Geodesic Patterns

for Freeform Architecture

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Objective

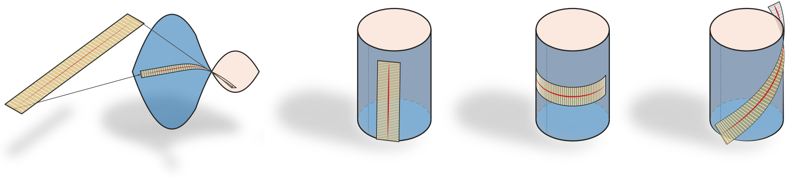
Objective

Discretize a given freeform surface into planks with the following properties:

1. Must be ***developable*** (Shelden 2002)
2. Should tend to have approximate ***equal width***
3. Should be ***as straight as possible***
4. Cannot bend by their strong axis but,
5. can have a twist and bend by their weak axis

Objective

Plank A plank is timber that is flat, elongated, and rectangular with parallel faces that are higher and longer than wide. (Wikipedia)



(a) **(b)**

Figure 1: Straight planks bent on a hyperbolic paraboloid (a) and a cylindrical surface (b).

Background

Background

The use of *straight developable planks* is widely used in:

Figure 2: Traditional boat building

Also common practice in naval engineering industry:

Figure 3: Connected developable patches for boat hull design

Background

The architecture studio NOX was one of the first to experiment with paper strips.

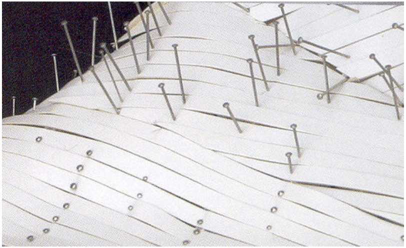


Figure 4: NOX Strips models

Figure 5: NOX render of strip model

Background

This techniques have also been used in the architecture world, mainly by **Frank Ghery**.

His façades are usually a collection of connected developable surfaces.

(a) **(b)** **(c)**

Figure 6: Famous Ghery projects (right to left): Guggenheim Museum (Bilbao, 1997), Walt Disney Concert Hall (Los Angeles, 2003) and the Guggenheim Abu Dhabi winning proposal (Abu Dhabi, –)

Background

Latest architectural work following similar techniques was:

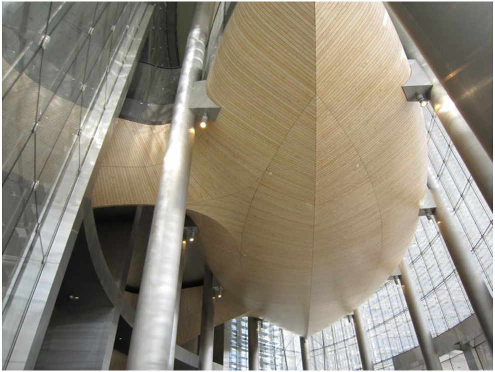


Figure 7: Burj Khalifa by Frank Ghery

Background

It was designed as a collection of:

- **Developable surfaces**
  - *Which can be covered by equal width planks*
- **Surfaces of constant curvature**
  - *Which can be covered by repeating the same profile*

Background

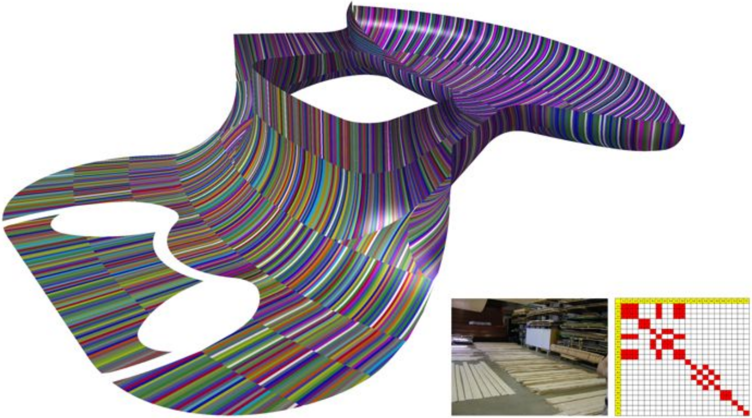


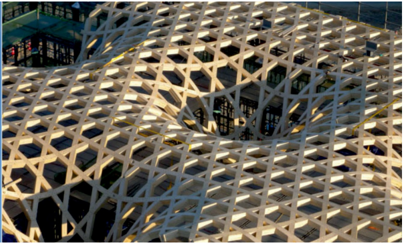
Figure 8: Burj Khalifa final panel solution

Other projects of interest are:

(a) **(b)**

Figure 9: Interior view of Toyo Ito's Mina No Mori wooden lattice roof (a) and exterior view of the lattice under construction (b).

(a)



(b)

Figure 10: Shigeru Ban's Yeoju Golf Resort. Right: Interior view of finished roof lattice. Left: Aerial view of lattice under construction.

