Integración por partes: $\int udv = uv - \int vdu$

1) $\int x \operatorname{sen} x dx \Rightarrow u = x, dv = \operatorname{sen} x dx \Rightarrow du = dx, v = \int \operatorname{sen} x dx = -\cos x \Longrightarrow \int x \operatorname{sen} x dx = x(-\cos x) - \int (-\cos x) dx = -x\cos x + \int \cos x dx = \boxed{-x\cos x + \operatorname{sen} x + C}$

2) $\int \ln x dx \Rightarrow u = \ln x, dv = dx \Rightarrow du = \frac{dx}{x}, v = \int dx = x \Longrightarrow \int \ln x dx = \ln x(x) - \int x \frac{dx}{x} = x\ln x - \int dx = x\ln x - x + C = \boxed{x(\ln x - 1) + C}$

3) $\int x^n \ln x dx \Rightarrow u = \ln x, dv = x^n dx \Rightarrow du = \frac{dx}{x}, v = \int x^n dx = \frac{x^{n+1}}{n+1} \Longrightarrow \int x^n \ln x dx = \ln x \frac{x^{n+1}}{n+1} - \int \frac{x^{n+1}}{n+1} \cdot \frac{dx}{x} = \frac{x^{n+1}}{n+1} \ln x - \frac{1}{n+1} \int \frac{x^{n+1}}{dx} = \frac{x^{n+1}}{n+1} \ln x - \frac{1}{n+1} \cdot \frac{x^{n+1}}{n+1} + C = \boxed{\frac{x^{n+1}}{n+1}(\ln x - \frac{1}{n+1}) + C}$

4) $\int xe^{ax} dx \Rightarrow u = x, dv = e^{ax} dx \Rightarrow du = dx, v = \int e^{ax} dx = \frac{e^{ax}}{a} \Longrightarrow \int xe^{ax} dx = x\frac{e^{ax}}{a} - \int \frac{e^{ax}}{a} dx = x\frac{e^{ax}}{a} - \frac{1}{a} \int e^{ax} dx = x\frac{e^{ax}}{a} - \frac{1}{a}\frac{e^{ax}}{a} + C = \boxed{\frac{e^{ax}}{a}(x - \frac{1}{a}) + C}$

5) $\int \arctan x dx \Rightarrow u = \arctan x, dv = dx \Rightarrow du = \frac{dx}{1+x^2}, v \int dx = x \Longrightarrow \int \arctan x dx = \arctan(x) - \int \frac{dx}{1+x^2} dx = x\arctan x - \int \frac{xdx}{1+x^2} = \boxed{x\arctan x - \frac{1}{2}\ln(1+x^2) + C}$

6) $\int e^{-2x} \operatorname{sen} e^{-x} dx \Rightarrow u = e^{-x} \Rightarrow du = -e^{-x} dx \Rightarrow dx = -\frac{du}{e^{-x}} = -\frac{du}{u} \Rightarrow dv = e^{-2x} \operatorname{sen} u dx \Rightarrow \int e^{-2x} \operatorname{sen} e^{-x} dx = \int e^{-x} \cdot e^{-x} \operatorname{sen} e^{-x} = u \cdot u \operatorname{sen} u(-\frac{du}{u}) = -\int u \operatorname{sen} u du \Longrightarrow I = \int e^{-2x} \operatorname{sen} e^{-x} dx = -(-u\cos u + \operatorname{sen} u + C_1) = \boxed{+e^{-x}\cos e^{-x} - \operatorname{sen} e^{-x} + C} \Rightarrow C = -C_1$

Integrales resolubles mediante integración por partes
Forma A: $\int x^n e^{ax} dx$**,** $\int x^n \operatorname{sen} ax dx$**, o** $\int x^n \cos ax dx$ **hacer** $u = x^n$ **y** $dv = e^{ax}dx, \operatorname{sen} ax dx, \cos ax dx$
Forma B: $\int x^n \ln x dx$**,** $\int x^n \operatorname{arc\,sen} ax dx$**, o** $\int x^n \arccos ax dx$ **hacer** $u = \ln x, \operatorname{arc\,sen} ax, \arccos ax$ **y** $dv = x^n dx$
Forma C: $\int e^{ax} \operatorname{sen} bx dx$ **o** $\int e^{ax} \cos bx dx$ **hacer** $u = e^{ax}$ **y** $dv = \operatorname{sen} bx dx, \cos bx dx$

