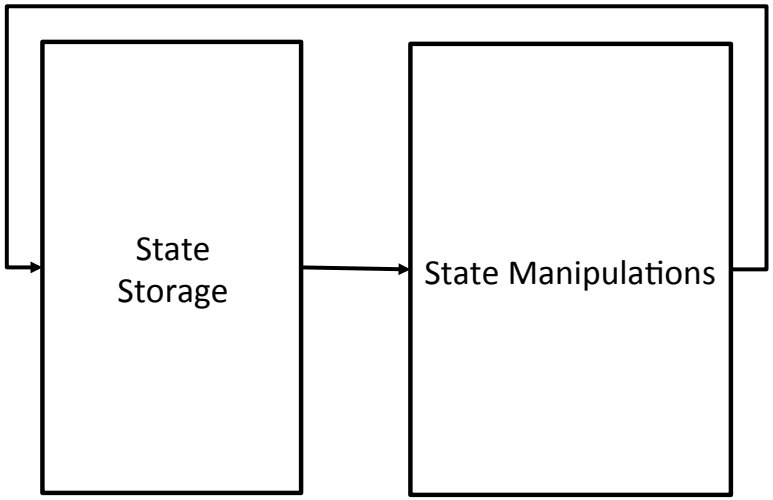
Building an ALU (Part 1):

An Arithmetic Logic Unit (ALU) is the primary manipulator of state information in computers

Computer can do 2 things

1) Store state
2) Manipulate state (Combine arithmetic and logical operations into one unit)



233 in one slide!

Today we will introduce how we use control bits to manipulate data

- The class consists roughly of 4 quarters: (Bolded words are the big ideas of the course, pay attention when you hear these words)
  1. You will build a simple computer processor
     Build and create **state** machines with **data**, **control**, and **indirection**
  2. You will learn how high-level language code executes on a processor
     Time limitations create **dependencies** in the **state** of the processor
  3. You will learn why computers perform the way they do
     Physical limitations require **locality** and **indirection** in how we access **state**
  4. You will learn about hardware mechanisms for parallelism
     Locality, dependencies, and **indirection** on performance enhancing drugs

- We will have a SPIMbot contest!

Today's lecture

- We start building our computer!
  - We'll start with the arithmetic/logic unit (ALU)

- Adding single bits
  - Half Adders and Full Adder
- Multi-bit Arithmetic
  - Hierarchical design
  - Subtraction
- Building a Logic Unit
  - Multiplexors

The computation in a computer processor takes place in the arithmetic logic unit (ALU)

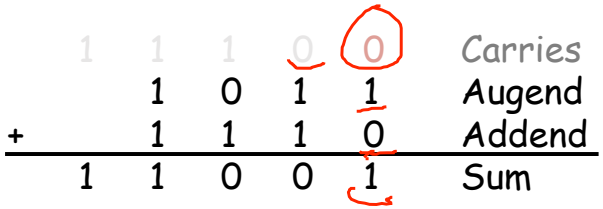
- Arithmetic Unit (AU) performs arithmetic operations
  - e.g., addition and subtraction
- Logic Unit (LU) performs bit-wise logical operations
  - e.g., AND, OR, NOR, XOR