Construction technique

Geodesic curves

A geodesic curve is the generalization of a *straight line* into *curved spaces*.

In this research, we concentrate on the concept of ***straightest geodesics***.

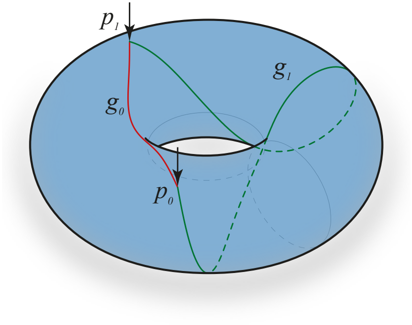


Figure 11: Geodesics on a torus: Although g_0 and g_1 are both geodesics, one doubles the length of the other

Developable surfaces

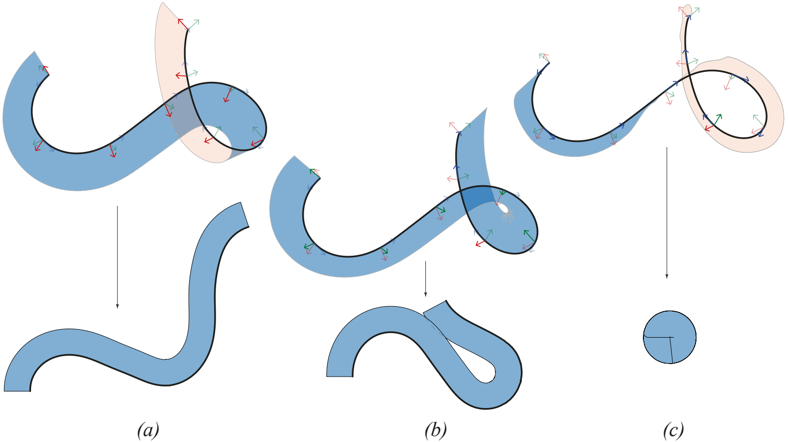


Figure 12: Surfaces with ***0 gaussian curvature***. Meaning, they can be flattened onto a plane ***without distortion***

Developable panels

We are interested in:

Developable surfaces

- *surfaces that can be flattened.*
- *can be generated by a single curve.*

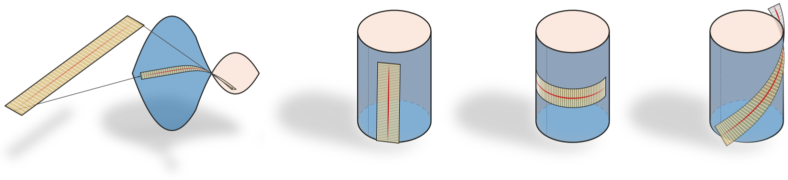
Geodesic curves

- *are straight lines in a curved space.*

Developable panels

If Panels are generated using geodesic curves on the surface

Then Resulting panels will be ***developable*** and mostly ***straight*** when flat.



(a) **(b)**

Figure 13: Straight planks bent on a hyperbolic paraboloid (a) and a cylindrical surface (b).

In other words

We wish to cover a given freeform surface with a pattern of **geodesic curves** with equal spacing.

This can only be achieved if the provided surface is already ***developable***.

A compromise exists between the ***curve spacing*** and the ***curve's geodesic curvature***

Properties we aim for

Geodesic property Look for ***straight*** or ***as straight as possible*** curves.

Constant width property Curves that are ***equally spaced*** on the surface

Developable property Surfaces generated from the curves ***MUST*** be developable.

Algorithmic strategies

Obtaining Geodesic Patterns

These are the main methods for the obtaining successful geodesic patterns:

1. The ***parallel transport*** method
2. The ***evolution*** method
   2.1 The ***piecewise geodesic*** evolution method
3. The ***level-set*** method

