Appendix C

Resonant Controller

C.1 Introduction

The resonant controller can be obtained either by applying the internal model principle or by using the coordinate transformation. The two methods are briefly described in the following.

C.2 Internal Model Principle

The internal model principle states that if the models of the reference and of the disturbance are included in the feedback control loop a good reference tracking and a good disturbance rejection capability is ensured. If the goal is to track and reject periodic signals that can be decomposed into sinusoidal components (harmonics), this procedure results in the design of controllers that have a pair of poles on the imaginary axis at the frequencies of the harmonics to track and/or to reject. In fact, the Laplace transform of a sinusoidal signal as the normalized grid voltage, i.e. a disturbance in the current control loop as shown in Figure C.1, is

$$E(s) = \frac{\omega}{s^2 + \omega^2} \tag{C.1}$$

The transfer function (C.1) will be used to demonstrate the effectiveness of a resonant controller tuned on the fundamental frequency to null the error due to the grid voltage.

Considering the block diagram as depicted in Figure C.1, the current, the consequence of the disturbance, will be defined as

$$I(s) = -\frac{G_f(s)}{1 + G_f(s) G_d(s) G_c(s)} E(s) \tag{C.2}$$

Considering that

$$G_f(s) = \frac{1}{Ls + R} \tag{C.3}$$

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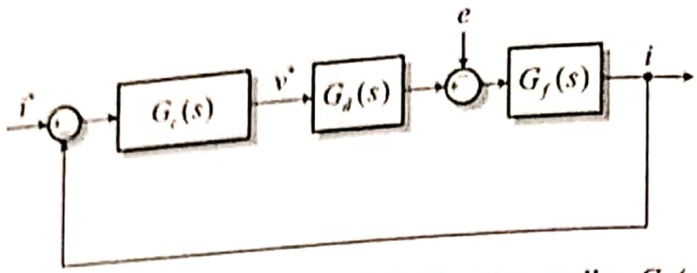

Figure C.1 The current control loop of an inverter: $G_c(s)$ is the controller, $G_d(s)$ is the delay of PWM and of the computational device and $G_f(s)$ models the filter and the grid

$$E(s) = \frac{\omega}{s^2 + \omega^2} \tag{C.4}$$

$$G_c(s) = \frac{k_P s^2 + k_I s + k_P \omega^2}{s^2 + \omega^2} \tag{C.5}$$

$$G_d(s) = \frac{1}{(1.5T_s)\, s + 1} \tag{C.6}$$

where L and R are respectively the total inductance and resistance of the grid and the filter, T is the sample time and k_P and k_I are the proportional and resonant gain. k_P is tuned in order to ensure that the overall system has a damping factor of 0.707. A high k_I is usually adopted in order to obtain sufficient attenuation of the tracking error in case the frequency of the grid is subjected to changes. However, a high k_I can cause too high an overshoot. The optimum gain k_I can be adopted considering that the grid frequency is stiff and it is only allowed to vary in a narrow range, typically $\pm$ 1 %

(C.2) results into:

$$\frac{\Delta I(s)}{E(s)} = \frac{(s^2 + \omega^2) \cdot (0.5T_s s + 1)}{(Ls + R) \cdot (s^2 + \omega^2) \cdot (0.5T_s s + 1) + \left(k_P s^2 + k_I s + k_P \omega^2\right)} \tag{C.7}$$

It can be proven that (C.7) is zero for $s = j\omega$.

C.3 Equivalence of the PI Controller in the *dq* Frame and the P+Resonant Controller in the $\alpha\beta$ Frame

The process can be derived by inverse transforming the synchronous controller back to the stationary $\alpha\beta$ frame $G_{dq}(s) \rightarrow G_{\alpha\beta}(s)$. The inverse transformation can be performed by using the following 2×2 matrix:

$$G_{\alpha\beta}(s) = \frac{1}{2} \begin{bmatrix} G_{dq1} + G_{dq2} & jG_{dq1} - jG_{dq2} \\ -jG_{dq1} + jG_{dq2} & G_{dq1} + G_{dq2} \end{bmatrix} \tag{C.8}$$

$$G_{dq1} = G_{dq}(s + j\omega) \tag{C.9}$$

$$G_{dq2} = G_{dq}(s - j\omega) \tag{C.10}$$

Given that $G_{dq}(s) = k_I/s$ and $G_{dq}(s) = k_I/(1 + (s/\omega_c))$, the equivalent controllers in the stationary frame for compensating positive-sequence feedback error are therefore expressed respectively as

$$G^{+}_{\alpha\beta}(s) = \frac{1}{2}\begin{bmatrix} \dfrac{2k_I s}{s^2+\omega^2} & \dfrac{2k_I \omega}{s^2+\omega^2} \\ -\dfrac{2k_I \omega}{s^2+\omega^2} & \dfrac{2k_I s}{s^2+\omega^2} \end{bmatrix} \tag{C.11}$$

$$G^{+}_{\alpha\beta}(s) \approx \frac{1}{2}\begin{bmatrix} \dfrac{2k_I \omega_c s}{s^2+2\omega_c s+\omega^2} & \dfrac{2k_I \omega_c \omega}{s^2+2\omega_c s+\omega^2} \\ -\dfrac{2k_I \omega_c \omega}{s^2+2\omega_c s+\omega^2} & \dfrac{2k_I \omega_c s}{s^2+2\omega_c s+\omega^2} \end{bmatrix} \tag{C.12}$$

