

## 1 Diversify Your Hand

You are dealt 13 cards from a standard 52 card deck. Let  $X$  be the number of distinct values in your hand (The 13 possible values are Ace, 2, 3, 4, ..., Jack, Queen, King). For instance, the hand (A, A, A, 2, 3, 4, 4, 5, 7, 9, 10, J, J) has 9 distinct values.

Calculate  $E[X]$ .

## 2 Combining Distributions

(a) Let  $X \sim \text{Pois}(\lambda)$ ,  $Y \sim \text{Pois}(\mu)$  be independent. Prove that  $X + Y \sim \text{Pois}(\lambda + \mu)$ .

*Hint:* Recall the binomial theorem, which states that

$$(a + b)^n = \sum_{i=0}^n \binom{n}{i} a^i b^{n-i}.$$

(b) Let  $X$  and  $Y$  be defined as in the previous part. Prove that the distribution of  $X$  conditional on  $X + Y$  is a binomial distribution, e.g. that  $X|X + Y$  is binomial. What are the parameters of the binomial distribution?

*Hint:* Your result from the previous part will be helpful.

## 3 Condition on an Event

The random variable  $X$  has the PDF

$$f_X(x) = \begin{cases} cx^{-2}, & \text{if } 1 \leq x \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Determine the value of  $c$ .

(b) Let  $A$  be the event  $\{X > 1.5\}$ . Calculate  $\mathbb{P}(A)$  and the conditional PDF of  $X$  given that  $A$  has occurred.

## 4 Bus Arrivals

Buses arrive at a bus stop according to a Poisson Arrival Process with  $\lambda = 20$  minutes starting time 0.

- (a) What's the probability there are no arrivals between times 1 and 3?
- (b) What's the probability that the fourth bus arrives within 5 minutes of the third?
- (c) You arrive at the bus stop some time later. What is the expected amount of time you wait?

## 5 Vegas

On the planet Vegas, everyone carries a coin. Many people are honest and carry a fair coin (heads on one side and tails on the other), but a fraction  $p$  of them cheat and carry a trick coin with heads on both sides. You want to estimate  $p$  with the following experiment: you pick a random sample of  $n$  people and ask each one to flip his or her coin. Assume that each person is independently likely to carry a fair or a trick coin.

- 1. Given the results of your experiment, how should you estimate  $p$ ?
- 2. How many people do you need to ask to be 95% sure that your answer is off by at most 0.05?

## 6 Three Tails

You flip a fair coin until you see three tails in a row. What is the average number of heads that you'll see until getting  $TTT$ ?