5 Optimal Partners

Proof by Contradiction.

Proof. We proceed by contradiction. Assume that the proposition is false, which means that for some two different men, M and M^*, they have the same optimal partner, W.

Due to the strict ordering of preferences, WLOG, let W prefer M to M^*. Then, by the definition of optimal partner, so there must exist a stable pairing T in which M^* and W are paired together. Suppose T looks like this: $\{..., (M^*, W), ..., (M, W'), ...\}$. We will argue that (M, W) is a rogue couple in T, thus contradicting stability.

First, by our assumption, W prefers M to M^*. Moreover, since W is M's optimal partner, by definition of optimal partner for a man, so M prefers W to W' (his partner in the stable pairing T).

Therefore, (M, W) will form a rogue couple in T, and so T is not stable. Thus, we have a contradiction, implying that no two men can have the same optimal partner.

Q.E.D.