4MM013 - Computational Mathematics

Mathematics Assignment-2

Full Marks: 20

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Submitted on : 4/21/2023

1. Using Cramer's rule obtain the solutions to the following set of equations:

$$2\,x_1 + x_2 - x_3 = 0$$

$$x_1 + x_3 = 4$$

$$x_1 + x_2 + x_3 = 0$$

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1. Given;

$2x_1 + x_2 - x_3 = 0$

$x_1 + x_3 = 4$

$x_1 + x_2 + x_3 = 0$

Now, in matrix form;

$$\begin{pmatrix} 2 & 1 & -1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix}$$

Here,

$$\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 2\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$$

$$= -2 + 0 - 1$$

$$= -3$$

$$\Delta x_1 = \begin{vmatrix} 0 & 1 & -1 \\ 4 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = -1\begin{vmatrix} -1 & 1 \\ 0 & 1 \end{vmatrix} - 1\begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix}$$

$$= \cancel{2\begin{vmatrix} -1 & 1 \\ 0 & 1 \end{vmatrix}} \cancel{- 0}\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} - 1\cancel{\begin{vmatrix} 1 & 4 \\ 1 & 0 \end{vmatrix}}$$

$$= \cancel{8 + 4}$$

$$= \cancel{12} \qquad = -4 - 8 \qquad = -8$$

$$\Delta x_2 = \begin{vmatrix} 2 & 0 & -1 \\ 1 & 4 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 2\begin{vmatrix} -1 & 1 \\ 0 & 1 \end{vmatrix} - 0\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 4 \\ 1 & 0 \end{vmatrix}$$

$$= 8 + 4$$

$$= 12$$

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$$\Delta x_3 = \begin{vmatrix} 2 & 1 & 0 \\ 1 & 0 & 4 \\ 1 & 1 & 0 \end{vmatrix} = 2\begin{vmatrix} 3 & 4 \\ 1 & 0 \end{vmatrix} - \begin{vmatrix} 1 & 4 \\ 1 & 0 \end{vmatrix} + 0$$

$$= -8+4+0$$

$$= -4$$

Now,

$$x_1 = \Delta x_1/\Delta = -8/-3 = 8/3$$

$$x_2 = \frac{\Delta x_2}{\Delta} = 12/-3 = -4$$

$$x_3 = \frac{\Delta x_3}{\Delta} = +4/+3 = 4/3$$

$\therefore x_1 = 8/3,\ x_2 = -4,\ x_3 = 4/3$ //

(4)

2.

a) Solve the following using Gauss elimination:

$$x_1 + x_2 + x_3 = 2$$

$$2x_1 + 3x_2 + 4x_3 = 3$$

$$x_1 - 2x_2 - x_3 = 1$$

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2\.a) Given eq^n;

$x_1 + x_2 + x_3 = 2$ — (i)

$2x_1 + 3x_2 + 4x_3 = 3$ — (ii)

$x_1 - 2x_2 - x_3 = 1$ — (iii)

Now, in matrix form,

$$[A \mid B] = \begin{bmatrix} 1 & 1 & 1 & : & 2 \\ 2 & 3 & 4 & : & 3 \\ 1 & -2 & -1 & : & 1 \end{bmatrix}$$

Here,

$$\begin{bmatrix} 1 & 1 & 1 & : & 2 \\ 2 & 3 & 4 & : & 3 \\ 1 & -2 & -1 & : & 1 \end{bmatrix} \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & 1 & 1 & : & 2 \\ 0 & 1 & 2 & : & -1 \\ 0 & -3 & 2 & : & 1 \end{bmatrix}$$

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$R_3 \to R_3 + 3R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 4 & -4 \end{array}\right]$$

Here,

$x_1 + x_2 + x_3 = 2$ — (iv)

$x_1 + 2x_3 = -1$ — (v)

$4x_3 = -4$

or, $x_3 = -1$ — (vi)

~~Keppi~~ Keeping the value of eqn (vi) in eqn (v)

So,

$x_2 + (-1).2 = -1$

$\therefore x_2 = 1$

Keeping the value of x_2 & x_3 in eqn (iv)

$x_1 + 1 - 1 = 2$

$\therefore x_1 = 2$

Hence,

$x_1 = 2, x_2 = 1, x_3 = -1$

(4)

b) Find the inverse of the matrix from (a) using elimination.

