A

Project Report

in

Control Engineering

for

Robotics, RA-602

offered by

Department of CICPS

Submitted to

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OBJECTIVE

In the dc-dc(buck) converter with L=10 mH, C=1 mF, R=100ohm, a 50 percent PWM duty cycle, and assuming the system's output is the voltage across the capacitor.

a) To write the converter's equation in the form of State Space model.

b) To Find the system's transfer function.

c) To Express the system's state equation in phase variable form.

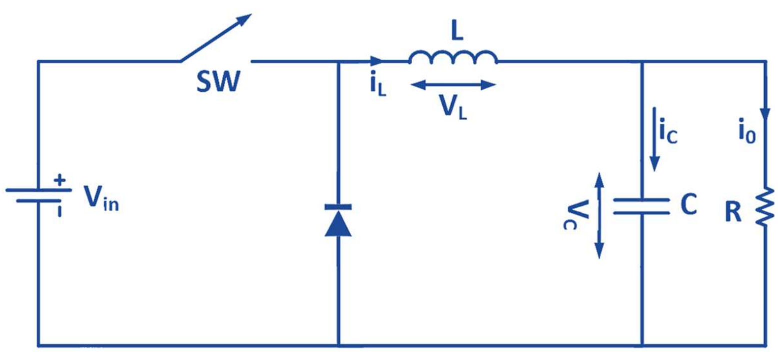
d) To Design a state feedback gain for the system with given closed loop poles s = -2+j2√ 3, s = −2 − j2 √ 3,

e) To Design an observer for the dc-dc converter with the given specification: overshoot = 40 percent, settling time = 0.3 sec.

f) To Simulate the system and observer for a unit step input using Simulink. Assuming that the initial conditions for the original system are x (0) = (2¦1). The observer should have initial conditions x' (0) = (0¦0).

➢ **SOFTWARE:** MATLAB R2022a

a) State Space model:



From the circuit of Buck converter,

When switch is ON:

$$V_L = V_{in} - V_c \Rightarrow L\frac{di_L}{dt} = V_{in} - V_c \Rightarrow \frac{di_L}{dt} = \frac{V_{in}}{L} - \frac{V_c}{L} \tag{1}$$

$$i_c = i_L - \frac{V_c}{R} \Rightarrow C\frac{dV_c}{dt} = i_L - \frac{V_c}{R} \Rightarrow \frac{dV_c}{dt} = \frac{i_L}{C} - \frac{V_c}{RC} \tag{2}$$

From equation 1 & 2

$$\begin{bmatrix} i_L' \\ V_c' \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{L} \\ \frac{1}{C} & \frac{-1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ V_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V_{in} \tag{3}$$

When switch is off:

$$V_L = -V_c \Rightarrow L\frac{di_L}{dt} = -V_c \Rightarrow \frac{di_L}{dt} = -\frac{V_c}{L} \tag{4}$$

$$i_c = i_L - \frac{V_c}{R} \Rightarrow C\frac{dV_c}{dt} = i_L - \frac{V_c}{R} \Rightarrow \frac{dV_c}{dt} = \frac{i_L}{C} - \frac{V_c}{RC} \tag{5}$$

Combining equation 4 & 5

$$\begin{bmatrix} i_L' \\ V_c' \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{L} \\ \frac{1}{C} & \frac{-1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ V_c \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} V_{in} \tag{6}$$

After deriving the buck converter state space, A and B matrix for its 'ON' and 'OFF' state. It is required to find its average A and B matrix with the account of switching duty cycle D.

So as,

$$A = D * A_{(ON)} + (1 - D) * A_{(OFF)} \tag{7}$$

$$A = D * \begin{bmatrix} 0 & \frac{-1}{L} \\ \frac{1}{C} & \frac{-1}{RC} \end{bmatrix} + (1 - D) * \begin{bmatrix} 0 & \frac{-1}{L} \\ \frac{1}{C} & \frac{-1}{RC} \end{bmatrix} \Rightarrow A = \begin{bmatrix} 0 & \frac{-1}{L} \\ \frac{1}{C} & \frac{-1}{RC} \end{bmatrix} \tag{8}$$

And

$$B = D * B_{(ON)} + (1 - D) * B_{(OFF)} \tag{9}$$

$$B = \mathrm{D} * \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} + (1 - D) * \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow B = \begin{bmatrix} \frac{D}{L} \\ 0 \end{bmatrix} \tag{10}$$

