N-Body Simulation

Figures:

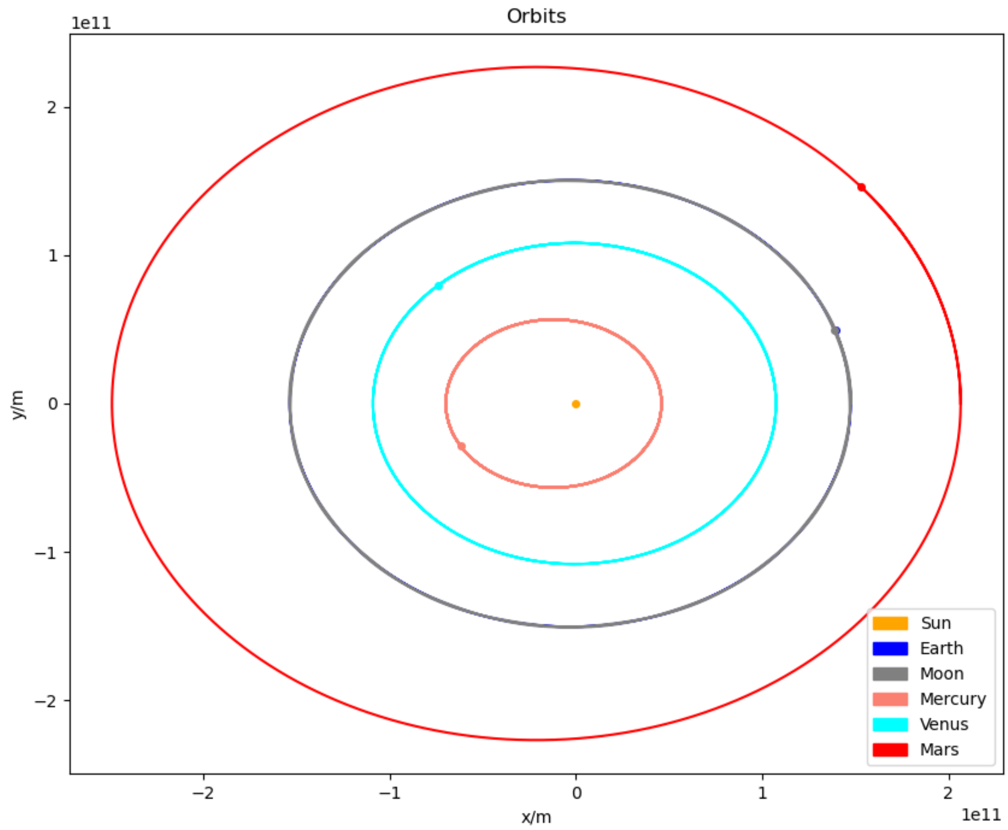

Figure 1: Animated plot of the orbits of the Sun, Earth, Moon, Mercury, Venus and Mars based on initial conditions for our solar system. This image has been taken at the end of the animation. The Moon and Earth follow roughly the same trajectory in the scale shown in this image, and so their orbits appear on top of each other, with the Earth's blue dot being just visible. The time period for these orbits was given as 2 years, with a year being 3.27×10^{7} seconds. The number of time points in this range was 10000.

