OPTIMIZATION

Assume the following indicative data for a Watershed:

		Costs per unit of usage	CO2 emissions per usage
Groundwater use allowance (renewable amount of groundwater that we can use, i.e. the groundwater recharge)	2000 m3	1.5 $/m3	0.15 kg CO2/m3
Surface water use allowance (renewable amount of surface water that we can use, depending on the infrastructure to collect it)	1000 m3	1.2 $/m3	0.05 kg CO2/m3
Energy production capacity from non-renewable sources (e.g. natural gas)	5000 kWh	0.2 $/kWh	0.25 kg CO2/kWh
Energy production capacity from renewable sources (e.g. wind, solar, hydropower)	3000 kWh	0.25 $/kWh	0.02 kg CO2/kWh

Water Demand (total)	**2700 m3** (= 700 for urban & 200 for agriculture)
Energy Demand (total)	**6500 kWh** (= 3500 for urban & 3000 for agriculture)

Budget (maximum money available to serve urban & agricultural uses) = 12000 $

Emissions (maximum allowable CO2 emissions) = 4500 kg CO2

We develop a simple optimization problem to achieve the following:

- Not exceed the water availability,
- Not exceed the energy availability,
- Meet the water demand,
- Meet the energy demand,
- Not exceed the CO2 emissions based on the predefined level of allowable emissions,
- Not exceed the costs based on the Budget we have,
- Not exceed the groundwater available resources (avoid over-exploitation of non-renewable water stocks),
- Not exceed the surface water available to use,
- Not exceed the energy production capacity from non-renewable sources,
- Not exceed the energy production capacity from renewable sources,

Variables Definition

Decision variables (unknowns, we want to find their optimum values).

- **GW**: amount of groundwater to use [m3] (>0)
- **SW**: amount of surface water to use [m3] (>0)
- **E**: amount of non-renewable energy to produce [kWh] (>0)
- **ER**: amount of renewable energy to produce [kWh] (>0)

Parameters (known values, we use them to estimate the decision variables). So, these are actually the symbols for the data we have in the tables above.

- **GWA**: Groundwater use allowance (Groundwater availability)
- **SWA**: Surface water use allowance (Surface water availability)
- **TWA**: Total Water Availability = GWA+SWA
- **EA**: Energy production capacity from non-renewable sources (Energy Availability)
- **ERA**: Energy production capacity from renewable sources (Energy Renewable Availability) =
- **TEA**: Total Energy Availability = EA+ERA
- **WD**: Water Demand (total)
- **ED**: Energy Demand (total)

- **GWC**: Cost for using groundwater (e.g. pumping, treatment, works)
- **SWC**: Cost for using surface water (e.g. storage and distribution works, treatment)
- **EC**: Cost for using non-renewable energy (e.g. works, production, distribution)
- **ERC**: Cost for using renewable energy (e.g. works, production, distribution)

- **GWE**: Groundwater emissions (from water treatment and distribution)
- **SWE**: Surface water emissions (from water treatment and distribution)
- **EE**: Non-renewable energy emissions (from production and distribution)
- **ERE**: Renewable energy emissions (from production and distribution)

- **GHG**: Emissions (maximum allowable CO_2 emissions – Green-House-Gases)
- **B**: Budget (maximum money available to serve urban & agricultural uses)

LINEAR PROBLEM FORMULATION

Objective Function (OF): Minimize total costs of using water and energy:

Zmin: **GWC*GW + SWC*SW + EC*E + ERC*ER**

Subject to - Constraints:

1. Water availability constraint: **GW + SW <= TWA** [m3]
2. Energy availability constraint: **E + ER <= TEA** [kWh]
3. Water demand constraint: **GW + SW = WD** [m3]
4. Energy demand constraint: **E + ER = ED** [kWh]
5. CO2 emissions constraint: **GWE*GW + SWE*SW + EE*E + ERE*ER <= GHG** [kg CO2]
6. Budget constraint: **GWC*GW + SWC*SW + EC*E + ERC*ER <= B** [$]
7. Groundwater capacity: **GW <= GWA** [m3]
8. Surface water capacity: **SW = SWA** [m3]
9. Non-renewable energy production capacity: **E <= EA** [kWh]
10. Renewable energy production capacity: **ER <= ERA** [kWh]

FUZZY OPTIMIZATION

Some of the parameters can be considered fuzzy. Fuzzy optimization allows us to deal with imprecise, vague, or uncertain parameters, which is common in real-world problems.