Integración por sustitución trigonométrica
$\sqrt{a^2 - u^2} \Rightarrow u = a\operatorname{sen}\theta \Rightarrow du = a\cos\theta d\theta \Rightarrow \sqrt{a^2-u^2} = a\cos\theta$
$\sqrt{a^2 + u^2} \Rightarrow u = a\tan\theta \Rightarrow du = a\sec^2\theta d\theta \Rightarrow \sqrt{a^2+u^2} = a\sec\theta$
$\sqrt{u^2 - a^2} \Rightarrow u = a\sec\theta \Rightarrow du = a\sec\theta\tan\theta d\theta \Rightarrow \sqrt{u^2-a^2} = a\tan\theta$

1) $\int \frac{du}{\sqrt{a^2-u^2}} \Rightarrow u = a\operatorname{sen}\theta \Rightarrow du = a\cos\theta d\theta \Rightarrow \theta = \operatorname{arc\,sen}\frac{u}{a} \Rightarrow \sqrt{a^2-u^2} = \sqrt{a^2 - a^2\operatorname{sen}^2\theta} = \sqrt{a^2(1-\operatorname{sen}^2\theta)} = \sqrt{a^2\cos^2\theta} = a\cos\theta \Longrightarrow I = \int \frac{du}{\sqrt{a^2-u^2}} = \int \frac{a\cos\theta d\theta}{a\cos\theta} = \int d\theta = \theta + C = \operatorname{arc\,sen}\frac{u}{a} + C \Rightarrow \int \frac{du}{\sqrt{a^2-u^2}} = \boxed{\operatorname{arc\,sen}\frac{u}{a} + C}$

2) $\int \frac{du}{u^2+a^2} \Rightarrow u = a\tan\theta \Rightarrow du = a\sec^2\theta d\theta \Rightarrow \theta = \arctan\frac{u}{a} \Rightarrow \sqrt{a^2+u^2} = \sqrt{a^2+a^2\tan^2\theta} = \sqrt{a^2(1+\tan^2\theta)} = \sqrt{a^2\sec^2\theta} = a\sec\theta \Longrightarrow I = \int \frac{du}{u^2+a^2} = \int \frac{du}{(\sqrt{u^2+a^2})^2} = \int \frac{a\sec^2\theta d\theta}{(a\sec\theta)^2} = \int \frac{a\sec^2\theta}{a^2\sec^2\theta} d\theta = \frac{1}{a}\int d\theta = \frac{1}{a}\theta + C = \frac{1}{a}\arctan\frac{u}{a} + C \Rightarrow \int \frac{du}{u^2+a^2} = \boxed{\frac{1}{a}\arctan\frac{u}{a} + C}$

3) $\int \frac{du}{u\sqrt{u^2-a^2}} \Rightarrow u = a\sec\theta \Rightarrow du = a\sec\theta\tan\theta d\theta \Rightarrow \theta = arcsec\frac{|u|}{a} \Rightarrow \sqrt{u^2+a^2} = \sqrt{a^2\sec^2\theta - a^2} = \sqrt{a^2(\sec^2\theta - 1)} = \sqrt{a^2\tan^2\theta} = a\tan\theta \Longrightarrow I = \int \frac{du}{u\sqrt{u^2-a^2}} = \int \frac{a\sec\theta\tan\theta d\theta}{(a\sec\theta)(a\tan\theta)} = \frac{1}{a}\int d\theta = \frac{1}{a}\theta + C = \frac{1}{a}arcsec\frac{|u|}{a} + C \Rightarrow \int \frac{du}{u\sqrt{u^2-a^2}} = \boxed{\frac{1}{a}arcsec\frac{|u|}{a} + C}$

4) $\int \sqrt{r^2-x^2}dx \Rightarrow a^2 = r^2 \Rightarrow a = r \Rightarrow u^2 = x^2 \Rightarrow u = x \Rightarrow x = r\operatorname{sen}\theta \Rightarrow dx = r\cos\theta d\theta \Rightarrow \operatorname{sen}\theta =$