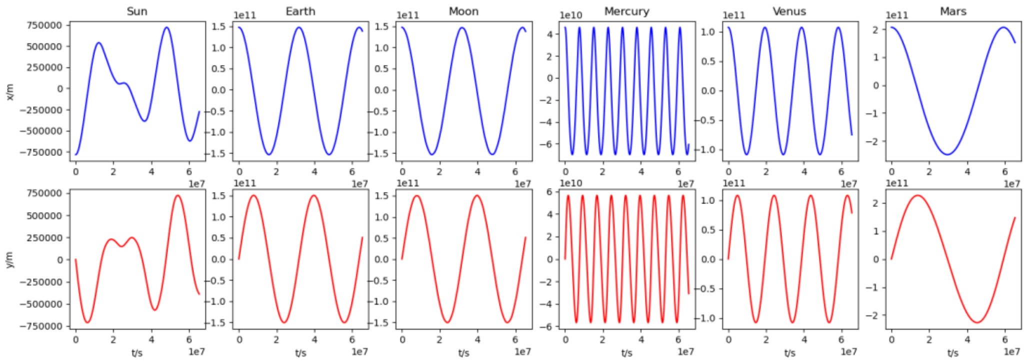

Figure 2: Plots of the x and y coordinates of the bodies from figure 1, taken from the same simulation. In this figure can be observed the sinusoidal nature of the orbits, as should be observed for bodies orbiting in this manner in an n-body simulation. The Earth's x and y coordinates are shown here to complete 2 full wavelengths in these graphs – this is in accordance with the actual solar system, as this simulation ran for 2 years with the period of Earth's orbit being 1 year. There is a small amount that the Earth's coordinates go over 2 full wavelengths – this is very small, however, and can be contributed to the rounding of initial conditions.

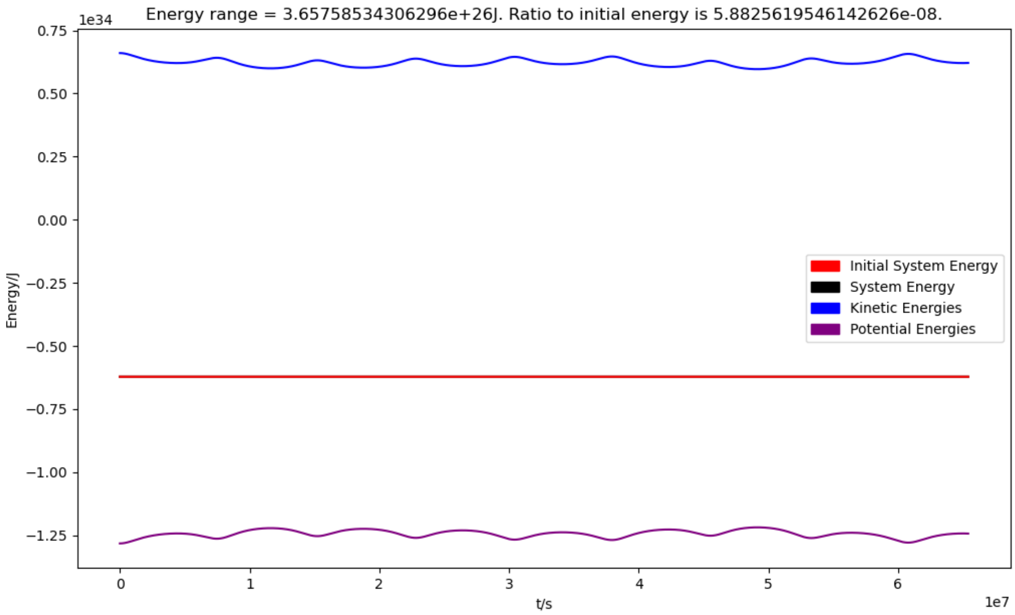

Figure 3: Plot of the energies of the system against time for the simulation shown in figure 1. The ratio of the ranges of energies to the initial energy is relatively very small, as can be seen above the figure, being of the order 10^{-8}. This is in accordance with the definition of a correct n-body system: the total energy of the system is always conserved, and this graph shows that the energy of the system deviates very little. The tiny deviations that do exist are due to the inaccuracies of the python odeint command using an approximation method to solve differential equations.