So, depending on natural phenomena, the following parameters can be fuzzy: GWA; SWA

WD and ED can also be uncertain, but usually engineers use upper-end values for safe design, so here we consider them as not fuzzy.

Practically, the fuzzy parameters can vary within a range of values (fuzzy sets), which we can define:

Fuzzy Parameter	Fuzzy Sets
GWA: Groundwater use allowance **[m3]**	Low: **1000** Medium: **2000** High: **4000**
SWA: Surface water use allowance **[m3]**	Low: **500** Medium: **1000** High: **3000**

The choice of the values and the size of the fuzzy sets depend on the problem and the available data. They must accurately represent the uncertainty and vagueness associated with the input variables.

This fuzzy optimization problem can be solved using fuzzy linear programming techniques, such as fuzzy simplex or fuzzy goal programming. The Decision Variables will be now also fuzzy sets.

So, the problem can be formulated as a fuzzy logic optimization problem. Fuzzy logic will express the equations affected by the fuzzy parameters:

The OF: **Zmin: GWC*GW + SWC*SW + EC*E + ERC*ER**

Subject to - Constraints:

1. Water availability constraint: **GW + SW <= TWA** [m3]
   where TWA is the total water availability, defined as the minimum of the fuzzy sets GWA and SWA. So, TWA is the minimum value of the alpha-cuts of GWA and SWA at the alpha-cut level of 0.5.
2. Energy availability constraint: **E + ER <= TEA** [kWh]
3. Water demand constraint: **GW + SW = WD** [m3]
4. Energy demand constraint: **E + ER = ED** [kWh]
5. CO2 emissions constraint: **GWE*GW + SWE*SW + EE*E + ERE*ER <= GHG** [kg CO2]
6. Budget constraint: **GWC*GW + SWC*SW + EC*E + ERC*ER <= B** [$]
7. Groundwater capacity: **GW <= GWA** [m3]
   where GWA is the fuzzy set representing the groundwater availability: namely, the upper (high) value of the GWA fuzzy set, which we cannot exceed.
8. Surface water capacity: **SW = SWA** [m3]
   where SWA is the fuzzy set representing the surface water availability: namely, upper (high) value of the SWA fuzzy set, which we cannot exceed.
9. Non-renewable energy production capacity: **E <= EA** [kWh]
10. Renewable energy production capacity: **ER <= ERA** [kWh]

DYNAMIC OPTIMIZATION

Several of the parameters used are not constant over time. They are changing every year, month, week, or even day. Let's see how to describe the problem, if we consider that the following parameters are time-dependent:

- **GWA**: Groundwater use allowance (Groundwater availability)
- **SWA**: Surface water use allowance (Surface water availability)
- **ERA**: Energy production capacity from renewable sources (Energy Renewable Availability)
- **WD**: Water Demand (total) (= 500 for urban & 1500 for agriculture)

We assume that **ED**: Energy Demand (total) stays stable around the year.

Time-dependent **Decision Variables**:

- GWt represent the amount of groundwater used at time t [m^3].
- SWt represent the amount of surface water used at time t [m^3].
- Et represent the amount of non-renewable energy produced at time t [kWh] (we assume this to be constant)
- ERt represent the amount of renewable energy produced at time t [kWh].

