Simple Reservoir Simulation Problem, and Different Optimization models

Angelos Alamanos

Model#5 considers the benefits from serving the urban, irrigation, and hydropower water uses, as well as the costs of the spills (as opportunity costs of the water that could have been used), and the cost of not meeting the minimum environmental flow requirements (using a penalty rate). The numbers provided are random and serve only the purpose of demonstrating the example model.

Decision Variables:

S_t represents the storage in the reservoir at the end of month t, where t=1,2,...,12.

R_{it} represents the releases to cover the demand of user i at month t=1,2,...,12.

i = our 3 uses: agriculture, hydropower, urban demand. So, i = 3.

$D_{i,t}$ demands for our 3 uses.

Sp_t represents the spills from the reservoir at month t, where t=1,2,...,12. (water spilled to avoid a flooding reservoir).

EF_t represents the environmental flows at month t, where t=1,2,...,12.

K is the reservoir capacity.

Objective Function: The objective is to maximize the total benefits derived from the reservoir over the 12-month period. This can be represented as:

$$max = \sum_{t=1}^{12} [\, B_R(R_{1t}, R_{2t}, R_{3t}) - C_{sp}(Sp_t) \; - C_{EF}(EF_t)]$$

Where

B_R = Benefits generated from the releases that will meet the user demands R_{1t}, R_{2t}, R_{3t} at each month.

C_{sp} = Cost associated with spills

C_{EF} = Cost associated with not meeting environmental flow requirements.

Constraints:

1. Storage Balance Equation:

$$S_t = S_{t-1} + I_t - E_t + P_t - \sum_{i}^{3}(R_{1t}, R_{2t}, R_{3t}) - Sp_t - EF_t\,, \qquad t = 1,2, \dots 12$$

This equation ensures that the storage at the end of each month is determined by the

- Existing storage in the previous time step S_{t-1},
- Inflows I_t,
- Evaporation losses E_t,
- Precipitation falling into the reservoir P_t
- Releases R_{it} to provide water to each user i=1,2,3,
- Spills Sp_t,
- Environmental flows EF_t,

at each month t, where t=1,2,...,12.

2. Storage Capacity Constraint:

$$0 < S_t \le K, \;\; t = 1,2, \dots 12$$

This constraint ensures that the storage in the reservoir does not exceed its capacity K.

3. Release Constraints:

$$\begin{gathered} R_{1t} \ge D_i \\ R_{2t} \ge D_i \\ R_{3t} \ge D_i \\ for \;\; t = 1,2, \dots 12 \end{gathered}$$

These constraints ensure that the releases will meet or exceed the user demands (i=1,2,3).

4. Spill Constraints:

$$0 \le Sp_t \le S_{t-1}, \;\; t = 1,2, \dots 12$$

This constraint limits the amount of spillage from the reservoir to prevent flooding.

5. Environmental Flow Constraints:

$$EF_t \ge MinEF_t$$

This constraint ensures that the minimum environmental flow requirements are met.

Necessary Data (known or predefined/ specified):

I. Reservoir Capacity (K): The maximum storage capacity of the reservoir. This is typically a known constant and represents the physical limit of how much water the reservoir can hold.

II. User Demands (D): The demands of each user i for each month t. In this example, we have agricultural, hydropower, and urban demands.

III. Inflows (I_t): The monthly inflow data, representing the amount of water flowing into the reservoir from various sources such as rivers, streams, or tributaries, for each month.

IV. Evaporation Losses (E_t): The monthly evaporation losses from the reservoir. These vary with seasonal changes, so data or estimates for each month are needed.

V. Precipitation (P_t): The monthly precipitation data, representing the amount of rainfall (or snowfall) that falls into the reservoir.

VI. Initial Storage (S_0): The initial storage level in the reservoir at the beginning of the time horizon (month 1). This may also be considered as an initial condition.

VII. Environmental Flow Requirements ($MinEF_t$): The minimum environmental flow requirements for each month t. These requirements are often based on ecological considerations (e.g. ecological flow assessments or legal requirements) and should be specified for each month.

VIII. Benefit Function ($B_R(R_{it})$): The function that quantifies the benefits generated by meeting user demands. This function should relate user demands to the benefits derived, and its parameters should be known or estimated.

