Simple Reservoir Simulation Problem, and Different Optimization models

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MODEL 1: MINIMUM SHORTAGES (or max water available)

Variables:

- S_t: Storage in the reservoir at time t. The amount of water stored in the reservoir.
- I_t: Inflows into the reservoir at time t. The amount of water flowing into the reservoir (from all sources, such as Precipitation, river or other inflows). Indicatively, for the sake of the example, the following random data are provided:

t=1 (JAN) = 3,000,000

t=2 (FEB) = 2,900,000

t=3 (MAR) = 2,700,000

t=4 (APR) = 2,600,000

t=5 (MAY) = 2,200,000

t=6 (JUN) = 2,000,000

t=7 (JUL) = 1,500,000

t=8 (AUG) = 900,000

t=9 (SEP) = 1,500,000

t=10 (OCT) = 1,800,000

t=11 (NOV) = 2,000,000

t=12 (DEC) = 2,500,000

- O_t: Outflows from the reservoir at time t. The total outflow from the reservoir, including spills and losses from evaporation or other unmanaged outflows. Indicatively, for the sake of the example, the following random data are provided:

t=1 (JAN) = 250,000

t=2 (FEB) = 250,000

t=3 (MAR) = 250,000

t=4 (APR) = 500,000

t=5 (MAY) = 500,000

t=6 (JUN) = 500,000

t=7 (JUL) = 500,000

t=8 (AUG) = 500,000

t=9 (SEP) = 500,000

t=10 (OCT) = 250,000

t=11 (NOV) = 250,000

t=12 (DEC) = 250,000

Decision Variables:

Releases (R): The amount of water released from the reservoir to meet the demands of three uses: urban, agricultural, and hydropower.

- $R_{u,t}$: Releases for urban use at time t.
- $R_{irr,t}$: Releases for agricultural use at time t.
- $R_{hydro,t}$: Releases for hydropower use at time t.

Parameters (indicative data for the sake of the example)**:**

- K: Reservoir capacity (maximum storage). K = 100,000,000 m3
- S_0: Initial storage in the reservoir at t=0. $S_0 =$ 50,000,000 m3
- $D_{u,t}$: Urban demand at time t. Data as follows:

t=1 (JAN) = 1,100,000

t=2 (FEB) =1,100,000

t=3 (MAR) =1,100,000

t=4 (APR) =1,200,000

t=5 (MAY) =1,500,000

t=6 (JUN) =1,700,000

t=7 (JUL) =1,800,000

t=8 (AUG) =1,700,000

t=9 (SEP) =1,200,000

t=10 (OCT) =1,100,000

t=11 (NOV) =1,100,000

t=12 (DEC) =1,100,000

- $D_{irr,t}$: Agricultural demand at time t. Data as follows:

t=1 (JAN) = 1,500,000

t=2 (FEB) = 1,500,000

t=3 (MAR) = 2,000,000

t=4 (APR) = 3,000,000

t=5 (MAY) = 5,000,000

t=6 (JUN) = 5,500,000

t=7 (JUL) = 5,800,000

t=8 (AUG) = 6,000,000

t=9 (SEP) = 4,500,000

t=10 (OCT) = 1,500,000

t=11 (NOV) = 1,500,000

t=12 (DEC) = 1,500,000

- $D_{hydro,t}$: Hydropower demand at time t. Data as follows:

t=1 (JAN) = 900,000

t=2 (FEB) = 900,000

t=3 (MAR) = 900,000

t=4 (APR) = 900,000

t=5 (MAY) = 900,000

t=6 (JUN) = 900,000

t=7 (JUL) = 900,000

t=8 (AUG) = 900,000

t=9 (SEP) = 900,000

t=10 (OCT) = 900,000

t=11 (NOV) = 900,000

t=12 (DEC) = 900,000

Objective: Minimize shortages in the reservoir (shortages are negative values of S_t) / or maximize the available water.

This can be represented in two equivalent ways:

$Zmin = \sum_t^{T=12}(-S_t)$ or $Zmax = \sum_t^{T=12}(S_t)$

Here we will keep the Zmax for simplicity, to avoid working with the negative values.

Objective Function: $Zmax = \sum_t^{T=12}(S_t)$

Constraints:

1. Storage balance equation: $S_t = S_{t-1} + I_t - O_t$, for all t.
2. Storage capacity constraint: $S_t \leq K$, for all t.
3. Release constraints:

$R_{u,t} \geq D_{u,t}$

$R_{irr,t} \geq D_{irr,t}$

$R_{hydro,t} \geq D_{hydro,t}$

, for all t.

4. Non-negativity constraints: $I_t , O_t, R_{i,t} \geq 0$

The solution of this problem will be that our decision variables (R) will be equal to the demands (D), to ensure a minimum level of releases, leading thus to the maximum storage.

The constraints (storage balance equation, storage capacity, release, and non-negativity constraints), are not restricting.

But they are designed to ensure that the system operates within physical and demand-related limits (for the sake of the correctness of the design).

The script provides the results (reservoir storage and releases per use) per each month of the simulation (t = 1,2,…, 12).