$\frac{x}{r} \Rightarrow \sqrt{r^2-x^2} = \sqrt{r^2-(r\operatorname{sen}\theta)^2} = \sqrt{r^2\cos^2\theta} = r\cos\theta \Rightarrow \cos\theta = \frac{\sqrt{r^2-x^2}}{r} \Rightarrow \theta = \arccos\frac{\sqrt{r^2-x^2}}{r}$ **o** $\theta = \operatorname{arc\,sen}\frac{x}{r} \Longrightarrow I = \int\sqrt{r^2-x^2}dx = \int r\cos\theta(r\cos\theta d\theta) = r^2\int\cos^2\theta d\theta = \frac{r^2}{2}\int(1+\cos 2\theta)d\theta = \frac{r^2}{2}\int d\theta + \frac{r^2}{2}\int\cos 2\theta d\theta = \frac{r^2}{2}\theta + \frac{r^2}{2}\frac{\operatorname{sen}2\theta}{2} + C \Rightarrow \operatorname{sen}2\theta = 2\operatorname{sen}\theta\cos\theta \Longrightarrow I = \frac{r^2}{2}\theta + \frac{r^2}{2}\frac{\operatorname{sen}2\theta}{2} + C = \frac{r^2}{2}\theta + \frac{r^2}{2}\frac{2\operatorname{sen}\theta\cos\theta}{2} + C = \frac{r^2}{2}\theta + \frac{r^2}{2}\operatorname{sen}\theta\cos\theta + C = \frac{r^2}{2}\operatorname{arc\,sen}\frac{x}{r} + \frac{r^2}{2}\frac{x}{r}\frac{\sqrt{r^2-x^2}}{r} + C = \boxed{\dfrac{r^2}{2}\operatorname{arc\,sen}\dfrac{x}{r} + \dfrac{x}{2}\sqrt{r^2-x^2} + C}$

5) $\int\sqrt{9x^2+1}dx \Rightarrow a = 1 \Rightarrow u = 3x \Rightarrow 3x = 1\cdot\tan\theta \Rightarrow x = \frac{1}{3}\tan\theta \Rightarrow dx = \frac{1}{3}\sec^2\theta d\theta \Rightarrow \sqrt{9x^2+1} = 1\cdot\sec\theta \Rightarrow \theta = \arctan 3x \Rightarrow \theta = arcsec\sqrt{9x^2+1} \Longrightarrow I = \int\sqrt{9x^2+1}dx = \int\sec\theta(\frac{1}{3}\sec^2\theta d\theta) = \frac{1}{3}\int\sec^3\theta d\theta = \frac{1}{3}[\frac{1}{2}(\sec\theta\cdot\tan\theta + \ln|\sec\theta+\tan\theta|)] + C = \frac{1}{6}(\sqrt{9x^2+1}\cdot 3x + \ln|\sqrt{9x^2+1}+3x|) + C = \frac{3x\sqrt{9x^2+1}}{6} + \frac{1}{6}\ln|\sqrt{9x^2+1} + 3x| + C = \frac{x\sqrt{9x^2+1}}{2} + \frac{1}{6}\ln|\sqrt{9x^2+1}+3x| + C = \boxed{\dfrac{x\sqrt{9x^2+1}}{2} + \ln|\sqrt{9x^2+1}+3x|^{\frac{1}{6}} + C}$

6)$\sqrt{x^2-4}dx \Rightarrow a = 2 \Rightarrow x = 2\sec\theta \Rightarrow dx = 2\sec\theta\tan\theta d\theta \Rightarrow \sqrt{x^2-4}dx = 2\tan\theta \Rightarrow \tan\theta = \frac{\sqrt{x^2-4}}{2} \Rightarrow \theta = \arctan\frac{\sqrt{x^2-4}}{2}$ **o** $\theta = arcsec\frac{x}{2} \Longrightarrow I = \int\sqrt{x^2-4}dx = \int 2\tan\theta\cdot(2\sec\theta\tan\theta d\theta) = 4\int\tan^2\theta\sec\theta d\theta = 4\int(\sec^2\theta-1)\sec\theta d\theta = 4\int\sec^3\theta d\theta - 4\int\sec\theta d\theta = 4(\frac{1}{2}(\sec\theta\cdot\tan\theta + \ln|\sec\theta+\tan\theta|)) - 4\ln|\sec\theta+\tan\theta| + C = 2\sec\theta\cdot\tan\theta + 2\ln|\sec\theta+\tan\theta| - 4\ln|\sec\theta+\tan\theta| + C = 2\sec\theta\cdot\tan\theta - 2\ln|\sec\theta+\tan\theta| + C = 2(\frac{x}{2})(\frac{\sqrt{x^2-4}}{2}) - 2\ln|\frac{x}{2} + \frac{\sqrt{x^2-4}}{2}| + C = \boxed{\dfrac{x\sqrt{x^2-4}}{2} - 2\ln|\dfrac{x+\sqrt{x^2-4}}{2}| + C}$

Integración por descomposición en fracciones racionales. Factores lineales: $\frac{A}{ax+b}$, $\frac{A}{(ax+b)^n}$

