Simple Reservoir Simulation Problem

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Mathematical description of the simulation problem:

Objective:

This simulation aims to model the operation of a reservoir over a 12-month period, considering various factors such as inflow, outflow, storage capacity, spills, and demand for urban use, agricultural use, and hydropower generation.

Parameters:

- `n_months`: The number of months in the simulation (12).
- `months`: A set of months, i.e., {1, 2, ..., 12}.
- `S[t]`: The storage level in the reservoir for month `t` (million cubic meters).
- `R_u[t]`: The releases from the reservoir for urban use in month `t` (million cubic meters).
- `R_irr[t]`: The releases from the reservoir for agricultural use in month `t` (million cubic meters).
- `R_hydro[t]`: The releases from the reservoir for hydropower generation in month `t` (million cubic meters).
- `Spills[t]`: The amount of water spilled from the reservoir in month `t` (million cubic meters).
- `K`: The reservoir capacity (maximum storage) (million cubic meters).
- `S0`: The initial storage level in the reservoir at the beginning of the simulation (million cubic meters).
- `S_min`: The minimum required storage level in the reservoir (million cubic meters).
- `D_u[t]`: The demand for water for urban use in month `t` (million cubic meters).
- `D_irr[t]`: The demand for water for agricultural use in month `t` (million cubic meters).
- `D_hydro[t]`: The demand for water for hydropower generation in month `t` (million cubic meters).
- `I[t]`: The inflow of water into the reservoir in month `t` (million cubic meters).
- `O[t]`: The outflow of water from the reservoir in month `t`, which includes losses such as evaporation (million cubic meters).

Constraints:

1. Initial Storage: The initial storage in the first month is calculated as follows:

S[1] = S0 + I[1] - O[1]

2. Storage Balance Equation: For all subsequent months (t > 1), the storage balance equation ensures that storage at the beginning of the month equals the storage at the end of the previous month, plus inflow, minus outflow, minus releases for different uses:

S[t] = S[t - 1] + I[t] - O[t] - R_u[t] - R_irr[t] - R_hydro[t]

3. Storage Capacity Constraints: The storage level `S[t]` is constrained to be between the minimum required storage `S_min` and the reservoir capacity `K`:

S_min <= S[t] <= K

4. Releases Constraints: Releases for different uses are determined by demand and available storage. Urban releases should meet urban demand, agricultural releases should meet agricultural demand, and hydropower releases should meet hydropower demand. Releases are subject to available storage:

R_u[t] = min(S[t], D_u[t])

R_irr[t] = min(S[t], D_irr[t])

R_hydro[t] = min(S[t], D_hydro[t])