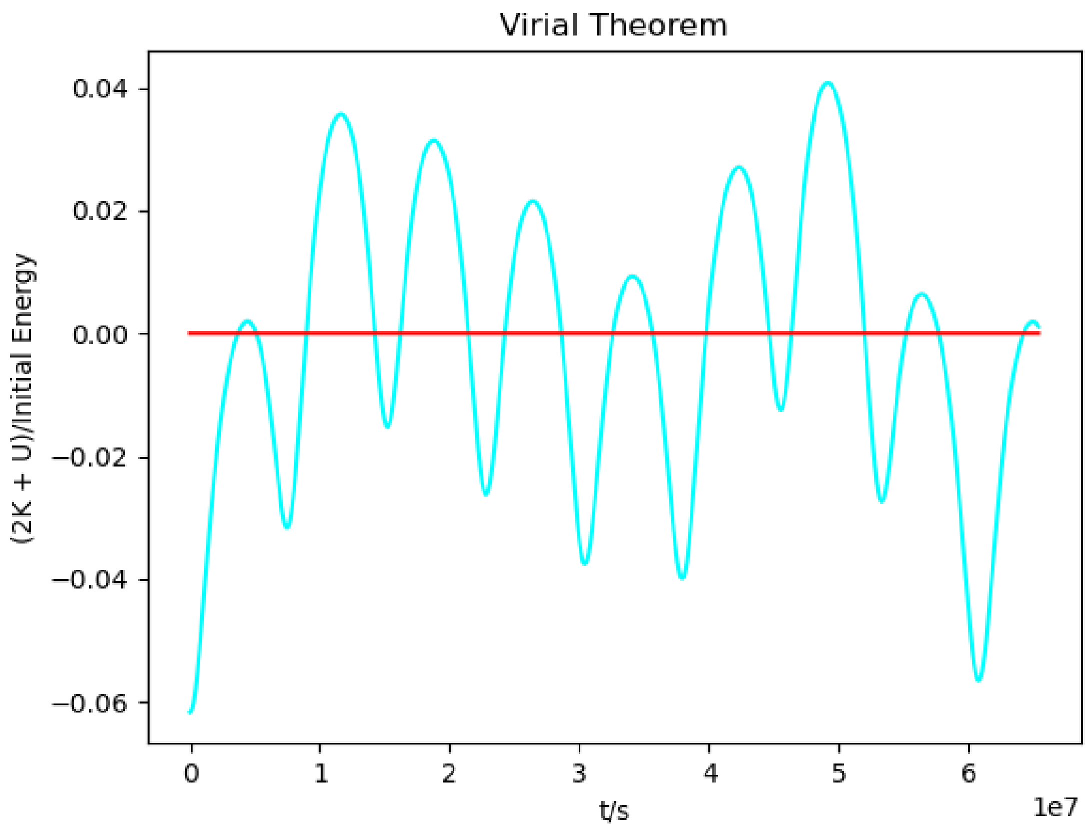

Figure 4: Plot of (2K+U)/Energy$_{Initial}$ for the simulation shown in figure 1. The red line is plotted at y = 0. The Virial Theorem states that $2K + U = 0$ *for bound orbits, where K is kinetic energy and U is potential energy. The orbits in the system should be bound orbits if the initial conditions for the system have been set up correctly. By plotting the ratio of the left hand side of this equation to the initial energy in this graph, we should be able to see if the left hand side of this equation is roughly equal to 0. The largest absolute deviation for the ratio is 0.06, which is quite close to 0. The reason it is not exactly 0 could be down to the inaccuracy of the python odeint function, or by the rounding of initial conditions making the orbits in the system not perfectly bound.*

