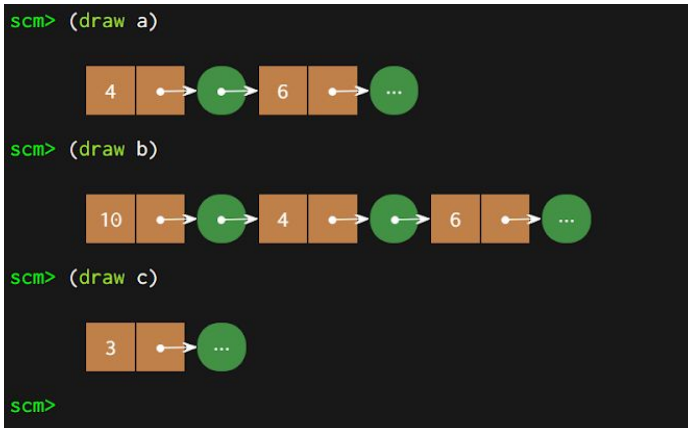
Streams

1. Streams WWSD

```
scm> (define a (cons-stream 4 (cons-stream 6 (cons-stream 8 a))))
scm> (car a)
4
scm> (cdr a)
#[promise (not forced)]
scm> (cdr-stream a)
(6 . #[promise (not forced)])
scm> (define b (cons-stream 10 a))
scm> (cdr b)
#[promise (not forced)]
scm> (cdr-stream b)
(4 . #[promise (forced)])
scm> (define c (cons-stream 3 (cons-stream 6)))
scm> (cdr-stream c)
Error: too few operands in form
```

What elements of a, b, and c have been evaluated thus far?



2. Write a function merge that takes in two sorted infinite streams and returns a new infinite stream containing all the elements from both streams, in sorted order.

```
(define (merge s1 s2)
      (if (null? s1)
            s2
            (if (<= (car s1) (car s2))
                  (cons-stream (car s1) (merge (cdr-stream s1) s2))
                  (cons-stream (car s2) (merge s1 (cdr-stream s2)))
            )
      )
)

; Alternate solution
(define (merge s1 s2)
      (if (null? s1)
            s2
            (if (<= (car s1) (car s2))
                  (cons-stream (car s1) (merge (cdr-stream s1) s2))
                  (merge s2 s1)
            )
      )
)
```

3. Write a function `half_twos_factorial` that returns a new stream containing all of the factorials that contain the digit 2 divided by two. Your solution must use only the following functions, without defining any additional ones. Likewise, any lambda expressions should contain only calls to the following functions or built in functions.

```
; Returns a new Stream where each new value is the result of calling
; fn on the value in the stream s
```

```
(define (map-stream s fn)
      (if (null? s) s
            (cons-stream (fn (car s)) (map-stream (cdr-stream s)
fn))))

; Returns a new Stream containing all values in the stream s that
; satisfy the predicate fn
(define (filter-stream s fn)
      (cond ((null? s) s)
            ((fn (car s)) (cons-stream (car s) (filter-stream
(cdr-stream s) fn)))
            (else (filter-stream (cdr-stream s) fn))))


; Returns True if n contains the digit 2. False otherwise
(define (contains-two n)
      (cond ((= n 0) #f)
            ((= (remainder n 10) 2) #t)
            (else (contains-two (quotient n 10)))))

; Returns the factorial n
(define (factorial n)
      (if (= n 0) 1 (* n (factorial (- n 1)))))

; Returns a stream of factorials
(define (factorial-stream)
      (define (helper n)
            (cons-stream (factorial n) (helper (+ n 1))))
      (helper 1))
```

Fill in the skeleton below.

```
(define (half-twos-factorial)
      (map-stream (filter-stream (factorial-stream) contains-two)
                  (lambda (x) (quotient x 2))))
```

Tail Recursion

1. For the following procedures, determine whether or not they are tail recursive. If they are not, write why they aren't and rewrite the function to be tail recursive to the right.

```
; Multiplies x by y
```

```
(define (mult x y)
     (if (= 0 y)
          0
          (+ x (mult x (- y 1)))))
```

```
(define (mult x y)
     (define (helper x y total)
          (if (= 0 y)
               total
               (helper x (- y 1) (+ total x))))
     (helper x y 0))
```

Notn tail recursive-after evaluating the recursive call, we still need to apply '+', so evaluating the recursive call is not the last thing we do in the frame. (Review non-tail recursive factorial, the reason that is not tail recursive applies to this procedure)

```
; Always evaluates to true
; assume n is positive
(define (true1 n)
     (if (= n 0)
          #t
          (and #t (true1 (- n 1)))))
```

