2 Airport Revisited

(a) $\frac{n}{4}$

Let X_n denote the number of empty airports after all planes have landed. Then, we can first write

$$X_n = I_1 + I_2 + \cdots + I_n$$

where $I_i = 1$ if neither of the planes from airports $i-1, i+1$ landed at airport i (i.e. both chose the other direction); and $I_i = 0$ otherwise.

Then, specifically, $\mathbb{E}[I_i] = 0 \cdot \mathbb{P}[I_i = 0] + 1 \cdot \mathbb{P}[I_i = 1] = \mathbb{P}[I_i = 1] = \mathbb{P}[$both planes next to airport i chose the other direction$] = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Thus, using Theorem 15.1, we have that $\mathbb{E}[X_n] = \mathbb{E}[I_1] + \mathbb{E}[I_2] + \cdots + \mathbb{E}[I_n] = \frac{1}{4} \cdot n = \frac{n}{4}$

(b) $\sum_{i=1}^{n} \prod_{a \in N(i)} \frac{\deg(a)-1}{\deg(a)}$

We proceed with a similar logic as part (a). Let X_n denote the number of empty airports after all planes have landed. Then, we can first write

$$X_n = I_1 + I_2 + \cdots + I_n$$

where $I_i = 1$ if none of the planes from $N(i)$ landed at airport i (i.e. all chose the other direction); and $I_i = 0$ otherwise.

Then, specifically, $\mathbb{E}[I_i] = 0 \cdot \mathbb{P}[I_i = 0] + 1 \cdot \mathbb{P}[I_i = 1] = \mathbb{P}[I_i = 1] = \mathbb{P}[$all planes from $N(i)$ chose another neighbor of theirs$] = \prod_{a \in N(i)} \frac{\deg(a)-1}{\deg(a)}$

Thus, using Theorem 15.1, we have that $\mathbb{E}[X_n] = \mathbb{E}[I_1] + \mathbb{E}[I_2] + \cdots + \mathbb{E}[I_n] =$

$$\sum_{i=1}^{n} \prod_{a \in N(i)} \frac{\deg(a) - 1}{\deg(a)}$$