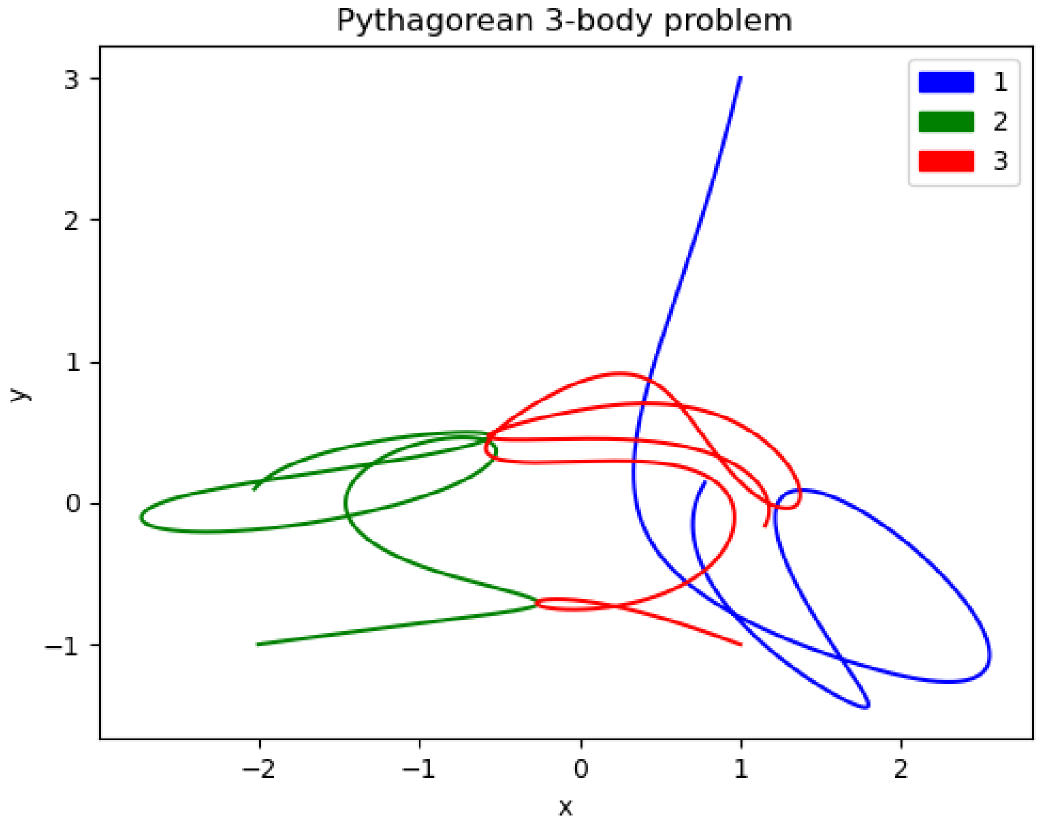

Figure 5: Graph of the Pythagorean 3-body problem, recreated using the n-body simulation. The initial conditions for the three bodies is that they were placed at the vertices of a 3-4-5 right triangle, with masses of 3, 4 and 5 respectively. The simulation ran until time = 10. The values for distance, mass and time are in arbitrary units, and for the case of this simulation the gravitational constant G was given a value of 1. The number of points used was 100000.