Caso 1. Los factores del denominador son todos de primer grado y distintos, de la forma $\frac{A}{ax+b}$

1) $\int\frac{4}{x^2-x}dx \Rightarrow \frac{4}{x^2-x} = \frac{4}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1} \Rightarrow 4 = \frac{(x)(x-1)A}{x} + \frac{(x)(x-1)B}{x-1} \Rightarrow 4 = A(x-1) + Bx = Ax - A + Bx = x(A+B) - A \Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 4 \end{bmatrix} \Rightarrow -A = 4 \Rightarrow \boxed{A = -4} \Rightarrow A+B = 0 \Rightarrow -4+B = 0 \Rightarrow \boxed{B = 4} \Longrightarrow \frac{4}{x^2-x} = \frac{-4}{x} + \frac{4}{x-1} \Longrightarrow I = \int\frac{4}{x^2-x}dx = \int\frac{-4}{x} + \int\frac{4}{x-1}dx = -4\ln x + 4\ln(x-1) + C = 4[\ln(x-1) - \ln x] + C = \boxed{\ln(\dfrac{x-1}{x})^4 + C}$

2) $\int\frac{dx}{4x^3-x} \Rightarrow \frac{1}{4x^3-x} = \frac{1}{x(4x^2-1)} = \frac{1}{x(2x+1)(2x-1)} = \frac{A}{x} + \frac{B}{2x+1} + \frac{C}{2x-1} \Rightarrow 1 = A(2x+1)(2x-1) + Bx(2x-1) + Cx(2x+1) = A(4x^2-1) + B(2x^2-x) + C(2x^2+x) \Rightarrow 1 = x^2(4A+2B+2C) + x(-B+C) - A \Rightarrow \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 4 & 2 & 2 & 0 \end{bmatrix} \sim 4R_1 + R_3 \sim \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 2 & 2 & 4 \end{bmatrix} \sim \frac{1}{2}R_3 \sim \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & 2 \end{bmatrix} \sim R_3 + R_2 \sim \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 1 & 1 & 2 \end{bmatrix} \Rightarrow 2C = 2 \Rightarrow \boxed{C = 1} \Rightarrow B + C = 2 \Rightarrow B + 1 = 2 \Rightarrow \boxed{B = 1} \Rightarrow -A = 1 \Rightarrow \boxed{A = -1} \Longrightarrow \int\frac{-1}{x}dx + \int\frac{1}{2x+1}dx + \int\frac{1}{2x-1}dx = -\ln x + \frac{1}{2}\ln(2x+1) + \frac{1}{2}\ln(2x-1) + C = \boxed{\ln\dfrac{\sqrt{(4x^2-1)}}{x} + C}$

Caso 2. Los factores del denominador son todos de primer grado y algunos es repiten, correspondiéndole la forma $\frac{A}{(ax+b)^n} + \frac{B}{(ax+b)^{n-1}} + ... + \frac{L}{(ax+b)}$

3) $\int\frac{x+5}{x^3-3x+2}dx \Rightarrow x^3 - 3x + 2 = (x+2)(x-1)^2 \Rightarrow \frac{x+5}{3x^2-3x+2} = \frac{x+5}{(x+2)(x-1)^2} = \frac{A}{x+2} + \frac{B}{(x-1)^2} + \frac{C}{x-1} \Rightarrow x + 5 = A(x-1)^2 + B(x+2) + C(x+2)(x-1) = A(x^2-2x+1) + B(x+2) + C(x^2+x-2) = x^2(A+C) + x(-2A+B+C) + (A+2B-2C) \Rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 \\ -2 & 1 & 1 & 1 \\ 1 & 2 & -2 & 5 \end{bmatrix} \sim 2R_1 + R_2, -1R_1 + R_3 \sim \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 2 & -3 & 5 \end{bmatrix} \sim -2R_2 + R_3 \sim \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -9 & 3 \end{bmatrix} \Rightarrow -9C = 3 \Rightarrow C = -3/9 \Rightarrow \boxed{C = -1/3} \Rightarrow B + 3C = 1 \Rightarrow B - 1 = 1 \Rightarrow$

$\boxed{B=2} \Rightarrow A+C=0 \Rightarrow A=-C \Rightarrow \boxed{A=1/3} \Longrightarrow I=\int \frac{x+5}{x^3-3x+2}dx=\int \frac{1/3}{x+2}dx+\int \frac{2}{(x-1)^2}dx+\int \frac{-1/3}{x-1}dx=$ $\frac{1}{3}\ln|x+2|-\frac{2}{x-1}-\frac{1}{3}\ln|x-1|+C=\frac{1}{3}\ln\frac{x+2}{x-1}-\frac{2}{x-1}+C=\boxed{\ln|\dfrac{x+2}{x-1}|^{\frac{1}{3}}-\dfrac{2}{x-1}+C}$

