Geodesic Patterns

for Freeform Architecture

Alan Rynne

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UPC - MPDA'18

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[1] Special thanks to . . . FILL IN LATER!

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Objective

Discretize a given freeform surface into planks with the following properties:

1. Must be ***developable*** (Shelden 2002)
2. Should tend to have approximate ***equal width***
3. Should be ***as straight as possible***
4. Cannot bend by their strong axis but,
5. can have a twist and bend by their weak axis

Plank A plank is timber that is flat, elongated, and rectangular with parallel faces that are higher and longer than wide. (Wikipedia)

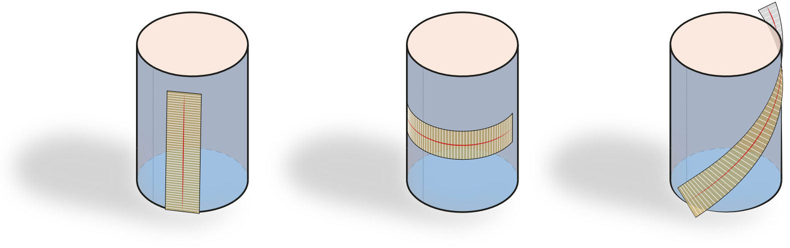

Figure 1: A straight plank laying on a cylinder on different directions.

Background

The use of *straight developable planks* is widely used in:

Figure 2: Traditional boat building

Also common practice in naval engineering industry:

Figure 3: Connected developable patches for boat hull design

The architecture studio NOX was one of the first to experiment with paper strips.

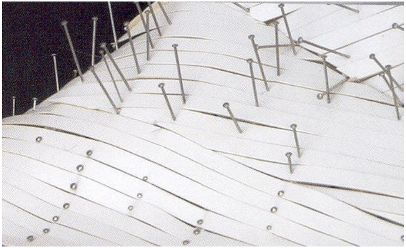

Figure 4: NOX Strips models

Figure 5: NOX render of strip model

MISSING BIB REFERENCE HERE!!!!

Frank Ghery

This techniques have also been used in the architecture world, mainly by **Frank Ghery**.

His façades are usually a collection of connected developable surfaces.

Latest architectural work following this techniques was:

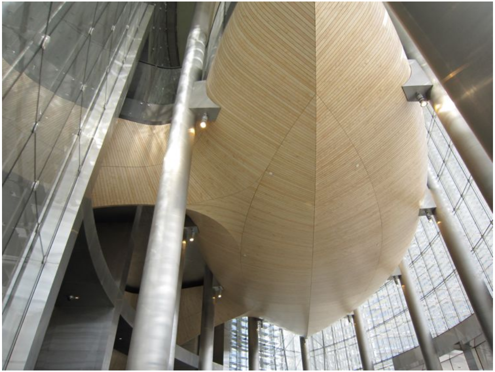

Figure 6: Burj Khalifa by Frank Ghery

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It was designed as a collection of:

- **Developable surfaces**
  - *Which can be covered by equal width planks*
- **Surfaces of constant curvature**
  - *Which can be covered by repeating the same profile*

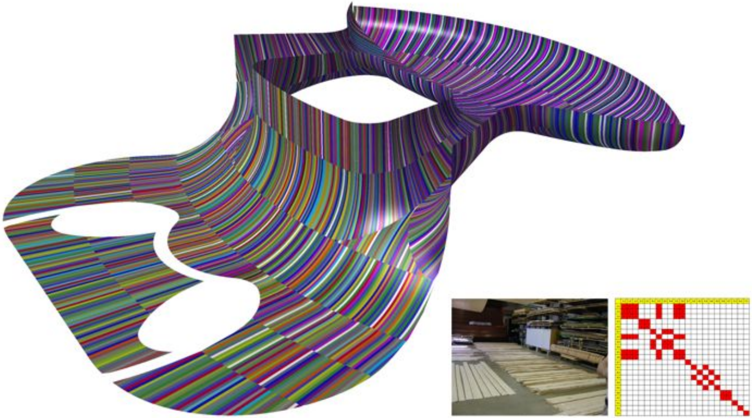

Figure 7: Burj Khalifa final panel solution

Construction technique

Geodesic curves

A geodesic curve is the generalization of a *straight line* into *curved spaces*.