Urban Demand:
To estimate the benefits generated by meeting urban water demand, we can consider factors such as the cost of alternative water sources (e.g., groundwater, desalination), the economic value of water to urban users (e.g., residential, commercial, industrial), and any societal benefits of reliable water supply (e.g., improved public health, increased property values). The benefit function in this case is represented as:

$$B_R(R_{urban,t}) = Economic\ Value\ of\ Water\ (\$/m3) \cdot R_{urban,t}(m3) - Cost\ of\ treatment\ (\$/m3) \cdot R_{urban,t}(m3)$$

Where
$Economic\ Value\ of\ Water = 1\ \$/m3$
$Cost\ of\ treatment = 0.2\ \$/m3$

Irrigation:
Estimating the benefits of meeting irrigation demand can involve factors such as crop yield improvement, crop value, and potential environmental benefits of efficient water use. The benefit function here is represented as:

$$B_R(R_{irr,t}) = Crop\ sales\ (\$/kg) \cdot Crop\ yields\ (\text{kg}) - Irrigation\ Costs\ (\text{tariffs in }\$)$$

Where
$Crop\ sales = \$2.50/kg$ for crop A, $\$2.00/kg$ for crop B, $\$1.50/kg$ for crop C, $\$1.10/kg$ for crop D.
$Crop\ yields = average\ monthly = 700\ kg$ for crop A, $950\ kg$ for crop B, $800\ kg$ for crop C, $600\ kg$ for crop D.
$Irrigation\ Costs = 0.30\ \$$ (tariff cost per unit of water)

Hydropower:
To estimate the benefits of hydropower generation, the function will associate it with factors like electricity generation capacity, electricity market prices, and the value of reliable power supply. The benefit function in our example is represented as:

$$B_R(R_{elect,t}) = Electricity\ produced\ (kWh) \cdot Price\ (\$/kWh) - HydropowerOperationCosts$$

Where

$Electricity\ produced\ (kWh/m3) = \dfrac{1}{0.068} = 14.705$ Hydroelectric plants typically use an average of 18 gal (68L or 0.068m3) of fresh water per kWh used by the consumer. $Price = 0.15\ \$/kWh$ $HydropowerOperationCosts$ = 0.03 $\$/m3$

IX. <u>Cost Function for Spills ($C_{sp}(Sp_t)$)</u>: The function that quantifies the cost associated with spills. This function should relate spill volumes to the associated costs, and its parameters should be known or estimated. In this example, it is considered to be the *Opportunity Cost,* meaning the lost opportunity to use the spilled water for beneficial purposes such as irrigation, hydropower generation, or municipal water supply. The cost is estimated based on the value of these foregone uses. For example, if the spilled water could have been used for irrigation, the cost might be calculated as the revenue that could have been generated from selling the water for irrigation, or for urban use. Same for lost Hydropower Generation: In cases where the reservoir is used for hydropower generation, spilled water represents lost electricity generation potential. The cost of spilled water can be calculated based on the revenue that would have been generated if it had been used for power generation. In our example, we assume that the water spilled is the weighted share (for month t = X% urban + Y% irrigation + Z% hydropower, where the %s are based on their respective monthly demands) from the following:

Urban Demand Opportunity Cost (Lost Water Tariffs): $C_{sp,urb}(Sp_t) = Economic\ Value\ of\ Water\ (\$/m3) \cdot\ Sp_{t,u}(m3)$
Opportunity Cost for Irrigation (Lost Revenue from Irrigation Water Sales): $C_{sp,irr}(Sp_t) =\ Irrigation\ Costs\ (\text{tariffs in \$/m3}) \cdot\ Sp_{t,irr}(m3)$
Hydropower Opportunity Cost (Lost Revenue from Electricity Sales): $C_{sp,hydro}(Sp_t) = Electricity\ that\ could\ have\ been\ produced\ (kWh\ with\ the\ spilled\ m3) \cdot Price\ (\$/kWh)$

X. <u>Cost Function for Environmental Flow Violations ($C_{EF}(EF_t)$)</u>: The function that quantifies the cost associated with not meeting environmental flow requirements. This function should relate the degree of violation to the associated costs, and its parameters should be known or estimated by environmental impact assessments (EIA) quantifying the damages and assigning them economic values (valuation). Then, usually a penalty value is set for violations of these flows. For the sake of our example, let's assume a linear cost function that penalizes deviations from the minimum environmental flow requirements (MinEFt):

- If $EF_t \geq MinEF_t$, then $Cost_{EF}(EF_t) = 0$
- If $EF_t < MinEF_t$, then $Cost_{EF}(EF_t) = PenaltyRate\ \cdot (MinEF_t - EF_t)$

Where:

PenaltyRate represents the cost per unit of flow deviation from the minimum requirement = 10 $/m3.

The model's script provides monthly results for the reservoir storage, spills, environmental flows, and releases for the three water uses (urban, irrigation, hydropower).