

# CS6491-2015 P2: Swirl

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## 1 Objective

We want to compute the mathematical formulation and the implementation of Steady Interpolating Similarity  $SIS(F_a, t, F_b)$ . and validate our solution by starting with user controlled frames  $F_0$  and  $F_1$ , by extrapolating a steady pattern of frames  $F_i$  such that the transformation  $T$  between  $F_i$  and  $F_{i+1}$  is independent of  $i$ , and by showing (visually) that  $F_i = SIS(F_0, i/k, F_k)$  for all  $i$ .

## 2 Definitions of Entities

A 3D frame  $\{I, J, K, O\}$  is a similarity frame if all its factors have the same magnitude, if each is orthogonal to the other two, and if  $(I \times J) \cdot K > 0$ .

A Similarity Motion is an similarity frame  $F(t)$  parameterize by time.

The Steady Interpolating Similarity  $SIS(F_a, t, F_b)$  is a Similarity Motion, such that:

- $SIS(F_a, 0, F_b) = F_a$ ,
- $SIS(F_a, 1, F_b) = F_b$ ,
- And the similarity transformation from  $SIS(F_a, t, F_b)$  to  $SIS(F_a, t+u, F_b)$  is independent of  $t$ ,

Figure 1 shows some of the 3D swirls in nature and man-made structures.

## 3 Approach

Steady interpolating similarity motion is a combination of rotation around a fixed axis, translation along that axis, and scaling. There are many kinds of SIS motions, where rotation, translation, and scaling are different combinations. For example we have,



Figure 1: (a) Swirl in nature

### 3.1 Screw Motion

Screw motion is the combination of rotation and translation. Assume the rotation is around axis  $(F, N)$ , initial point  $P_0$ , unit time rotation angle  $\alpha$ , speed along the axis is constant  $d$ , we can describe the screw motion as:

$$P(t) = F + FP_0^o(t\alpha) + dtN \quad (1)$$

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### 3.2 Spiral Motion

Spiral motion is the combination of rotation and scaling. One common spiral motion is log spiral. Assume the the rotation is around axis  $(F, N)$ , initial point  $P_0$ , unit time rotation angle  $\alpha$ , scaling factor is exponential of  $m$ , we can describe the log spiral motion as:

$$P(t) = F + m^t FP_0^o(t\alpha) \quad (2)$$

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### 3.3 Swirl Motion

Swirl motion is the combination (“commutatie composition”) of log spiral motion around an axis and exponential translation along the axis. Assume the the rotation is around axis  $(F, N)$ , initial point  $P_0$ , unit time rotation angle  $\alpha$ , scaling factor is exponential of  $m$ , velocity along the axis is exponential of  $d$ , we can describe the log spiral motion as:

$$P(t) = F + m^t FP_0^o(t\alpha, N) + d^t N \quad (3)$$

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## 4 Implementation

### 4.1 Computing $N$

For computing the  $N$  from  $F_0$  and  $F_1$ , suppose we are given the input of triangle  $ABC$  and  $DEF$ , where  $I_0 = AB, K_0 = AB \times AC, J_0 = K_0 \times I_0, I_1 = DE, K_1 = DE \times DF, J_1 = K_1 \times I_1$ , we can

derive  $N$  as

$$N = \frac{\Delta I \times \Delta J + \Delta J \times \Delta K + \Delta K \times \Delta I}{3} \quad (4)$$

where  $\Delta I = I_1 - I_0$ ,  $\Delta J = J_1 - J_0$ ,  $\Delta K = K_1 - K_0$ .

## 4.2 Computing $F$

For computing the  $F$  from  $F_0$  and  $F_1$ , suppose we are given the input the same as above, we get

$$A = F + FA \quad (5)$$

$$D = F + FA^o(t\alpha, N)m^t + d^t N \quad (6)$$

Therefore we can derive  $F$  by

$$A^o(t\alpha, N)m^t + d^t N - D = F^o(t\alpha, N)m^t - F \quad (7)$$

where

$$d^t = FD \cdot N - FA \cdot N \quad (8)$$

$$m^t = \frac{FD - (FD \cdot N)N}{FA - (FA \cdot N)N} \quad (9)$$

And rotating an arbitrary point around an axis can be done by ([1]):

- (1) Translate space so that the rotation axis passes through the origin.
- (2) Rotate space about the z axis so that the rotation axis lies in the xz plane.
- (3) Rotate space about the y axis so that the rotation axis lies along the z axis.
- (4) Perform the desired rotation by about the z axis.
- (5) Apply the inverse of step (3).
- (6) Apply the inverse of step (2).
- (7) Apply the inverse of step (1).

## 4.3 Computing $F_i$ from $F_0$ and $F_1$ in 3D

If we know  $F$ ,  $F_0$ , and  $F_1$ , we know the swirl motion description  $P(t) = F + m^t F P_0^o(t\alpha, N) + d^t N$ , so we can interpolate each frame simply by changing  $t$  to  $2t, 3t, \dots, kt$ .

## 5 Analysis

## 6 Discussion

## References

- [1] Glenn Murray, *Rotation About an Arbitrary Axis in 3 Dimensions*, [http://inside.mines.edu/fs\\_home/gmurray/ArbitraryAxisRotation/](http://inside.mines.edu/fs_home/gmurray/ArbitraryAxisRotation/).