Finally, we have:

$$\begin{bmatrix} i_L' \\ V_c' \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{L} \\ \frac{1}{C} & \frac{-1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ V_c \end{bmatrix} + \begin{bmatrix} \frac{D}{L} \\ 0 \end{bmatrix} V_{in} \tag{11}$$

To obtain the output of system as the voltage across capacitor (V_c)

$$Y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i_L \\ V_c \end{bmatrix} \tag{12}$$

Let's assume state variable as

$i_L = x_1$, $V_c = x_2$ and output of system =Y

The Buck converter state space equation will be given as:

$$\begin{bmatrix} x_1{}' \\ x_2{}' \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{L} \\ \frac{1}{C} & \frac{-1}{RC} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{D}{L} \\ 0 \end{bmatrix} V_{in} \tag{13}$$

$$Y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{14}$$

For the above system i.e. (13) & (14) using the given values as

L=10mH, c=1mF, R=100-ohm, D=0.5,

state space equation can be obtained as:

$$\begin{bmatrix} x_1{}' \\ x_2{}' \end{bmatrix} = \begin{bmatrix} 0 & -100 \\ 1000 & -10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 50 \\ 0 \end{bmatrix} V_{in} \tag{15}$$

$$Y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{16}$$

b) Transfer Function:

Now the Transfer function of this system is can be obtained using:

$$\frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D \tag{17}$$

Where $\quad A = \begin{bmatrix} 0 & -100 \\ 1000 & -10 \end{bmatrix}, \quad B = \begin{bmatrix} 50 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad \& \quad D = 0$

So now,

$$(SI - A)^{-1} = \left[s * \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & -100 \\ 1000 & -10 \end{pmatrix} \right]^{-1} = \begin{bmatrix} s & 100 \\ -1000 & s+10 \end{bmatrix}^{-1}$$

$$= \frac{1}{s(s+10) + 10^5} \begin{bmatrix} s+10 & -100 \\ 1000 & s \end{bmatrix}$$

And $C(sI - A)^{-1}B = \begin{bmatrix} 0 & 1 \end{bmatrix} * \frac{1}{s(s+10)+10^5} \begin{bmatrix} s+10 & -100 \\ 1000 & s \end{bmatrix} * \begin{bmatrix} 50 \\ 0 \end{bmatrix}$

$$= \frac{1}{s(s+10) + 10^5} * \begin{bmatrix} 0 & 1 \end{bmatrix} * \begin{bmatrix} 50(s+10) \\ 50000 \end{bmatrix}$$

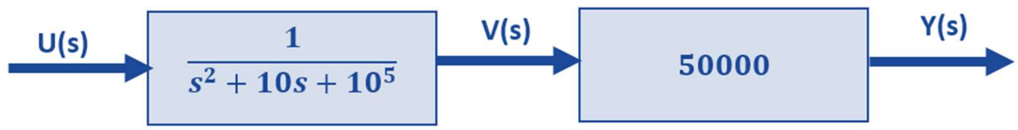
$$= \frac{50000}{s(s+10)+10^5}$$

So, the Transfer function of the system is

$$\boldsymbol{T(s) = \frac{50000}{s(s+10)+10^5}} \tag{18}$$

c) Phase Variable Canonical Form:

$$T(s) = \frac{Y(s)}{U(s)} = \frac{50000}{s^2+10s+10^5} \Rightarrow \frac{V(s)}{U(s)} * \frac{Y(s)}{V(s)} = \frac{1}{s^2+10s+10^5} * 50000$$



$$\frac{V(s)}{U(s)} = \frac{1}{s^2+10s+10^5} \tag{19}$$

$$(s^2+10s+10^5)V(s) = U(s) \Rightarrow s^2V(s) + 10sV(s) + 10^5V(s) = U(s)$$

$$\Rightarrow \ddot{V}(t) + 10sV(\dot{t}) + 10^5V(t) = U(s)$$

Let $x_1 = V(t), x_2 = V(\dot{t}),\ \ \dot{x}_2 = V(\ddot{t})$, now we'll get:

$$\boldsymbol{\dot{x}_1 = x_2} \tag{20}$$

$$\boldsymbol{\dot{x}_2 = -10^5x_1 - 10x_2 + U(s)} \tag{21}$$

$$\frac{Y(s)}{V(s)} = 50000 \Rightarrow Y(s) = 50000V(s) \Rightarrow Y(t) = 50000V(t)$$

$$\boldsymbol{\Rightarrow Y(t) = 50000x_1} \tag{22}$$

Using equation 20, 21 & 22 we can write the phase variable form representation as:

$$\boldsymbol{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -10^5 & -10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)} \tag{23}$$

$$\boldsymbol{\mathrm{Y(t)} = [50000\ \ 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} \tag{24}$$

