I worked with Jessica (jexicagjr@berkeley.edu), mainly on Q3.

1 Family Planning

(a)

The sample space is $(G, C) = \{(1,1), (1,2), (1,3), (0,3)\}$, where we can calculate the probability of each sample to be:

$\mathbb{P}[(1,1)] = \frac{1}{2}$ since it just represents the probability of their first child being a girl.

$\mathbb{P}[(1,2)] = \frac{1}{4}$ i.e. the probability of first child being a boy and second child being a girl.

$\mathbb{P}[(1,3)] = \frac{1}{8}$ with similar logic.

$\mathbb{P}[(0,3)] = \frac{1}{8}$ with similar logic.

(b)

	$C=1$	$C=2$	$C=3$
$G=0$	0	0	$\frac{1}{8}$
$G=1$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

(c)

$\mathbb{P}(G=0)$	$\frac{1}{8}$
$\mathbb{P}(G=1)$	$\frac{7}{8}$

$\mathbb{P}(C=1)$	$\mathbb{P}(C=2)$	$\mathbb{P}(C=3)$
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

The probability of the Browns having 0 girls is equivalent to them having 3 boys in a row, which is $\mathbb{P}(G=0) = (\frac{1}{2})^3 = \frac{1}{8}$, so we have $\mathbb{P}(G=1) = \mathbb{P}(\overline{G=0}) = \frac{7}{8}$.

Results confirmed since we could calculate the probability of them having 1 child, 2 children, 3 children, respectively could be done in a similar way to get: $\mathbb{P}(C=1) = \frac{1}{2}\mathbb{P}(C=2) = \frac{1}{4}\mathbb{P}(C=3) = \frac{1}{4}$, which confirms our result.

(d) No, they aren't.

Consider the case when the Browns have 0 girls and 3 children in total, so we have $\mathbb{P}(G=0, C=3) = \frac{1}{8}$. On the other hand, $\mathbb{P}(G=0)\mathbb{P}(C=3) = \frac{1}{8} \cdot \frac{1}{4} = \frac{1}{32}$, which gives that $\mathbb{P}(G=0, C=3) \neq \mathbb{P}(G=0)\mathbb{P}(C=3)$, which implies that G and C aren't independent.

(e) $\mathbb{E}[G] = \frac{7}{8}, \mathbb{E}[C] = \frac{7}{4}$

We can calculate that:

$$\mathbb{E}(G) = \frac{1}{8} \cdot 0 + \frac{7}{8} \cdot 1 = \frac{7}{8}$$

$$\mathbb{E}(C) = \frac{1}{2} \cdot 1 + \frac{1}{4} \cdot 2 + \frac{1}{4} \cdot 3 = \frac{7}{4}$$