Geodesic Patterns for Free-form Architecture

MPDA'18 Master Thesis —— UPC-ETSAV

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Abstract

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Contents

1 Introduction

This section explains the different algorithmic aproaches that can be taken in order to completely cover any given freeform surface with panels, ideally wood or metal, which are of approximately the same width and rectantular (or nearly rectangular) when flat and that achieve a surface paneling that is not only cost-effective but also watertight.

> To be continued…

2 Background

> There is very little backgrounnd on this topic without entering direclty into Orlando's topic `Ruled Surfaces`. Some background that must be included:

1. Burj Khalifa interior panelling (Meredith and Kotronis 2013)
2. Ghery's architecture in general uses same width metal sheets to cover entire buildings, although I am not shure if that is not Orlando's subject either…
3. Denis Shelden thesis on constructability of gherys architecture (Shelden 2002)
4. MAYBE?? Include non-optimized builidng examples to demonstrate the method's usefulness.
5. Looking for other built examples or previous/further research on the subject.

3 Geodesic curves

In differential geometry, a *geodesic curve* is the generalization of a straight line into curved spaces (see fig. 1).

Also, in the presence of an *affine connection*, a geodesic is defined to be a curve whose tangent vectors remain parallel if they are transported along it. We will explore the notion of vector *parallel transport* in the following sections.

For triangle meshes, shortest polylines cross edges at ***equal angles***.

Finding the truly shortest geodesic paths requires the computation of distance fields (see Carmo 2016; Kimmel and Sethian 1998)

3.1 Algorithmic ways of generating geodesics

The computation of geodesics on smooth surfaces is aclassical topic, and can be reduced to two different solutions, depending on the initial conditions of the problem, you can either generate a geodesic on a surface given ***a starting point and a direction*** *or* given ***the starting point and end point*** of the desired geodesic.

3.1.1 Start point + Directon problem

Finding a geodesic on a surface given a start point and a direction is equivalent to solving an initial value problem for a 2nd order ODE (???)

> MUST COME UP WITH A BETTER EXPLANATION FOR THIS

3.1.2 Start point + End point problem

This method is equivalent to solving a *boundary value problem.* # Geodesic surfaces

> This is very well explained in p.170 of Denis Shelden thesis (Gerard's suggestion). Explanation is inspired by that section.