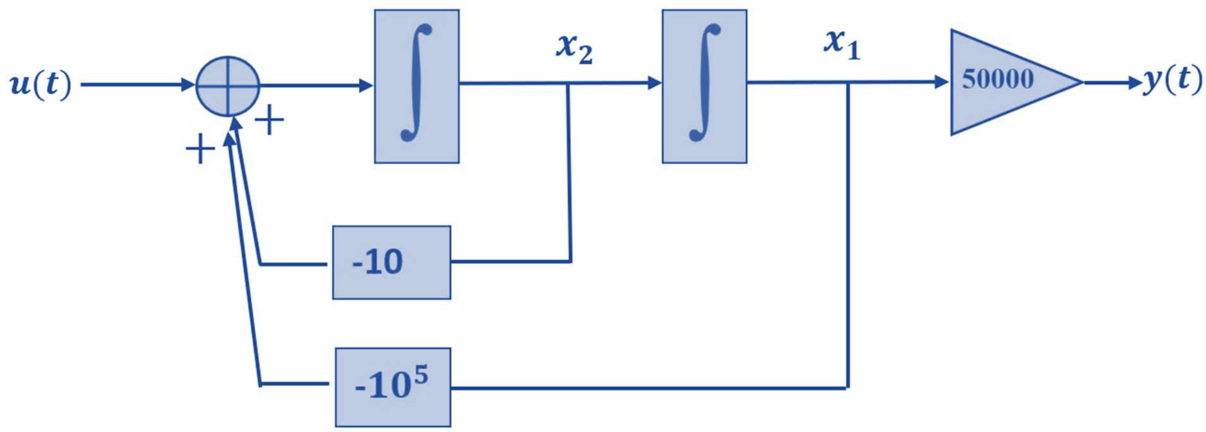


Fig:2 Block diagram representation of Phase variable form

d) Design of state feedback Gain:

➢ MATLAB CODE:

```
clc;
clear all;
close all;
% Given state model
A = [0 -100; 1000 -10]; B = [50; 0]; C = [0 1]; D = 0;
%% Close loop pole to design state feedback model
J = [-2+1i*3.464 -2-1i*3.464];
% Calculating State feedback gain
K = acker (A, B, J)
```

➢ OUTPUT:

```
K =

   -0.1200   -1.9985
```

e) Observer Design:

For the given system we have,

- Overshoot, $\mathbf{M_p = 40\ \%}$ &
- settling time $\mathbf{t_s = 0.3}$ **sec.**

So now,

$$t_s = \frac{4}{\zeta\omega_n} = 0.3 \Rightarrow \zeta\omega_n = 13.33$$

$$M_p = e^{\{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\}} = 0.4 \Rightarrow \zeta = 0.28087$$

$$\omega_n = \frac{13.33}{0.28087} = 47.4596$$

Now using the standard characteristic equation for second order system as

$$s^2 + 2\zeta\omega_n s + {\omega_n}^2 = 0$$

$$\Rightarrow s^2 + 26.6599s + 2252.4136 = 0$$

$$\Rightarrow \boldsymbol{s = -13.32995 \pm j45.5491} \quad \mathbf{(25)}$$

From (25) we have got the closed loop poles as:

$$(\boldsymbol{-13.32995 + j45.5491}) \text{ \& } (\boldsymbol{-13.32995 - j45.5491})$$

