EECE 5644

HOMEWORK-2

A1.
$$\lambda(\alpha_i|w_j) = \begin{cases} 0 & i=j \quad i,j = 1,2,\ldots c \\ \lambda_r & i = c+1 \\ \lambda_s & \text{otherwise,} \end{cases}$$

The cost associated with classifying a pattern as class i, when its true class is j is given by the following expression.

$\lambda(\alpha_i|w_j) = 0$ for the case of correct classification the cost associated with it is 0.

$\lambda(\alpha_i|w_j) = \lambda_r$ for $i = c+1$, is defined as the rejection cost, when the input is classified as none of the 'c' patterns.

$\lambda(\alpha_i/w_j) = \lambda_s$, for the remaining cases where $i \neq j$, $i,j = 1,2,\ldots c$
This is defined as the substitution or the missclassification cost.

The equation for Risk is given by,

For the case $i = 1,2,3 \ldots c$

$$R(\alpha_i/x) = \sum_{j=1}^{c} \lambda(\alpha_i/w_j) \cdot P(w_j/x)$$

$$R(\alpha_i/x) = \sum_{i=j} 0 \cdot P(\omega_j/x) + \sum_{j=1, j\neq i}^{c} \lambda_s \cdot P(\omega_j/x)$$

$$R(\alpha_i/x) = \lambda_s \sum_{j=1, j\neq i}^{c} P(\omega_j/x)$$

Now $\sum_{j=1}^{c} P(\omega_j/x) = 1$

$$R(\alpha_i/x) = \lambda_s [1 - P(\omega_i/x)]$$

This is the risk for cases where the input pattern is classified as one of the c patterns

Now

For $i = c+1$

$$R(\alpha_{c+1}/x) = \sum_{j=1}^{c} \lambda_r \cdot P(\omega_j/x)$$

$$= \lambda_r \sum_{j=1}^{c} P(\omega_j/x)$$

$$R(\alpha_{c+1}/x) = \underline{\underline{\lambda_r}}$$

Now minimum risk is acheived if the risk associated with classifying it as one of the 'c' patterns, which includes correct classification and misclassification is less than or equal to the risk associated with rejecting the input pattern.

$$R(\alpha_i/x) \leq R(\alpha_{c+1}/x)$$

$$\lambda_s [1 - P(\omega_i/x)] \leq \lambda_r$$

$$P(\omega_i/x) \geq 1 - \frac{\lambda_r}{\lambda_s}$$

If this condition is not satisfied, we reject otherwise.

Now if the cost associated with rejection, $\lambda r = 0$, then there is less risk in rejecting a pattern than in missclassifying it and in this case rejection will always be preferred to substitution.

For the case when $\lambda r > \lambda s$, or the cost associated with rejection is greater than that for substitution. Rejection never occurs as less risk is associated with substitution.

Q2

To generate
$$p[x/w_i] \sim N(\mu_i, C_i)$$

from $Y \sim N(0, I)$

mean 0, Identity covariance matrix

From Linear transformation,

$$X = AY + B$$

$\mu_i = A . \mu_y + B$

$\mu_i = A.(0) + B$

$$\boxed{\mu_i = B}$$

$\cancel{C_i = A^T . I^{-1} . A}$

$$\boxed{C_i = A . A^T},$$

since $A . A^T$ is symmetrical

$$A = (C_i)^{1/2}$$

To acquire samples of Y with given mean μ_i and C_i as covariance matrix.

First generate required no of iid samples with zero mean and Identity covariance matrix

Translate the Y points using
$$X = AY + B$$
where $B = \mu_i$ & $A = (C_i)^{1/2}$

Class priors p_1 & p_2 are then used to divide the ~~\$~~ X sample between 2 classes.