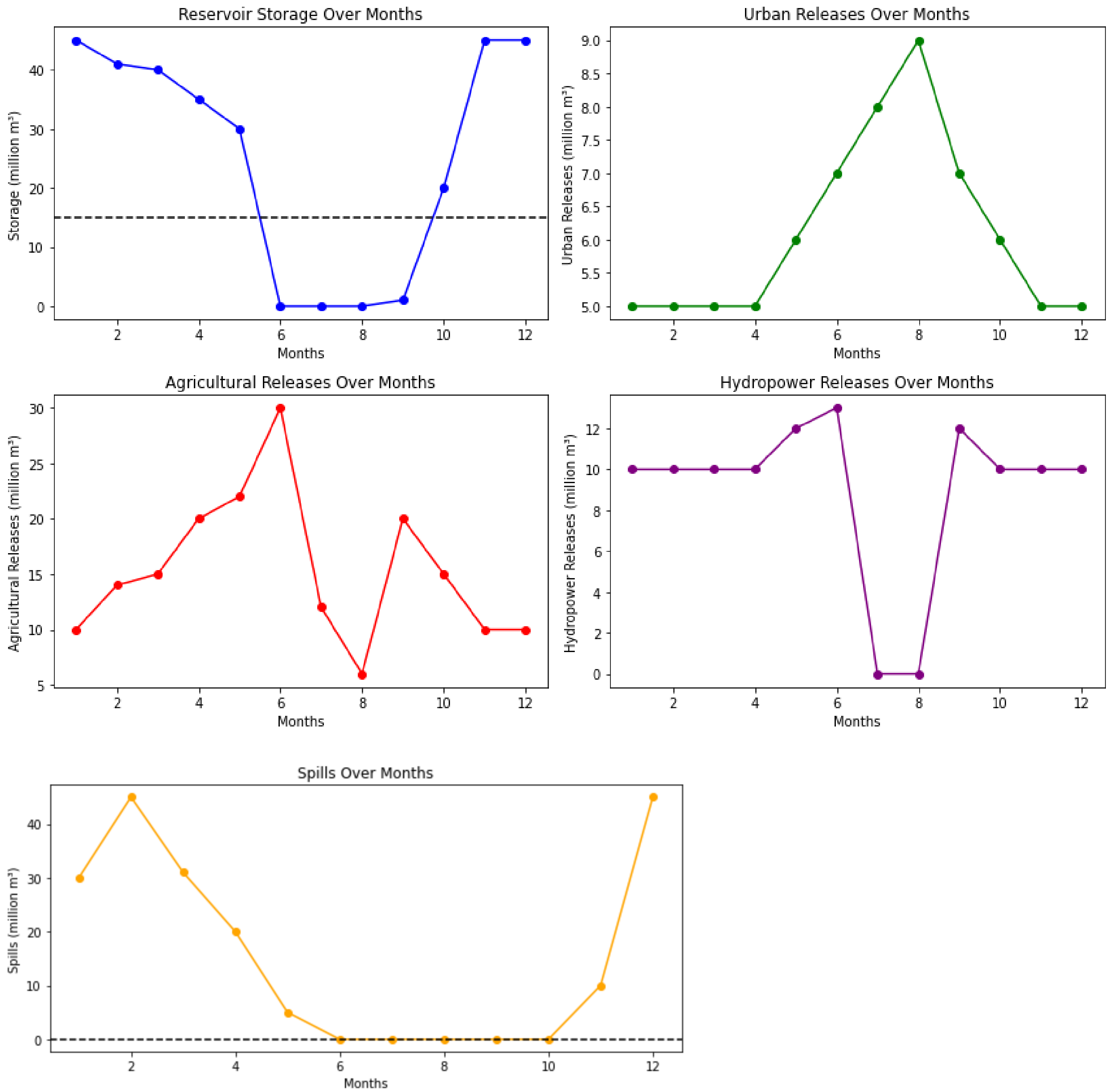
5. Spills Calculation: Spills from the reservoir occur when the storage level exceeds the capacity `K`. Spills are calculated as the excess storage above `K`:

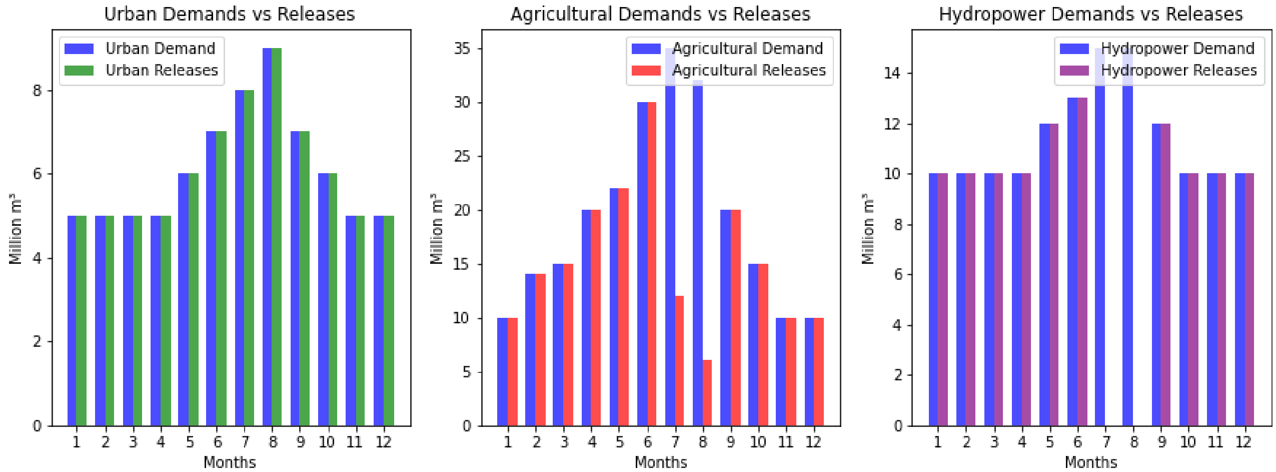
Spills[t] = max(0, S[t] - K)