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2.b) Given eqn;

$x_1 + x_2 + x_3 = 2$ — (i)

$2x_1 + 3x_2 + 4x_3 = 3$ — (ii)

$x_1 - 2x_2 - x_3 = 1$ — (iii)

In matrix form;

let,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 1 & -2 & -1 \end{bmatrix}$$

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Qo,

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 3 & 4 & 0 & 1 & 0 \\ 1 & -2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 1 & -2 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_3 \to R_3 - R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & -3 & -2 & -1 & 0 & 1 \end{array}\right]$$

$R_3 \to R_3 + 3R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & 0 & 4 & -7 & 3 & 1 \end{array}\right]$$

$R_3 \to R_3/4$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & 0 & 1 & -7/4 & 3/4 & 1/4 \end{array}\right]$$

$R_2 \to R_2 - 2R_3$ & $R_1 \to R_1 - R_3$

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$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 11/4 & -3/4 & -1/4 \\ 0 & 1 & 0 & 3/2 & -1/2 & -1/2 \\ 0 & 0 & 1 & -7/4 & 3/4 & 1/4 \end{array}\right)$$

$R_1 \to R_1 - R_2$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/4 & -1/4 & 1/4 \\ 0 & 1 & 0 & 3/2 & -1/2 & -1/2 \\ 0 & 0 & 1 & -7/4 & 3/4 & 1/4 \end{array}\right)$$

Now,

$$B^{-1} = \begin{pmatrix} 3/4 & -1/4 & 1/4 \\ 3/2 & -1/2 & -1/2 \\ -2/4 & 3/4 & 1/4 \end{pmatrix}$$

(4)

3. Determine whether the following sequence converges or diverges.

$$t_n = (-1)^{n+1}\,\frac{n+1}{n^2+3}$$

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3. Soln.

$$t_n = (-1)^{n+1} \frac{n+1}{n^2+3}$$

Putting lim on t_n;

$$= \lim_{n\to\infty} (-1)^{n+1} \frac{n+1}{n^2+3}$$

$$= \lim_{n\to\infty} \frac{n+1}{n^2+3}$$

$$= \lim_{n\to\infty} \frac{n^2\left(1/n + 1/n^2\right)}{n^2\left(1+3/n^2\right)}$$

$$= \frac{1/\infty + 1/\infty}{1 + 3/\infty}$$

$$= 0/1$$

$\therefore$ It is converges & its converged.

(4)

4. Find the Maclaurin series expansion of **Sinx,** also calculate the radius of convergence. (4).

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4. Soln:

$f(x) = \sin(x)$

Now, derivatives of $f(x) = \sin(x)$

$f(x) = \sin(x) = f(0) = 0$

$f'(x) = \cos(x) = f'(0) = 1$

$f''(x) = -\sin(x) = f''(0) = 0$

$f'''(x) = -\cos(x) = f'''(0) = -1$

$f^{iv}(x) = \sin(x) = f^{iv}(0) = 0$

$f^{v}(x) = \cancel{\cos 0}\ \cos(x) = f^{v}(0) = 1$

Maclaurin series expansion,

$$f(x) = f(0) + f'(0)\frac{x}{1!} + f''(0)\frac{x^2}{2!} + f'''(0)\frac{x^3}{3!} + f^{iv}(0)\frac{x^4}{4!} + \frac{f^{v}(0)}{5!}$$

$$f(x) = \sin x = 0 + x + 0 - \frac{x^3}{3!} + 0 + \frac{x^5}{5!} + \cdots$$

So,

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

Now,

$$\sum_{n=0}^{\infty} a_n x^{2n+1} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$\therefore\ a_n = (-1)^{n+1} \dfrac{1}{(2n+1)!}$

$\therefore\ a = 2$

Radius of covergence $(R) = \sqrt{\dfrac{1}{\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|}}$

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$$= \sqrt{\frac{1}{\lim_{n\to\infty} \left|\frac{1}{(2(n+1)+1)!} \cdot (2n+1)!\right|}}$$

$$= \sqrt{\frac{1}{\lim_{n\to\infty} \left|\frac{1}{(2n+3)(2n+2)(2n+1)!} \cdot (2n+1)!\right|}}$$

$$= \sqrt{1/0}$$

$$= \sqrt{\infty}$$

$$= \infty \ //$$

The End