3 Grid Induction

Proof. We proceed by induction on n, where $n = i + j$, and we call it the "distance" between Pacman and $(0, 0)$.

Base case ($n = 0$): Considering the restraints, so $i = j = 0$, so Pacman ends at $(0, 0)$
Thus, the base case is correct.

Inductive Hypothesis: Assume that, for arbitrary $n = k \geq 0$, the claim, Pacman would reach $(0, 0)$ in finite time, is true.

Inductive Step: We prove the claim for $n = k+1 \geq 1$. Let Pacman be at position $(i_1, j_1), i_1+j_1 = k+1$. Since Pacman only has two options, either walk one step down or walk one step to the left, which means that his position after one unit time is either $(i_1, j_1 - 1)$ or $(i_1 - 1, j_1)$. Moreover, Pacman's constraints tell us that he has to stay in the first quadrant, which means that at any time, let his location be (i^*, j^*), then $i^*, j^* >= 0$. So, after one unit time, his "distance" is always $i_1 + j_1 - 1 = k + 1 - 1 = k$. Thus, the Inductive Hypothesis implies that he'll reach $(0, 0)$ from here within finite time. Therefore, Pacman would reach $(0, 0)$ in finite time for $n = k + 1$.

Thus, by the principle of mathematical induction, the claim holds.

Q.E.D.