In this research, we concentrate on the concept of ***straightest geodesics***.

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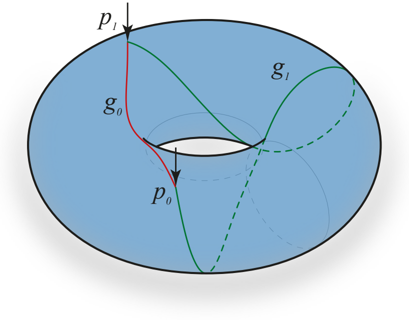

Figure 8: Shortest geodesic on a torus.

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Developable surfaces

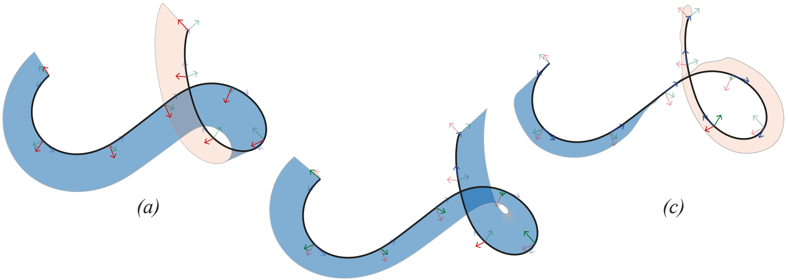

Figure 9: Surfaces with ***0 gaussian curvature***. Meaning, they can be flattened onto a plane ***without distortion***

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Developable surfaces

- *surfaces that can be flattened.*
- *can be generated by a single curve.*

Geodesic curves

- *are straight lines in a curved space.*

If Panels are generated using geodesic curves on the surface

Then Resulting panels will be ***developable*** and mostly ***straight*** when flat.

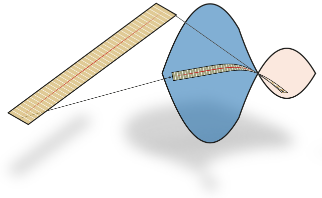

Figure 10: A straight plank laying on a hyperbolic paraboloid

In other words

We wish to cover a given freeform surface with a pattern of **geodesic curves** with equal spacing.

This can only be achieved if the provided surface is already *developable*.

A compromise exists between the *curve spacing* and the *curve's geodesic curvature*

Algorithmic strategies

Obtaining Geodesic Patterns

These are the main methods for the obtaining successful geodesic patterns:

1. The ***parallel transport*** method
2. The ***evolution*** method
    2.1 The ***piecewise geodesic*** evolution method
3. The ***level-set*** method

The parallel transport method

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Vector parallel transport

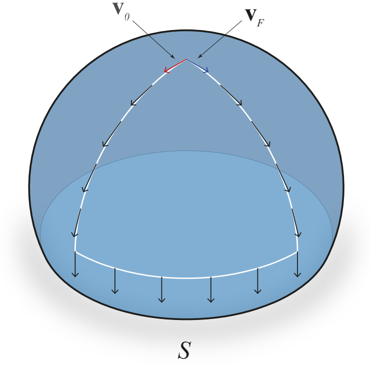

Figure 11: Parallel transport of a vector over a path on a sphere

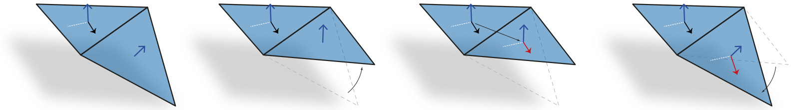

Figure 12: Parallel transport over two adjacent mesh faces

P.T. Implementation

Input: A surface Φ, represented as a triangular mesh (V,E,F)
Output: Set of geodesic curves $g_i, \quad where \quad i = 0, ..., M$

```
Place a geodesic curve g_x along S such that it divides the surface completely in 2.
Divide the curve into N equally spaced points p with distance W.
Place an vector v onto p_0
Parallel transport that vector along g_x as described in [@fig:parTransProc].
for all points p_i where i = 0,...,M do
    Generate geodesic curve +g_i and −g_i using vector v_i and −v_i respectively.
    Join +g_i and −g_i together to obtain g_i
    Add g_i to output.
end for
```

