Cheat Sheet, Financial Intermediation

Depository Institutions

- A standard measure of profitability is income over total assets, or $ROA = \frac{\text{net profit after taxes}}{\text{total assets}}$
- The bank' return on equity (ROE) is $ROE = \frac{\text{net profit after taxes}}{\text{total equity capital}}$
- Between ROA and ROE stands leverage: $ROE = ROA \times EM, EM = \frac{\text{total assets}}{\text{total equity capital}}$
- We can break down ROA into:

$$ROA = \frac{\text{Net income}}{\text{Total operating income}} \times \frac{\text{Total operating income}}{\text{Total assets}} = PM \times AU$$

- We can break down PM into:

$$\text{Interest expense Ratio} = \frac{\text{Interest expense}}{\text{Total operating income}}$$

$$\text{Provision for loss ratio} = \frac{\text{Provision for loan losses}}{\text{Total operating income}}$$

$$\text{Non-interest expense Ratio} = \frac{\text{Non-interest expense}}{\text{Total operating income}}$$

$$\text{Tax Ratio} = \frac{\text{Income taxes}}{\text{Total operating income}}$$

- Net interest margin measures the net return on earning assets (inv. securities, loans, leases):

$$NIM = \frac{\text{interest income - interest expenses}}{\text{earning assets}}$$

- The Spread measures the differential in yield of A and cost of L:

$$Spread = \frac{\text{interest income}}{\text{earning assets}} - \frac{\text{interest expense}}{\text{interest-bearing liabilities}}$$

Financial Services: Mutual Funds and Hedge Funds

- If a fund is kept for T years,

$$total\,cost = \left(\text{Total expense ratio} + \frac{\text{Front-end load}}{T} + \frac{\text{Back-end load}}{T}\right)$$

where Total expense ratio $= \frac{\text{annual fees charged per share}}{\text{share value}}$

Interest rate risk

- For a given bucket i and a change of rates ΔR_i,

$$\Delta NII_i = (RSA_i - RSL_i) \times \Delta R_i = GAP_i \times \Delta R_i$$

- Gap scaled as a percentage of assets is $\frac{CGAP}{A}$
- In case of unequal changes in interest rates to A&L

$$\Delta NII = (RSA \times \Delta R_{RSA}) - (RSL \times \Delta R_{RSL})$$

- For a security that pays annual coupons CF_t at times $t = 1, ..., N$,

$$D = \frac{\sum_{t=1}^{N} \frac{CF_t}{(1+R)^t} \times t}{\sum_{t=1}^{N} \frac{CF_t}{(1+R)^t}} = \frac{\sum_{t=1}^{N} \frac{CF_t}{(1+R)^t} \times t}{P} = \sum_{t=1}^{N} \frac{CF_t \times (1+R)^{-t}}{P} \times t = \sum_{t=1}^{N} \frac{PV_t}{P} \times t$$

- If coupons are paid semi-annually at $t = 0.5, 1, 1.5, ..., N$, then

$$D = \frac{\sum_{t=1/2}^{N} \frac{CF_t}{(1+R/2)^{2t}} \times t}{\sum_{t=1/2}^{N} \frac{CF_t}{(1+R/2)^{2t}}}$$

- Approximately (i.e. for small ΔR), we have that

$$\frac{\Delta P}{P} = -D \left[\frac{\Delta R}{1+R} \right]$$

- Define $MD \equiv D/(1+R)$, the modified duration:

$$\frac{\Delta P}{P} = -MD \times \Delta R \Longrightarrow \Delta P = - \underbrace{MD \times P}_{\text{Dollar duration}} \times \Delta R$$

- Call X_{iA} the proportion of asset i, $i = 1, ..., n$, in the asset portfolio at market values. If asset i has duration D_1^A, the duration of A is simply the assets' weighted avg duration:

$$D_A = X_{1A} D_1^A + X_{2A} D_2^A + ... + X_{nA} D_n^A$$

- Call X_{iL} the proportion of liability i, $i = 1, ..., m$, in the liability portfolio (excl. E) at market values. If liability i has duration D_1^L, the duration of L is simply the liabilities' weighted avg duration:

$$D_L = X_{1L} D_1^L + X_{2L} D_2^L + ... + X_{mL} D_m^L$$

- $\Delta A = \Delta L + \Delta E \;\; \Rightarrow \Delta E = \Delta A - \Delta L$
- Rearranging terms, if $k = L/A$ we obtain

$$\Delta E = -[D_A - D_L k] \times A \times \frac{\Delta R}{1+R}$$

Liquidity risk

- Liquidity index is defined as:

$$I = \sum_{i=1}^{N} w_i (P_i / P_i^*) \qquad \sum_{i=1}^{N} w_i = 1$$

  - Weighted sum of "fire sale price" P_i to "fair market price" P_i^*
  - weights w_i are the percent of the portfolio value formed by the individual assets

