Geodesic Patterns for Free-form Architecture*

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Abstract

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1 Introduction

1.1 Geodesic patterns

What are geodesic patterns?

- Patterns made of panels (wood or metal).
- Bent by their weak axis.
- Mounted on a free-form surface.
- Rectangular or cuasi-rectangular when layed flat.
- Water-tight.
- Overall shape is achieved by pure bending. See Fig. 1

Figure 1: Geodesic pattern examples & previous work

1.2 Properties to aim for in panels

1.2.1 Geodesic property

- Long Thin panels that bend about their weak axis
- Zero geodesic curvature
- Represent the shortest path between two points on a surface

1.2.2 Constant width property[1]

- Panels whose original, unfolded shape is a rectangle.
- The only way this can happen is if the entire surface is developable.
- For all other surfaces:
  - Assuming no gaps between panels
  - Panels will not be exactly rectangular when unfolded
  - ***Requirement:*** Geodesic curves that guide the panels must have approximately constant distance from thier neighbourhood curves.

1.2.3 Developable (or 'pure-bending') property

- Bending panels on surfaces changes the distances in points only by a small amount so,
- A certain amount of twiting is also present in this aplications.
  Some methods in this chapter do not take into account this property.

1.3 Problem Statement

1.3.1 Problem 1

Look for a system of geodesic curves that covers a freeform surface in a way that:

1. They have approximate constant distance with it's neighbours.
2. This curves will serve as guiding curves for the panels.
3. The panels are to cover the surface with ***no overlap*** and ***only small gaps***

1.3.2 Problem 2

Look for a system of geodesic curves in a freeform surface which:

1. Serve as the boundaries of wooden panels.
2. The panel's deveopment is ***nearly straight***.
3. Those panels cover the surface with ***no gaps***

2 Algorithmic ways of generating geodesics

2.1 Geodesic curves on a surface

2.2 Geodesic curves on a mesh

3 Design strategies for geodesic systems

3.1 Design by parallel transport

This method, described in (Pottmann et al. 2010), allows for the generation of a system of geodesic curves where either the maximum distance or the minimum distance between adjacent points ocurrs at a prescribed location.

In differential geometry, the concept of *parallel transport* (see fig. 2) of a vector $\boldsymbol{V}$ along a curve $\boldsymbol{S}$ contained in a surface means moving that vector along $\boldsymbol{S}$ such that:

1. It remains tangent to the surface
2. It changes as little as possible in direction

[1]This is just some random footnote in the paper.

3. It is a known fact that the length of the vector remains unchanged

Figure 2: Parallel Transport

MISSING PARALLEL TRANSPORT PROCEDURE

Following this procedure, *extremal distances between adjacent geodesics occur near the chosen curve.*

3.2 Design by evolution & segmentation

Two main concepts are covered in this section, both proposed by (Pottmann et al. 2010): the first, what is called the *evolution method*, and a second method based on *piecewise-geodesic* vector fields.

3.2.1 The *evolution method*

As depicted in Fig. 3: Starting from a source geodesic somewhere in the surface:

- Evolve a pattern of geodesics iteratively computing ‘next’ geodesics.
- ‘Next’ geodesics must fullfil the condition of being at approximately constant distance from its predecesor.
- If the deviation from its predecesor is too great, breakpoints are introduced and continued as a *‘piecewise geodesic’.*

Figure 3: The evolution method

3.2.2 Piecewise-geodesic vectorfields

XXX

4 Panels from curve patterns

In this section, we will discuss several ways to generate panels from a system of 1-geodesic curves.

4.1 Tangent-developable method

The notion of ***Conjugate tangents*** on smooth surfaces must be defined:

- Strictly related to the ***Dupin Indicatrix***
- In negatively curved areas, the Dupin Indicatrix is an hyperbola whose asymptotic directions (A1, A2)
- Any parallelogram tangentialy circumscribed to the indicatrix defines two conjugate tangents **T** and **U**.
- The asymptotic directions of the dupin indicatrix are the diagonals of any such parallelogram.

Figure 4: Tangent developable method for panels

Initial algorithm is as follows:

For all geodesics s_i in a given pattern:

1. Compute the *tangent developable surfaces* $\rightarrow \Psi_i$
2. Trim Ψ_i along the intersection curves with their respective neighbours.
3. Unfold the trimed Ψ_i, obtaining the panels in flat state.

Unfortunately, this method needs to be refined in order to work in practice because:

1. The rulings of tangent developables may behave in weird ways
2. The intersection of the neighbouring Ψ_i's is often *ill-defined*.