The parallel transport method

Vector parallel transport

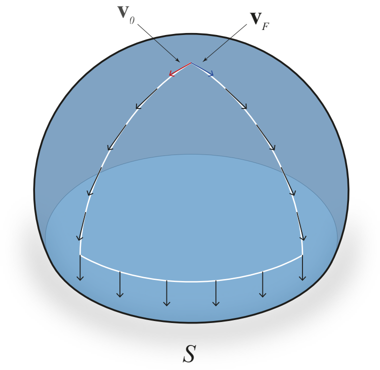


Figure 14: Parallel transport of a vector over a path on a sphere

Vector parallel transport

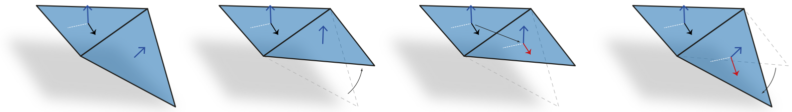


Figure 15: Parallel transport over two adjacent mesh faces

The parallel transport method

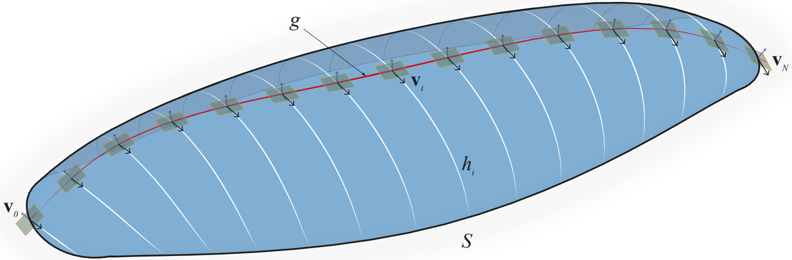


Figure 16: Parallel transport method over a positive curvature surface

The parallel transport method

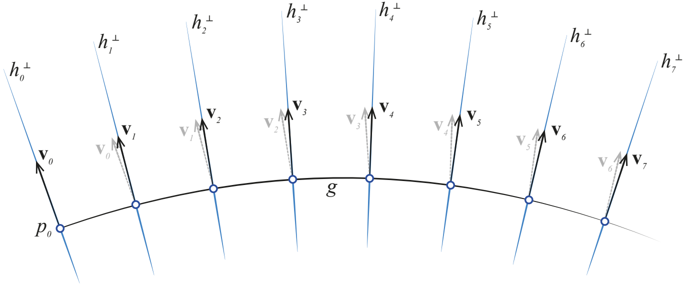


Figure 17: Parallel transport method diagram

The parallel transport method

Input: A surface Φ, represented as a triangular mesh (V,E,F)

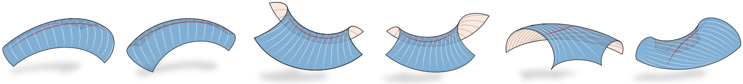
Place a geodesic curve g_x along S such that it divides the surface completely in 2.
Divide the curve into N equally spaced points p with distance W.
Place an vector $\mathbf{v}$ onto p_0
Parallel transport that vector along g_x as described in [@fig:parTransProc].
for all points p_i where i = 0,...,M **do**
    Generate geodesic curve $+g_i$ and $-g_i$ using vector $\mathbf{v}_i$ and $-\mathbf{v}_i$ respectively.
    Join $+g_i$ and $-g_i$ together to obtain g_i
    Add g_i to output.
end for

Output: Set of geodesic curves $g_i, \quad where \quad i = 0, ..., M$

The parallel transport method

There are **three** *extreme* cases depending on the ***local gaussian curvature*** where g lies on Φ:

Positive curvature Panels will have **Maximum width** on g
Negative curvature Panels will have **Minimum width** on g
0 gaussian curvature: Panels will be of equal width



The parallel transport results

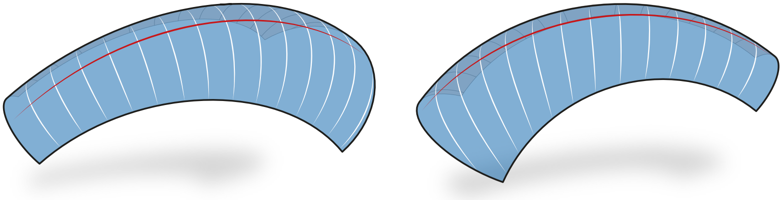


Figure 18: Parallel transport results over a positive curvature surface. Starting geodesic (red) and geodesic pattern (white). **Máximum** distance between curves occur on g.

The parallel transport results

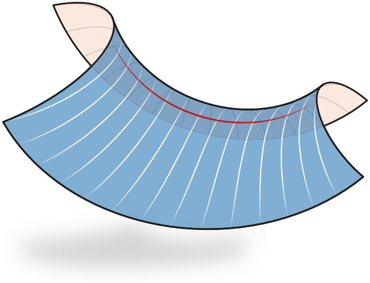


Figure 19: Parallel transport results over a negative curvature surface. Starting geodesic g (red) and geodesic pattern (white). **Minimum** distance between panels occur on g

The parallel transport results

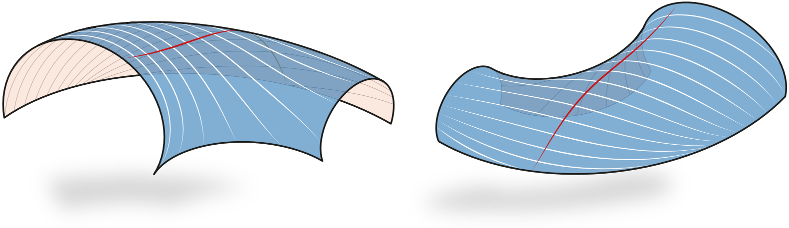


Figure 20: Parallel transport results over a doubly curved surface. Starting geodesic (red) and geodesic pattern (white).