Simulation:

The simulation iterates through each month, updating storage, releases, and spills based on the defined constraints and parameters. It calculates the storage level, releases, and spills for each month, providing insights into the reservoir's operation over the entire year. For the sake of this example, indicative numbers are provided in the script, in order to show how the visualization looks like (Figs below).

The simulation results can be visualized through plots, showing the changes in storage, releases, and spills over the 12-month period.


Reservoir Storage Over Months
Storage (million m³)
Months
Urban Releases Over Months
Urban Releases (million m³)
Months
Agricultural Releases Over Months
Agricultural Releases (million m³)
Months
Hydropower Releases Over Months
Hydropower Releases (million m³)
Months
Spills Over Months
Spills (million m³)
Months



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In the provided script, the model prioritizes meeting the urban demand before allocating water to agricultural and hydropower uses. This prioritization is evident in the simulation logic, where the releases for each type of demand are determined as follows:

1. Urban Releases (`R_u[t]`): The model first allocates water to meet the urban demand (`D_u[t]`) if there is sufficient water available in the reservoir. If the reservoir storage is not enough to meet the urban demand, it releases the available storage, and any excess urban demand is not met.

2. Agricultural Releases (`R_irr[t]`): After allocating water to urban demand, the model checks if there is any surplus storage available. If there is a surplus, it allocates water to meet the agricultural demand (`D_irr[t]`). If the surplus storage is not sufficient to meet the agricultural demand, it releases the available surplus, and any excess agricultural demand is not met.

3. Hydropower Releases (`R_hydro[t]`): Similar to agricultural releases, after meeting urban and agricultural demands, any remaining surplus storage is allocated to meet the hydropower demand (`D_hydro[t]`). If the surplus storage is insufficient, it releases the available surplus, and any excess hydropower demand is not met.

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Each action and water allocation policy has also economic implications, and policymakers are more likely to understand economic indicators when deciding.

Benefit Function ($B_R(R_{it})$): The function that quantifies the benefits generated by meeting user demands. This function should relate user demands to the benefits derived.

Benefit Functions:

$B_R(R_{urban,t}) = \text{Economic Value of Water } (\$/m3) \cdot R_{urban,t}(m3) - \text{Cost of treatment } (\$/m3) \cdot R_{urban,t}(m3)$ Where $\text{Economic Value of Water} = 1\ \$/m3$ $\text{Cost of treatment} = 0.2\ \$/m3$
$B_R(R_{irr,t}) = \text{Crop sales } (\$/kg) \cdot \text{ Crop yields (kg)} - \text{Irrigation Costs (tariffs in \$)}$ Where $\text{Crop sales} = \$2.50/kg$ for crop A, $\$2.00/kg$ for crop B, $\$1.50/kg$ for crop C, $\$1.10/kg$ for crop D. $\text{Crop yields} = \text{average monthly} = 700\ kg$ for crop A, $950\ kg$ for crop B, $800\ kg$ for crop C, $600\ kg$ for crop D. $\text{Irrigation Costs} = 0.30\ \$$ (tariff cost per unit of water)
$B_R(R_{hydro,t}) = \text{Electricity produced } (kWh/m3) \cdot R_{hydro,t} \cdot \text{Price } (\$/kWh) - \text{HydropowerOperationCosts} \cdot R_{hydro,t}$ Where $\text{Electricity produced } (kWh/m3) = \frac{1}{0.068} = 14.705$ Hydroelectric plants use an average of 18 gal (68L or 0.068m3) of fresh water per kWh used by the consumer. $\text{Price} = 0.15\ \$/kWh$ $\text{HydropowerOperationCosts} = 0.03\ \$/m3$

Cost Function for Spills ($C_{sp}(Sp_t)$): The function that quantifies the cost associated with spills. This function should relate spill volumes to the associated costs.

