5 Clothing Argument

(a) $10,000$

Based on the given information, we can use the First Rule of Counting, so there are $10 \cdot 10 \cdot 10 \cdot 10 = 10,000$ distinct outfits.

(b) 600

Since we want to wear exactly 2 out of the 4 categories, so we first use the Second Rule of Counting, and we're sampling without replacement, which gives us $\frac{4!}{(4-2)! \cdot 2!} = \frac{24}{2 \cdot 2} = 6$ ways.

Thus, we can then use the First Rule of Counting, multiplying by the previous number, so there are $6 \cdot 10 \cdot 10 = 600$ distinct outfits when choosing from exactly two categories.

(c) 5040

Since we want to pick exactly 4 out of the 10 hats, so we first use the Second Rule of Counting, and we're sampling without replacement, which gives us $\binom{10}{4} = \frac{10!}{(10-4)! \cdot 4!} = 210$ ways.

Thus, we can then use the First Rule of Counting to line them up on a wall, and we're sampling without replacement, multiplying by the previous number, so there are $210 \cdot 4! = 5040$ of hanging four of the ten hats.

(d) 210

Since we want to pack exactly 4 out of the 10 hats, so we use the Second Rule of Counting, and we're sampling without replacement, which gives us $\binom{10}{4} = \frac{10!}{(10-4)! \cdot 4!} = 210$ different ways.

We can explain this number in terms of our answer in part (c) since there exists an 24-to-1 function f from hanging hats (part c) to packing hats (part d) as we discussed in the second paragraph of part (c), which shows that after picking out the 4 out of 10 hats, then there're $4! = 24$ ways for hanging them up on a wall. Thus, f will map 24 elements in the domain (the set of hanging ordered 4-hat subsets of the 10 hats) of the function to 1 element in the range (the set of packing 4-hat subsets of the 10 hats) of the function, so this number is $\frac{1}{24}$ of our answer to part (c), which is correct with $5040 \cdot \frac{1}{24} = 210$.

(e) 10

Since we want to pack exactly 3 hats, and for each color, we have three or more of them (that are indistinguishable), so this is a problem of sampling with replacement, but where order does not matter. We are picking 3 hats out of 3 different colors of hats, and using the formula from Note 12, there are $\binom{3+3-1}{3} = \frac{5!}{(5-3)! \cdot 3!} = \frac{120}{2 \cdot 6} = 10$ distinct sets of 3-hat subsets.