2 Miscellaneous Logic

(a)

(i) Possibly true.

false example:

Let $G(x, y) : y = x + 2$, so $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})$ is true for $G(x, y)$.
However, since $3 + 2 = 5 \neq 4$, so $G(3, 4)$ is false.

true example:

Let $G(x, y) : y = x + 1$, so $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})$ is true for $G(x, y)$.
And, since $3 + 2 = 4$, so $G(3, 4)$ is true.

(ii) Possibly true.

false example:

Let $G(x, y) : y = x + 2$, so $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})$ is true for $G(x, y)$.
However, consider $x = 0$.
Since $0 + 2 = 2 \neq 3$, so $(\forall x \in \mathbb{R})\ G(x, 3)$ is false.

true example:

Let $G(x, y) : y = 3$, so $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})$ is true for $G(x, y)$.
Since the statement given indicates that y is always 3,
So, $(\forall x \in \mathbb{R})\ G(x, 3)$ is true.

(iii) Certainly true.

Since the statement given, $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})\ G(x, y)$, is true, and since $3 \in \mathbb{R}$, so there must exist a $y \in \mathbb{R}$ such that $G(3, y)$ is true. Thus, $\exists y\ G(3, y)$ is a true statement.

(iv) Certainly false.

Since the statement given, $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})\ G(x, y)$, is true, and since $3 \in \mathbb{R}$, so there must exist a $y \in \mathbb{R}$ such that $G(3, y)$ is true, which means that $\forall y\ \neg G(3, y)$ is a false statement.

(v) Possibly true.

false example:

Let $G(x, y) : y = 3$, so $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})$ is true for $G(x, y)$.
So $\forall x \in \mathbb{R},\ y = 3$, which means that there's no x such that $y = 4$.
So $\exists x\ G(x, 4)$ is false.

true example:

Let $G(x, y) : y = x + 2$, so $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})$ is true for $G(x, y)$.
Consider $x = 2, x \in \mathbb{R}$.
Since $2 + 2 = 4$, so $\exists x\ G(x, 4)$ is true.

(b)

$$\Big(X \vee Y \vee Z\Big) \wedge \Big(\neg\ (X \wedge Y) \vee (Y \wedge Z) \vee (Z \wedge X)\Big)$$