

5 Weathermen

(a) 60.9%

We represent snowing as “S”, not snowing as “NS”, Tom predicting snow as “TS”, Tom predicting no snowing as “TNS”. So, using the given information, we have that the probability of actually snowing in New York is $\mathbb{P}[S] = 0.1$, and $\mathbb{P}[TS|S] = 0.7$, $\mathbb{P}[TNS|NS] = 0.95$.

Since days that snow and days that don’t snow form a partition, and the days that Tom predicts to snow and the days that Tom predicts to not snow form another partition, so we have that

$$\mathbb{P}[NS] = 1 - \mathbb{P}[S] = 0.9$$

$$\mathbb{P}[TNS|S] = \mathbb{P}[\overline{TS}|S] = 1 - \mathbb{P}[TS|S] = 0.3$$

$$\mathbb{P}[TS|NS] = \mathbb{P}[\overline{TNS}|NS] = 1 - \mathbb{P}[TNS|NS] = 0.05$$

So, $\mathbb{P}[TS] = \mathbb{P}[TS|S]\mathbb{P}[S] + \mathbb{P}[TS|\overline{S}](1 - \mathbb{P}[S]) = \mathbb{P}[TS|S]\mathbb{P}[S] + \mathbb{P}[TS|NS](1 - \mathbb{P}[S]) = 0.7 \cdot 0.1 + 0.05 \cdot 0.9 = 0.115$

Thus, this problem is asking for the probability $\mathbb{P}[S|TS] = \frac{\mathbb{P}[TS|S] \cdot \mathbb{P}[S]}{\mathbb{P}[TS]} = \frac{0.7 \cdot 0.1}{0.115} = 0.609 = 60.9\%$, which means that if Tom says that it is going to snow, then the probability it will actually snow is 60.9%.

(b) 92.5%

We first have that $\mathbb{P}[TNS] = \mathbb{P}[\overline{TS}] = 1 - \mathbb{P}[TS] = 1 - 0.115 = 0.885$. Then, using the same strategy, we could calculate Tom’s accuracy when he predicts no snow:

$$\mathbb{P}[NS|TNS] = \frac{\mathbb{P}[TNS|NS] \cdot \mathbb{P}[NS]}{\mathbb{P}[TNS]} = \frac{0.95 \cdot 0.9}{0.885} = 0.966$$

Thus, $\mathbb{P}[\text{Tom}] = \mathbb{P}[S|TS] \cdot \mathbb{P}[TS] + \mathbb{P}[S|\overline{TS}](1 - \mathbb{P}[TS]) = \mathbb{P}[S|TS] \cdot \mathbb{P}[TS] + \mathbb{P}[S|TNS](1 - \mathbb{P}[TS]) = 0.609 \cdot 0.115 + 0.966 \cdot 0.885 = 0.925 = 92.5\%$, which means that Tom’s overall accuracy is 92.5%.

(c) Example

Consider this situation for Jerry: Let Alaska be snowy 90% of the days, and that she predicts snow correctly 75% of the actual snowy days, and she predicts no snow (sunny) correctly 100% of the actual no snow (sunny) days.

So for Jerry, $\mathbb{P}[S] = 0.9$, $\mathbb{P}[JS|S] = 0.75 > \mathbb{P}[TS|S] = 0.7$, $\mathbb{P}[JNS|NS] = 1 > \mathbb{P}[TNS|NS] = 0.95$, which means that Jerry is indeed better than Tom at predicting snow on snowy days and sun on sunny days. Yet, let’s now calculate Jerry’s overall accuracy.

Using similar logic as we did in parts (a) and (b), so

$$\mathbb{P}[NS] = \mathbb{P}[\overline{S}] = 1 - \mathbb{P}[S] = 0.1$$

$$\mathbb{P}[JNS|S] = \mathbb{P}[\overline{JS}|S] = 1 - \mathbb{P}[JS|S] = 1 - 0.75 = 0.25$$

$$\mathbb{P}[JS|NS] = \mathbb{P}[\overline{JNS}|NS] = 1 - \mathbb{P}[JNS|NS] = 1 - 1 = 0$$

Thus, we have that: $\mathbb{P}[JS] = \mathbb{P}[JS|S] \cdot \mathbb{P}[S] + \mathbb{P}[JS|NS] \cdot \mathbb{P}[NS] = 0.75 \cdot 0.9 + 0 \cdot 0.1 = 0.675$, which means that when Jerry says it's going to snow, the probability that it will actually snow is $\mathbb{P}[S|JS] = \frac{\mathbb{P}[JS|S] \cdot \mathbb{P}[S]}{\mathbb{P}[JS]} = \frac{0.75 \cdot 0.9}{0.675} = 1$.

Similarly, with $\mathbb{P}[JNS] = 1 - \mathbb{P}[JS] = 0.325$, so when Jerry says it's going to not snow, the probability that it will actually not snow is $\mathbb{P}[NS|JNS] = \frac{\mathbb{P}[JNS|NS] \cdot \mathbb{P}[NS]}{\mathbb{P}[JNS]} = \frac{1 \cdot 0.1}{0.325} = 0.308$.

Thus, Jerry's overall accuracy is

$$\mathbb{P}[\text{Jerry}] = \mathbb{P}[S|JS] \cdot \mathbb{P}[JS] + \mathbb{P}[NS|JNS] \cdot \mathbb{P}[JNS] = 1 \cdot 0.675 + 0.308 \cdot 0.325 = 0.775 = 77.5\%$$

Here, we could see that $\mathbb{P}[\text{Jerry}] < \mathbb{P}[\text{Tom}]$ even though $\mathbb{P}[JS|S] > \mathbb{P}[TS|S]$ and $\mathbb{P}[JNS|NS] > \mathbb{P}[TNS|NS]$, which gives an examples to the situation described in the problem.

More generally, since both weatherman predicts no snow much better than snow days, so a weatherman in a region that snows a lot (i.e. Alaska compared to New York) will have much lower overall accuracy (given the weight of abundant snowy days) even if she/he is better at predicting snow on snow days and no snow on sunny days.