Tail recursive--the recursive call to "true1" is the final sub-expression of the `and` special form. Therefore, we will not need to perform any additional work after getting the result of the recursive call.

```
; Always evaluates to true
; assume n is positive
(define (true2 n)
     (if (= n 0)
          #t
          (or (true2 (- n 1)) #f)))
```

```
(define (true2 n)
     (if (= n 0)
          #t
          (true2 (- n 1))))
```

Not tail recursive--the recursive call to "true2" is not the final sub-expression of the `or` special form. Even though it will always evaluate to `true` and short-circuit, the interpreter does not take that into account when determining whether to evaluate it in a tail context or not.

```
; Returns true if x is in lst
(define (contains lst x)
      (cond ((null? lst) #f)
            ((equal? (car lst) x) #t)
            ((contains (cdr lst) x) #t)
            (else #f)))
```

Not tail recursive--the recursive call to "contains" is in a predicate sub-expression. That means we will have to evaluate another expression if it evaluates to true, so it is not the final thing we evaluate.

```
(define (contains lst x)
      (cond ((null? lst) #f)
            ((equal? (car lst) x) #t)
            (else (contains (cdr lst) x))))
```

2. Rewrite this function tail-recursively.

```
; Returns a list of pairs, the ith pair has item as its car and the
; ith element of lst as its cdr
(define (add-to-all item lst)
  (if (null? lst)
    lst
    (cons (cons item (car lst))
          (add-to-all item (cdr lst)))))
```

```
(define (add-to-all item lst)
      (define (helper item lst so-far)
            (if (null? lst)
            so-far
            (helper item (cdr lst) (append so-far (list (cons item
(car lst)))))))
      (helper item lst nil))
```

3. Implement `sum-satisfied-k` which, given an input list lst, a predicate procedure `f` which takes in one argument, and an integer `k`, will return the sum of the first `k` elements that satisfy `f`. If there are not `k` such elements, return 0.

```
; Doctests
scm> (define lst `(1 2 3 4 5 6))
scm> (sum-satisfied-k lst even? 2)  ; 2 + 4
6
scm> (sum-satisfied-k lst (lambda (x) (= 0 (modulo x 3))) 10)
0
scm> (sum-satisfied-k lst (lambda (x) #t) 0)
0
```

Implement `sum-satisfied-k` tail recursively.

```
(define (sum-satisfied-k lst f k)
      (define (sum-helper lst k total)
            (cond ((= 0 k) total)
                  ((null? lst) 0)
                  ((f (car lst)) (sum-helper (cdr lst) (- k 1)
                                             (+ total (car lst))))
                  (else (sum-helper (cdr lst) k total))))
      (sum-helper lst k 0))
```

4. Implement remove-range which, given one input list `lst`, and two nonnegative integers `i` and `j`, returns a new list containing the elements of `lst` in order, without the elements from index `i` to index `j` inclusive. For example, given the list `(0 1 2 3 4)`, with `i = 1` and `j = 3`, we would return the list `(0 4)`. You may assume `j > i`, and `j` is less than the length of the list. (Hint: you may want to use the built-in `append` function, which returns the result of appending the items of all lists in order into a single well-formed list.)

```
; Doctests
scm> (remove-range '(0 1 2 3 4) 1 3)
(0 4)

(define (remove-range lst i j)
      (define (helper lst index)
            (cond ((> index j) lst)
                  ((>= index i) (helper (cdr lst) (+ index 1)))
                  (else (cons (car lst) (helper (cdr lst) (+ index
1))))))
      (helper lst 0))
```

Now implement `remove-range` tail recursively.

```
(define (remove-range lst i j)
      (define (remove-tail lst index so-far)
            (cond ((> index j) (append so-far lst))
                  ((>= index i)
                        (remove-tail (cdr lst) (+ index 1) so-far))
                  (else (remove-tail
                              (cdr lst)
                              (+ index 1)
                              (append so-far (list (car lst)))))))
      (remove-tail lst 0 nil)))
```

Interpreters

1. For the following questions, circle the number of calls to scheme_eval and the number of calls to scheme_apply:

```
scm> (+ 1 2)
3
```

Calls to `scheme_eval`:	1 | 3 | 4 | 6

Calls to `scheme_apply`:	1 | 2 | 3 | 4

1. Evaluate entire expression
    a. Evaluate +
    b. Evaluate 1
    c. Evaluate 2
    d. **Apply** + to 1 and 2

```
scm> (if 1 (+ 2 3) (/ 1 0))
5
```

Calls to `scheme_eval`:	1 | 3 | 4 | 6
Calls to `scheme_apply`:	1 | 2 | 3 | 4

1. Evaluate entire expression
    a. Evaluate predicate 1
    b. Since 1 is true, evaluate <if-true> sub-expression
        i. Evaluate +
        ii. Evaluate 2
        iii. Evaluate 3
        iv. **Apply** + to 2 and 3

