3 Geometric and Poisson

(a) $e^{-\lambda p}$

Since $X \sim \text{Geo}(p), Y \sim \text{Poisson}(\lambda)$, so we have that $\mathbb{P}(X > Y) = \sum_{i=0}^{\infty} \mathbb{P}[Y = i]\mathbb{P}[X > i]$. Now, for any $i \in \mathbb{N}$, we have that

$$\mathbb{P}[Y = i] = \frac{\lambda^i}{i!}e^{-\lambda}$$

and with $p \neq 0$, we have

$$\mathbb{P}[X > i] = \sum_{j=i+1}^{\infty} \mathbb{P}[X = j] = \sum_{j=i+1}^{\infty} (1-p)^{j-1}p = \frac{(1-p)^i p}{1-(1-p)} = (1-p)^i$$

Then, using the Taylor series expansion of e^x, we have that:

$$\mathbb{P}(X > Y) = \sum_{i=0}^{\infty} \frac{\lambda^i}{i!}e^{-\lambda} \cdot (1-p)^i = e^{-\lambda} \cdot \sum_{i=o}^{\infty} \frac{(\lambda(1-p))^i}{i!} = e^{-\lambda} \cdot e^{\lambda(1-p)}$$

Thus,

$$\mathbb{P}(X > Y) = e^{-\lambda+\lambda-\lambda p} = e^{-\lambda p}$$

(b) 1

Since Z is defined as $Z = \max(X, Y)$, so by definition, we have that $\forall i,\ Z \geq X$. Therefore, $\mathbb{P}(Z \geq X) = 1$.

(c) $1 - e^{-\lambda p}$

Again, since $Z = \max(X, Y)$, so we have that $\mathbb{P}(Z \leq Y) = \mathbb{P}(Z = Y) = \mathbb{P}(X \leq Y) = \mathbb{P}(\overline{X > Y}) = 1 - \mathbb{P}(X > Y)$. Using our result from part (a), so:

$$\mathbb{P}(Z \leq Y) = 1 - \mathbb{P}(X > Y) = 1 - e^{-\lambda p}$$