The Evolution Method

The Evolution Method

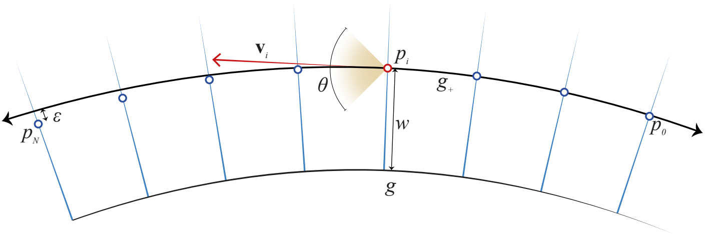


Figure 21: Calculating the best-fit geodesic

Evolution Implementation

Input: Curve g_i,perp geodesics $h^{\perp}$, desired width W and an angle threshold α.

1: Obtain a point p_i at distance W from the starting point of each $g_i^{\perp}$
2: Select any point p_i as the start point and name it p_0.
3: Select the tangent vector $\mathbf{v}_T$ of $g_i^{\perp}$ at p_0.
4: Rotate $\mathbf{v}_T$ 90º around the normal of Φ at p_0.
5: Generate an initial geodesic g_i.
6: Obtain error measure ϵ as the least squares difference between the desired distance W and the actual distance D
7: Find the geodesic that has the least error by rotating v_T by a small amount each step.

Output: The next geodesic curve that best fits W.

Input: A surface Φ, represented as a triangular mesh (V,E,F), a desired width W, and a starting geodesic curve g_0.

1: Place a geodesic curve g_i along Φ and name it g_0.
2: Divide the curve into equally N number of sample points.
3: **for all** points p_i where $i = 0, ..., N$ **do**
4:     Find tangent vector $\mathbf{v}_i^T$ of g
5:     Rotate $\mathbf{v}_i^T$ by $\frac{\pi}{4}$
6:     Generate geodesic $h_i^{\perp}$ from p_i and $\mathbf{v}_g^T$
7: **end for**
8: Compute BEST FIT GEODESIC g_{i+1}**??**
9: **if** No best fit is found **then**
10:     BREAK
11: **end if**
12: Make g_{i+1} the current geodesic for next iteration.

Output: Set G of geodesic curves $g_i, \quad where \quad i = 0, ..., M$

Evolution Method Results

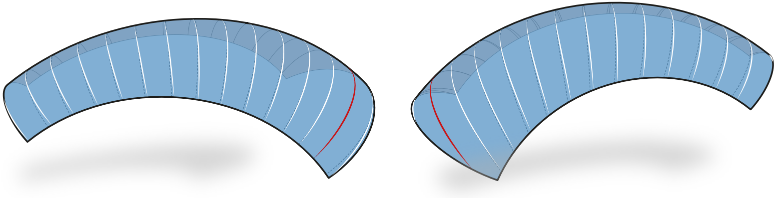


Figure 22: Evolution method results over a positive curvature surface. Starting geodesic (red), geodesic pattern (white) and results from fig. 18 (dashed gray lines).

Evolution Method Results

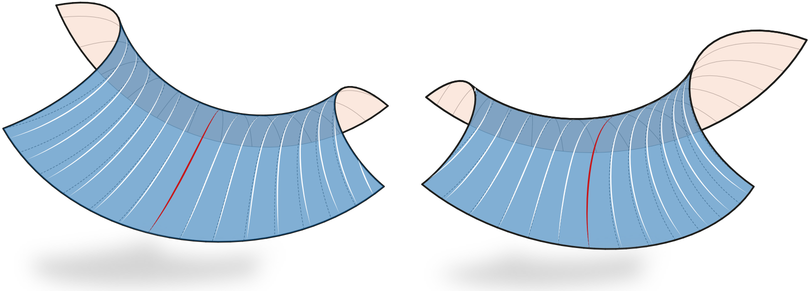


Figure 23: Evolution method results over a negative curvature surface. Starting geodesic (red), geodesic pattern (white) and results from fig. 19 (dashed gray lines).

Evolution Method Results

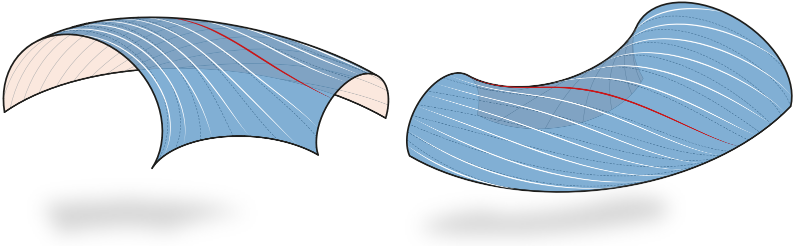
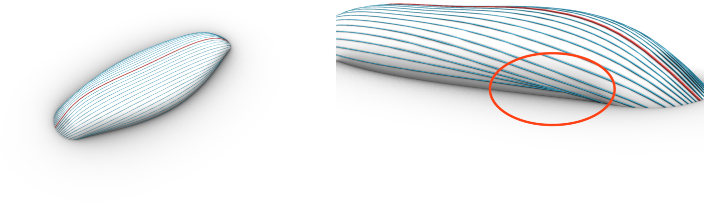


Figure 24: Evolution method results over a doubly curved surface. Starting geodesic (red), geodesic pattern (white) and results from fig. 20 (dashed gray lines).