Objective Function: Minimize the total costs of using water and energy over a time horizon T (e.g. T = 12 months):

Zmin: $\sum_{t=1}^{T=12}(\mathbf{GWC} * \mathbf{GWt} + \mathbf{SWC} * \mathbf{SWt} + \mathbf{EC} * \mathbf{Et} + \mathbf{ERC} * \mathbf{ERt})$

Subject to - Constraints: $\forall$ t $\in${1,2,…,T=12}

1. Water availability constraint: **GWt + SWt <= TWAt** [m3]
2. Energy availability constraint: **Et + ERt <= TEAt** [kWh]
3. Water demand constraint: **GWt + SWt = WDt** [m3]
4. Energy demand constraint: **Et + ERt = EDt** [kWh]
5. CO2 emissions constraint: **GWE*GWt + SWE*SWt + EE*Et + ERE*ERt <= GHG** [kg CO2]
6. Budget constraint (ensures feasibility of the OF's solution and consistency with real-life budget constraints): **GWC*GWt + SWC*SWt + EC*Et + ERC*ERt <= B** [$]
7. Groundwater capacity: **GWt <= GWAt** [m3]
8. Surface water capacity: **SWt = SWAt** [m3]
9. Non-renewable energy production capacity: **Et <= EAt** [kWh]
10. Renewable energy production capacity: **ERt <= ERAt** [kWh] (we assume this to be constant)

For T = 12 the model will provide a set of 12 solutions.

MULTI-OBJECTIVE OPTIMIZATION – GOAL PROGRAMMING (GP)

Decision variables

- **GW**: amount of groundwater to use [m3] (>0)
- **SW**: amount of surface water to use [m3] (>0)
- **E**: amount of non-renewable energy to produce [kWh] (>0)
- **ER**: amount of renewable energy to produce [kWh] (>0)

Parameters

- **GWA**: Groundwater use allowance (Groundwater availability)
- **SWA**: Surface water use allowance (Surface water availability)
- **TWA**: Total Water Availability = GWA+SWA
- **EA**: Energy production capacity from non-renewable sources (Energy Availability)
- **ERA**: Energy production capacity from renewable sources (Energy Renewable Availability)
- **TEA**: Total Energy Availability = EA+ERA
- **WD**: Water Demand (total)
- **ED**: Energy Demand (total)
- **GWC**: Cost for using groundwater (e.g. pumping, treatment, works)
- **SWC**: Cost for using surface water (e.g. storage and distribution works, treatment)
- **EC**: Cost for using non-renewable energy (e.g. works, production, distribution)
- **ERC**: Cost for using renewable energy (e.g. works, production, distribution)
- **GWE**: Groundwater emissions (from water treatment and distribution)
- **SWE**: Surface water emissions (from water treatment and distribution)
- **EE**: Non-renewable energy emissions (from production and distribution)
- **ERE**: Renewable energy emissions (from production and distribution)
- **GHG**: Emissions (maximum allowable CO2 emissions – Green-House-Gases)
- **B**: Budget (maximum money available to serve urban & agricultural uses)

Constraints (Goal Programming Targets):

1. Minimize costs [$]: **GWC*GW + SWC*SW + EC*E + ERC*ER** $- \boldsymbol{d_c^+}$ **<= B**
2. Minimize CO2 emissions [kg CO2]: **GWE*GW + SWE*SW + EE*E + ERE*ER** $- d_{CO2}^+$ **<= GHG**
3. Minimize Groundwater use [m3]: **GW** $- \boldsymbol{d_{gw}^+}$ **<= GWA**
4. Reach Surface water availability [m3]: **SW** $- \boldsymbol{d_{sw}^+} + \boldsymbol{d_{sw}^-}$ **< = > SWA**
5. Minimize non-renewable energy production capacity [kWh]: **E** $- \boldsymbol{d_e^+}$ **<= EA**
6. Maximize renewable energy production capacity [kWh]: **ER** $+ \boldsymbol{d_{er}^-}$ **>= ERA**

Don't exceed water availability (TWA), energy availability (TEA), meet water demand (WD), meet energy demand (ED):

7. Water availability [m3]: **GW + SW** $- \boldsymbol{d_w^+}$ **<= TWA**
8. Energy availability [kWh]: **E + ER** $- \boldsymbol{d_{en}^+}$ **<= TEA**
9. Meet Water Demand [m3]: **GW + SW** $- \boldsymbol{d_{wd}^+} + \boldsymbol{d_{wd}^-}$ **< = > WD**

10. Meet Energy Demand [kWh]: **E + ER** $- \boldsymbol{d_{ed}^{+}} + \boldsymbol{d_{ed}^{-}}$ **< = > ED**

<u>Objective</u>: Min deviation from the predefined targets, so minimize the positive and negative deviation variables (d +, d-), which will be weighted depending their importance by weights w+ and w-, respectively.