Similarly, for compensating negative sequence feedback error, the required transfer functions are expressed as

$$G^{-}_{\alpha\beta}(s) = \frac{1}{2}\begin{bmatrix} \dfrac{2k_I s}{s^2+\omega^2} & -\dfrac{2k_I \omega}{s^2+\omega^2} \\ \dfrac{2k_I \omega}{s^2+\omega^2} & \dfrac{2k_I s}{s^2+\omega^2} \end{bmatrix} \tag{C.13}$$

$$G^{-}_{\alpha\beta}(s) \approx \frac{1}{2}\begin{bmatrix} \dfrac{2k_I \omega_c s}{s^2+2\omega_c s+\omega^2} & -\dfrac{2k_I \omega_c \omega}{s^2+2\omega_c s+\omega^2} \\ \dfrac{2k_I \omega_c \omega}{s^2+2\omega_c s+\omega^2} & \dfrac{2k_I \omega_c s}{s^2+2\omega_c s+\omega^2} \end{bmatrix} \tag{C.14}$$

Comparing (C.11) and (C.12) with (C.13) and (C.14), it is noted that the diagonal terms of $G^{+}_{\alpha\beta}(s)$ and $G^{-}_{\alpha\beta}(s)$ are identical, but their nondiagonal terms are opposite in polarity. This inversion of polarity can be viewed as equivalent to the reversal of rotating direction between the positive- and negative-sequence synchronous frames.

Combining the above equations, the resulting controllers for compensating both positive- and negative-sequence feedback errors are expressed as

$$G_{\alpha\beta}(s) = \frac{1}{2}\begin{bmatrix} \dfrac{2k_I s}{s^2+\omega^2} & 0 \\ 0 & \dfrac{2k_I s}{s^2+\omega^2} \end{bmatrix} \tag{C.15}$$

$$G_{\alpha\beta}(s) \approx \frac{1}{2}\begin{bmatrix} \dfrac{2k_I \omega_c s}{s^2+2\omega_c s+\omega^2} & 0 \\ 0 & \dfrac{2k_I \omega_c s}{s^2+2\omega_c s+\omega^2} \end{bmatrix} \tag{C.16}$$

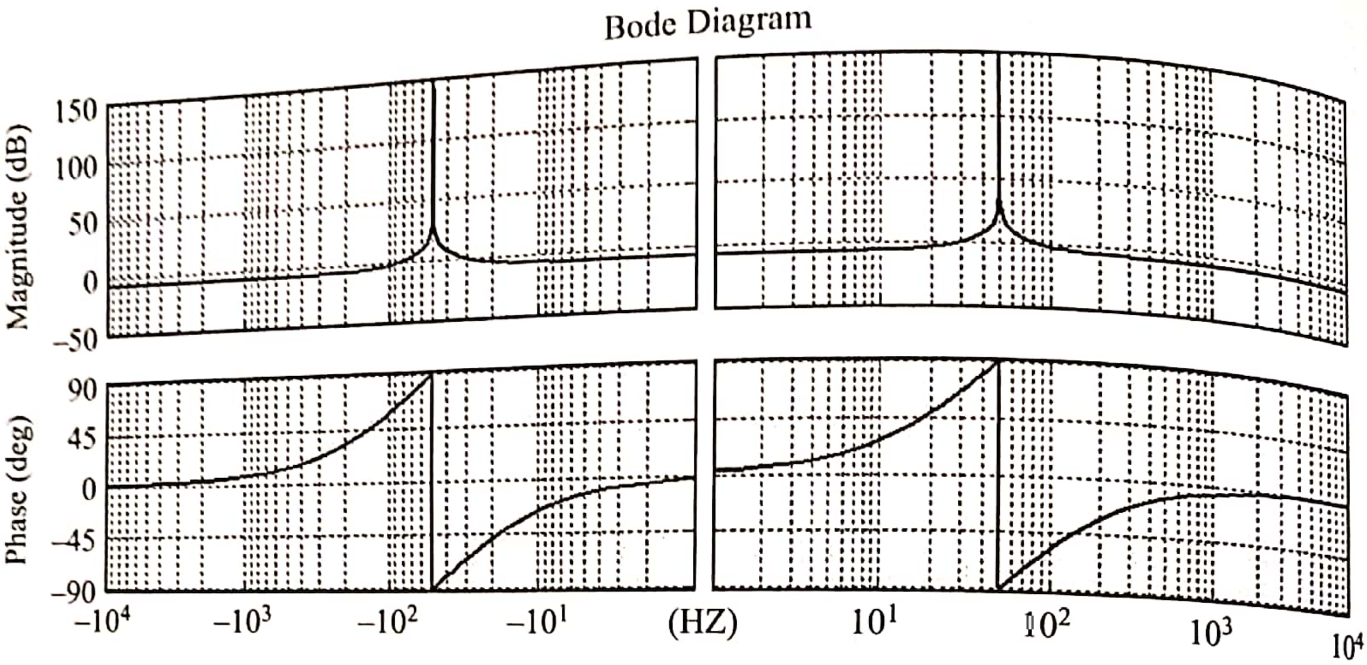

Figure C.2 Positive- and negative-sequence Bode diagrams of a PR controller

Bode plots representing (C.15) are drawn in Figure C.2, where their error-eliminating ability is clearly reflected by the presence of two resonant peaks at the positive frequency ω and negative frequency $-\omega$. Note that if (C.11) or (C.12) ((C.13) or (C.14)) is used instead, only the resonant peak at ω ($-\omega$) is present since those equations represent PI control only in the positive-sequence (negative-sequence) synchronous frame. Another feature of (C.15) and (C.16) is that they have no cross-coupling nondiagonal terms, implying that each of the α and β stationary axes can be treated as a single-phase system. Therefore, the theoretical knowledge described earlier for single-phase PR control is equally applicable to the three-phase functions expressed in (C.15) and (C.16).