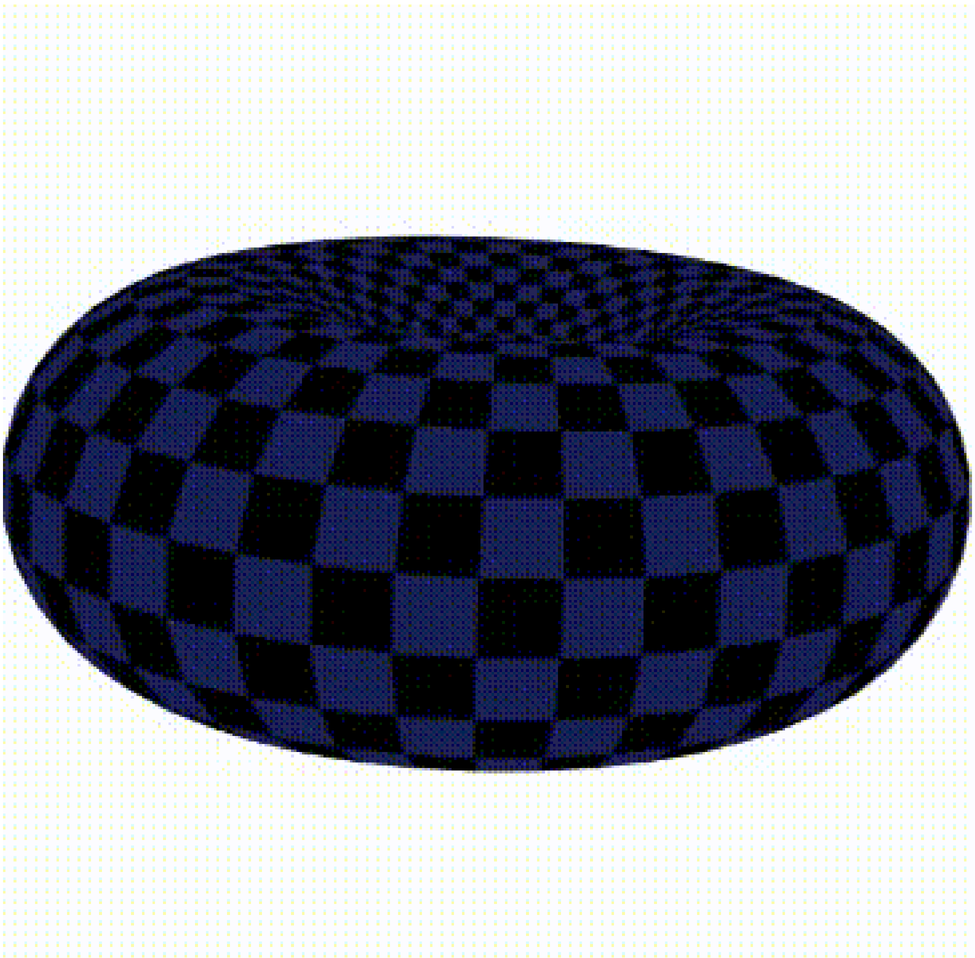


Fig. 1: If an insect is placed on a surface and continually walks "forward", by definition it will trace out a geodesic (image taken from Wikipedia).

4 Geodesic patterns

What are geodesic patterns?

- Patterns made of panels (wood or metal).
- Bent by their weak axis.
- Mounted on a free-form surface.
- Rectangular or cuasi-rectangular when layed flat.
- Water-tight.
- Overall shape is achieved by pure bending. See Fig. 2

Fig. 2: Geodesic pattern examples & previous work

4.1 Properties to aim for in panels

4.1.1 Geodesic property

- Long Thin panels that bend about their weak axis
- Zero geodesic curvature
- Represent the shortest path between two points on a surface

4.1.2 Constant width property

- Panels whose original, unfolded shape is a rectangle.
- The only way this can happen is if the entire surface is developable.
- For all other surfaces:
  - Assuming no gaps between panels
  - Panels will not be exactly rectangular when unfolded
  - ***Requirement:*** Geodesic curves that guide the panels must have approximately constant distance from thier neighbourhood curves.

4.1.3 Developable (or 'pure-bending') property

- Bending panels on surfaces changes the distances in points only by a small amount so,
- A certain amount of twiting is also present in this aplications.
  Some methods in this chapter do not take into account this property.

4.2 Problem Statement

Problem 1 Look for a system of geodesic curves that covers a freeform surface in a way that:

1. They have approximate constant distance with it's neighbours.
2. This curves will serve as guiding curves for the panels.
3. The panels are to cover the surface with ***no overlap*** and ***only small gaps***

Problem 2 Look for a system of geodesic curves in a freeform surface which:

1. Serve as the boundaries of wooden panels.
2. The panel's deveopment is ***nearly straight***.
3. Those panels cover the surface with ***no gaps***

5 Design strategies for geodesic systems

5.1 Design by parallel transport

This method, described in (Pottmann et al. 2010), allows for the generation of a system of geodesic curves where either the maximum distance or the minimum distance between adjacent points ocurrs at a prescribed location.

> In differential geometry, the concept of *parallel transport* (see fig. 3) of a vector $\boldsymbol{V}$ along a curve $\boldsymbol{S}$ contained in a surface means moving that vector along $\boldsymbol{S}$ such that:
>
> 1. It remains tangent to the surface
> 2. It changes as little as possible in direction
> 3. It is a known fact that the length of the vector remains unchanged

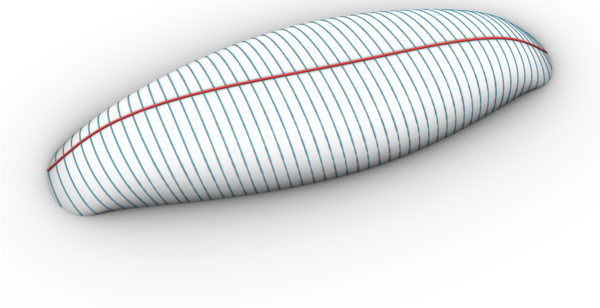


Fig. 3: Example of parallel transport method. Generatrix geodesic g (red) and geodesics $g^{\perp}$ generated from a parallel transported vector (blue) computed given a point and a vector $\mathbf{v}$ tangent to the surface, in both positive and negative directions.

Fig. 4: Parallel transport along a curve g lying on surface S is equivalent to projecting $\mathbf{v}_{i-1}$ onto the tangent plane on p_i and subsequently normalizing $\mathbf{v}_i$.

5.1.1 Procedure

Given a surface S represented as a triangular mesh (V,E,F):

1. Place a geodesic curve g_x along S such that it divides the surface completely in 2 .

2. Divide the curve into equaly spaced points p with distance W.
3. Place an vector $\mathbf{v}$ onto p_0
4. Parallel transport that vector along g_x as described in fig. 4.
5. For each point p_i, generate geodesic curves g_i and g_-i using vector $\mathbf{v}_i$ and $-\mathbf{v}_i$ respectively.
6. Both geodesics form a single, continuous geodesic flowing on the surface.

