角速度绝对性

$O:\vec{R}$ $O':\vec{R'}$ $P:\vec{r}$

$\vec{v_P} = \frac{d\vec{R}}{dt} + \omega(\vec{R}-\vec{r})$

$\vec{v_P} = \frac{d\vec{R'}}{dt} + \omega'(\vec{R'}-\vec{r})$

$\frac{d\vec{R'}}{dt} = \frac{d\vec{R}}{dt} + \omega(\vec{R}-\vec{R'})$

$\Rightarrow \omega = \omega'$

质心运动定理

$\frac{d\vec{P_i}}{dt} = \vec{F_i}外 + \sum_{j=1}^{N} \vec{f_{ji}}$

$j=i$ $\vec{f_{ii}}=0$

$\sum \frac{d\vec{P_i}}{dt} = \sum \vec{F_i}外 + \sum\sum \vec{f_{ji}}$

$\therefore \vec{F}外 = \frac{d\vec{P}}{dt}$

$\vec{M} = \frac{d\vec{L}}{dt}$ $\vec{L} = \vec{r}\times\vec{P}$

$I = \sum m r_i^2$ 转动惯量

$I\frac{d\omega}{dt} = M外$

圆环 mR^2 圆面 $\frac{1}{2}mR^2$

$\frac{1}{2}m(R_1^2+R_2^2)$

$\frac{1}{12}m(a^2+b^2)$

$\frac{1}{12}mL^2$ } 棍

$\frac{1}{3}mL^2$

$\frac{2}{5}mR^2$ 球

$\frac{2}{3}mR^2$ 壳球

刚体动能

$T = \sum_{i=1}^{n}\frac{1}{2}m_i v_i^2$

$\vec{v_i} = \vec{v_c} + \vec{v_i'}$

$T = \sum_{i=1}^{n}\frac{1}{2}m_i(\vec{v_c}^2 + \vec{v_i'}^2 + 2\vec{v_c}\vec{v_i'})$

$= \frac{1}{2}M\vec{v_c}^2 + \frac{1}{2}\sum_{i=1}^{n} m_i\vec{v_i'}^2$

$= T_{平动} + T_{转动}$

$= \frac{1}{2}M\vec{v_c}^2 + \frac{1}{2}J_c\omega^2$

$\frac{d(I\omega)}{dt}\omega = M\omega$

$\frac{d(\frac{1}{2}I\omega^2)}{dt} = M\cdot\frac{d\varphi}{dt}$

$\Delta(I\omega^2) = \int M\,d\varphi$

平行轴 $I_P = I_0 + md^2$ I_0质心

垂直轴 $I_z = I_x + I_y$ 平面

进动&章动

$dL = L\sin\theta\cdot d\alpha$

$dL = M\cdot dt$

$\Omega = \frac{d\alpha}{dt} = \frac{M}{L\sin\theta} = \frac{M}{I\omega\sin\theta}$

