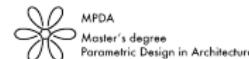


Geodesic Patterns

for Freeform Architecture

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¹Special thanks to ... FILL IN LATER!

Objective

Discretize a given freeform surface into planks with the following properties:

1. Must be *developable* (Shelden 2002)
2. Should tend to have approximate *equal width*
3. Should be *as straight as possible*
4. Cannot bend by their strong axis but,
5. can have a twist and bend by their weak axis

Plank A plank is timber that is flat, elongated, and rectangular with parallel faces that are higher and longer than wide. (Wikipedia)

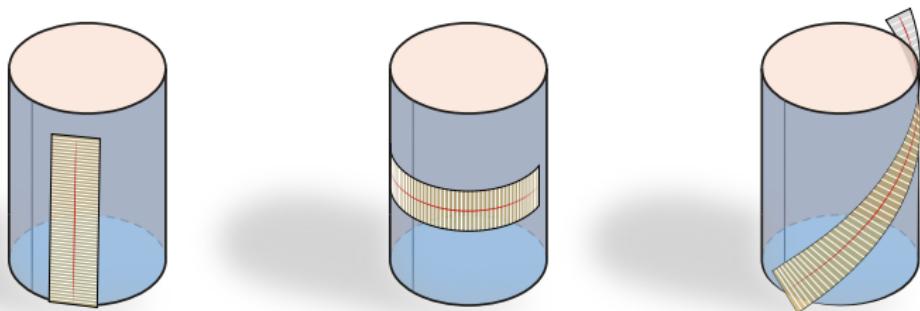


Figure 1: A straight plank laying on a cylinder on different directions.

Background

The use of *straight developable planks* is widely used in:



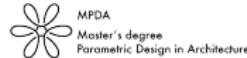
Figure 2: Traditional boat building

Also common practice in naval engineering industry:



Figure 3: Connected developable patches for boat hull design

NOX was one of the first to experiment with paper strips.
MISSING NOX IMAGE/REFERENCE!



Frank Ghery

This techniques have also been used in the architecture world, mainly by **Frank Ghery**.

His façades are usually a collection of connected developable surfaces.

Latest architectural work following this techniques was:

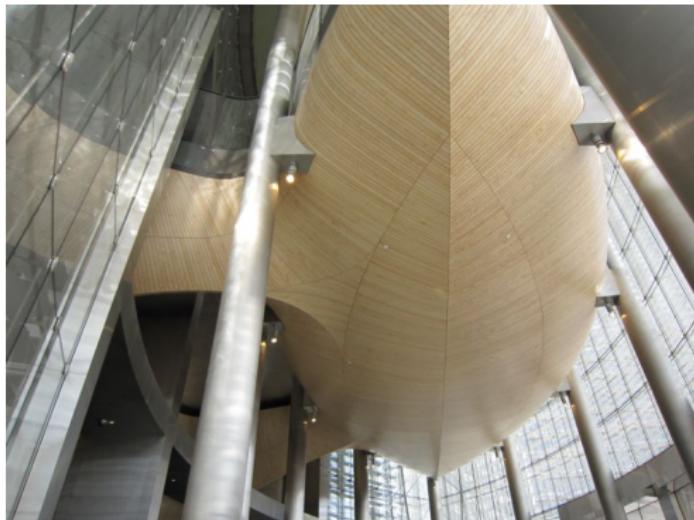


Figure 4: Burj Khalifa by Frank Ghery

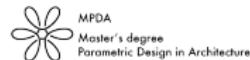
It was designed as a collection of:

- **Developable surfaces**
 - Which can be covered by equal width planks
- **Surfaces of constant curvature**
 - Which can be covered by repeating the same profile





Figure 5: Burj Khalifa final panel solution



Construction technique

Geodesic curves

A geodesic curve is the generalization of a *straight line* into *curved spaces*.

In this research, we concentrate on the concept of *straightest geodesics*.

