Project: 4" Flanged Pipe 90° Elbow
Engineer: Alan C. Longfellow
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Calculation Type: Internal Pressure, Hoop Stress Verification
References: ASME B31.3, Lamé equations

Given:

- Pipe ID = 4.000 in

- Wall thickness $(t) = 0.500$ in

- Max design internal pressure $(P_{\text{inner}}) = 5.000$ ksi

- External pressure $(P_{\text{outer}}) = \text{STP} = 0.0147$ ksi

- Safety factor (FS) = 1.5

Material:

- ASTM A106 Grade B

- Yield strength $(\sigma_y) = 35.0$ ksi

Calculated geometry:

- $r_{\text{inner}} = 2.000\,\text{in}$

- $r_{\text{outer}} = 2.500\,\text{in}$

- $r_{\text{mean}} = 0.5 \cdot (2.000\,\text{in} + 2.500\,\text{in}) = 2.250\,\text{in}$

Lamé equation for hoop stress σ_θ in a thick-walled vessel ($r_{\text{outer}}\,/\,t \,<\, 10$):

$$\sigma_\theta = \frac{P_{\text{inner}}r_{\text{inner}}^2 - P_{\text{outer}}r_{\text{outer}}^2}{r_{\text{outer}}^2 - r_{\text{inner}}^2} - \frac{r_{\text{inner}}^2 r_{\text{outer}}^2 \left(P_{\text{inner}} - P_{\text{outer}}\right)}{\left(r_{\text{outer}}^2 - r_{\text{inner}}^2\right) r_{\text{mean}}^2}$$

Solution for hoop stress:

$$\sigma_\theta = \frac{(5.000\,\text{ksi}) \cdot (2.000\,\text{in})^2 - (0.0147\,\text{ksi}) \cdot (2.500\,\text{in})^2}{(2.500\,\text{in})^2 - (2.000\,\text{in})^2} -$$

$$\frac{(2.000\,\text{in})^2 \cdot (2.500\,\text{in})^2 \cdot (5.000\,\text{ksi} - 0.0147\,\text{ksi})}{\left((2.500\,\text{in})^2 - (2.000\,\text{in})^2\right) \cdot (2.250\,\text{in})^2} = 19.790\,\text{ksi}$$

Safety factor verification:

$$\sigma_{\text{max}} = \frac{\sigma_y}{\text{FS}} = \frac{35.0\,\text{ksi}}{1.5} = 23.333\,\text{ksi}$$

$$\frac{\sigma_\theta}{\sigma_{\text{max}}} = \frac{23.333\,\text{ksi}}{19.790\,\text{ksi}} = 117.9\%\ \text{FS}, \quad \textbf{ACCEPTABLE}$$