples were previously classified.
- Each node specifies a unique attribute which is used as test.
- N – node N
- ASET – Atribute Set
- ISET – Instance Set

If Se the ISET is empty then the terminal node N is an unknown class  
if not

If all the examples of ISET are of the same class  
then the terminal node N has the name of the class  
if not

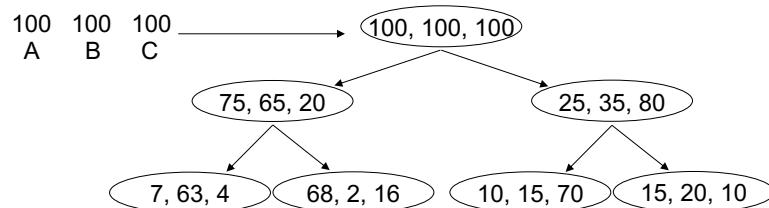
For each attribute A of the set of attribute ASET  
Evaluate A according to its capability to discriminate a class  
Select the attribute B which has the best discriminate value  
For each value V of the best attribute B  
Create a new node C from node N  
Place the par attribute value (B, V) in C  
Let JSET be the set of examples of ISET with value V in B  
Let KSET be the set of attributes of ASET with B removed  
DDT(C, KSET, JSET)

## Worked-Example

### Tree Accuracy

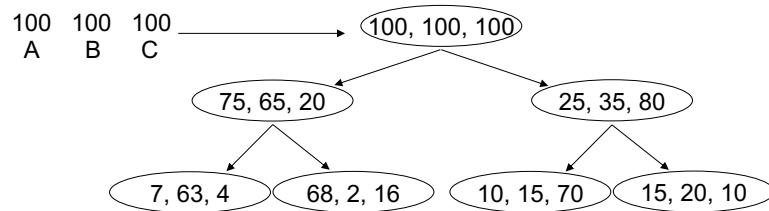
## Quality of the results

### Error rate



## Quality of the results

### Error rate



### Error Calculation

$$TEA = \frac{(74 * 14,9\% + 86 * 20,9\% + 95 * 26,3\% + 45 * 55,5\%)}{300} = 26,3\%$$

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