Some markdown content

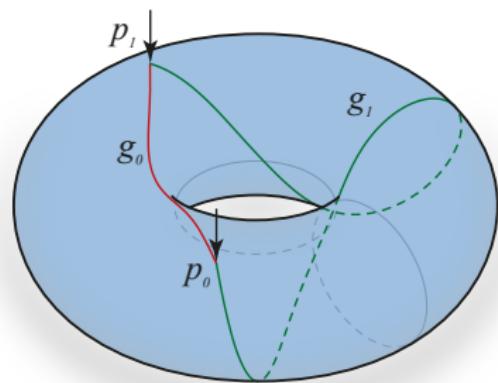


Figure 6: Shortest geodesic on a torus

Developable surfaces

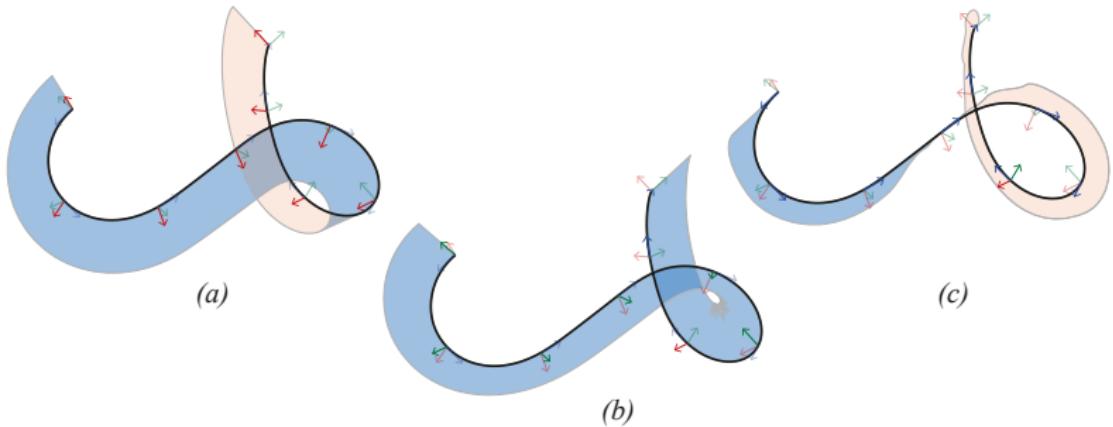


Figure 7: Surfaces with *0 gaussian curvature*. Meaning, they can be flattened onto a plane *without distortion*

Developable surfaces

- *surfaces that can be flattened.*
- *can be generated by a single curve.*

Geodesic curves

- *are straight lines in a curved space.*

If Panels are generated using geodesic curves on the surface
Then Resulting panels will be *developable* and mostly *straight* when flat.

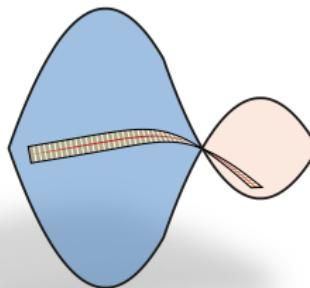


Figure 8: A straight plank laying on a hyperbolic paraboloid

In other words

We wish to cover a given freeform surface with a pattern of **geodesic curves** with equal spacing.

This can only be achieved if the provided surface is already *developable*.

A compromise exists between the *curve spacing* and the *curve's geodesic curvature*

Algorithmic strategies

Obtaining Geodesic Patterns

These are the main methods for the obtaining successful geodesic patterns:

1. The *parallel transport* method
2. The *evolution* method
 - 2.1 The *piecewise geodesic* evolution method
3. The *level-set* method