Evolution Method Results

Local changes in curvature produce:

- Sharp panel endings in positive curvature areas
- Panel width increase in negative curvature areas



Evolution method example: top view (a), perspective view (b) with highlighted unwanted results.

The Piecewise Evolution Method

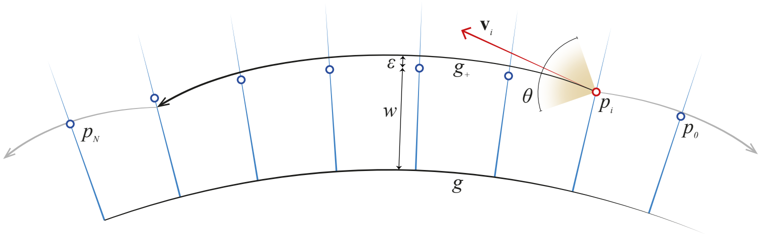


Figure 25: Calculating the best piece-wise geodesic

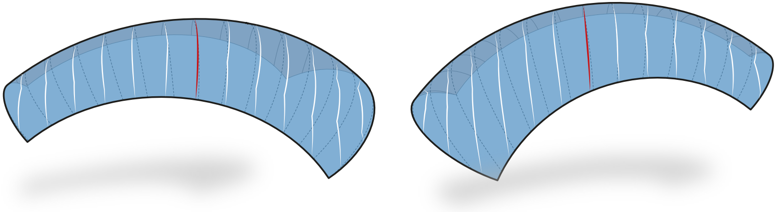
Piecewise Ev. Implementation

Start from: Step ?? of Algorithm 3.

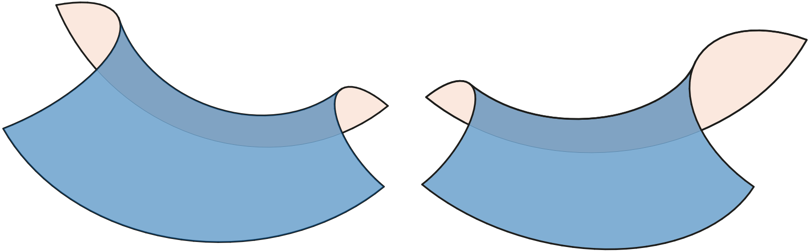
Find the largest interval I of g_{i+1} that does not exceed ϵ by a given
if g_i satisfies the ϵ for all $h^{\perp}$'s **then**
    BREAK
end if
Split g_i
Remove used $h^{\perp}$ from list.
Start Algorithm 3 again using the new $h^{\perp}$, and the start/end point of g_i

On end: Continue from Step ?? of Algorithm 3.

