Mathematical Description – MaritimeGCH

Sets and Indices

- years: Set of years, indexed by y (2020 to 2050 for this example).
- ship_types: Set of ship types, indexed by s (Container, Tanker, Bulk, Cargo, Other).
- fuel_types: Set of fuel types, indexed by f (Oil, LNG, LPG, MET, MeOH, NH3, AllNoLNG, RefPO, H2).
- engine_types: Set of engine types, indexed by eng [ME-C engine, ME-GI (high pressure gas engine), ME-LGI (liquid gas injection)]

Parameters

- invest_cost_s: Investment cost of ship type s (in million Euros).
- op_cost_s: Operational cost of ship type s per year (in million Euros).
- fuel_cost_f: Fuel cost of fuel type f (in Euros per tonne).
- tax_co2_y: CO2 tax in year y (in Euros per tonne of CO2).
- $\text{emissions\_factor}_f$: Emission factor of fuel type f (tonnes of CO2 per tonne of fuel).
- $\text{prod\_capacity}_{y,s}$: Production capacity of ship type s in year y (number of ships).
- lifetime_s: Lifetime of ship type s (in years).
- $\text{fuel\_consumption}_{s,f,eng}$: Fuel consumption of ship type s using fuel type f (tonnes of fuel per year) per engine type eng.
- $\text{demand\_shipping}_{y,s}$: Demand for shipping services in year y [Gross Tonnage per Nautical Mile (Gt*NM)] of ship type s in year y.
- init_capacity_fleet: Initial capacity of fleet of ship type s in the year 2020 (number of ships).
- $\text{fuel\_avail}_{f,y}$: Available amount of fuel type f that can be used per year y (tonnes)
- cap_s: Capacity, namely the weight of each ship types' load (GtNM - Gross Tonnage*Nautical Mile)
- $\text{CII}_{desired,s}$: Desired value of Carbon Intensity Indicator of ship type s.

Decision Variables

- new_ship_y: Number of new ships of type s to make in year y.
- stock_ship_y: Stock of ships of type s in year y.
- $\text{fuel\_demand}_{f,y}$: Fuel demand of fuel type f in year y (tonnes).
- co2_emissions_y: CO2 emissions in year y (tonnes of CO2).

Objective Function = Minimize the total cost over the planning horizon:

$min \sum_{y=2020}^{2050} (total\_cost_y)$ Total cost in year y (in million Euros)

Where:

$$total\_cost_y = \sum_s (\text{new\_ship}_{y,s} \times \text{invest\_cost}_s) + \sum_s (\text{stock\_ship}_{y,s} \times \text{op\_cost}_s) + \sum_s (\text{fuel\_demand}_{y,f} \times \text{fuel\_cost}_f) + (\text{co2\_emissions}_y \times \text{tax\_co2}_y)$$

Constraints:

Fleet Capacity Constraint: The total stock of ships each year must be sufficient to meet the demand for shipping services:

$$\sum_s (\text{stock\_ship}_{y,s} * \text{cap}_s) \geq \text{demand\_shipping}_y \quad \forall y$$

Ship Production Constraint: The number of new ships built each year is limited by their production capacity:

$$\text{new\_ship}_{y,s} \leq \text{prod\_capacity}_{y,s} \quad \forall y, s$$

Fleet Stock Update Constraint: The stock of ships of each type in a given year is the sum of new ships built and surviving ships from previous years:

If y=2020,

$$\text{stock\_ship}_{y,s} = \text{init\_capacity\_fleet}_s$$

Else:

$$\text{stock\_ship}_{y,s} \leq \text{new\_ship}_{y,s} + \text{stock\_ship}_{y-1,s} \times \left(1 - \frac{1}{lifetime_s}\right) \quad \forall y, s$$

Fuel Demand and Availability Constraints: The fuel demand is derived from the operational needs of the ships, which however, cannot exceed the available amount of each fuel type this year:

$$\text{fuel\_demand}_{y,f} = \sum_s \text{stock\_ship}_{y,s} \times \text{fuel\_consumption}_{s,f,eng} \quad \forall y, f, s, eng$$

And $\text{fuel\_demand}_{y,f} \leq fuel\_avail_{f,y} \; \forall y, f$

Where: $\text{fuel\_consumption}_{s,f,eng} = fixed\ number\ per\ each\ combination\ of\ s, f, eng$

Emissions: The total CO2 emissions are calculated based on fuel consumption:

$$\text{co2\_emissions}_y = \sum_f \text{fuel\_demand}_{y,f} \times \text{emissions\_factor}_f \quad \forall y$$

Calculation of the CII:
$CII_{s,y}$ = Carbon Intensity Indicator of ship type s per year[1]

$$\text{CII}_{s,y} = \frac{\text{co2\_emissions}_y}{\text{cap}_s}$$

Carbon Intensity Indicator Constraint: It should not exceed a performance defined by regulations, or the user/ owner ($\text{CII}_{desired\ per\ ship\ type\ s}$):

$$\text{CII}_{s,y} \leq \text{CII}_{desired,s}$$

<u>The script saves the following results in the working directory (change it with yours):</u>

- maritimeLP.txt (= model details)
- maritime_results.xlsx (= table with the outputs)
- saved .png plots in the working directory

[1] Simplified form (tailored to this example) of the typical Equation:

CII = [Fuel $\text{Consumption}_{ship,\ eng}$ * CO2 emission factor_{fuel} / dist_trav_s * Load Capacity_{ship}] * Correction factor