4 Proof by?

(a)

We proceed by contradiction. Assume that the proposition is false, which means that for some $x, y \in \mathbb{Z}, (10 \nmid xy)$, and that $((10 \mid x)$ or $(10 \mid y))$. Let our assertion R state that $10 \nmid xy$.

Without loss of generality, let $10 \mid x$, so let $x = 10k,\ k \in \mathbb{Z}$. So $xy = 10ky = 10(ky)$ where $ky \in \mathbb{Z}$, which by definition, means that $10 \mid xy$. This implies $\neg R$.

We conclude that $R \wedge \neg R$ holds; thus, we have a contradiction, as desired.

Q.E.D.

I used Proof by Contradiction.

(b)

Prove. The contrapositive proposition is: $(\forall x, y \in \mathbb{Z})\ ((10 \mid x) \vee (10 \mid y)) \implies (10 \mid xy)$.

As shown in part (a) above, if $((10 \mid x) \vee (10 \mid y))$, then $10 \mid xy$. Thus, the contrapositive is true.

Q.E.D.

(c)

Disprove. The converse proposition is: $(\forall x, y \in \mathbb{Z})\ ((10 \nmid x) \wedge (10 \nmid y)) \implies (10 \nmid xy)$.

Consider $x = 2, y = 5$, so $x, y \in \mathbb{Z}$. Since there's no such integer m, n such that $10 * m = 2$ or $10 * n = 5$, so by definition, x, y is not divisible by 10. So $((10 \nmid x) \wedge (10 \nmid y))$ is true.

Now, $xy = 2 * 5 = 10 = 1 * 10$ where $1 \in \mathbb{Z}$, so by definition, $10 \mid xy$. So $(10 \nmid xy)$ is false.//[.1cm] So we have $True \implies False$, which shows that the converse is false.

Q.E.D.