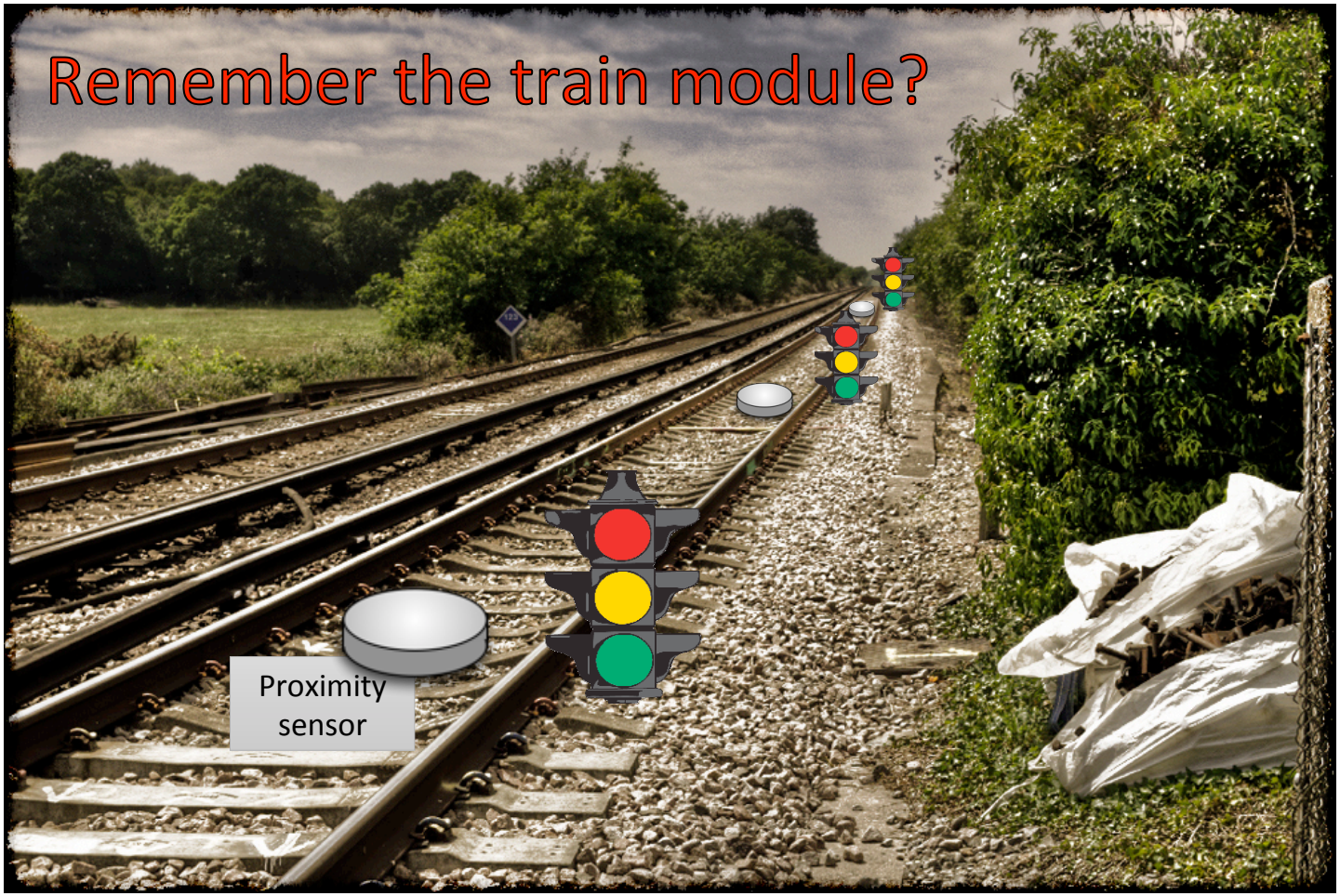
Put 'em together and what do you get?

- Typically these operations are performed on multi-bit words
  - The MIPS-subset processor we will build uses 32-bit words

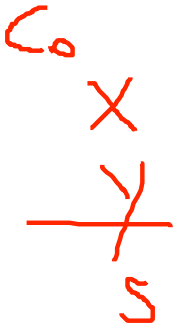
In Lab 3 you will build a 32-bit ALU with the above operations

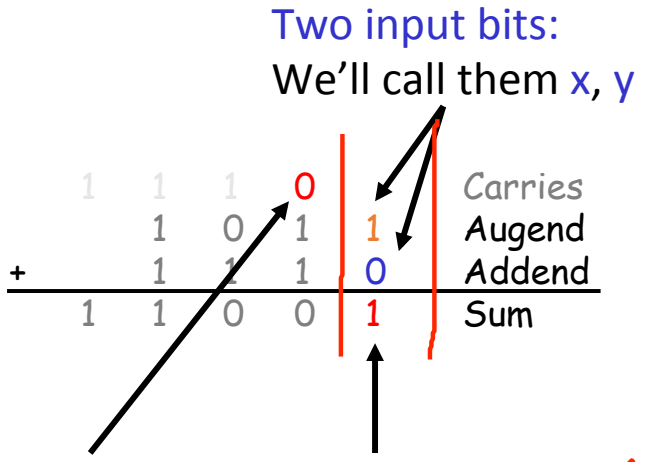
Binary Addition Review




Remember the train module?
Proximity sensor

First bit position receives two input bits to produce two output bits



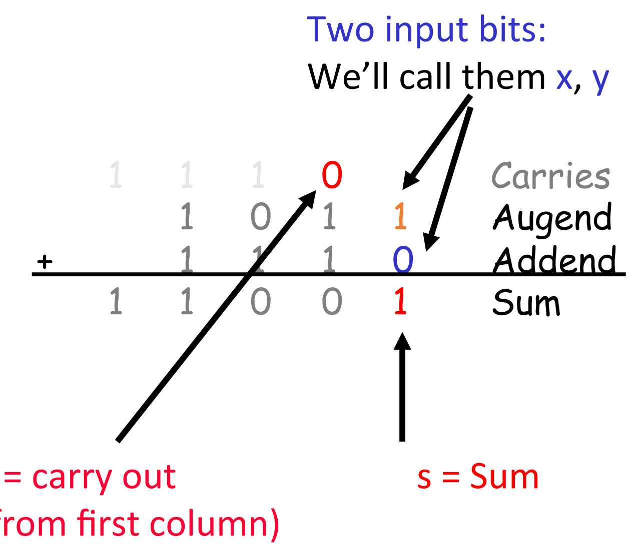


Two output bits: c = carry out (from first column)

s = Sum = 1

Specify the first bit position's behavior with a truth table

X	Y	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	+ 1	1	0



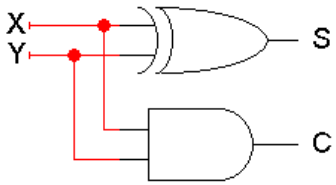
This truth table specifies a circuit we call a half adder

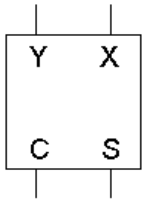
- Adds two input bits to produce a sum and carry out.