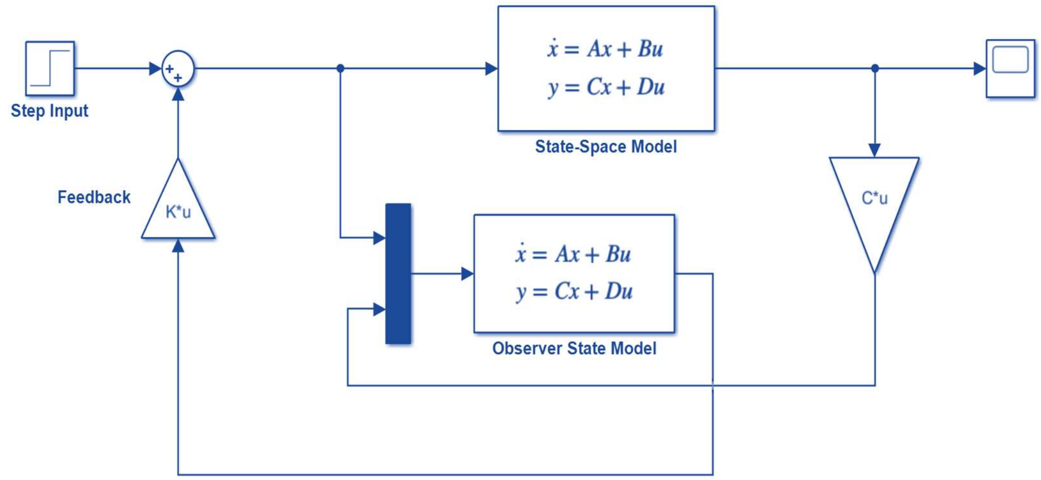
Now with these closed loops pole we will find the observer gain **L** using **acker ()** command.

f) Simulation of Observer:

➢ MATLAB CODE:

```
clc;
clear all;
close all;
% Given state model
A = [0 -100; 1000 -10]; B = [50; 0]; C = [0 1]; D = 0;
%% Close loop pole to design state feedback model
J = [-2+1i*3.464 -2-1i*3.464]
% Calculating State feedback gain
K = acker (A, B, J)
%% Calculated close loop pole to design observer
P = [-13.3298+1i*45.5489 -13.3298-1i*45.5489];
% calculating observer gain matrix
l = acker (A', C', P);
L = transpose(l)
%% calling Simulink file from .m file
sim('Observer.slx')
% ploting
figure (1)
step (ss (A-L*C, B, C, D))
```