   Following this procedure, *extremal distances between adjacent geodesics occur near the chosen curve.* Meaning:

   1. For surfaces of **positive curvature**, the parallel transport method will yield a 1-geodesic pattern on which the *maximum distance* between curves will be W.
   2. On the other hand, for surfaces of **negative curvature**, the method will yield a 1-geodesic pattern with W being the *closest (or minimum)* distance between them.

The placement of the first geodesic curve and the selection of the initial vector are not trivial tasks. For surfaces with high variations of surfaces, the results might be unpredictable and, as such, this method is only suitable for surfaces with nearly constant curvature. Other solutions might involve cutting the surface into patches of nearly-constant curvature, and applying the *parallel transport method* independently on each patch.

5.2 Design by evolution & segmentation

Two main concepts are covered in this section, both proposed by (Pottmann et al. 2010): the first, what is called the *evolution method*, and a second method based on *piecewise-geodesic* vector fields.

5.2.1 The *evolution method*

As depicted in: Starting from a source geodesic somewhere in the surface:

- Evolve a pattern of geodesics iteratively computing 'next' geodesics.
- 'Next' geodesics must fullfil the condition of being at approximately constant distance from its predecesor.
- If the deviation from its predecesor is too great, breakpoints are introduced and continued as a *'piecewise geodesic'*.
- 'Next geodesics' are computed using Jacobi Fields

5.2.2 Distances between geodesics

1. No straight forward solution.
   1. Only for rotational surfaces (surfaces with evenly distriuted meridian curves).
2. **But** a first-order aproximation of this distance can be approximated:

   Starting at time $t = 0$ with a geodesic curve $g(s)$, parametrized by arc-length s, and let it move within the surface.
   A snapshot at time $t = \varepsilon$ yields a geodesic g^+ near g.

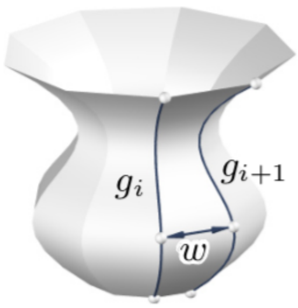
$$g^+(s) = g(s) + \varepsilon\mathbf{v}(s) + \varepsilon^2(\dots) \tag{1}$$

The derivative vector field $\mathbf{v}$ is called a *Jacobi field*. We may asume it is orthogonal to $g(s)$ and it is expressed in terms of the geodesic tangent vector g' as:

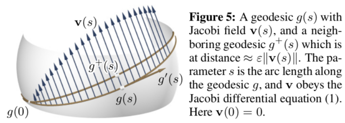
$$\mathbf{v}(s) = w(s) \cdot R_{\pi/2}(g'(s)), \quad \text{where } w'' + Kw = 0 \tag{2}$$

Since the distance between infinitesimally close geodesics are governed by Eq. 2, that equation also goberns the width of a strip bounded by two geodesics at a small finite distance.

Using this principle, you can develop strips whose width $w(s)$ fulfills the Jacobi equation $w(s) = \alpha \cosh(s\sqrt{|K|})$[1] for some value $K < 0$.
Gluing them together will result in a surface of approximate Gaussian curvature.



(a) Distances between geodesics



(b) Distances between geodesics

Fig. 5: Geodesic distances on sphere

5.2.2.1 Algorithm pseudocode

PENDING

5.2.3 Piecewise-geodesic vectorfields

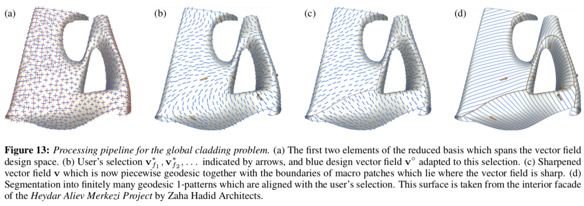


Fig. 6: Geodesic Vector Fields

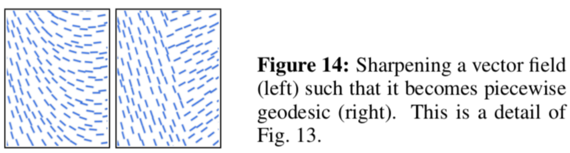


Fig. 7: Geodesic Vector Field sharpening

6 Panels from curve patterns

In this section, we will discuss several ways to generate panels from a system of 1-geodesic curves.

