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Project: 4" Flanged Pipe 90° Elbow

Engineer: Alan C. Longfellow

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Calculation Type: Internal Pressure, Hoop Stress Verification

References: ASME B31.3, Lamé equations

## Given:

• Pipe ID = 4.000 in

• Wall thickness (t) = 0.500 in

• Max design internal pressure  $(P_{inner}) = 5.000 \text{ ksi}$ 

• External pressure  $(P_{\text{outer}}) = \text{STP} = 0.0147 \text{ ksi}$ 

• Safety factor (FS) = 1.5

## Material:

• ASTM A106 Grade B

• Yield strength  $(\sigma_y) = 35.0 \text{ ksi}$ 

Calculated geometry:

•  $r_{\text{inner}} = 2.000 \, \text{in}$ 

•  $r_{\text{outer}} = 2.500 \, \text{in}$ 

•  $r_{\text{mean}} = 0.5 \cdot (2.000 \,\text{in} + 2.500 \,\text{in}) = 2.250 \,\text{in}$ 

Lamé equation for hoop stress  $\sigma_{\theta}$  in a thick-walled vessel ( $r_{\text{outer}}/t < 10$ ):

$$\sigma_{\theta} = \frac{P_{\text{inner}}r_{\text{inner}}^2 - P_{\text{outer}}r_{\text{outer}}^2}{r_{\text{outer}}^2 - r_{\text{inner}}^2} - \frac{r_{\text{inner}}^2r_{\text{outer}}^2 \left(P_{\text{inner}} - P_{\text{outer}}\right)}{\left(r_{\text{outer}}^2 - r_{\text{inner}}^2\right)r_{\text{mean}}^2}$$

Solution for hoop stress:

$$\sigma_{\theta} = \frac{(5.000 \,\mathrm{ksi}) \cdot (2.000 \,\mathrm{in})^2 - (0.0147 \,\mathrm{ksi}) \cdot (2.500 \,\mathrm{in})^2}{(2.500 \,\mathrm{in})^2 - (2.000 \,\mathrm{in})^2} - \frac{(2.000 \,\mathrm{in})^2 \cdot (2.500 \,\mathrm{in})^2 \cdot (5.000 \,\mathrm{ksi} - 0.0147 \,\mathrm{ksi})}{((2.500 \,\mathrm{in})^2 - (2.000 \,\mathrm{in})^2) \cdot (2.250 \,\mathrm{in})^2} = 19.790 \,\mathrm{ksi}$$

Safety factor verification:

$$\begin{split} \sigma_{\max} &= \frac{\sigma_y}{\mathrm{FS}} = \frac{35.0\,\mathrm{ksi}}{1.5} = 23.333\,\mathrm{ksi} \\ \frac{\sigma_{\theta}}{\sigma_{\max}} &= \frac{23.333\,\mathrm{ksi}}{19.790\,\mathrm{ksi}} = 117.9\%\,\mathrm{FS}, \quad \mathbf{ACCEPTABLE} \end{split}$$