There are **three** *extreme* cases depending on the ***local gaussian curvature*** where g lies on Φ:

Positive curvature Panels will have **Maximum width** on g
Negative curvature Panels will have **Minimum width** on g
0 gaussian curvature: Panels will be of equal width

P.T. Example

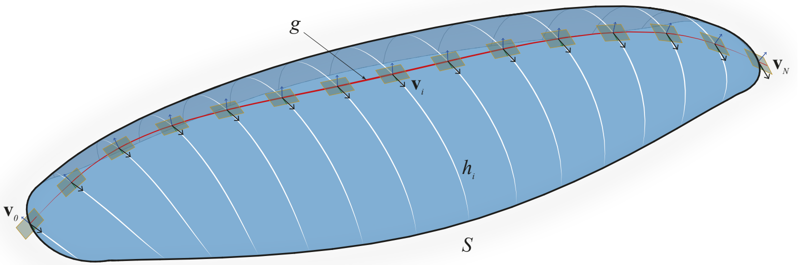

Figure 13: Parallel transport method over a positive curvature surface

P.T. Results

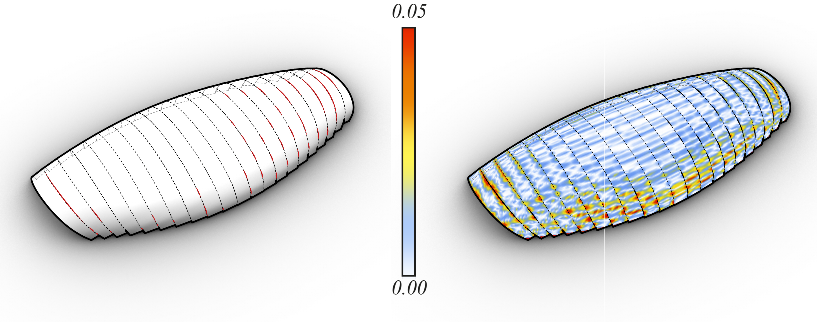

Figure 14: TNB generated panels & distance to original mesh

The Evolution Method

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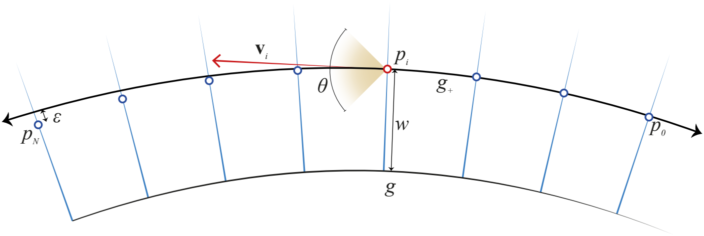

Figure 15: Calculating the best-fit geodesic

Evolution Implementation

Input: A surface Φ, represented as a triangular mesh (V,E,F)
Output: Set of geodesic curves $g_i, \quad where \quad i = 0, ..., M$

Place a geodesic curve g_x along S such that it divides the surface completely in 2.
Divide the curve into N equally spaced points p with distance W.
Place an vector $\mathbf{v}$ onto p_0
Parallel transport that vector along g_x as described in [@fig:parTransProc].
for all points p_i where i = 0,...,M **do**
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    Join $+g_i$ and $-g_i$ together to obtain g_i
    Add g_i to output.
end for

Evolution Method Results

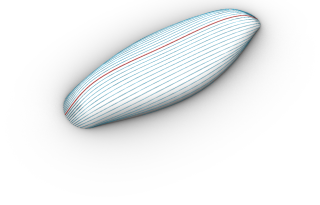

Figure 16: Evolution method example

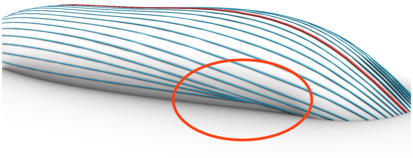

Figure 17: Evolution method problems

Local changes in curvature produce:

- Sharp panel endings in positive curvature areas
- Panel width increase in negative curvature areas

The Piecewise Evolution Method

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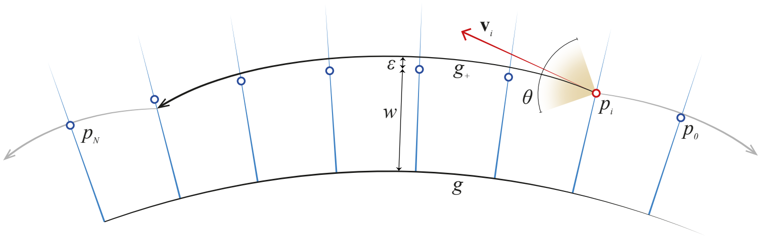

Figure 18: Calculating the best piece-wise geodesic

Piecewise Ev. Implementation

INSERT ALGORITHM HERE

Piecewise Ev. Results

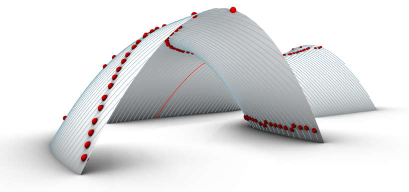

Figure 19: Piecewise Test

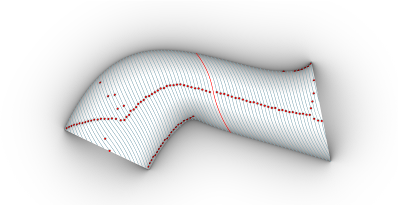

Figure 20: Piecewise Test

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The level set method

Mesh Level-sets

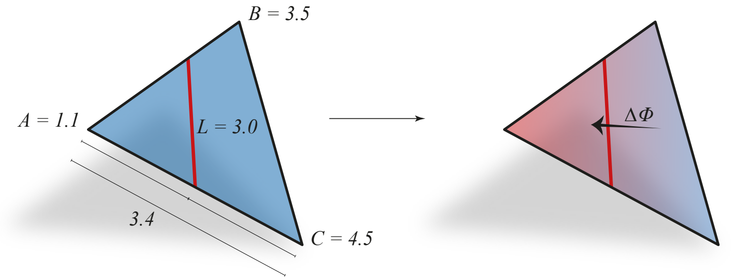

Figure 21: Level set on a single mesh face

Level-set Implementation

INSERT ALGORITHM HERE

Results

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Modeling planks

Tangent-developable method

Given any point in g:

1. Assuming $T(x)$ is tangent g.
2. Compute $U(x)$ as $T(x) \times N_{\Phi}(x)$
   The union of all $U(x)$ *is a developable ruled surface* Ψ.

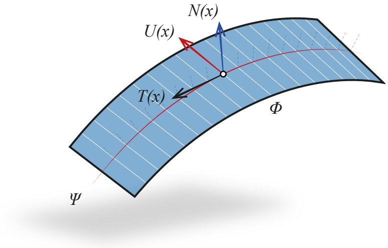

Figure 22: Tangent developable method for panels

Initial algorithm is as follows:

For all geodesics s_i in a given pattern:

1. Compute the *tangent developable surfaces* $\rightarrow \Psi_i$
2. Trim Ψ_i along the intersection curves with their respective neighbours.
3. Unfold the trimmed Ψ_i, obtaining the panels in flat state.

Unfortunately, this method needs to be refined in order to work in practice because:

1. The rulings of tangent developable may behave in weird ways
2. The intersection of the neighboring Ψ_i's is often *ill-defined*.