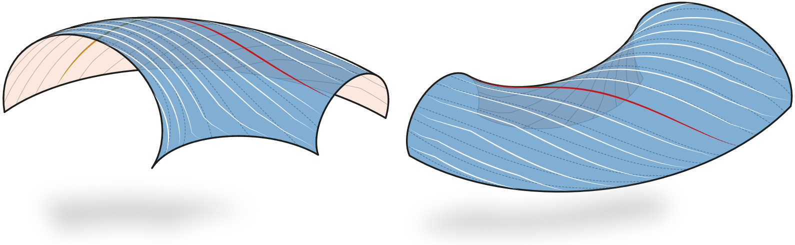
Piecewise Ev. Results



Piecewise Ev. Results



Piecewise Ev. Results



Piecewise Ev. Results

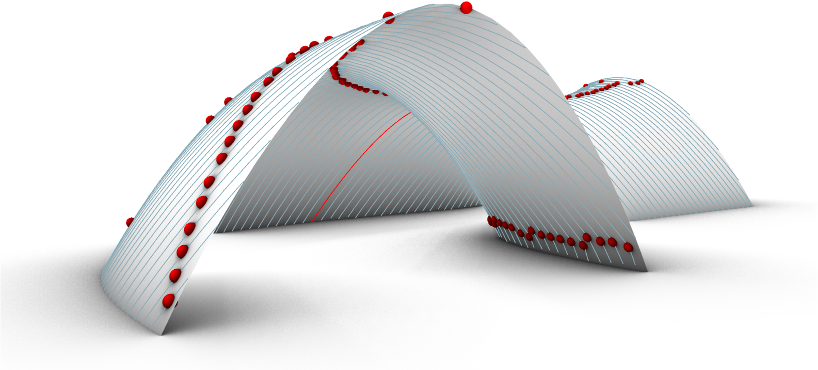


Figure 26: Piecewise Test

Piecewise Ev. Results

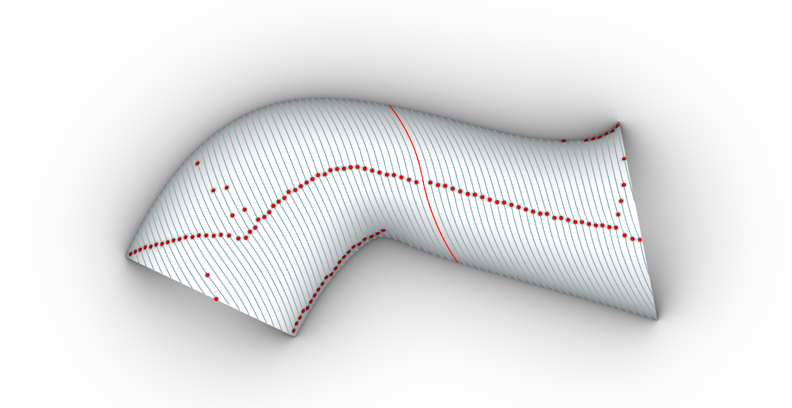


Figure 27: Piecewise Test

The level set method

Mesh Level-sets

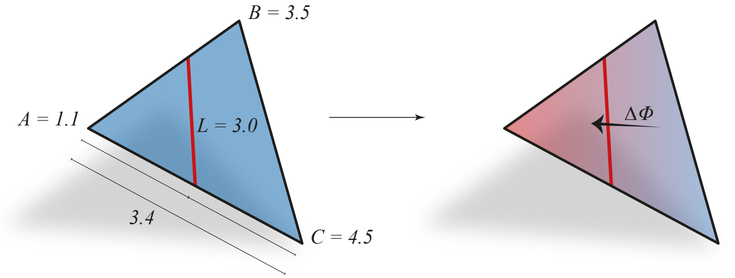


Figure 28: Level set on a single mesh face

Level-set Start Condition

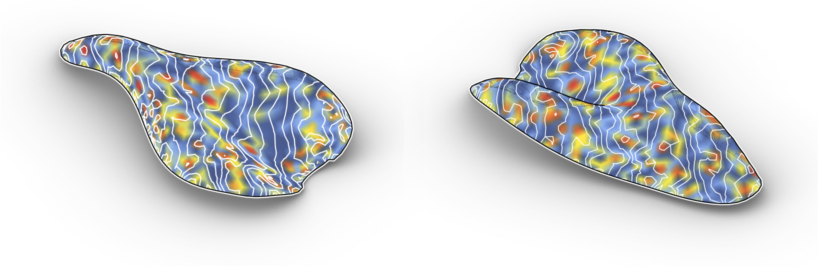


Figure 29: Starting conditions for the level set method on a double curve surface

Computed level sets

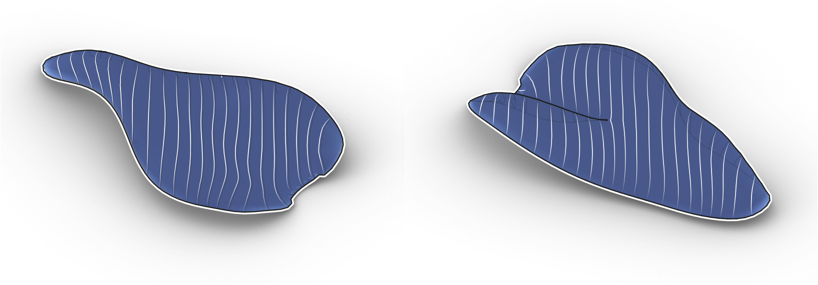


Figure 30: Final result of the level set method over a complex double curve surface.

Piecewise geodesic vector-fields

Piecewise geodesic vector-fields

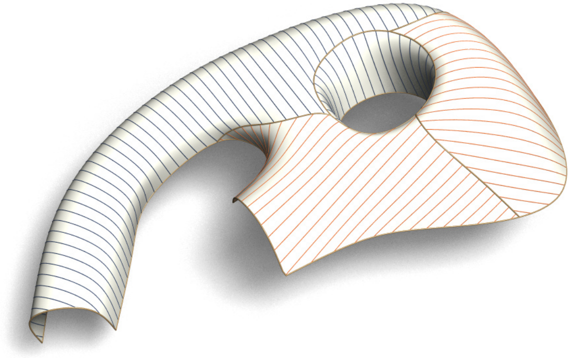


Figure 31: Geodesic pattern example[1]

[1] Image taken from Pottmann et al. (2010)

Piecewise geodesic vector-fields

The objective is to divide or *cut* the mesh into areas that will be easilly covered by a 1-pattern of geodesics.[2]

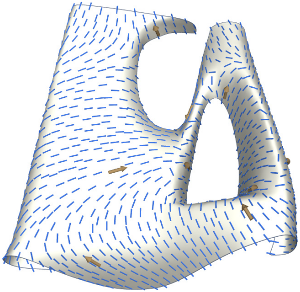


Figure 32: User specified directions

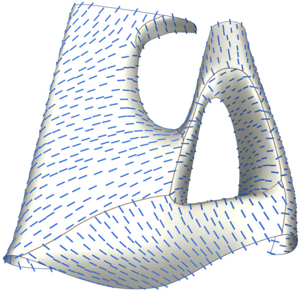


Figure 33: Computed geodesic vector field & cuts

This is done by computing a *geodesic vector field* over the surface, in a way that it aligns with several user specified directions.