The parallel transport method

Vector parallel transport

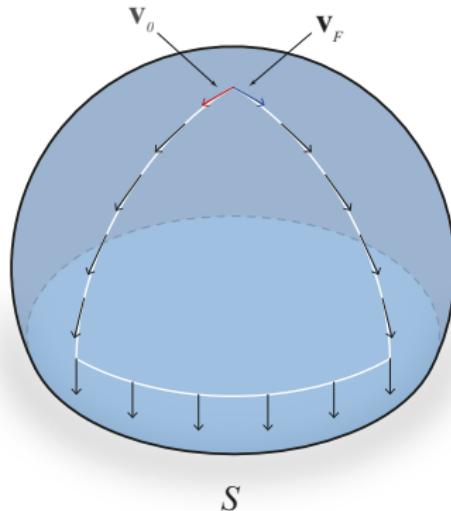


Figure 9: Parallel transport of a vector over a path on a sphere

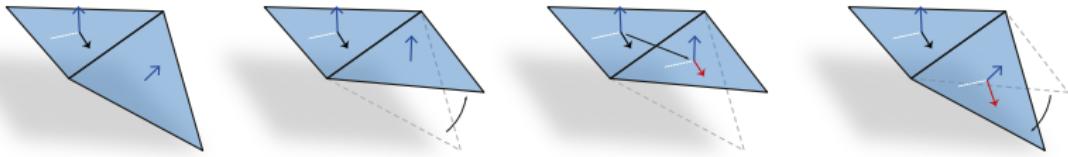


Figure 10: Parallel transport over two adjacent mesh faces

P.T. Implementation

Input: A surface Φ , represented as a triangular mesh (V,E,F)

Output: Set of geodesic curves g_i , where $i = 0, \dots, M$

- 1: Place a geodesic curve g_x along S such that it divides the surface completely in 2.
- 2: Divide the curve into N equally spaced points p with distance W .
- 3: Place a vector \mathbf{v} onto p_0
- 4: Parallel transport that vector along g_x as described in [@fig:parTransProc].
- 5: **for all** points p_i where $i = 0, \dots, M$ **do**
- 6: Generate geodesic curve $+g_i$ and $-g_i$ using vector \mathbf{v}_i and $-\mathbf{v}_i$ respectively.
- 7: Join $+g_i$ and $-g_i$ together to obtain g_i
- 8: Add g_i to output.
- 9: **end for**

There are **three extreme** cases depending on the *local gaussian curvature* where g lies on Φ :