Therefore, the procedure was modified in the following way:

1. Compute the *tangent developable surfaces* Ψ_i for all surfaces $s_i \to i =$ even numbers
2. Delete all rulings where the angle enclose with the tangent α is smaller than a certain threshold (i.e. 20º).
3. Fill the holes in the rulings by interpolation (???)
4. On each ruling:
5. Determine points $A_i(x)$ and $B_i(x)$ which are the closest to geodesics s_{i-1} and s_{i+1}. This serves for trimming the surface Ψ_i.
6. Optimize globally the positions of points $A_i(x)$ and $B_i(x)$ such that
7. Trim curves are *smooth*
8. $A_i(x)$ and $B_i(x)$ are close to geodesics s_{i-1} and s_{i+1}
9. The ruling segments $A_i(x)B_i(x)$ lies close to the *original surface* Φ

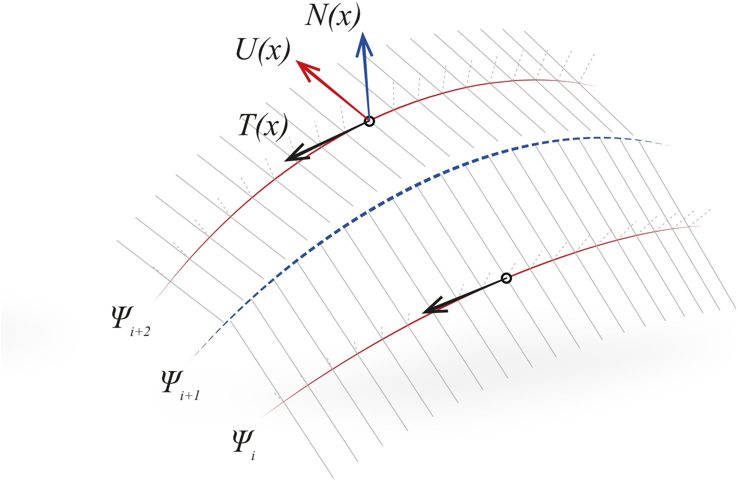

Figure 23: Panels computed using the using the modified tangent developable method.

Optimization

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Piecewise geodesic vector-fields

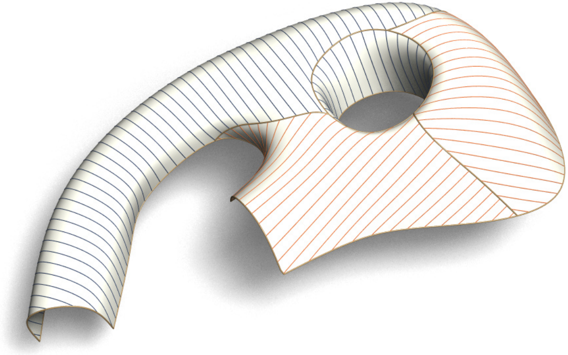

Figure 24: Geodesic pattern example[2]

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[2] Image taken from Pottmann et al. (2010)

The objective is to divide or *cut* the mesh into areas that will be easilly covered by a 1-pattern of geodesics.[3]

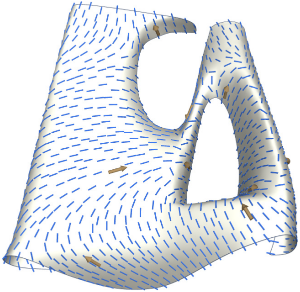

Figure 25: User specified directions

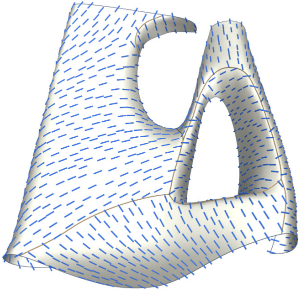

Figure 26: Computed geodesic vector field & cuts

This is done by computing a *geodesic vector field* over the surface, in a way that it aligns with several user specified directions.

