I worked alone without any help.

1 Random Cuckoo Hashing

(a) $\mathbb{P}[\text{No Collision}] = \frac{n!}{n^n}; \longrightarrow 0$

Since the size of the event space of such a situation is $|\omega| = n!$, and the size of the probability space is $|\Omega| = n^n$, so the probability of such a situation is $\mathbb{P}[\text{No Collision}] = \frac{|\omega|}{|\Omega|} = \frac{n!}{n^n}$, and as $n \to \infty$, we can see that $\mathbb{P}[\text{No Collision}] \to 0$.

We can represent $\mathbb{P}[\text{No Collision}]$ in another method: $\mathbb{P}[\text{No Collision}] = \frac{n}{n}\frac{n-1}{n}\cdots\frac{1}{n}$, which will tend toward 0 (all terms of this product are smaller than or equal to 1, and $\lim_{n\to\infty}\frac{1}{n} = 0$) as n grows very large.

(b) $\mathbb{E}[\text{Collisions}] = n - 1$

Let the expected number of collisions that we'll see while hashing D_n be $\mathbb{E}[\text{Collisions for } D_n] = X$.

Since we have already hashed $D_1, \ldots, D_{n-1}$, and they each occupy their own bucket, so in this situation, the probability of D_n not getting a collision is $\frac{1}{n}$ (which is equivalent to having 0 collisions); then, in other words, the probability of D_n getting a first collision is $1 - \frac{1}{n} = \frac{n-1}{n}$.

Now, let D_n take the i^{th} bucket, the bucket of D_i. So now, we reached the same situation where $(n-1)$ pieces of data have occupied their own buckets, and a single piece of data D_i needs to be rehashed, and thus, the expected number of collisions we'll see hashing D_i would be X again, because it's an identical situation. This implies that the total number of collision of hashing D_n in this situation would be $\mathbb{E}[\text{First Collision}] = 1 + X$.

Thus, looking back at $\mathbb{E}[\text{Collisions for } D_n]$, we have this equation:

$$X = \frac{1}{n} \cdot 0 + \frac{n-1}{n} \cdot (1 + X)$$

Thus, we can calculate that:

$$X = n - 1$$