Positive curvature Panels will have **Maximum width** on g

Negative curvature Panels will have **Minimum width** on g

0 gaussian curvature: Panels will be of equal width

P.T. Example

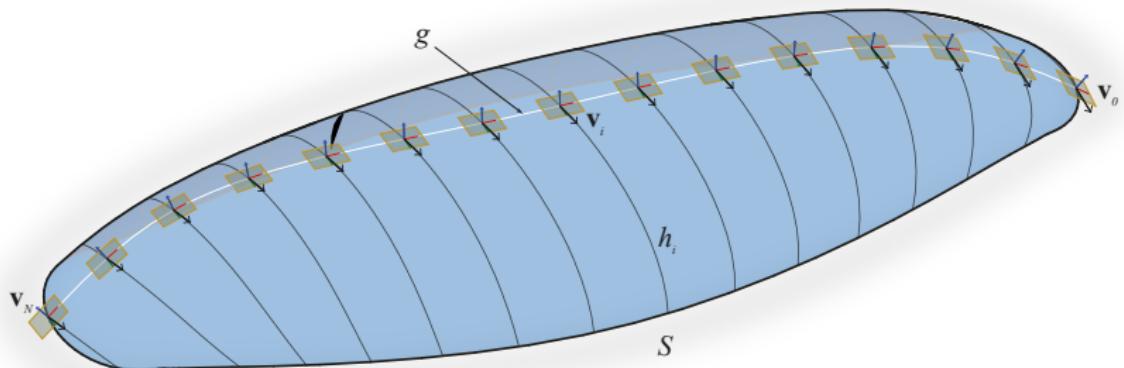


Figure 11: Parallel transport method over a positive curvature surface

P.T. Results

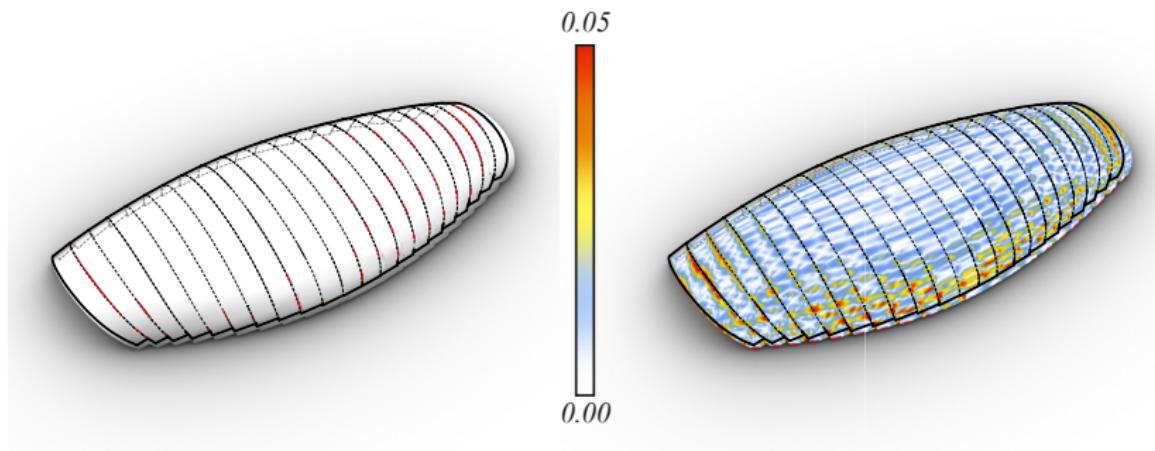


Figure 12: TNB generated panels & distance to original mesh

The Evolution Method

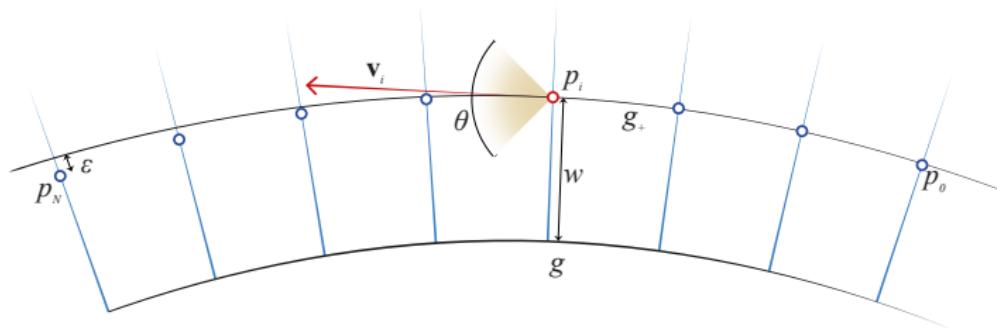


Figure 13: Calculating the best-fit geodesic

Evolution Implementation

Input: A surface Φ , represented as a triangular mesh (V,E,F)

Output: Set of geodesic curves g_i , where $i = 0, \dots, M$

- 1: Place a geodesic curve g_x along S such that it divides the surface completely in 2.
- 2: Divide the curve into N equally spaced points p with distance W .
- 3: Place a vector \mathbf{v} onto p_0
- 4: Parallel transport that vector along g_x as described in [@fig:parTransProc].
- 5: **for all** points p_i where $i = 0, \dots, M$ **do**
- 6: Generate geodesic curve $+g_i$ and $-g_i$ using vector \mathbf{v}_i and $-\mathbf{v}_i$ respectively.
- 7: Join $+g_i$ and $-g_i$ together to obtain g_i
- 8: Add g_i to output.
- 9: **end for**

Evolution Method Results

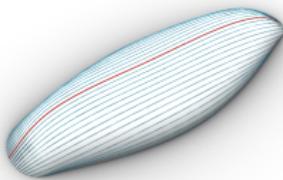


Figure 14: Evolution method example

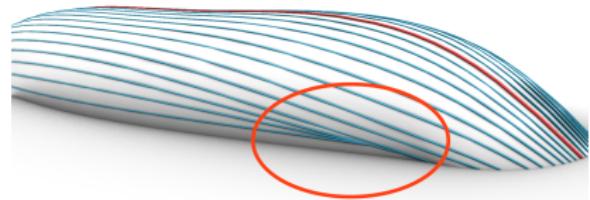


Figure 15: Evolution method problems

Local changes in curvature produce:

