3 Modular Exponentiation

(a) 1

Since $13 = 1 * 12 + 1$, so we have that $13 \equiv 1 \pmod{12}$, and that $1^{2018} = 1$.
So, $13^{2018} \equiv 1^{2018} \equiv 1 \pmod{12}$.

(b) 8

Since $8^2 = 64 = 7 * 9 + 1$, so we have that $8^2 \equiv 1 \pmod{9}$, and that again, any power of 1 is 1.
So, $8^{11111} = 8^{2*5550+1} = (8^2)^{5550} * 8 \equiv 1 * 8 \equiv 8 \pmod{9}$.

(c) 4

Since we have that:
$7^2 = 49 \equiv 5 \pmod{11}$,
$7^4 \equiv 5^2 = 25 \equiv 3 \pmod{11}$,
$7^8 \equiv 3^2 = 9 \equiv 9 \pmod{11}$,
$7^{16} \equiv 9^2 = 81 \equiv 4 \pmod{11}$,
$7^{32} \equiv 4^2 = 16 \equiv 5 \pmod{11}$,
$7^{64} \equiv 5^2 = 25 \equiv 3 \pmod{11}$,
$7^{128} \equiv 3^2 = 9 \equiv 9 \pmod{11}$,
Thus, $7^{256} \equiv 9^2 = 81 \equiv 4 \pmod{11}$.

(d) 16

Since we have that:
$3^2 = 9 \equiv 9 \pmod{23}$,
$3^4 = 9^2 = 81 \equiv 12 \pmod{23}$,
$3^8 = 12^2 = 144 \equiv 6 \pmod{23}$,
$3^{16} = 6^2 = 36 \equiv 13 \pmod{23}$,
$3^{32} = 13^2 = 169 \equiv 8 \pmod{23}$,
$3^{64} = 8^2 = 64 \equiv 18 \pmod{23}$,
$3^{128} = 18^2 = 324 = 14 * 23 + 2 \equiv 2 \pmod{23}$.
Thus, $3^{160} = 3^{128+32} = 3^{128} * 3^{32} \equiv 2 * 8 \equiv 16 \pmod{23}$.