X	Y	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

$x'y$

xy'





$C = XY$

$S = X'Y + XY'$

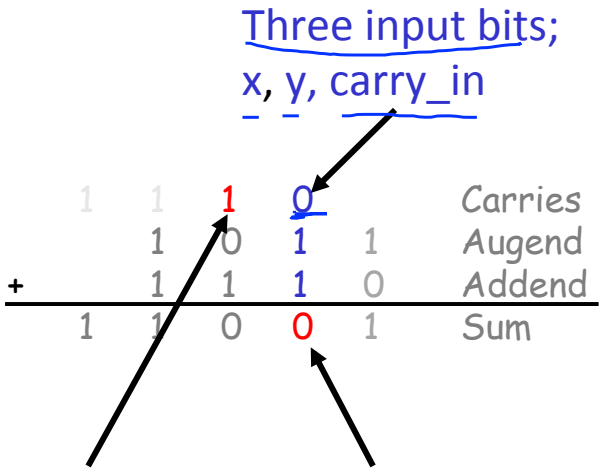
$= X \oplus Y$

XOR

- The carry-out bit has twice the magnitude of the sum bit

Second bit position receives three input bits to produce two output bits

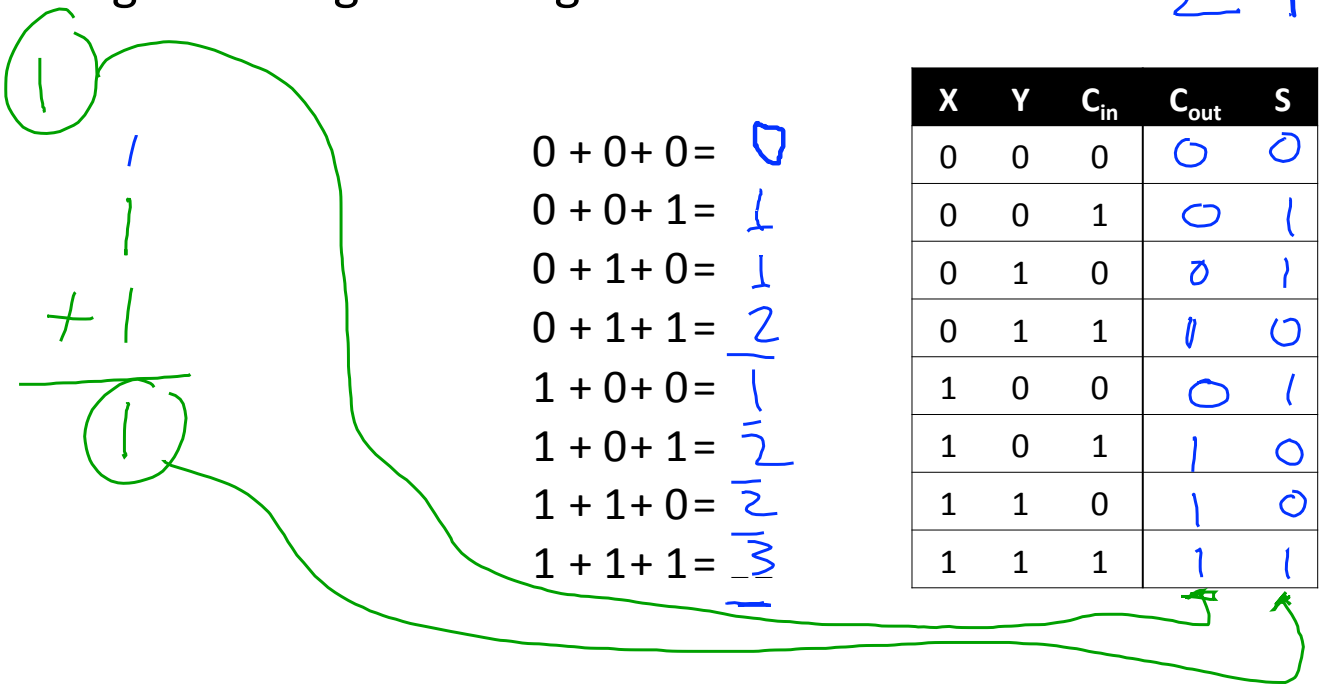
- (and every subsequent position)



