Quiz 17 Solutions

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This quiz does not count towards your grade. It exists to simply gauge your understanding. Treat this as though it were a portion of your midterm or final exam. "Intuition Practice" might be tricky; watch out for subtleties. "Proofs" will be challenging to start; develop an arsenal of *approaches* to starting a problem.

1 Intuition Practice

In this problem, consider three random variables X, Y, and Z, where X is the outcome of a dice roll, Y is the outcome of another dice roll, and $Z = Y + Z$ (Z be the sum of the two dice roll outcomes).

For each question, circle "True" or "False," and provide a brief justification.

1. (True or False) $E[Z] = 7$

   Solution: True

$$E[Z] = E[X + Y]$$
$$= E[X] + E[Y]$$
$$= \frac{7}{2} + \frac{7}{2}$$
$$= 7$$

2. (True or False) $E[X^2] = \frac{91}{6}$

   Solution: True.

$$E[X^2] = \sum_x x^2 Pr[X = x]$$
$$= (1^2)(\frac{1}{6}) + (2^2)(\frac{1}{6}) + ...(6^2)(\frac{1}{6})$$
$$= \frac{1}{6}(1^2 + 2^2 + ...6^2)$$

$$= \frac{91}{6}$$

3. (True or False) $E[Z^2] < 49$

Solution: **True** **False**

First, $E[Z^2]$ is not guaranteed to be 49. $E[Z^2] \neq E[Z]^2$ *necessarily*. Whereas it is possible, it is not guaranteed. Whereas the intuition is thus to say "False," we have to compute the actual value of $E[Z^2]$ to see if the bound is valid.

$$\begin{aligned} E[Z^2] &= E[(X+Y)^2] \\ &= E[X^2 + 2XY + Y^2] \\ &= E[X^2] + 2E[XY] + E[Y^2] \end{aligned}$$

We will compute each term separately, to provide clarity.

First, we compute $2E[XY]$. Since, X and Y are independent, we can rewrite this as

$$2E[X]E[Y] = 2(\frac{7}{2})(\frac{7}{2}) = \frac{49}{2}$$

Second, we compute $E[X^2]$. This is given in the previous part.

Third, note that $E[X^2] = E[Y^2]$, since these are both dice rolls. Thus,

$$\begin{aligned} &E[X^2] + 2E[XY] + E[Y^2] \\ &= \frac{91}{6} + \frac{49}{2} + \frac{91}{6} \\ &= \frac{231}{6} = \frac{77}{2} \end{aligned}$$

= (91+147+91)/6 = 329/6 > 49

This is 38.5, which is less than 49.