Cost Functions for Spills. This we will assume that is the weighted share (for month t = X% urban + Y% irrigation + Z% hydropower, where the %s are based on their respective monthly demands) from the following:

Urban Demand Opportunity Cost (Lost Water Tariffs): $$C_{sp,urb}(Sp_{u,t}) = Economic\ Value\ of\ Water\ (\$/m3) \cdot Sp_{t,u}(m3)$$
Opportunity Cost for Irrigation (Lost Revenue from Irrigation Water Sales): $$C_{sp,irr}(Sp_{irr,t}) = \ Irrigation\ Costs\ (\text{tariffs in \$/m3}) \cdot Sp_{t,irr}(m3)$$
Hydropower Opportunity Cost (Lost Revenue from Electricity Sales): $$C_{sp,hydro}(Sp_{hydro,t}) = Electricity\ that\ could\ have\ been\ produced\ (kWh\ with\ the\ spilled\ m3) \cdot Sp_{t,hydro}(m3) \cdot Price\ (\$/kWh)$$

In our case the Spills for urban, irrigation, and hydropower are their %s of the total demand (based on their respective monthly demands – just for the sake of estimating an average cost):

D_u	D_irr	D_hydro	D_tot	%of Urban	%of Irr	%of Hydro
5	10	10	25	20.00	40.00	40.00
5	14	10	29	17.24	48.28	34.48
5	15	10	30	16.67	50.00	33.33
5	20	10	35	14.29	57.14	28.57
6	22	10	38	15.79	57.89	26.32
7	30	10	47	14.89	63.83	21.28
8	35	10	53	15.09	66.04	18.87
9	32	10	51	17.65	62.75	19.61
7	20	10	37	18.92	54.05	27.03
6	15	10	31	19.35	48.39	32.26
5	10	10	25	20.00	40.00	40.00
5	10	10	25	20.00	40.00	40.00
			Averages:	17	52	30