Still two output bits: c = carry out (from first column) s = Sum

Specify the remaining bit positions' behaviors with a truth table

- Adding 3 bits together to get a two bit number



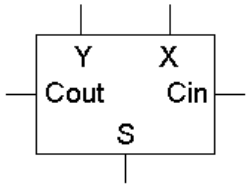
0 + 0+ 0= 0
0 + 0+ 1= 1
0 + 1+ 0= 1
0 + 1+ 1= 2
1 + 0+ 0= 1
1 + 0+ 1= 2
1 + 1+ 0= 2
1 + 1+ 1= 3

X	Y	C_{in}	C_{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

This truth table specifies a circuit we call a Full Adder

- Adds three input bits to produce a sum and carry out.

$$S = X \oplus Y \oplus C_{in}$$
$$C_{out} = XY + (X \oplus Y)C_{in}$$



X	Y	C_{in}	C_{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

We can use hierarchical design to build a full adder from two half adders

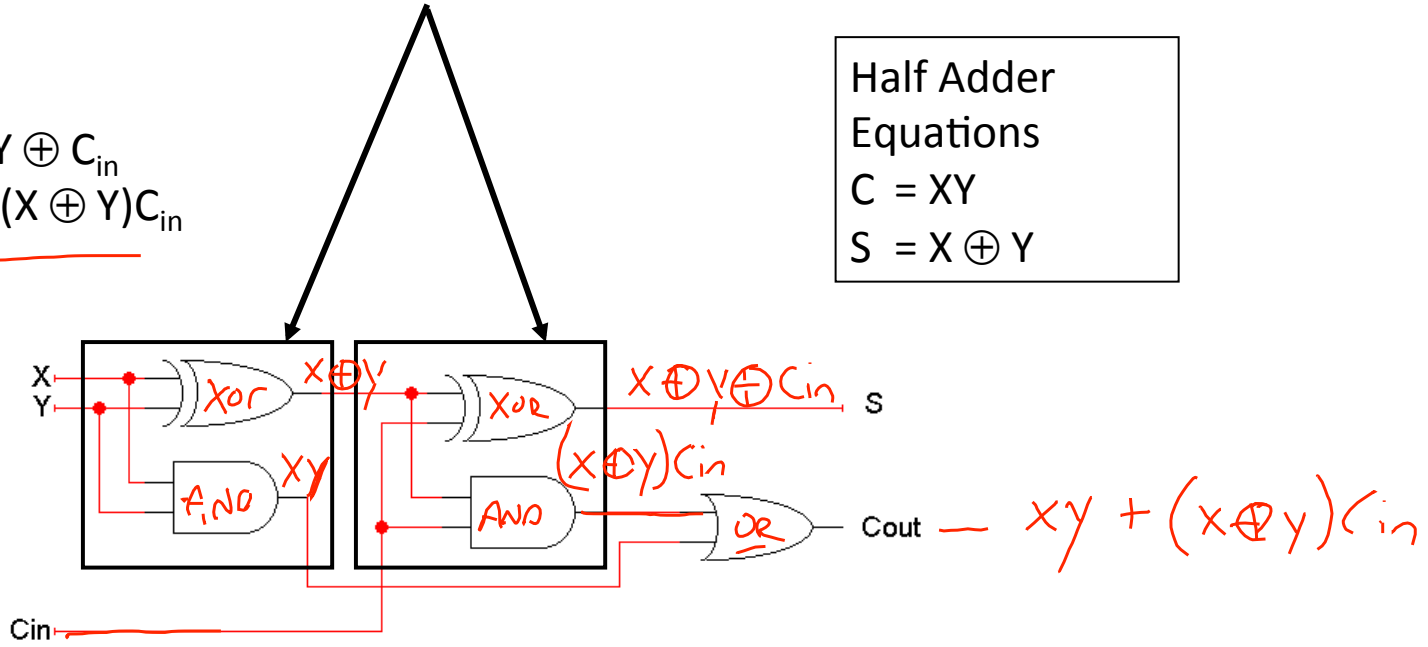
$S = X \oplus Y \oplus C_{in}$

$C_{out} = XY + (X \oplus Y)C_{in}$

Half Adder Equations

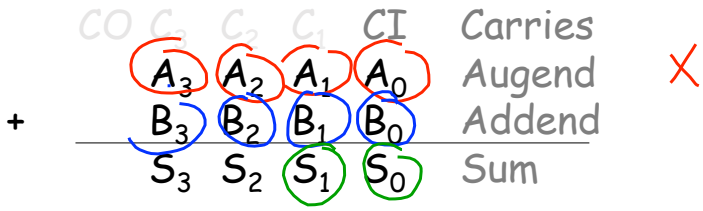