- **Simulink file used in MATLAB code:** `'Observer.slx'`



- **OUTPUT**

```
L =
      -97.7476
       16.6596
```

- **Output Response:**

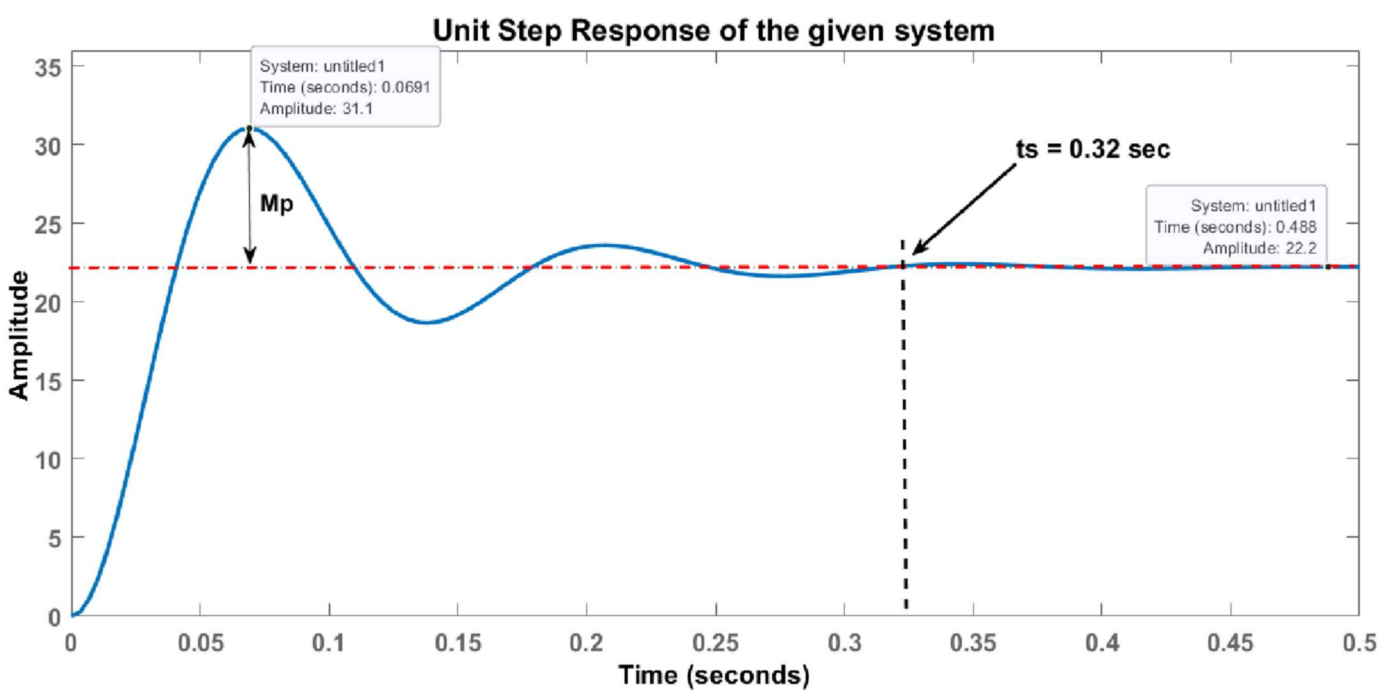


Fig1: step response

➢ System states:

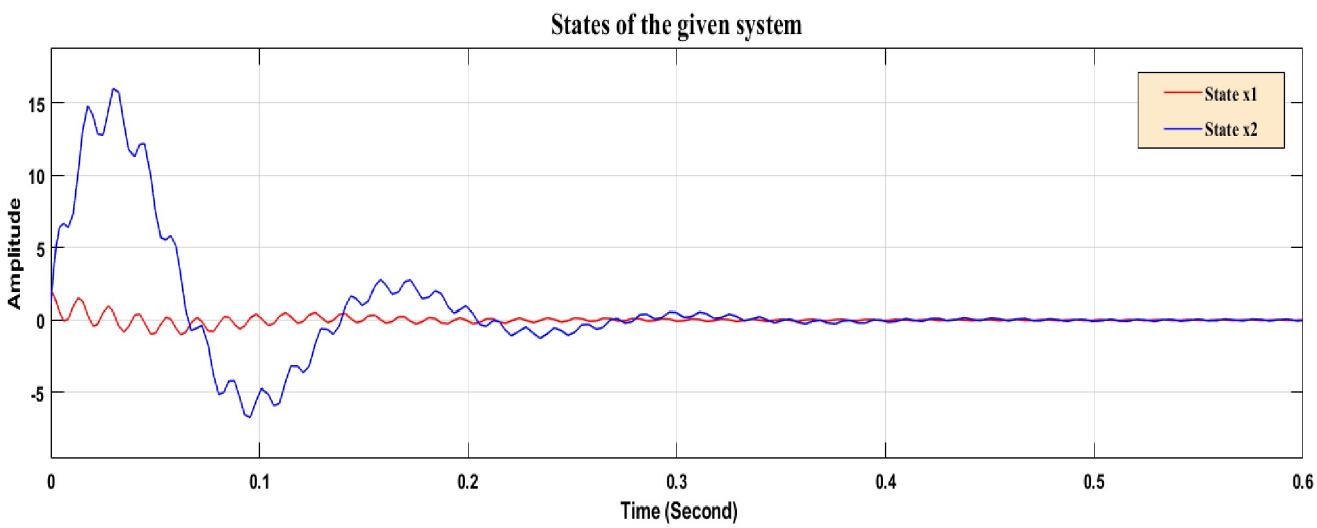


Fig2: System states

➢ CALCULATIONS:

☞ Peak Overshoot:

From the obtained output response, we found that the peak value of the response is:

$$C(peak) = 31.1$$

And the steady state value is

$$C(\infty) = 22.2$$

So now we can calculate the peak overshoot as

$$M_p = \frac{C(peak) - C(\infty)}{C(\infty)} * 100\%$$

$$= \mathbf{40.09}\%$$

☞ Settling time:

As we can see from the step response, the settling times comes out to be

$$\boldsymbol{t_s = 0.32s}$$

➢ CONCLUSION:

We have simulated the system & observer with given specifications and we obtained the step response which has the similar overshoot (40.09%) and settling time (0.32s) as it was given 40% & 0.3s respectively.