4) $\int \frac{x^4-2x^2+4x+1}{x^3-x^2-x+1}dx \Rightarrow \frac{x^4-2x^2+4x+1}{x^3-x^2-x+1}=x+1+\frac{4x}{x^3-x^2-x+1} \Rightarrow x^3-x^2-x+1=x^2(x+1)-1(x-1)=$ $(x-1)(x^2-1)=(x-1)(x-1)(x+1)=(x-1)^2(x+1) \Rightarrow \frac{4x}{x^3-x^2-x+1}=\frac{4x}{(x-1)^2(x+1)}=\frac{A}{(x-1)^2}+\frac{B}{x-1}+\frac{C}{x+1} \Rightarrow$ $4x=A(x+1)+B(x-1)(x+1)+C(x-1)^2=A(x+1)+B(x^2-1)+C(x^2-2x+1)=x^2(B+C)+x(A-2C)+$ $(A-B+C)\begin{bmatrix}0&1&1&0\\1&0&-2&4\\1&-1&1&0\end{bmatrix}\sim -1R_2+R_3\sim\begin{bmatrix}0&1&1&0\\1&0&-2&4\\0&-1&3&-4\end{bmatrix}\sim 1R_1+R_3\sim\begin{bmatrix}0&1&1&0\\1&0&-2&4\\0&0&4&-4\end{bmatrix}\Rightarrow 4C=$ $-4\Rightarrow\boxed{C=-1}\Rightarrow A-2C=4\Rightarrow A=4-2\Rightarrow\boxed{A=2}\Rightarrow B+C=0\Rightarrow\boxed{B=1}\Longrightarrow I=\int\frac{x^4-2x^2+4x+1}{x^3-x^2-x+1}dx=$ $\int[x+1+\frac{2}{(x-1)^2}+\frac{1}{x-1}+\frac{-1}{x+1}]dx=\frac{x^2}{2}+x-\frac{2}{x-1}+\ln|x-1|-\ln|x+1|+C=\boxed{\dfrac{x^2}{2}+x-\dfrac{2}{x-1}+\ln\dfrac{|x-1|}{|x+1|}+C}$

Área entre dos curvas que se intersectan en dos puntos
Encontrar el área limitada por la hipérbola $xy=4$ y la recta $x+y=5$.
$f(x)=y=5-x\Rightarrow g(x)=y=\frac{4}{x}$
Puntos de intersección: $f(x)=g(x)\Rightarrow 5-x=4/x\Rightarrow 5x-x^2=4\Rightarrow x^2-5x+4=0\Rightarrow(x-4)(x-1)=0$ $\Rightarrow\underline{(x-1)=0}\Rightarrow x_1=1\Rightarrow y=5-(1)\Rightarrow(1,4)\Longrightarrow\underline{(x-4)=0}\Rightarrow x_2=4\Rightarrow y=5-(4)\Rightarrow(4,1)\Longrightarrow\boxed{[1,4]}$
Para saber que función esta arriba: para $x=2$
$f(x)=y=\underline{5-x}\Rightarrow f(2)=5-2=3\Longrightarrow g(x)=y=\underline{\frac{4}{x}}\Rightarrow g(2)=\frac{4}{2}=2\Longrightarrow$
$\boxed{f(x)\text{ esta arriba y }g(x)\text{ esta debajo: }g(x)\le f(x)}$
Por lo tanto, el área entre las funciones f y g en el intervalo, es: $A=\int_1^4[f(x)-g(x)]\,dx\int_1^4[(5-x)-(\frac{4}{x})]\,dx=5\int_1^4 dx-\int_1^4 x\,dx-4\int_1^4\frac{dx}{x}=[5x-\frac{x^2}{2}-4\ln|x|]_1^4=(5(4)-\frac{4^2}{2}-4\ln|4|)-(5(1)-\frac{1^2}{2}-4\ln|1|)=$ $20-8-4\ln|4|-5+\frac{1}{2}+0=7+\frac{1}{2}-4\ln|4|=\frac{15}{2}-4\ln|4|=7.5-4\ln|4|=1.954822556\,\mathrm{u}^2$

Área entre dos curvas que se intersectan en más de dos puntos
Encontrar el área de la región comprendida entre las gráficas de $f(x)=x^3+2$ y $g(x)=x+2$.