$C = XY$

$S = X \oplus Y$

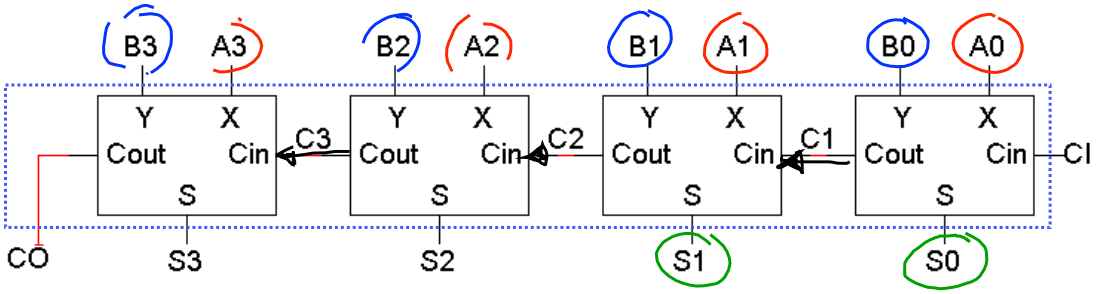


Use hierarchical design to build multi-bit adders

- Recall our discussion about hierarchical design
  - *(The stop lights to prevent train collisions…)*

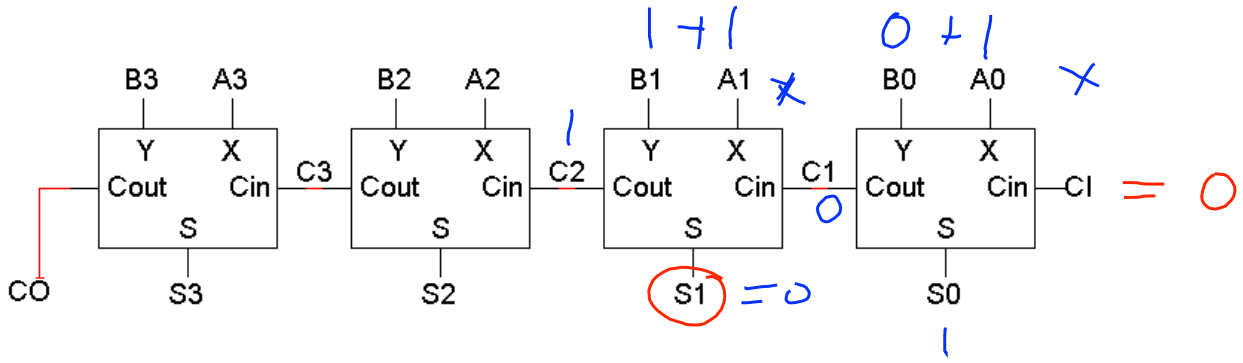


- Example: 4-bit adder



An example of 4-bit addition

- Let's try our initial example: A=1011 (eleven), B=1110 (fourteen).

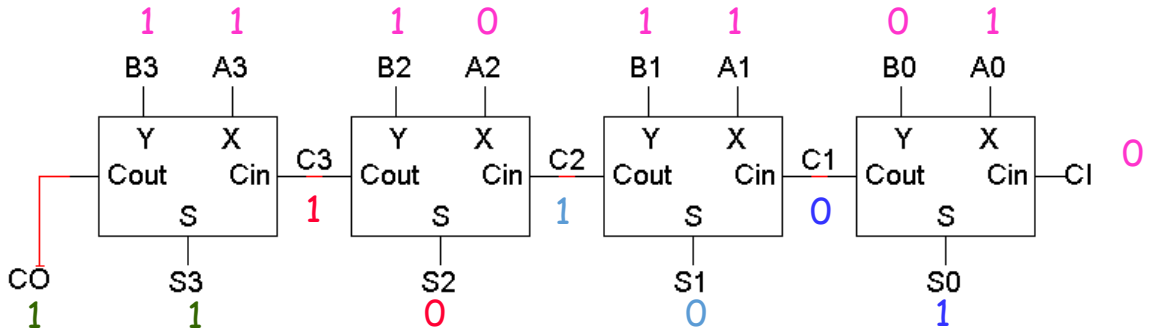


What is the value of S1?

a) 0
b) 1

An example of 4-bit addition

- Let's try our initial example: A=1011 (eleven), B=1110 (fourteen).



1. Fill in all the inputs, including CI=0
2. The circuit produces C1 and S0 (1 + 0 + 0 = 01)
3. Use C1 to find C2 and S1 (1 + 1 + 0 = 10)
4. Use C2 to compute C3 and S2 (0 + 1 + 1 = 10)
5. Use C3 to compute CO and S3 (1 + 1 + 1 = 11)

Woohoo! The final answer is 11001 (twenty-five) *if we consider it a 5-bit output*.