w+ and w- are the weights or penalties the user defines for each deviation. For example, the degree that we do not want something to happen (e.g. penalty 1 if we exceed CO2 emissions, etc.)

$$\begin{aligned}\text{Min Z} = {} & w_c^{+} d_c^{+} + w_{CO2}^{+} d_{CO2}^{+} + w_{gw}^{+} d_{gw}^{+} + w_{sw}^{+} d_{sw}^{+} + w_{sw}^{-} d_{sw}^{-} + w_e^{+} d_e^{+} + w_{er}^{-} d_{er}^{-} + w_w^{+} d_w^{+} + w_{en}^{+} d_{en}^{+} \\ & + w_{wd}^{+} d_{wd}^{+} + w_{wd}^{-} d_{wd}^{-} + w_{ed}^{+} d_{ed}^{+} + w_{ed}^{-} d_{ed}^{-}\end{aligned}$$

Each d+, d- is a deviation we want to minimize, “as much as its importance” which is defined by the weight w. So, each term of the Obj.Function corresponds to each goal:

$\text{Min Z} = w_c^{+} d_c^{+}$ (goal 1. Minimize Costs)

$+w_{CO2}^{+} d_{CO2}^{+}$ (goal 2. Minimize Emissions)

$+w_{gw}^{+} d_{gw}^{+}$ (goal 3. Minimize Groundwater use)

$+w_{sw}^{+} d_{sw}^{+} + w_{sw}^{-} d_{sw}^{-}$ (goal 4. Reach surface water use)

$+ w_e^{+} d_e^{+}$ (goal 5. Minimize non-renewable energy use)

$+w_{er}^{-} d_{er}^{-}$ (goal 6. Maximize renewable energy use)

$+w_w^{+} d_w^{+}$ (goal 7. Don’t exceed water availability)

$+w_{en}^{+} d_{en}^{+}$ (goal 8. Don’t exceed energy availability)

$+w_{wd}^{+} d_{wd}^{+} + w_{wd}^{-} d_{wd}^{-}$ (goal 9. Meet Water Demand)

$+w_{ed}^{+} d_{ed}^{+} + w_{ed}^{-} d_{ed}^{-}$ (goal 10. Meet Energy Demand)

We assumed 3 scenarios of weights' preferences (custom scale), as follows:

Description	**w**	**"Intensive Economy"**	**"Middle solution"**	**"Environmentalist"**
Minimize Costs	w_c^+	0.9	0.5	0.2
Minimize Emissions	w_{CO2}^+	0.1	0.4	1
Minimize Groundwater use	w_{gw}^+	0.1	0.4	0.8
Reach surface water use	w_{sw}^+	0.1	0.5	0.7
Reach surface water use	w_{sw}^-	0.1	0.5	0.7
Minimize non-renewable energy use	w_e^+	0.3	0.6	0.8
Maximize renewable energy use	w_{er}^-	0.5	0.7	0.8
Don't exceed water availability	w_w^+	0.2	0.5	0.7
Don't exceed energy availability	w_{en}^+	0.2	0.3	0.5
Meet Water Demand	w_{wd}^+	0.8	0.7	0.5
Meet Water Demand	w_{wd}^-	0.8	0.7	0.5
Meet Energy Demand	w_{ed}^+	0.8	0.6	0.4
Meet Energy Demand	w_{ed}^-	0.8	0.6	0.4

NON-LINEAR PROGRAMMING (NLP)

The formulation of our initial linear problem assumes that the costs and CO_2 emissions for using water and energy sources are linear with respect to the amounts used. It also assumes that the energy production from non-renewable and renewable sources are directly proportional to the amount produced.

Here we will examine what happens if these assumptions do not hold. Then, the linear programming (LP) formulation becomes non-linear (NLP) and it needs to be adjusted accordingly.