- Liquidity Coverage Ratio (LCR) is defined as:

$$LCR = \frac{\textit{Stock of high-quality liquid assets}}{\textit{Total net cash outflows over the next 30 calendar days}}$$

- Stock of high-quality liquid assets (LCR numerator):
  - A minimum 15 percent "*haircut*" to each level 2 asset
  - Level 2B no more than 15% of a bank's stock of high-quality liquid assets
  - Level 2 in aggregate no more than 40% of a bank's stock of high-quality liquid assets. Level 1 amount has no cap.

- Total net cash outflows over the next 30 calendar days (LCR denominator):

$$\textit{Total net cash outflows} = \textit{Outflows} - \textit{Min(inflows; 75\% of outflows)}$$

- Net Stable Funding Ratio (NSFR) is defined as:

$$NSFR = \frac{\textit{Avaliable amount of stable funding}}{\textit{Required amount of stable funding}}$$

note: This part not relevant for Financial Intermediation I (stream "Digital Finance')

`Approach based on balance sheet data (Credit Scoring)`

- The optimal vector of coefficients is:

$$\widehat{\gamma} = \Sigma^{-1}(\mu_H - \mu_U)$$

- A cutoff point or **score** α can then be the mid-point of the groups averages:

$$\alpha = \frac{\overline{Z}_H + \overline{Z}_U}{2}$$

- If the indicator variables are jointly normally distributed, then the posterior probability that a firm will default is

$$\pi_i = prob(U|x_i) = \frac{1}{1 + \frac{1-\pi_U}{\pi_U} e^{Z_i - \alpha}}$$

- The bank may choose to refuse the loan when:

$$C_I \cdot \pi_i > C_{II} \cdot (1 - \pi_i) \ \rightarrow \ Z_i < \alpha + \ln \frac{\pi_U C_I}{(1 - \pi_U) C_{II}} \equiv \alpha^*$$

Approach based on option theory: the Merton (1974) model and its extension (KMV)

- The value of the firm $V_t = E_t + B_t$ follows a continuous time stochastic process (GBM)

$$dV_t = \mu_v V_t dt + \sigma_v V_t dW_t$$

- The value of Equity at t is then the B&S price of this call option:

$$E_t = call_t(V_t, D, r, \sigma_v, T) = V_t N(d_1) - De^{-r(T-t)} N(d_2)$$

where

$$d_1 = \frac{\ln(V_t/D) + \left(r + \frac{\sigma_v^2}{2}\right)(T-t)}{\sigma_v \sqrt{T-t}} \qquad d_2 = d_1 - \sigma_v \sqrt{T-t}$$

- It can be proved that the volatility of Equity returns σ_E equals:

$$\sigma_E = \frac{V_t}{E_t} N(d_1) \sigma_v$$

- The implied risk-adjusted probability of default between time t and T is:

$$Pr[V_T < D] = N\left(-\frac{\ln(V_t/D) + (r - \frac{1}{2}\sigma_v^2)(T-t)}{\sigma_v \sqrt{T-t}}\right) = N(-d_2)$$

- The credit spread in the Merton model is:

$$cs(T) = y(T) - r = -\frac{1}{T} \ln\left[\frac{1}{L} N(-d_1) + N(d_2)\right]$$

where $L \equiv \frac{De^{-rT}}{V_0}$ represents a quasi-leverage ratio

- KMV approach is based on the "distance to default" DD:

$$DD = \frac{V_0 - D}{\sigma_v V_0}$$

where the default point D is calculated over an interval of **1 year** as:

$$D = 0.5 \times \text{ long-term debt} + \text{short term debt}$$

Approach based on credit spreads

- Q_t: the cumulative default probability of defaulting by year t
- V_t: the probability of surviving up to year t is simply $V_t = 1 - Q_t$
- UDP_t: the probability of defaulting between $t-1$ and t based on the information available at 0 is $UDP_t = Q_t - Q_{t-1}$
- CDP_t: the probability of defaulting in year t conditional of not having defaulted by year $t-1$ is $CDP_t = \frac{Q_t - Q_{t-1}}{1 - Q_{t-1}} = \frac{UDP_t}{V_{t-1}}$
- If we assume a given recovery rate RR expected on the corporate bond in the case of a default event, we can extract from observed prices and spreads $s(t)$ the risk-neutral probability of default by date t, $Q(t)$. In equilibrium,

$$Q(t) = \frac{1 - e^{-s(t) \cdot t}}{1 - RR}$$