- Sharp panel endings in positive curvature areas
- Panel width increase in negative curvature areas

The Piecewise Evolution Method

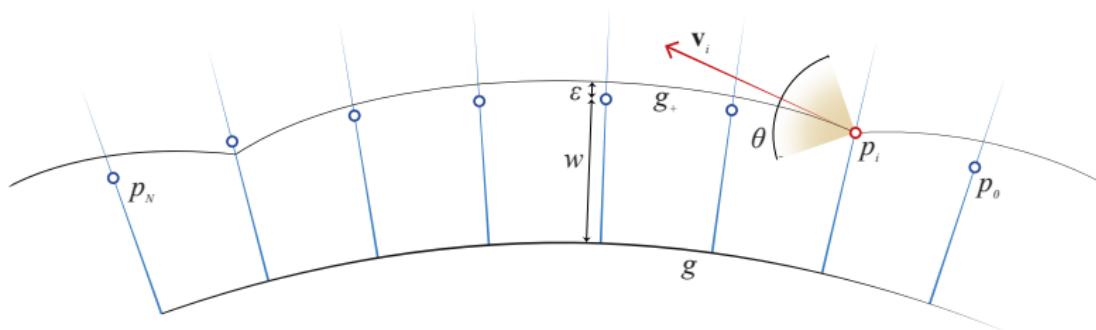


Figure 16: Calculating the best piece-wise geodesic

Piecewise Ev. Implementation

INSERT ALGORITHM HERE

Piecewise Ev. Results

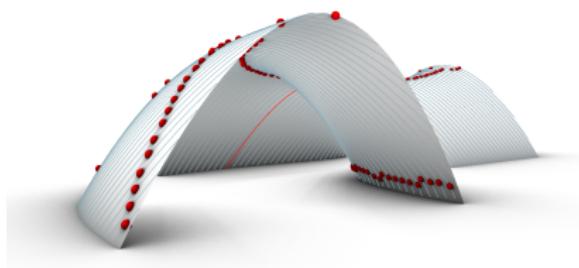


Figure 17: Piecewise Test

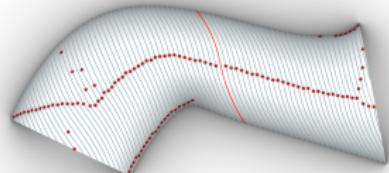


Figure 18: Piecewise Test

The level set method

Mesh Level-sets

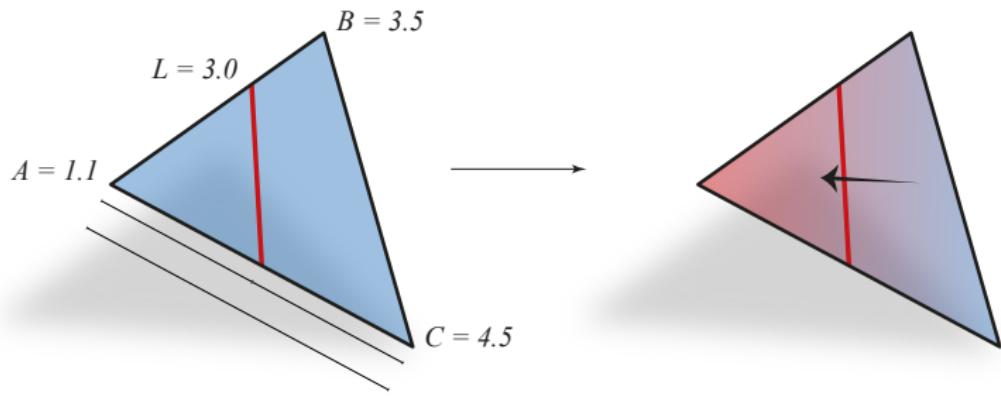


Figure 19: Level set on a single mesh face

Level-set Implementation

INSERT ALGORITHM HERE

Results

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Modeling planks

Given any point in g :

1. Assuming $T(x)$ is tangent g .
2. Compute $U(x)$ as $T(x) \times N_\Phi(x)$
The union of all $U(x)$ is a developable ruled surface Ψ .