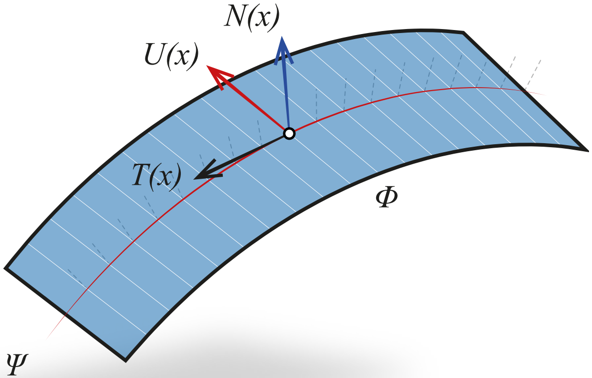
[2]Image taken from Pottmann et al. (2010)

Modeling planks

Tangent-developable method

Given any point in g:

1. Assuming $T(x)$ is tangent g.
2. Compute $U(x)$ as $T(x) \times N_{\Phi}(x)$
   The union of all $U(x)$ is a developable ruled surface Ψ.



Tangent-developable method

Initial algorithm is as follows:

For all geodesics s_i in a given pattern:

1. Compute the *tangent developable surfaces* $\rightarrow \Psi_i$
2. Trim Ψ_i along the intersection curves with their respective neighbours.
3. Unfold the trimmed Ψ_i, obtaining the panels in flat state.

Unfortunately, this method needs to be refined in order to work in practice because:

1. The rulings of tangent developable may behave in weird ways
2. The intersection of the neighboring Ψ_i's is often *ill-defined*.

Tangent-developable method

Therefore, the procedure was modified in the following way:

Input: The reference surface Ψ, represented as a mesh (V,E,F) and a geodesic pattern of curves $g_{0 \dots N}$

1: Compute the *tangent developable surfaces* Ψ_i for all surfaces $s_i \rightarrow i =$ even numbers

2: Delete all rulings where the angle enclose with the tangent α is smaller than a certain threshold (i.e. 20º).

3: **for all** Rulings **do**

4: Determine points $A_i(x)$ and $B_i(x)$ which are the closest to geodesics s_{i-1} and s_{i+1}. This serves for trimming the surface Ψ_i.

5: **end for**

6: Trim Ψ_i with the curve generated from all points $A_i(x)$ and $B_i(x)$ respectively.

Output: Trimmed developable surface Ψ_i.

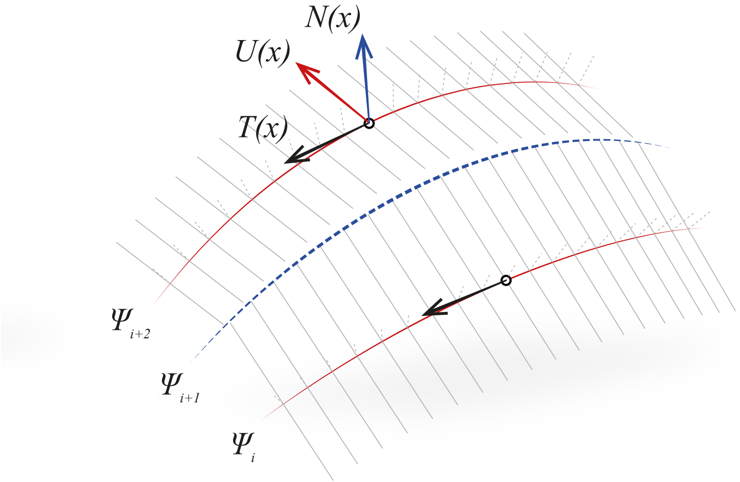


Figure 35: Panels computed using the using the modified tangent developable method.

Analysis

Stress in panels

Assuming the material:

- is bent to the shape of a ruled surface Ψ
- the central line m of the plank follows the *'middle geodesic'* in Ψ.

Then:

- Lines parallel to m at distance $d/2$ are not only bent but also stretched.