[1] **Question:** What is α in this formula? Missing image

6.1 Tangent-developable method

The notion of ***Conjugate tangents*** on smooth surfaces must be defined:

- Strictly related to the ***Dupin Indicatrix***
- In negatively curved areas, the Dupin Indicatrix is an hyperbola whose asymptotic directions (A1, A2)
- Any parallelogram tangentialy circumscribed to the indicatrix defines two conjugate tangents **T** and **U**.
- The asymptotic directions of the dupin indicatrix are the diagonals of any such parallelogram.

Fig. 8: Tangent developable method for panels

Initial algorithm is as follows:

For all geodesics s_i in a given pattern:

1. Compute the *tangent developable surfaces* $\rightarrow \Psi_i$
2. Trim Ψ_i along the intersection curves with their respective neighbours.
3. Unfold the trimed Ψ_i, obtaining the panels in flat state.

Unfortunately, this method needs to be refined in order to work in practice because:

1. The rulings of tangent developables may behave in weird ways
2. The intersection of the neighbouring Ψ_i's is often *ill-defined*.

Therefore, the procedure was modified in the following way:

1. Compute the *tangent developable surfaces* Ψ_i for all surfaces $s_i \rightarrow i =$ even numbers
2. Delete all rulings where the angle enclose with the tangent α is smaller than a certain threshold (i.e. 20º).
3. Fill the holes in the rulings by interpolation (???)
4. On each ruling:
   1. Determine points $A_i(x)$ and $B_i(x)$ which are the closest to geodesics s_{i-1} and s_{i+1}. This serves for trimming the surface Ψ_i.
5. Optimize globaly the positions of points $A_i(x)$ and $B_i(x)$ such that
   1. Trim curves are *smooth*
   2. $A_i(x)$ and $B_i(x)$ are close to geodesics s_{i-1} and s_{i+1}
   3. The ruling segments $A_i(x)B_i(x)$ lies close to the *original surface* Φ

6.2 The Bi-Normal Method

The second method for defining panels, once an appropriate system of geodesics has been found on Φ, works directly with the geodesic curves.

Assume that a point $P(t)$ traverses a geodesic s with unit speed, where t is the time parameter. For each time t there is:

- a velocity vector $T(t)$
- the normal vector $N(t)$
- a third vector $B(t)$, *the binormal vector*

Fig. 9: Tangent developable method examples

This makes $T.N.B$ a ***moving orthogonal right-handed frame***

The surface Φ is represented as a triangle mesh and s is given as a polyline. For each geodesic, the associated surface is constructed according to Fig. 10. Points $L(t)$ and $R(t)$ represent the border of the panel, whose distance from $P(t)$ is half the panel width.

Fig. 10: Binormal Method for panels & T.N.B. frame

6.3 Method Comparison

See tbl. 1 for more info...

7 Stress and strain in panels

The following section investigates the behaviour of a rectangular strip of elastic material when it is bent to the shape of a ruled surface Ψ un such way that:

The central line m of the strip follows the *'middle geodesic'* s in Ψ

This applies to both methods defining panels.fig. 11

Fig. 11: Stress in panels

7.1 Stress formulas

$$\rho = 1/\sqrt{K}, \tag{3}$$

$$d/2\rho \leq C, \quad with \quad C = \sqrt{\sigma_{max}/E}, \tag{4}$$

$$\varepsilon = \frac{1}{2}(d/2\rho)^2 + \cdots \tag{5}$$

MISSING MORE INFO ON STRESS ANALYISIS

8 Final analysis cost, quality

All strategies must be compared against cost & quality of the different solutions.

Fig. 12: Cost/Quality Final Assesment

8.1 Frequent measures used in the topic

- Bounding-box diagonal of the panels

8.2 Cost variables

Cost should be defined as:

1. ???
2. ???
3. ???

8.3 Quality variables

Quality should be defined as:

1. ???
2. ???
3. ???

8.4 Variable weighting method

Explanation of the weighting of variables?

9 Math Section

Some nomenclature and formula clarification for the non-mathematicians!?

9.1 Nomenclature guide

1. ρ
2. τ
3. Φ
4. Ψ
5. s
6. V, T, N, B
7. θ
8. σ
9. Add more...

9.2 Formulas & referencing guide

LaTeX formulas and reference them (like Eq. 3 or multiple at once like Eqns. 4, 5) can be inserted using `$$` and formated using Symbols.PDF found in the 'resources' folder.