Note that we never needed to evaluate the `if` because it's one of our special forms!

```
scm> (or #f (and (+ 1 2) 'apple) (- 5 2))
apple
```

Calls to `scheme_eval`:	6 | 8 | 9 | 10
Calls to `scheme_apply`:	1 | 2 | 3 | 4

1. Evaluate entire expression
    a. Evaluate first sub-expression of or
    b. Evaluate second sub-expression of or
        i. Evaluate first sub-expression of and

1. Evaluate +
2. Evaluate 1
3. Evaluate 2
4. **Apply** + to 1 and 2

ii. Evaluate 'apple

iii. Since the and expression evaluates to true, we short circuit

Note that we never needed to evaluate the `or` or the `and` because they are special forms!

```
scm> (define (add x y) (+ x y))
add
scm> (add (- 5 3) (or 0 2))
2
```

Calls to `scheme_eval`:	12 | 13 | 14 | 15
Calls to `scheme_apply`:	1 | 2 | 3 | 4

1. Evaluate entire define expression

2. Evaluate call to add function
    a. Evaluate add function
    b. Evaluate first argument
        i. Evaluate -
        ii. Evaluate 5
        iii. Evaluate 3
        iv. **Apply** - to and 3
    c. Evaluate second argument (the or expression)
        i. Evaluate 0
        ii. Since 0 is #t, we short circuit
    d. **Apply** add function to 2 and 0 (enter body of the function)
        i. Evaluate the body of the function
            1. Evaluate +
            2. Evaluate x to be 2
            3. Evaluate y to be 0
            4. **Apply** + to 2 and 0

Macros

Question 0

What will Scheme output? If you think it errors, write Error.

```
scm> (define-macro (doierror) (/ 1 0))
doierror
scm>(doierror)
Error
scm> (define x 5)
x
```

```
scm> (define-macro (evaller y) (list (list 'lambda '(x) x)) y)
evaller
scm> (evaller 2)
2
```

Question 1

Consider a new special form, **when**, that has the following structure:
(when <condition
        <expr1> <expr2> <expr3> ...)
If the condition is not false (a truthy expression), all the subsequent operands are evaluated in order and the value of the last expression is returned. Otherwise, the entire **when** expression evaluates to *okay*.

```
scm> (when (= 1 0)(/1 0) 'error)
okay
scm> (when (= 1 1) (print 6) (print 1) 'a)
6
1
a
```

Create this new special form using a macro. Recall that putting a dot before the last formal parameter allows you to pass any number of arguments to a procedure, a list of which will be bound to the parameter, similar to (*args) in Python.
a) Fill in the skeleton below to implement this without using quasiquotes.
(define-macro (when condition . exprs)
(list 'if `condition (cons 'begin exprs) ''okay`))

b) Now, implement the macro using quasiquotes.
(define-macro (when condition . exprs)
`(if ,condition ,(cons 'begin exprs) 'okay))

Question 2

Define a macro while that processes a while loop by converting it to a tail recursive function
The goal of this question is to define a macro that represents a while loop. Since this is a difficult task we will break it into parts.

2a
Write tail-recursive factorial:

```
(define (fact n)
      (define (fact-tail n result)
            (if (= n 0)
            result
```

```
            (fact-tail (- n 1) (* n result))))
      (fact-tail n 1)
  )
```

2b

Using the above problem to assist implementation, create the while macro. This macro will accept 4 arguments:

- initial-bindings: this will represent initialization values for variables in the loop
- condition: this will represent the condition which the while loop should continue to check to see if the loop should continue
- return: after the loop has ended this represents the value that should be returned

You may find the built-in map function useful for this problem:

```
scm > (map (lambda (x) (* 2 x)) '(1 2 3))
(2 4 6)
```

And here's an example of the while macro being used to calculate the factorial:

```
scm > (define (fact n)
      (while
            ((acc 1) (n n))
            (> n 0)
            ((* acc n) (- n 1))
            acc))
fact
scm> (fact 4)
24
```

Fill in the following macro definition:

```
(define (cadr lst) (car (cdr lst)))

(define-macro (while initial-bindings condition updates return)
    (define helper-vars (map car initial-bindings))
    (define initial-vals (map cadr initial-bindings))
    (list 'begin
        (list 'define (cons 'helper helper-vars)
            `(if ,condition
                ,(cons 'helper updates)
                ,return))
        (cons 'helper initial-vals)))
```

CONGRATULATIONS!

You made it to the end of the worksheet! Great work :)