Implementing Subtraction

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- Subtraction is technically negating the second input and then adding

$$A - B = A + (-B)$$

- Negating in 2's complement is inverting the bits and adding one

$$-B = \sim B + 1$$

- Substituting in:

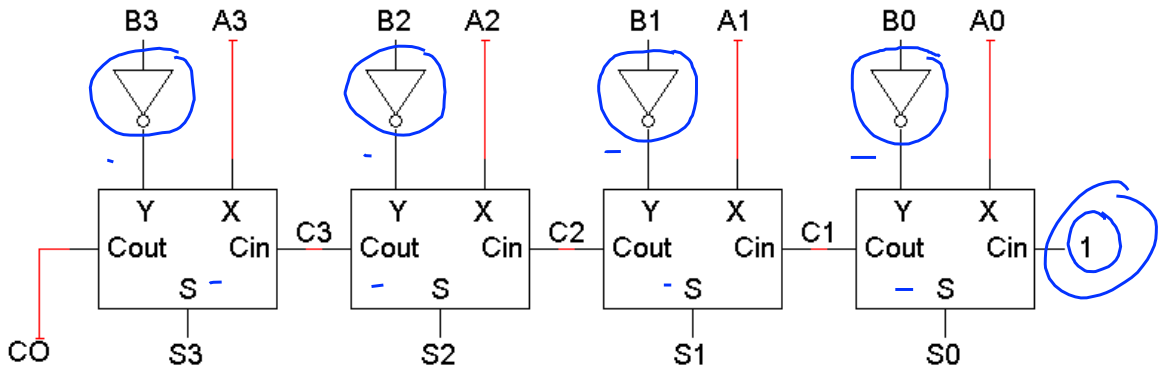
$A - B = A + (-B) =$

A:	**A – ~B + 1**
B:	**A + ~B + 1**
C:	**A – (~B + 1)**
D:	**A + ~B – 1**
E:	**none of the above**

Implementing Subtraction, cont.

- Let's try an example: A=0011 (three), B=1110 (negative 2).

$A + \sim B + 1$



What is the value of S3?

a) 0
b) 1

Use XOR gates to implement Addition + Subtraction in one circuit

- XOR gates let us selectively complement the B input.

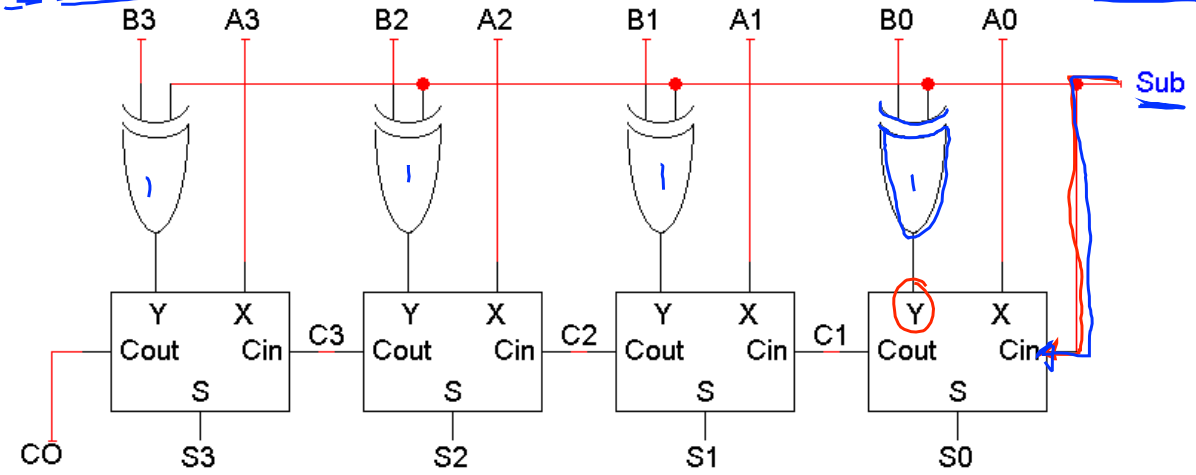
Control bit

$X \oplus 0 = X$ $\quad$ $X \oplus 1 = X'$

Data bit

- When Sub = 0, Y = B and Cin = 0. Result = A + B + 0 = A + B.
- When Sub = 1, Y = ~B and Cin = 1. Result = A + ~B + 1 = A – B.

