3 Easy A's

$\mathbb{E} = 35; \text{var} = 25$

Since each of the first 3 problems (and the last 4 problems) are graded independently with the same range, expectation and variance, so the two sets are i.i.d., which implies that I could calculate desired results separately. Let the grading of one of the first three problems be I_1 and let the grading of one of the last four problems be I_2, and denote my total score as the random variable X, where $X = 3I_1 + 4I_2$. As analyzed earlier, we have that the I_i's are mutually independent:

Using given information, we have that $\mathbb{E}[I_i] = 5$ and $\text{var}(I_i) = 1$. Thus, using results from the Notes (Theorem 15.1 and 16.3), we have that:

$$\mathbb{E}[X] = \mathbb{E}[3I_1 + 4I_2] = 3\mathbb{E}[I_1] + 4\mathbb{E}[I_2] = 3 \cdot 5 + 4 \cdot 5 = 35$$

$$\text{var}[X] = \text{var}[3I_1 + 4I_2] = \text{var}[3I_1] + \text{var}[4I_2] = 3^2\text{var}[I_1] + 4^2\text{var}[I_2] = 9 \cdot 1 + 16 \cdot 1 = 25$$

Now, using Chebyshev's inequality, we can calculate that:

$$\mathbb{P}\big[|X - \mathbb{E}[X]| \geq 25\big] \;\leq\; \frac{\text{var}(X)}{25^2}$$

$$\implies \mathbb{P}\big[|X - 35| \geq 25\big] \;\leq\; \frac{25}{625} = \frac{1}{25} < 5\%$$

In other words, I have less than a 5% chance of getting an A when the grades are randomly chosen this way. Q.E.D.