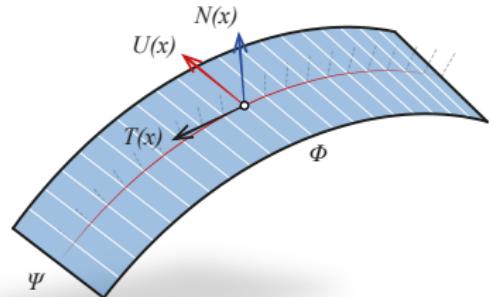


Figure 20: Tangent developable method for panels

Initial algorithm is as follows:

For all geodesics s_i in a given pattern:

1. Compute the *tangent developable surfaces* $\rightarrow \Psi_i$
2. Trim Ψ_i along the intersection curves with their respective neighbours.
3. Unfold the trimmed Ψ_i , obtaining the panels in flat state.

Unfortunately, this method needs to be refined in order to work in practice because:

1. The rulings of tangent developable may behave in weird ways
2. The intersection of the neighboring Ψ_i 's is often *ill-defined*.

Therefore, the procedure was modified in the following way:

1. Compute the *tangent developable surfaces* Ψ_i for all surfaces
 $s_i \rightarrow i = \text{even numbers}$
2. Delete all rulings where the angle enclose with the tangent α is smaller than a certain threshold (i.e. 20°).
3. Fill the holes in the rulings by interpolation (???)
4. On each ruling:
 5. Determine points $A_i(x)$ and $B_i(x)$ which are the closest to geodesics s_{i-1} and s_{i+1} . This serves for trimming the surface Ψ_i .
 6. Optimize globally the positions of points $A_i(x)$ and $B_i(x)$ such that
 7. Trim curves are *smooth*
 8. $A_i(x)$ and $B_i(x)$ are close to geodesics s_{i-1} and s_{i+1}
 9. The ruling segments $A_i(x)B_i(x)$ lies close to the *original surface* Φ

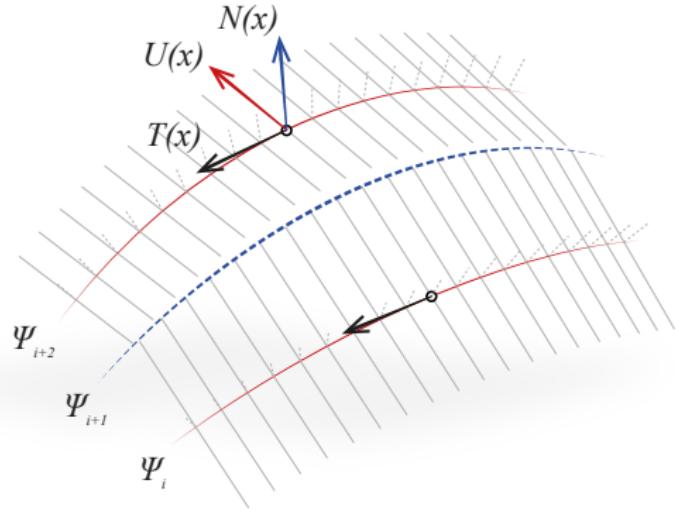


Figure 21: Panels computed using the using the modified tangent developable method.

Optimization

Piecewise geodesic vector-fields

Missing PG Vector Field Images!

Developability of triangle meshes

MISSING K.CRANE'S IMPLEMENTATION IMAGES!

Analysis

Gaps in panelization

???