Addition
Subtraction



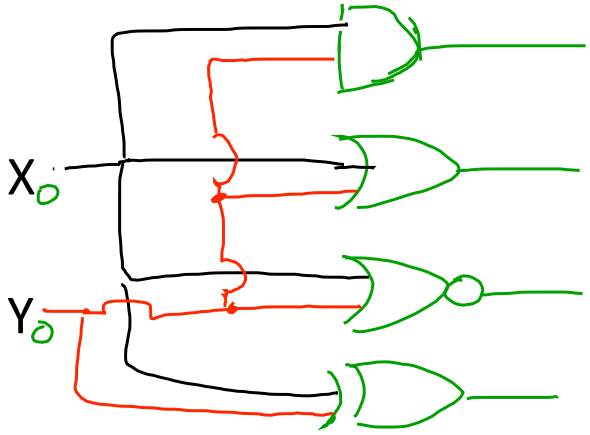
We conceptually distinguish two types of signal in hardware: Data and Control

- Datapath
  - *These generally carry the numbers we're crunching*
  - *E.g., the X and Y inputs and the output S*

- Control
  - *These generally control how data flows and what operations are performed*
  - *E.g., the SUB signal.*

Logical Operations

- In addition to ADD and SUBTRACT, we want our ALU to perform bit-wise AND, OR, NOR, and XOR.
- This should be straight forward.
  - We have gates that perform each of these operations.

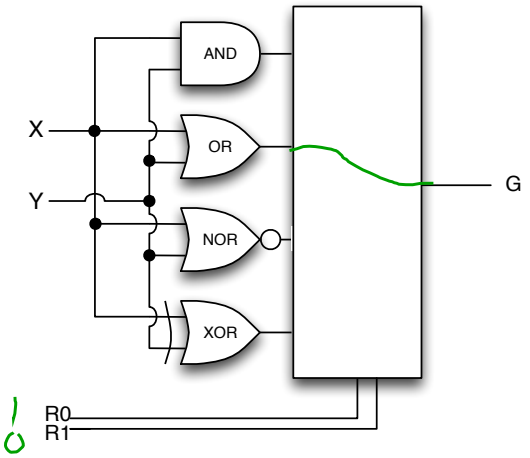


Selecting the desired logical operation

- We need a control signal to specify the desired operation:
  - We'll call that signal R
  - 4 operations means R is 2 bits

	R_1	R_0	Output	
0	0	0	$G_i = X_i Y_i$	AND
1	0	1	$G_i = X_i + Y_i$	OR
2	1	0	$G_i = (X_i + Y_i)'$	NOR
3	1	1	$G_i = X_i \oplus Y_i$	XOR

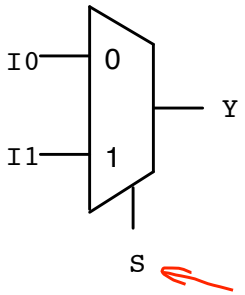
- We need a circuit to perform the selection:



Multiplexors use control bits to select data

- A multiplexor is a circuit that (logically) selects one of its data inputs to connect to its data output

- Consider a 2-to-1 multiplexor. It has:
  - 2 data input bits (I_0, I_1)
  - a 1-bit control input bit (S)
  - 1 data output bit (Y)



S	Y
0	I0
1	I1

- The control input selects which data input is output:

$$Y = S'I_0 + SI_1$$

Multiplexors, cont.

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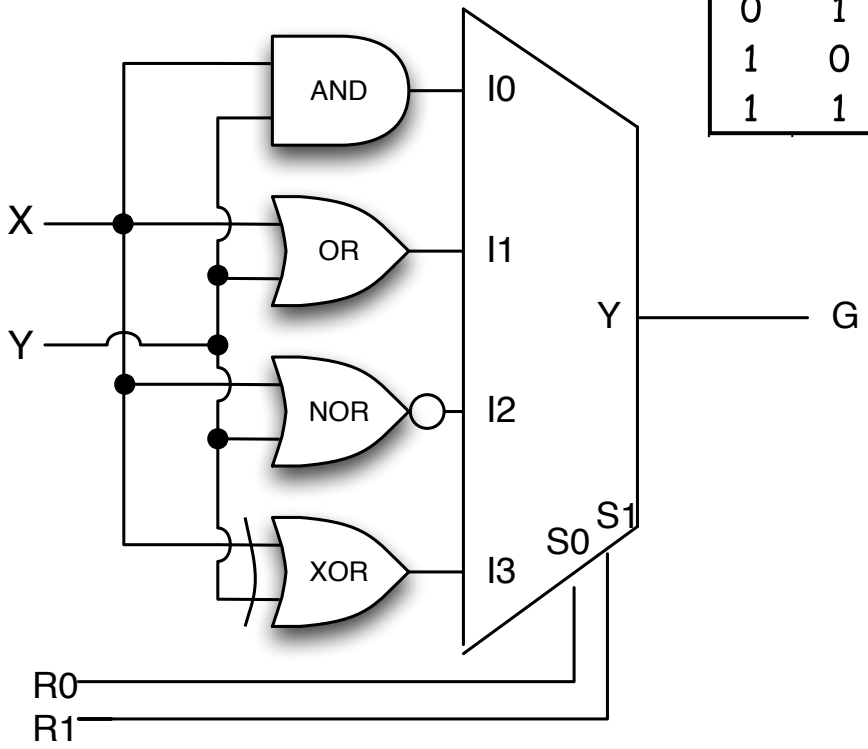
- In general, a multiplexor (mux) has:
  - 2^N data input bits (I_0—I_{2^N-1})
  - an *N*-bit control input (S)
  - 1 data output bit (Y)
- If S = K then Y = I_K