Stress in panels

If we introduce the radius of Gaussian curvature $\rho = 1/\sqrt{|K|}$

the relative increment in length ε (strain) of the strip is:

$$\varepsilon = \frac{1}{2}(d/2\rho)^2 + \cdots \tag{1}$$

Where:

- d is the plank width
- ρ is the radius of gausian curvature

Tensile stress

Tensile stress can be expressed as $\sigma = E\varepsilon$[3], which yields:

$$d/2\rho \leq C, \quad with \quad C = \sqrt{\sigma_{max}/E}$$

$$\sigma_{max} = \text{maximum admissible stress}$$

$$E = \text{Young's modulus}$$

[3] Since this calculation is an approximation, a safety factor must be used when choosing σ_{max}.

Admissible panel width

$$d/2\rho \leq C, \quad with \quad C = \sqrt{\sigma_{max}/E}$$

Since C is a material constant. We obtain the maximum admissible width with:

$$d_{max} = 2\rho_{min}C$$

C constant

Maybe missing an image here?

Table 1: Example calculation of constant C for some of the most suitable materials.

Material	Young Modulus	Max. stress	C
Wood	200000		
Steel	13000		
Others?	?		

Admissibility for models

INSERT IMAGE!!!

Bending and shear stress

Only for panels with thin rectangular cross-sections ***(**$h/d \ll 1$**)***

Bending (σ) and shear stresses (τ) depend on:

Panel thickness h but ***not*** on the panel width d.

Temp Title

Maximum values occur on the outer surface of the panel (Wallner et al. 2010) and depend on:

- the curvature κ of the central geodesic
- the rate of torsion θ of the panel.

$$\sigma = E\kappa h/2 \quad and \quad \tau = hG\theta \tag{2}$$

Where G is the shear modulus.

τ, measured by arc per meter, does not exceed $\sqrt{|K|} = 1/\rho$,

maximum value ocurs if:

- the central geodesic's tangent is an asymptotic direction of the panel surface (Carmo 2016).

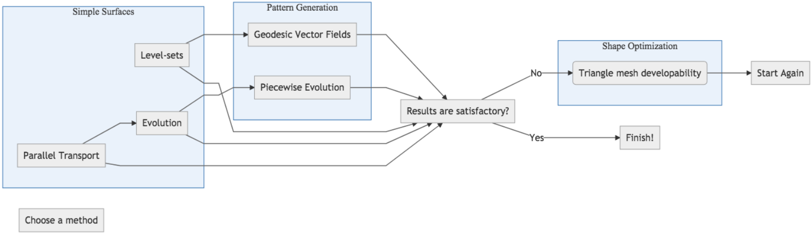
Temp Title

It is standard procedure to combine all streses (tension, shear, bending) and use this information for checking panel admissibility.

INSERT RESULTS!!!

Conclusion

Temp Title


Simple Surfaces
Level-sets
Evolution
Parallel Transport
Pattern Generation
Geodesic Vector Fields
Piecewise Evolution
Results are satisfactory?
No
Yes
Shape Optimization
Triangle mesh developability
Start Again
Finish!
Choose a method

Temp Title

- Parallel transport method is ONLY usefull when surfaces are developable or *nearly* developable.
- Evolution Method improves upon it's predecessor but still lacks the ability to maintain equal thickness over complex surfaces.
- *Piecewise* evolution method gives the best results overall.
- *Level-set method* can be used to calculate geodesic webs.
  - To cover freeform surfaces it need to be coupled with the geodesic-vector field technique.
- *Geodesic vector fields* is an introductory step to cut the mesh into pieces that will be easily covered by a 1-pattern of geodesic curves.
- *Developalizing* the surface is an extreme measure, since during the process, the overall smoothness of the surface will be lost. It can still be done in a controlled manner to reduce areas of high curvature.

Thanks[4]

[4]Special thanks to . . . FILL IN LATER!

Appendix

Resources

PUT LINKS TO GH COMPONENTS HERE + OTHER NICE SOFTWARE!

References i

Carmo, Manfredo P do. 2016. *Differential Geometry of Curves and Surfaces: Revised and Updated Second Edition*. Courier Dover Publications.

Pottmann, Helmut, Qixing Huang, Bailin Deng, Alexander Schiftner, Martin Kilian, Leonidas Guibas, and Johannes Wallner. 2010. "Geodesic Patterns." In *ACM SIGGRAPH 2010 Papers on - SIGGRAPH '10*. https://doi.org/10.1145/1833349.1778780.

Shelden, Dennis Robert. 2002. "Digital Surface Representation and the Constructibility of Gehry's Architecture." PhD thesis, Massachusetts Institute of Technology. http://hdl.handle.net/1721.1/16899.

Wallner, Johannes, Alexander Schiftner, Martin Kilian, Simon Flöry, Mathias Höbinger, Bailin Deng, Qixing Huang, and Helmut Pottmann. 2010. "Tiling Freeform Shapes with Straight Panels: Algorithmic Methods." In *Advances in Architectural Geometry 2010*, edited by Cristiano Ceccato, Lars Hesselgren, Mark Pauly, Helmut Pottmann, and Johannes Wallner. Vienna: Springer Vienna. https://doi.org/10.1007/978-3-7091-0309-8_5.