[3]Image taken from Pottmann et al. (2010)

Developability of triangle meshes

Different methods for *developalizing* meshes have been developed over the years:

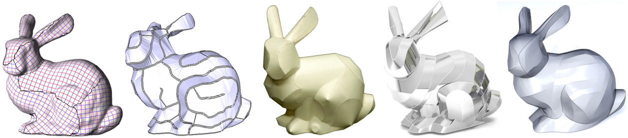

Figure 27: Left to right: Julius, Kraevoy, and Sheffer (2005), Mitani and Suzuki (2004), Shatz, Tal, and Leifman (2006) Liu et al. (2006) and Stein, Grinspun, and Crane (2018)[4]

[4]Image taken from Stein, Grinspun, and Crane (2018)

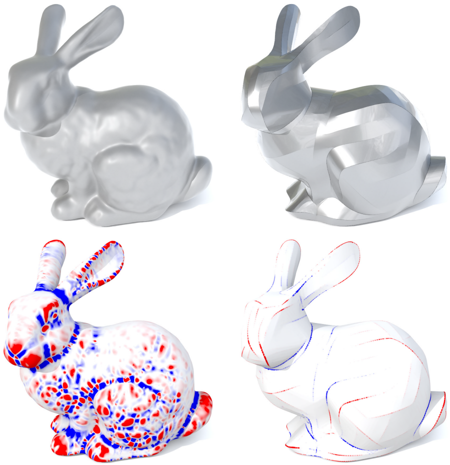

Figure 28: Developability of the Stanford Bunny (Stein, Grinspun, and Crane 2018)[5]

[5] Image taken from Stein, Grinspun, and Crane (2018)

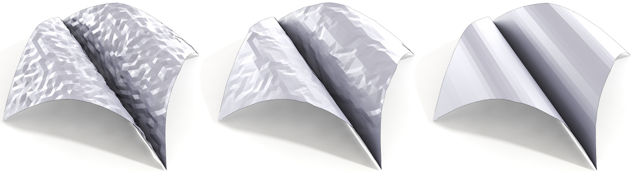

Figure 29: Noisy to smooth sheet (Stein, Grinspun, and Crane 2018)[6]

[6]Image taken from Stein, Grinspun, and Crane (2018)

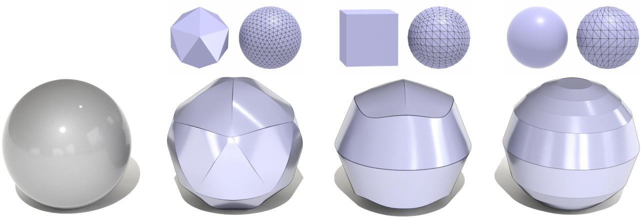

Figure 30: Developalizing a sphere. Results highly depend on the initial mesh topology[7]

[7]Image taken from Stein, Grinspun, and Crane (2018)

Energy applied is equivalent to forcing all vertices *angle defect* to be 0.

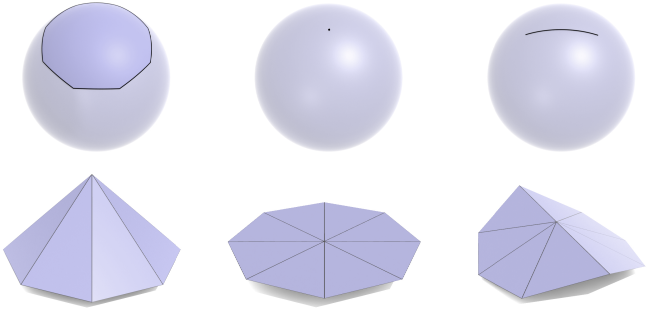

Figure 31: Any given vertex on a mesh (left) will become either flat (centre) or a hinge (right)[8]

This will automatically create *hinges* in the vertices where it is not possible to be flat.

[8] Image taken from Stein, Grinspun, and Crane (2018)

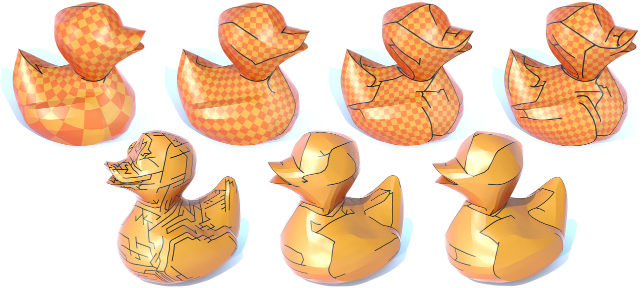

Figure 32: Different threshold configurations[9]

[9] Image taken from Stein, Grinspun, and Crane (2018)

Analysis

Gaps in panelization

???