A:	S_1S_0
B:	S_2S_0
C:	S_1S_0'
D:	S_2S_0'
E:	$S_1'S_0$

- Examples:
  - 4-to-1 mux: 4 data input bits, 2-bit control input
    - $Y = S_1'S_0'I_0 + S_1'S_0I_1 + \_\_\_\_I_2 + S_1S_0I_3$
  - 16-to-1 mux: 16 data input bits, 4-bit control input

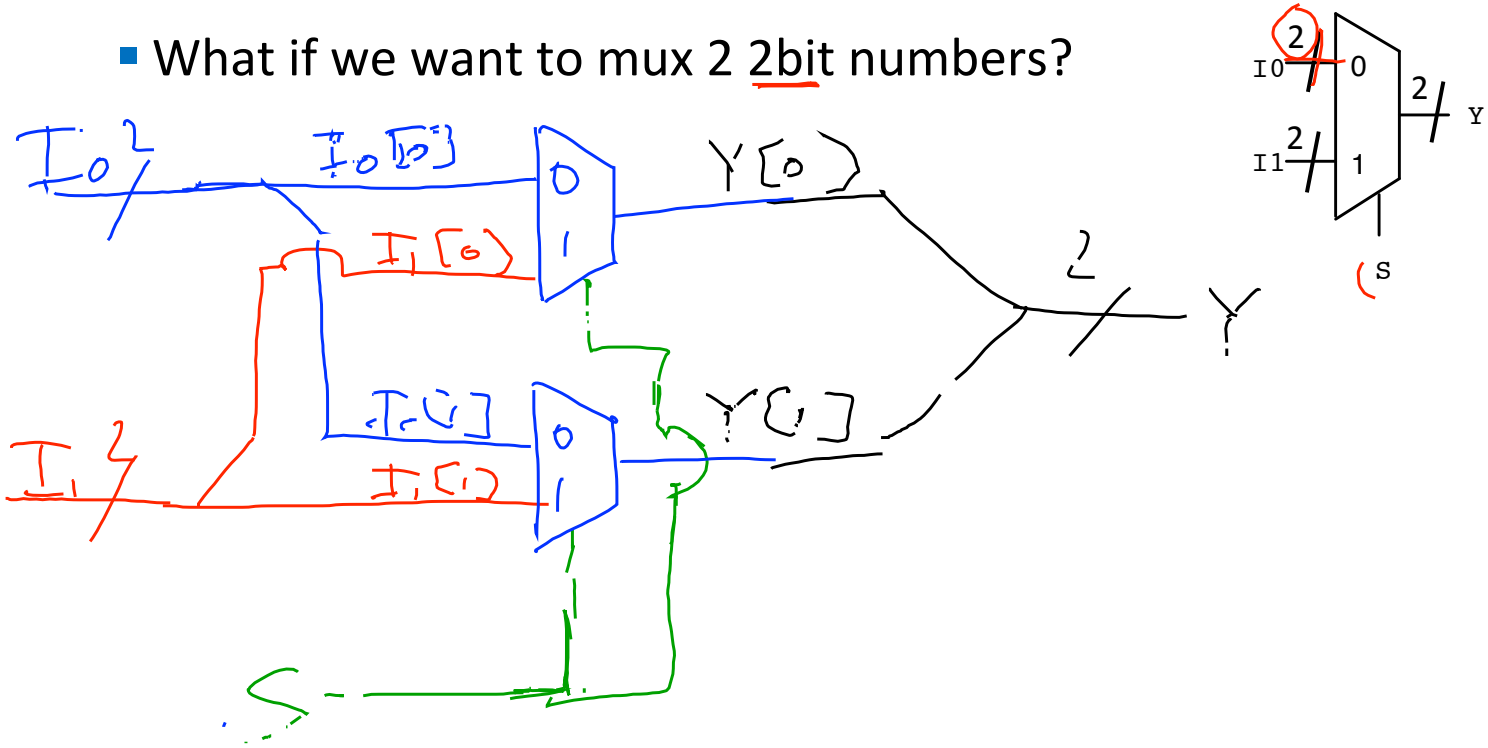
Complete 1-bit Logic Unit

R_1	R_0	Output
0	0	$G_i = X_i Y_i$
0	1	$G_i = X_i + Y_i$
1	0	$G_i = (X_i + Y_i)'$
1	1	$G_i = X_i \oplus Y_i$



Mux Hierarchical Design (operand width)

- What if we want to mux 2 2bit numbers?



Mux Hierarchical Design (more inputs)

- How do we build a mux with 4 inputs?