2 Binomial Beads

(a) $\binom{n}{k}$

First we'll make all the beads unique, which means that there are $n!$ unique keychains by such definition, and then we consider all the duplicates as the blue/golden beads are all the same, which gives us $\frac{n!}{k!(n-k!)} = \binom{n}{k}$ unique keychains.

(b) $x^k y^{n-k}$

By definition given on the question, the price of a keychain with exactly k blue beads and thus $n-k$ gold beads is: $x^k y^{n-k}$

(c) $\sum\limits_{0}^{n} \binom{n}{k} x^k y^{n-k}$

Using our results from parts (a) and (b), his total revenue is:

$$\sum_{0}^{n} \binom{n}{k} x^k y^{n-k}$$

(d)

On the one hand $(x+y)^n$ is the sum of all different combination of the product of choosing a total of n numbers from x and y, sampling with replacement.

On the other hand, $\sum\limits_{0}^{n} \binom{n}{k} x^k y^{n-k}$ is the sum of: the product of 0 x's and n y's, the product of 1 x's and $(n-1)$ y's, the product of 2 x's and $(n-2)$ y's, $\ldots$, the product of $(n-1)$ x's and 1 y's, and the product of n x's and 0 y's. Thus, $\sum\limits_{0}^{n} \binom{n}{k} x^k y^{n-k}$ also represents the sum of all the different combinations of the product of choosing a total of n numbers from x and y, sampling with replacement, which gives the connection between that and $(x+y)^n$.