Stress in panels

Assuming the material:

- is bent to the shape of a ruled surface Ψ
- the central line m of the plank follows the *'middle geodesic'* in Ψ.

Then:

- Lines parallel to m at distance $d/2$ are not only bent but also stretched.

If we introduce the radius of Gaussian curvature $\rho = 1/\sqrt{|K|}$ the relative increment in length ε (strain) of the strip is:

$$\varepsilon = \frac{1}{2}(d/2\rho)^2 + \cdots \tag{1}$$

where d is the planks width.

Tensile stress
Tensile stress can be expressed as $\sigma = E\varepsilon$[10], which yields:

$$d/2\rho \leq C, \quad with \quad C = \sqrt{\sigma_{max}/E}$$

$$\sigma_{max} = \text{maximum admissible stress}$$

$$E = \text{Young's modulus}$$

[10] Since this calculation is an approximation, a safety factor must be used when choosing σ_{max}.

$$d/2\rho \leq C, \quad with \quad C = \sqrt{\sigma_{max}/E}$$

Since C is a material constant. We obtain the maximum admissible width with:

$$d_{max} = 2\rho_{min}C$$

Maybe missing an image here?

Table 1: Example calculation of constant C for some of the most suitable materials.

Material	Young Modulus	Max. stress	C
Wood	200000		
Steel	13000		
Others?	?		

Bending and shear stress

Only for panels with thin rectangular cross-sections ($h/d \ll 1$***)***

Bending (σ) and shear stresses (τ) depend on:

Panel thickness h but ***not*** on the panel width d.

Maximum values occur on the outer surface of the panel (Wallner et al. 2010) and depend on:

- the curvature κ of the central geodesic
- the rate of torsion θ of the panel.

$$\sigma = E\kappa h/2 \quad and \quad \tau = hG\theta \tag{2}$$

Where G is the shear modulus.

τ, measured by arc per meter, does not exceed $\sqrt{|K|} = 1/\rho$,

maximum value ocurs if:

- the central geodesic's tangent is an asymptotic direction of the panel surface (Carmo 2016).

It is standard procedure to combine all streses (tension, shear, bending) and use this information for checking panel admissibility.

INSERT RESULTS!!!

Conclusion

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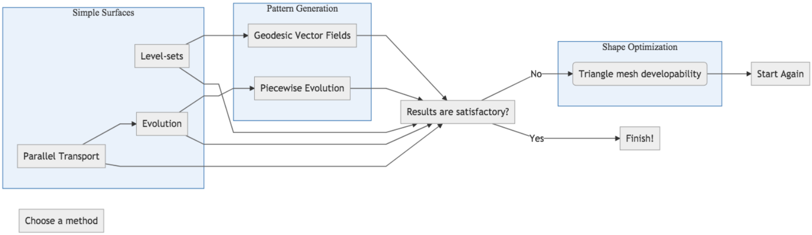

Simple Surfaces
Level-sets
Evolution
Parallel Transport
Pattern Generation
Geodesic Vector Fields
Piecewise Evolution
Results are satisfactory?
No
Yes
Shape Optimization
Triangle mesh developability
Start Again
Finish!
Choose a method

- Parallel transport method is ONLY usefull when surfaces are developable or *nearly* developable.

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- Evolution Method improves upon it's predecessor but still lacks the ability to maintain equal thickness over complex surfaces.

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- *Level-set method* can be used to calculate geodesic webs.

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- *Level-set method* can be used to calculate geodesic webs.
  - To cover freeform surfaces it need to be coupled with the geodesic-vector field technique.
- *Geodesic vector fields* is an introductory step to cut the mesh into pieces that will be easily covered by a 1-pattern of geodesic curves.
- *Developalizing* the surface is an extreme measure, since during the process, the overall smoothness of the surface will be lost. It can still be done in a controlled manner to reduce areas of high curvature.

Thanks[11]

[11] Special thanks to . . . FILL IN LATER!

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Appendix

Resources

PUT LINKS TO GH COMPONENTS HERE + OTHER NICE SOFTWARE!

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