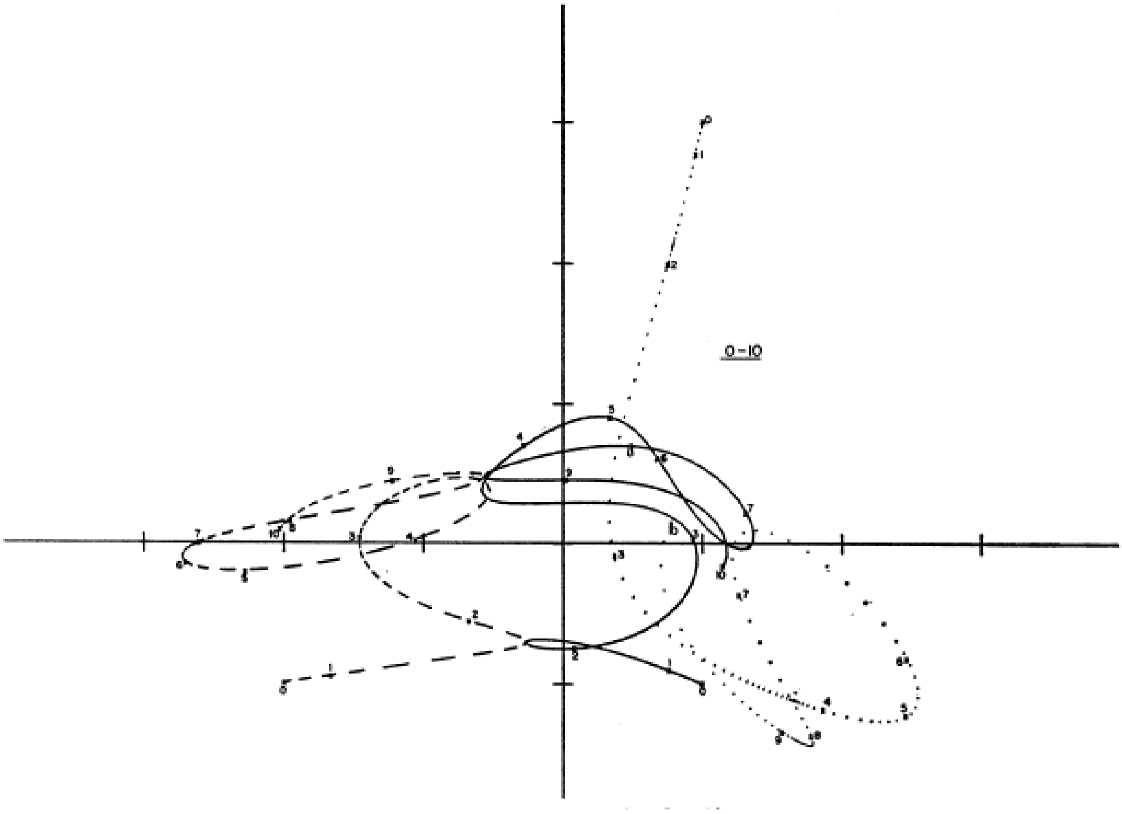

Figure 6: Solution to the Pythagorean 3-body problem for time from 0 to 10, taken from 'Complete Solution of a General Problem of Three Bodies' by Victor Szebehely and C. Frederick Peters 16/05/1967. This image matches very closely to the plot generated using the n-body simulation in figure 5, giving reason to believe that the simulation is an accurate representation of an n-body system.