So,

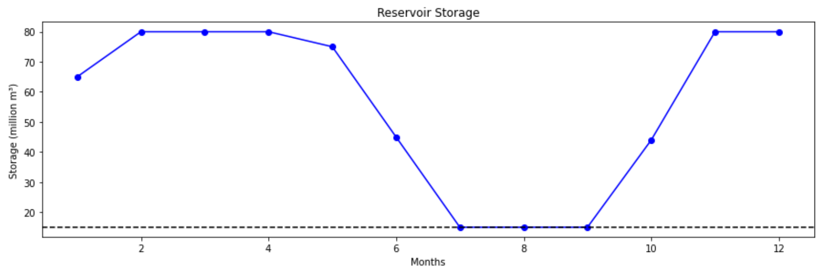
$$Sp_{t,u} = 0.17 * \ Spills_t$$

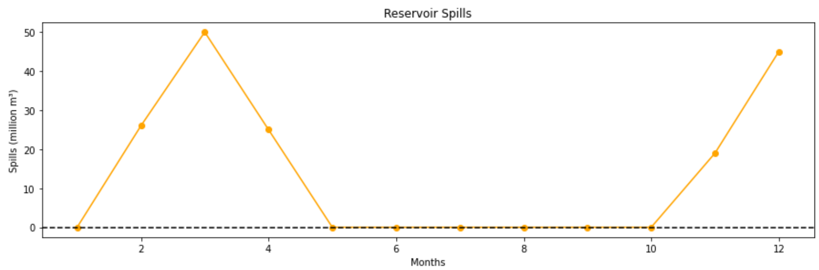
$$Sp_{t,irr} = 0.52 * \ Spills_t$$

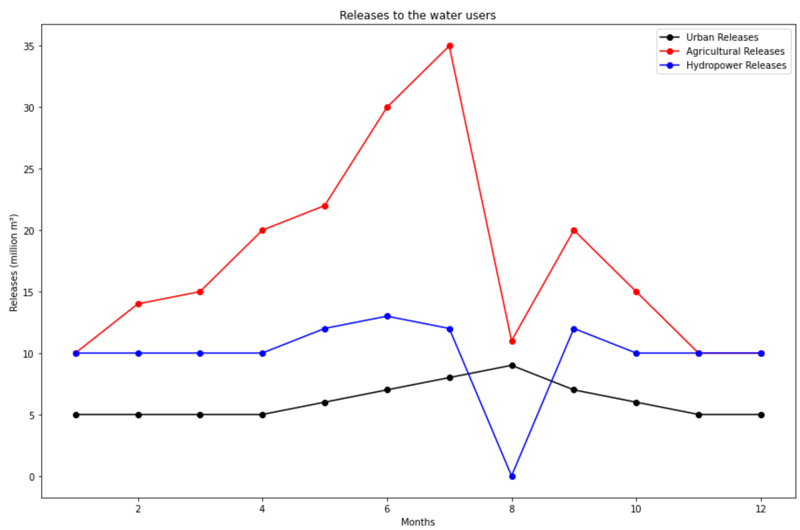
$$Sp_{t,hydro} = 0.3 * \ Spills_t$$

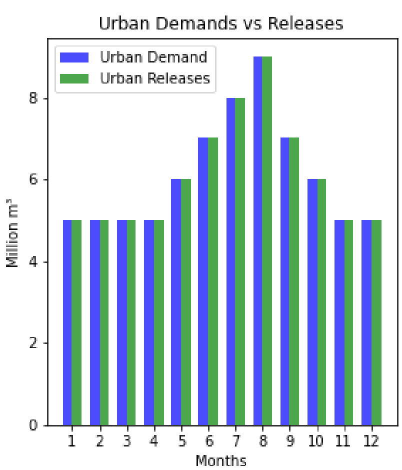
Note that other approaches could be also followed, depending on the problem (e.g. assuming the cost will be the minimum or maximum opportunity cost of one of these uses).

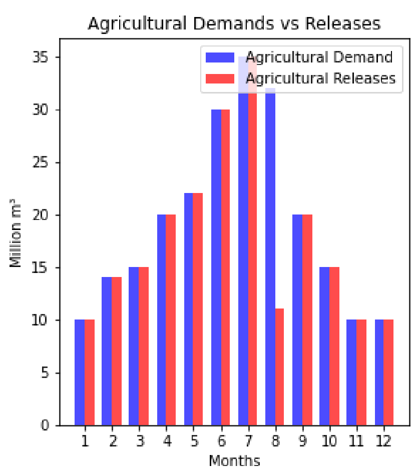
INDICATIVE EXAMPLE RESULTS:

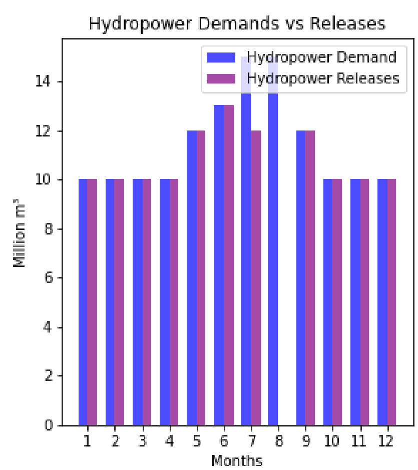


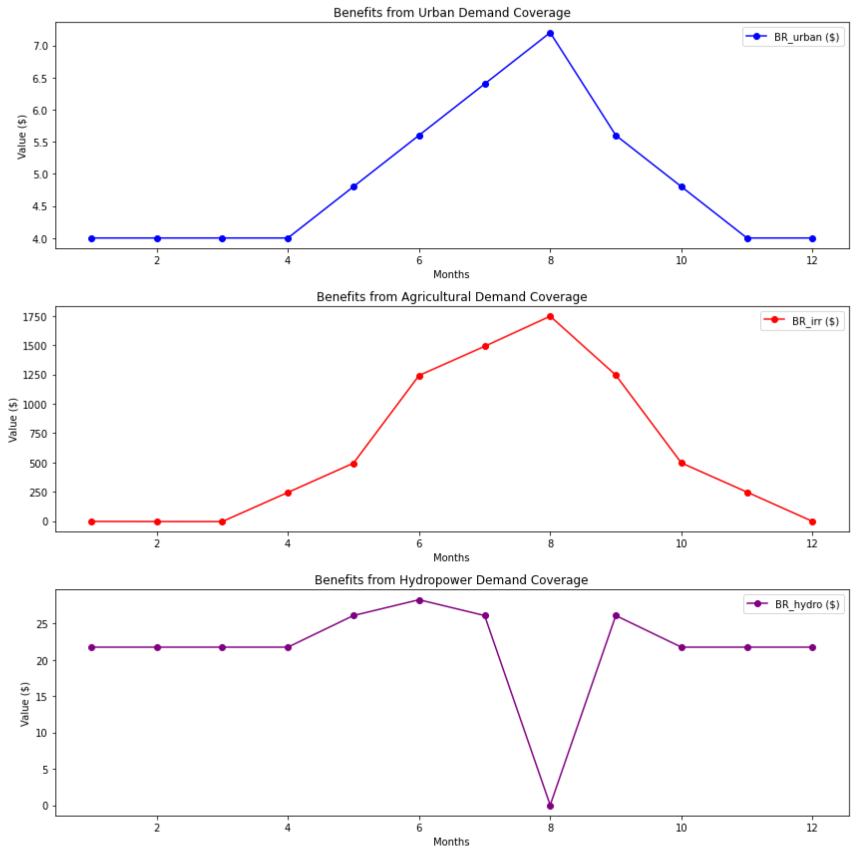


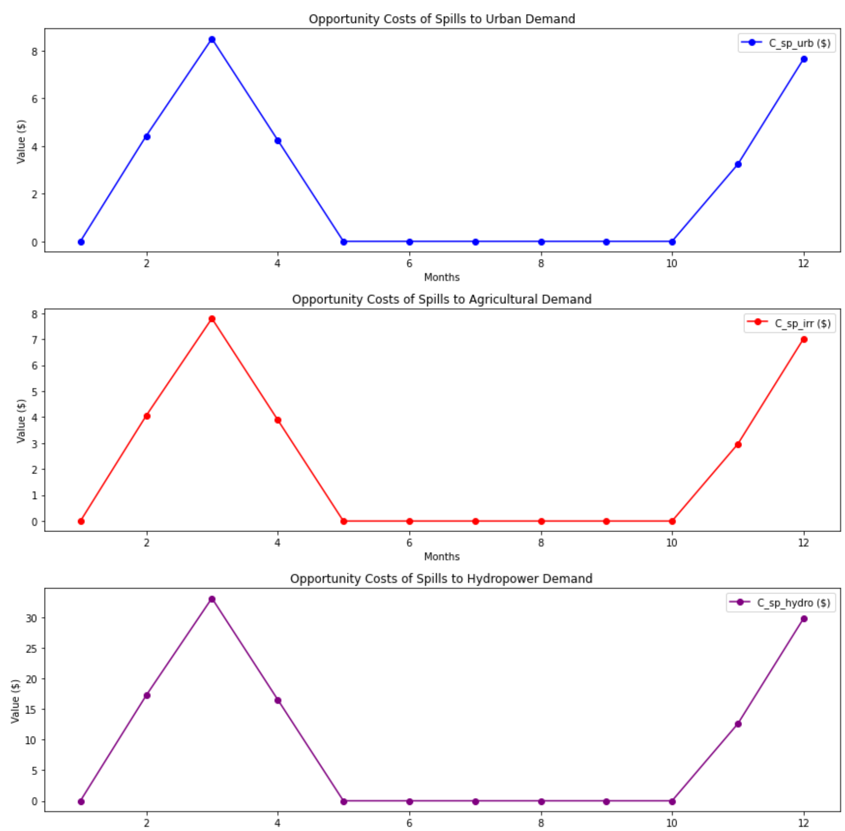









Benefits from Urban Demand Coverage
BR_urban ($)
Value ($)
Months
Benefits from Agricultural Demand Coverage
BR_irr ($)
Value ($)
Months
Benefits from Hydropower Demand Coverage
BR_hydro ($)
Value ($)
Months


Opportunity Costs of Spills to Urban Demand
C_sp_urb ($)
Value ($)
Months
Opportunity Costs of Spills to Agricultural Demand
C_sp_irr ($)
Value ($)
Months
Opportunity Costs of Spills to Hydropower Demand
C_sp_hydro ($)
Value ($)
Months