Decision Variables:

- GW: amount of groundwater to use [m3] (GW > 0)
- SW: amount of surface water to use [m3] (SW > 0)
- E: amount of non-renewable energy to produce [kWh] (E > 0)
- ER: amount of renewable energy to produce [kWh] (ER > 0)

Parameters:

- GWA: Groundwater use allowance (Groundwater availability)
- SWA: Surface water use allowance (Surface water availability)
- TWA: Total Water Availability = GWA + SWA
- EA: Energy production capacity from non-renewable sources (Energy Availability)
- ERA: Energy production capacity from renewable sources (Energy Renewable Availability)
- TEA: Total Energy Availability = EA + ERA
- WD: Water Demand (total)
- ED: Energy Demand (total)
- GWC: Cost for using groundwater (e.g., pumping, treatment, works)
- SWC: Cost for using surface water (e.g., storage and distribution works, treatment)
- EC: Cost for using non-renewable energy (e.g., works, production, distribution)
- ERC: Cost for using renewable energy (e.g., works, production, distribution)
- GWE: Groundwater emissions (from water treatment and distribution)
- SWE: Surface water emissions (from water treatment and distribution)
- EE: Non-renewable energy emissions (from production and distribution)
- ERE: Renewable energy emissions (from production and distribution)

- GHG: Emissions (maximum allowable CO2 emissions – Green-House-Gases)
- B: Budget (maximum money available)

Objective Function: Minimize the total costs of using water and energy:

Zmin = GWC * GW + SWC * SW + EC * E + ERC * ER

Here,

GWC = f(GW),

SWC = f(SW),

EC = f(E),

ERC = f(ER)

are nonlinear functions that represent the cost as a function of the amount used. These functions should capture the nonlinear relationships between the costs and the decision variables. For example, you can use polynomial functions, exponential functions, or other nonlinear relationships to model the cost functions based on your data.

Constraints:

1. Water availability constraint: GW + SW <= TWA [m3]
2. Energy availability constraint: E + ER <= TEA [kWh]
3. Water demand constraint: GW + SW = WD [m3]
4. Energy demand constraint: E + ER = ED [kWh]
5. CO2 emissions constraint: GWE * GW + SWE * SW + EE * E + ERE * ER <= GHG [kg CO2]

Here,

GWE = g(GW),

SWE = g(SW),

EE = g(E),

ERE = g(ER)

are nonlinear functions that represent the emissions as a function of the amounts used.

6. Budget constraint: GWC * GW + SWC * SW + EC * E + ERC * ER <= B [$]
7. Groundwater capacity: GW <= GWA [m3]
8. Surface water capacity: SW = SWA [m3]

9. Non-renewable energy production capacity: E <= EA [kWh]
10. Renewable energy production capacity: ER <= ERA [kWh]

In this formulation, the nonlinearity comes from the functions f() and g().

We need to define these functions based on your data and the actual relationships between the costs and the amounts used.

The choice of specific nonlinear functions will depend on the nature of the cost data and the relationships observed.

For example, regression techniques or other data-driven methods will determine the appropriate nonlinear functions, which will be then incorporated into the NLP problem formulation.

Let's assume we have collected some data and performed linear regression to model the cost as a function of the respective decision variables. The linear regression equations might look like this:

1. Cost for using groundwater GWC = f(GW) = a1 * GW + b1
2. Cost for using surface water SWC = f(SW) = a2 * SW + b2
3. Cost for using non-renewable energy EC = f(E) = a3 * E + b3
4. Cost for using renewable energy ERC = f(ER) = a4 * ER + b4

Here, a1, a2, a3, and a4 are the coefficients obtained from linear regression, and b1, b2, b3, and b4 are the intercepts.

1. Emissions from groundwater GWE = g(GW) = c1 * GW + d1
2. Emissions from surface water SWE = g(SW) = c2 * SW + d2
3. Emissions from non-renewable energy EC = g(E) = c3 * E + d3
4. Emissions from renewable energy ERE = g(ER) = c4 * ER + d4

Here, c1, c2, c3, and c4 are the coefficients obtained from linear regression, and d1, d2, d3, and d4 are the intercepts.

These functions represent the cost of using groundwater, surface water, non-renewable energy, and renewable energy as nonlinear functions of their respective usage amounts; and same for their associated emissions.