3 Propositional Practice

(a) $(\exists x \in \mathbb{R})\ (x \notin \mathbb{Q})$

True.
Consider $x = \pi$. $\pi \in \mathbb{R}$, and $\pi \notin \mathbb{Q}$, so the proposition is true.

(b) $(\forall x \in \mathbb{Z})\ \Big(\big((x \in \mathbb{N}) \vee (x < 0)\big) \wedge \big(\neg\big((x \in \mathbb{N}) \wedge (x < 0)\big)\big)\Big)$

True.
Let $x \in \mathbb{Z}$, so $x >= 0$ or $x < 0$, but not both.
If $x >= 0$, then x is a natural number; if $x < 0$, then x is negative; x can't be both.
Thus, the proposition is true.

(c) $(\forall x \in \mathbb{N})\ \big((6 \mid x) \implies ((2 \mid x) \vee (3 \mid x))\big)$

True.
Let $x \in \mathbb{N}$, $x = 6 * k$, so $k \in \mathbb{N}$
So $x = 2 * (3k)$ where $3k \in \mathbb{N}$, which means that $2 \mid x$
So $((2 \mid x) \vee (3 \mid x))$ is true, which means that the proposition is true.

(d) All real numbers are complex numbers.

True.
Let $x \in \mathbb{R}$, so $x = x + 0 * i$, and since $x, 0 \in \mathbb{R}$,
So by definition of complex numbers, x is a complex number.

(e) If an integer is divisible by 2 or is divisble by 3, then it is divisible by 6.

False.
Consider $x = 2$, so x is an integer.
Since x is divisible by 2, so it is divisible by 2 or by 3.
However, there's no such integer a such that $2 * a = 6$
So by definition, x is not divisble by 6, so the proposition is false.

(f) If a natural number is greater than 7, then it can be expressed as the sum of two natural numbers.

True.
Let $x \in \mathbb{N}, x > 7$
Consider $a = 0, b = x$, so $a, b \in \mathbb{N}$
Thus, since $a + b = 0 + x = x$, so the proposition is true.