I worked alone without getting any help, except asking questions on Piazza and reading the Notes of this course.

1 Modular Arithmetic Solutions

(a) $x = 10$

Since $2, 15 \in \mathbb{Z}^+$ and $\gcd(2,\ 15) = 1$, using Theorem 6.2, we have that 2 has a multiplicative inverse, $2^{-1} \pmod{15}$, and it is unique (in the modular setting). Since we have that $2 * 8 = 16 \equiv 1 \pmod{15}$, so $2^{-1} = 8 \pmod{15}$. Then, to compute the solution to $2x \equiv 5 \pmod{15}$, we have that $x = 2^{-1} * 5 = 8 * 5 = 40 \equiv 10 \pmod{15}$, and this solution would be unique modulo 15 as well. Thus, $x = 10$ is the only solution.

(b) No solution.

We proceed by contradiction. Assume that $x \in \mathbb{Z}$ is a solution to the equation, such that $2x = y \equiv 5 \pmod{16}$, so $y \in \mathbb{Z}$.

Since $x \in \mathbb{Z}$, so we have that $2x$ is an even number. On the other hand, consider the right side of the equation, since $y \equiv 5 \pmod{16}$, so $y = 16k + 5, k \in \mathbb{Z}$. Then, since $y = 16k + 5 = 2(8k + 2) + 1$ where $8k + 2 \in \mathbb{Z}$, so we have that y is an odd number, which implies that $x \neq y$, and we reach a contradiction. Therefore, we conclude that there is no solution to this equation.

(c) $x = 2, 7, 12, 17, 22$

Let x be a solution to the equation. By definition of modular arithmetic, so we have that $0 \leq x < 25$ and $x \in \mathbb{N}$. Using the given equation, let $5x = y \equiv 10 \pmod{25}$, so $y \in \mathbb{N}$. Then, let $y = 25k + 10$, so $k \in \mathbb{N}$. Now, we have that $5x = 25k + 10$, and dividing both sides by 5, we would get $x = 5k + 2$. Since $x < 25$, so $x = 5k + 2 < 25$, so $k < \frac{23}{5}$. Since $k \in \mathbb{N}$, so k can only be $= 0$, 1, 2, 3 and 4, with x being 2, 7, 12, 17 and 22, respectively. Thus, the only solutions are $x = 2, 7, 12, 17, 22$.