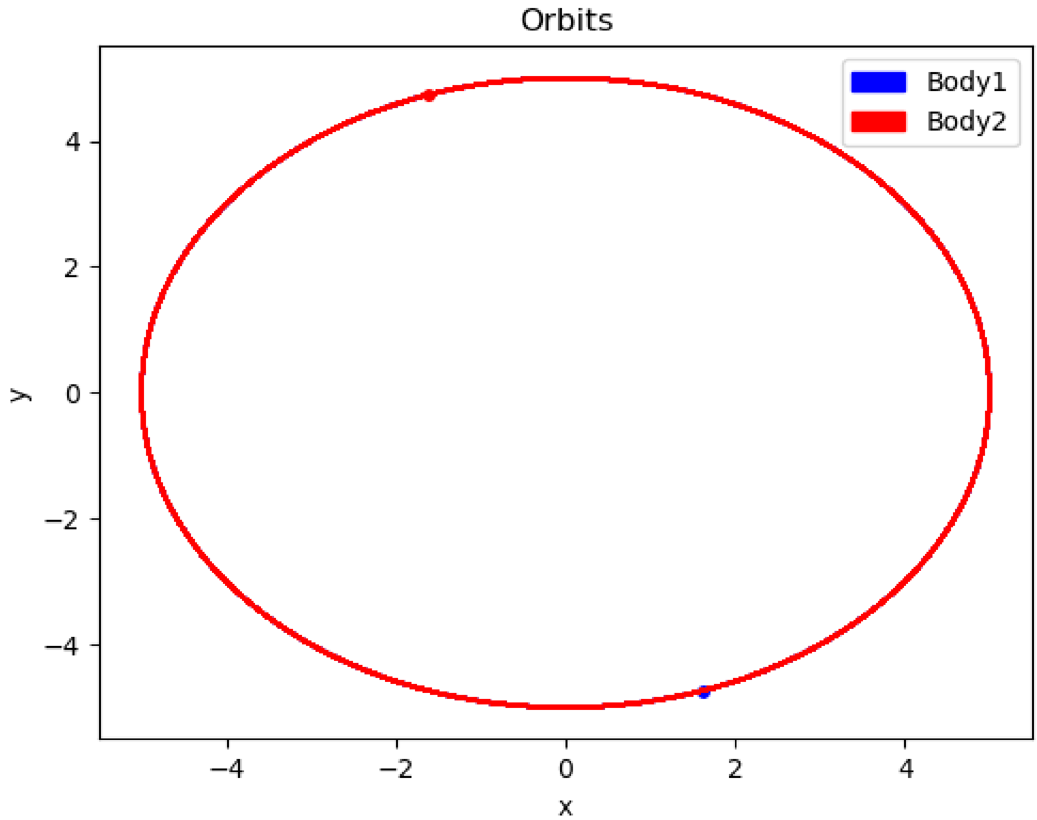

Figure 7: Plot of the n-body simulation run with two equally spaced bodies, to prove that they can perfectly orbit. The simulation to create this plot ran to time = 1000, with 10000 points. The initial conditions were set by inputting values such that they satisfied the equation $\frac{mv^2}{r} = \frac{GMm}{(2r)^2}$ *which equates gravitational force to the centripetal force. The initial conditions chosen were* $v_{orbital} = 1$, $M = m = 20$ *and* $r = 5$*. The gravitational constant G was set to 1, hence the units on all the values are arbitrary. As can be seen in the figure, the two bodies follow the same orbital path directly opposite to each other, as they should for a true n-body simulation given these conditions.*