Stress in panels

Assuming the material:

- is bent to the shape of a ruled surface Ψ
- the central line m of the plank follows the ‘middle geodesic’ in Ψ .

Then:

- Lines parallel to m at distance $d/2$ are not only bent but also stretched.

If we introduce the radius of Gaussian curvature $\rho = 1/\sqrt{|K|}$
the relative increment in length ε (strain) of the strip is:

$$\varepsilon = \frac{1}{2}(d/2\rho)^2 + \dots \quad (1)$$

where d is the planks width.

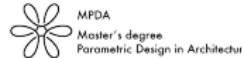
Tensile stress

Tensile stress can be expressed as $\sigma = E\varepsilon^2$, which yields:

$$d/2\rho \leq C, \quad \text{with} \quad C = \sqrt{\sigma_{max}/E}$$

σ_{max} = maximum admissible stress

E = Young's modulus



²Since this calculation is an approximation, a safety factor must be used when choosing σ_{max} .

$$d/2\rho \leq C, \quad \text{with} \quad C = \sqrt{\sigma_{max}/E}$$

Since C is a material constant. We obtain the maximum admissible width with:

$$d_{max} = 2\rho_{min}C$$

Maybe missing an image here?

Table 1: Example calculation of constant C for some of the most suitable materials.

Material	Young Modulus	Max. stress	C
Wood	200000		
Steel	13000		
Others?	?		

Bending and shear stress

Only for panels with thin rectangular cross-sections ($h/d \ll 1$)

Bending (σ) and shear stresses (τ) depend on:

Panel thickness h but **not** on the panel width d .

Maximum values occur on the outer surface of the panel (Wallner et al. 2010) and depend on:

- the curvature κ of the central geodesic
- the rate of torsion θ of the panel.

$$\sigma = E\kappa h/2 \quad \text{and} \quad \tau = hG\theta \quad (2)$$

Where G is the shear modulus.

τ , measured by arc per meter, does not exceed $\sqrt{|K|} = 1/\rho$,

maximum value occurs if:

- the central geodesic's tangent is an asymptotic direction of the panel surface (Carmo 2016).

It is standard procedure to combine all stresses (tension, shear, bending) and use this information for checking panel admissibility.

INSERT RESULTS!!!

Conclusion

- Parallel transport method is ONLY usefull when surfaces are developable or *nearly* developable.
- Evolution Method improves upon it's predecessor but still lacks the ability to maintain equal thickness over complex surfaces.
- *Piecewise* evolution method gives the best results overall.
- *Level-set method* can be used to calculate geodesic webs.
 - To cover freeform surfaces it need to be coupled with the geodesic-vector field technique.
- *Geodesic vector fields* is an introductory step to cut the mesh into pieces that will be easily covered by a 1-pattern of geodesic curves.
- *Developalizing* the surface is an extreme measure, since during the process, the overall smoothness of the surface will be lost. It can still be done in a controlled manner to reduce areas of high curvature.

Thanks³

³Special thanks to . . . FILL IN LATER!



Appendix

Resources

PUT LINKS TO GH COMPONENTS HERE + OTHER NICE SOFTWARE!



References i

- Carmo, Manfredo P do. 2016. *Differential Geometry of Curves and Surfaces: Revised and Updated Second Edition*. Courier Dover Publications.
- Shelden, Dennis Robert. 2002. “Digital Surface Representation and the Constructibility of Gehry’s Architecture.” PhD thesis, Massachusetts Institute of Technology. <http://hdl.handle.net/1721.1/16899>.
- Wallner, Johannes, Alexander Schiftner, Martin Kilian, Simon Flöry, Mathias Höbinger, Bailin Deng, Qixing Huang, and Helmut Pottmann. 2010. “Tiling Freeform Shapes with Straight Panels: Algorithmic Methods.” In *Advances in Architectural Geometry 2010*, edited by Cristiano Ceccato, Lars Hesselgren, Mark Pauly, Helmut Pottmann, and Johannes Wallner. Vienna: Springer Vienna. https://doi.org/10.1007/978-3-7091-0309-8_5.