References are placed using the format `[@type:label]`, being `label` the unique name of the desired reference on the format, and `type` the type of reference, in the following format:

- Images: `{#fig:LABEL}`
- Tables: `{#tbl:LABEL}`v
- Equations: `{#eq:LABEL}`
- Sections: `{#sec:LABEL}`
  - If sections are added, they will change all the reference names to include their corresponding sectionsd
- Code blocks: `{#lst:LABEL}`

9.2.1 Distances between geodesics (Eqns. 6, 7)

$$g^{+}(s) = g(s) + \varepsilon \mathbf{v}(s) + \varepsilon^{2}(\ldots) \tag{6}$$

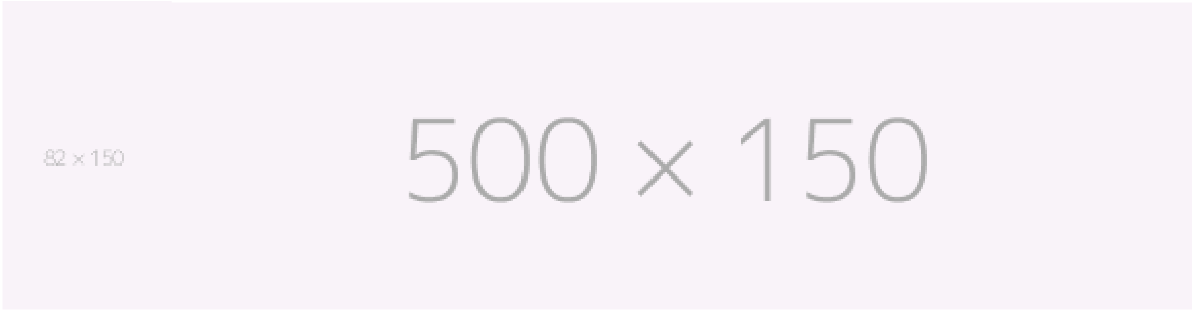
$$\mathbf{v}(s) = \omega(s) \cdot R_{\pi/2}(g'(s)), \quad where \quad \omega'' + K\omega = 0. \tag{7}$$

Tables are also an option:

Tab. 1: Comparisson between panel generation methods

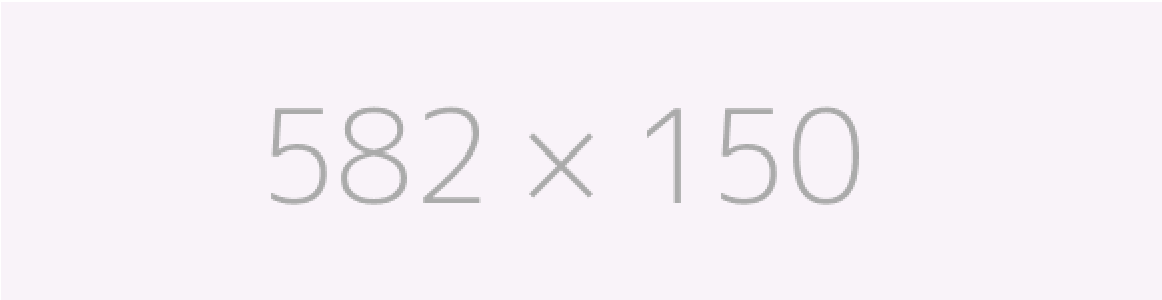
Tangent-Developable Method	**Bi-Normal Method**
Tries tor reproduce panels achievable by pure bending	Simple, obvious way of mathematically defining panels
Panels produced remain tangent to the surface	**Unclear** if the panels should follow this shape.
Follows a manufacturing goal	Panel surfaces are mathematically exact
	Panels are admissible from the viewpoint of stresses and strain

HTML figure disposition is also available, with customization options like width, per image captions, etc. . .



(a) (b)

Fig. 13: Difference between width-settings:



And some very nice diagrams too, using the Mermaid library

10 References that must be used

- (Eigensatz et al. 2010)
- (Chen and Han 1996)
- (Kahlert, Olson, and Zhang 2010)
- (Surazhsky et al. 2005)
- (Arsan and Özdeger 2015)
- (Pottmann et al. 2010)
- (Polthier and Schmies 1998)
- (Carmo 2016)
- (Kimmel and Sethian 1998)
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- (Weinand and Pirazzi 2006)
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- (Pottmann, Schiftner, and Wallner 2008)
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- (Kensek, Leuppi, and Noble 2000)
- (Deng 2011)
- (Jia 2017)
- (Deng, Pottmann, and Johannes, n.d.)
- Geodesic Lines Grasshopper implementation
- Non-optimized geodesic planks building
- Non-optimized geodesic planks stairwell
- Discrete Geodesic Nets for Modeling Developable Surfaces
- Video
- Add this paper to bib: Discrete Geodesic Nets

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