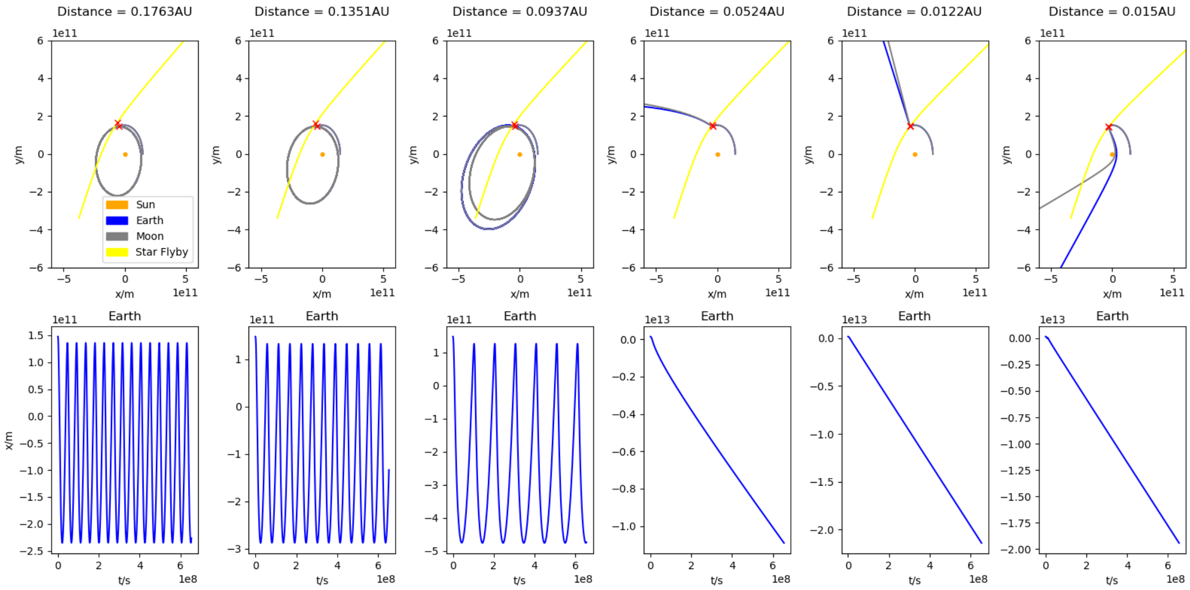

Figure 8: Investigation of the effects of a flyby star with 10% the mass of the sun on Earth's orbit. The orbit plots shown have been centred at the Sun. The star was initially positioned at (-3.4x10^{11}m, -3.4x10^{11}m) with x and y velocities (26500m/s, 53000m/s) respectively, and was shown to displace the Earth and Moon's orbits (see – top right). The initial x position of the star was multiplied by 6 factors between 1.2 and 1 to produce the plots seen in the figure (from left to right). The top set of plots shows how the flyby star affected the orbits in each case. The red 'x's mark the positions where the flyby star and Earth were closest to each other. Above the plots is listed the closest distance between the Earth and the flyby star, which shows the distance at this trajectory which would displace the Earth. The lower set of plots shows the Earth's x-positions against time for each simulation – the first three are sinusoidal, showing that the Earth's orbit around the Sun is maintained even though affected, whereas the last three are straight lines, showing that the Earth has been displaced from it's orbital nature. This simulation ran for time 20 years.

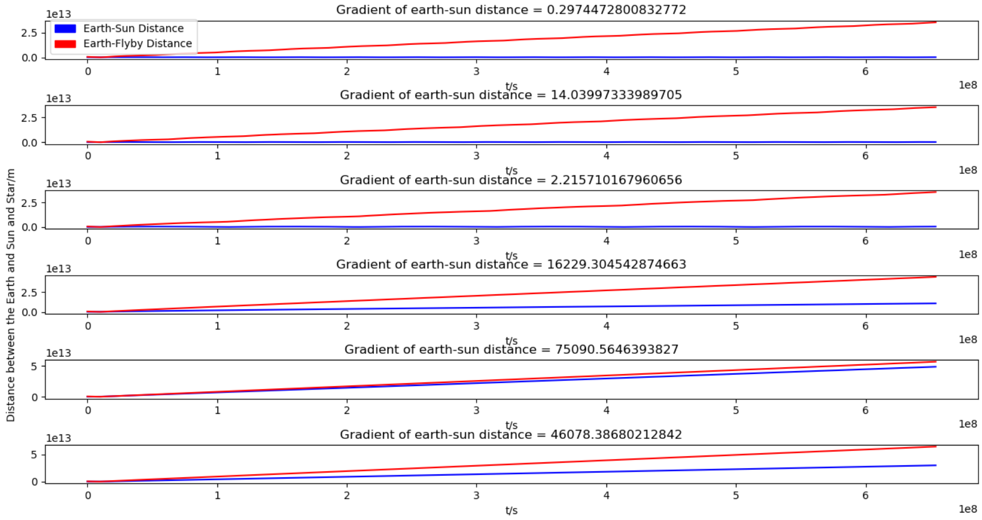

Figure 9: Plots of the distances between the Earth and the Sun and between the Earth and the flyby star against time from the simulations that were ran in figure 8. The plots are in descending order, with the initial x-position of the flyby star being 1.2(-3.4x10^{11}m) in the top plot, and 1*(-3.4x10^{11}m) in the final plot. The gradients of the Earth-Sun lines are displayed above the plots. If the Earth maintained its orbit around the Sun, the gradient of the Earth-Sun line should be close to 0 as the distance should only change a very small amount due to its elliptical orbit. It can be seen in this figure that the gradients for the final three simulations are far greater than the gradients in the first three simulations, which are negligible in comparison. This is consistent with figure 8, as in the final three simulations the Earth is ejected from its orbit around the Sun and will begin moving away linearly after a time, causing a linear increase in Earth-Sun distance.*