Therefore, the procedure was modified in the following way:

1. Compute the *tangent developable surfaces* Ψ_i for all surfaces $s_i \rightarrow i =$ even numbers
2. Delete all rulings where the angle enclose with the tangent α is smaller than a certain threshold (i.e. 20º).
3. Fill the holes in the rulings by interpolation (???)
4. On each ruling:
   1. Determine points $A_i(x)$ and $B_i(x)$ which are the closest to eodesics s_{i-1} and s_{i+1}. This serves for trimming the surface Ψ_i.
5. Optimize globaly the positions of points $A_i(x)$ and $B_i(x)$ such that
   1. Trim curves are *smooth*
   2. $A_i(x)$ and $B_i(x)$ are close to geodesics s_{i-1} and s_{i+1}
   3. The ruling segments $A_i(x)B_i(x)$ lies close to the *original surface* Φ

4.2 The Bi-Normal Method

The second method for defining panels, once an appropriate system of geodesics has been found on Φ, works directly with the geodesic curves.

Assume that a point $P(t)$ traverses a geodesic s with unit speed, where t is the time parameter. For each time t there is:

- a velocity vector $T(t)$
- the normal vector $N(t)$
- a third vector $B(t)$, *the binormal vector*

Figure 5: Tangent developable method examples

This makes $T.N.B$ a ***moving orthogonal right-handed frame***

The surface Φ is represented as a triangle mesh and s is given as a polyline. For each geodesic, the associated surface is constructed according to Fig. 6. Points $L(t)$ and $R(t)$ represent the border of the panel, whose distance from $P(t)$ is half the panel width.

Figure 6: Binormal Method for panels & T.N.B. frame

4.3 Method Comparison

See tbl. 1 for more info…

5 Stress and strain in panels

The following section investigates the behaviour of a rectangular strip of elastic material when it is bent to the shape of a ruled surface Ψ un such way that:

The central line m of the strip follows the *'middle geodesic'* s in Ψ

This applies to both methods defining panels.fig. 7

Figure 7: Stress in panels

5.1 Stress formulas

$$\rho = 1/\sqrt{K}, \tag{1}$$

$$d/2\rho \leq C, \quad with \quad C = \sqrt{\sigma_{max}/E}, \tag{2}$$

$$\varepsilon = \frac{1}{2}(d/2\rho)^2 + \cdots . \tag{3}$$

MISSING MORE INFO ON STRESS ANALYISIS

6 Final analysis cost, quality

All strategies must be compared against cost & quality of the different solutions.

Figure 8: Cost/Quality Final Assesment

6.1 Frequent measures used in the topic

- Bounding-box diagonal of the panels

6.2 Cost variables

Cost should be defined as:

1. ???
2. ???
3. ???

6.3 Quality variables

Quality should be defined as:

1. ???
2. ???
3. ???

6.4 Variable weighting method

Explanation of the weighting of variables?

7 Math Section

Some nomenclature and formula clarification for the non-mathematicians!?

7.1 Nomenclature guide

1. ρ
2. τ
3. Φ
4. Ψ
5. s
6. V, T, N, B
7. θ
8. σ
9. Add more...

7.2 Formulas & referencing guide

LaTeX formulas and reference them (like Eq. 1 or multiple at once like Eqns. 2, 3) can be inserted using `$$` and formated using Symbols.PDF found in the 'resources' folder.

References are placed using the format `[@type:label]`, being `label` the unique name of the desired reference on the format, and `type` the type of reference, in the following format:

- Images: `{#fig:LABEL}`
- Tables: `{#tbl:LABEL}`
- Equations: `{#eq:LABEL}`
- Sections: `{#sec:LABEL}`
  - If sections are added, they will change all the reference names to include their corresponding sectionsd
- Code blocks: `{#lst:LABEL}`

7.2.1 Distances between geodesics (Eqns. 4, 5)

$$g^{+}(s) = g(s) + \varepsilon \mathbf{v}(s) + \varepsilon^{2}(\ldots) \tag{4}$$

$$\mathbf{v}(s) = \omega(s) \cdot R_{\pi/2}(g'(s)), \quad where \quad \omega'' + K\omega = 0. \tag{5}$$

Tables are also an option:

Table 1: Comparisson between panel generation methods

Tangent-Developable Method	**Bi-Normal Method**
Tries tor reproduce panels achievable by pure bending	Simple, obvious way of mathematically defining panels
Panels produced remain tangent to the surface	**Unclear** if the panels should follow this shape.
Follows a manufacturing goal	Panel surfaces are mathematically exact
	Panels are admissible from the viewpoint of stresses and strain

HTML figure disposition is also available, with customization options like width, per image captions, etc. . .

(a) (b) (c)

Figure 9: Difference between width-settings, nocaption option, etc… Fig. 9 is a full figure reference, but you can also reference just one of the images, like [Fig. 9a;[Fig. 9b;Fig. 9c.

8 References that must be used

- (Eigensatz et al. 2010)
- (Chen and Han 1996)
- (Kahlert, Olson, and Zhang 2010)
- (Surazhsky et al. 2005)
- (Arsan and Özdeger 2015)
- (Pottmann et al. 2010)
- (Polthier and Schmies 1998)
- (Carmo 2016)
- (Kimmel and Sethian 1998)
- (Rose et al. 2007)
- (Weinand and Pirazzi 2006)
- (Wallner et al. 2010)

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