Ω 为进动角速度

进动同时陀螺转，其上部下沿速度方向相同，因而在进动转动过程中，科氏力方向左右，上面向左，下面向右，形成回转力矩。

在进动过程中，回转轴交替向下倾倒和向上翻升，章动。

月球、太阳引力

地球不完美球形。

$F/S = Y\Delta l/l$

$\frac{\Delta\omega}{\omega} = \frac{\Delta h}{h} = -\nu\frac{\Delta l}{l}$

$\frac{\Delta a}{a} = \frac{P}{Y}$

P_1 $\frac{\Delta b}{b} = \frac{P}{Y}(-\nu)$

$\frac{\Delta c}{c} = \frac{P}{Y}(-\nu)$

$\frac{\Delta a}{a} = \frac{P}{Y} - \nu(\frac{P_2+P_3}{Y})$

$\frac{\Delta V}{V} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}$

$= \frac{1}{Y}(1-2\nu)(P_1+P_2+P_3)$

$K = \frac{Y}{3(1-2\nu)}$

剪切应变

$\frac{\Delta x}{L} = \frac{\Delta y}{Y}\frac{F}{S}$

底面 S

底面 F_s

$\frac{\Delta L}{\Delta L}$ $\frac{1+\nu}{Y}\cdot\frac{F}{S}$

$\Delta l = \frac{\Delta l}{\sqrt{2}}$

$\frac{1+\nu}{Y}\cdot\frac{F}{S}\cdot\Delta x = D\Delta\frac{F}{S}$

$G = \frac{F}{S\theta} = \frac{Y}{2(1+\nu)}$

圆柱扭转

沿半径 $r(r<a)$ Δr

$\theta = \frac{r\varphi}{L}$ $\tau = G\theta = \frac{G r\varphi}{L}$

$\tau = \frac{\Delta F}{\Delta A r}$ $\Delta M = r\Delta F = r\tau\Delta A r$

$\therefore M = r\tau\Delta r\,S\Delta l$

$= 2\pi r^2\tau\Delta r$

$= 2\pi G\cdot\frac{\Delta r\varphi}{L}r^3$

实心 $M = \frac{G\pi a^4}{2L}\varphi$

剪切应(横)

$M\propto \varphi/L$

$M = G\cdot\frac{\pi a^4}{2}\frac{\partial\varphi}{\partial x}$

$M(x+\Delta x) = M(x) + (\frac{\partial M}{\partial x})\Delta x$

$\Delta M = (\frac{\partial M}{\partial x})\Delta x = G\frac{\pi a^4}{2}\frac{\partial^2\varphi}{\partial x^2}\Delta x$

$\Delta m = (\pi a^2\Delta x)\rho$

$\Delta I = \frac{1}{2}\Delta m a^2 = \frac{\pi}{2}\rho a^4\Delta x$

$\because \Delta M = \Delta J\frac{\partial^2\varphi}{\partial t^2}$

$\therefore G\cdot\frac{\pi a^4}{2}\frac{\partial^2\varphi}{\partial x^2}\Delta x = (\frac{\pi}{2}\rho a^4\Delta x)\frac{\partial^2\varphi}{\partial t^2}$

$\frac{\partial^2\varphi}{\partial t^2} - \frac{G}{\rho}\frac{\partial^2\varphi}{\partial x^2} = 0$

胀缩波(纵)

$F(x+\Delta x) - F(x) = S[P(x+\Delta x) - P(x)]$

$= SY\frac{\partial^2 y}{\partial x^2}\Delta x$

$\Delta m = \rho S\Delta x$

$SY\frac{\partial^2 y}{\partial x^2}\Delta x = \rho S\Delta x\cdot\frac{\partial^2 y}{\partial t^2}$

$\therefore \frac{\partial^2 y}{\partial t^2} = \frac{Y}{\rho}\frac{\partial^2 y}{\partial x^2}$

某些特殊条件下 纵波入射 只反射横波

横波入射，只反射纵波

偏振交换 or

波型转换

超声波探测 成像

CT

地震波反演

理想流体中某点处压强大小与方向无关

$P_x\Delta x\Delta z - P_n\Delta n\Delta z\cos\alpha = 0$

$P_y\Delta y\Delta z - P_n\Delta n\Delta z\cos\alpha = 0$

$\Delta x = \Delta n\sin\alpha$

$\Delta y = \Delta n\cos\alpha$

$\Rightarrow P_x = P_y = P$ 同理 $P_z = P$

Δmg 3阶小量

$p+dp$ S dz

$PS = (P+dP)S + \rho g\,dz$

$P_2 - P_1 = -\rho g(z_2 - z_1)$

$-\nabla P - \rho\Delta\varphi = 0$

$P\Delta x\Delta z - (P + \frac{\partial P}{\partial y})\Delta x\Delta z$

$= -\frac{\partial P}{\partial y}\Delta x\Delta y\Delta z$

合力 $= -\Delta x\Delta y\Delta z(\vec{i}\frac{\partial P}{\partial x} + \vec{j}\frac{\partial P}{\partial y} + \vec{k}\frac{\partial P}{\partial z})$

$= -\nabla P(\Delta x\Delta y\Delta z)$

连续性方程

$\rho\vec{v}\cdot\Delta\vec{S}$

$\lim_{\Delta S\to 0}\sum\rho\vec{v}\cdot\Delta\vec{S} = \oint_S\rho\vec{v}\cdot d\vec{S}$

$-\frac{dm}{dt} = -\frac{d}{dt}\iiint_V\rho dV$

$\oint_S\rho\vec{v}\cdot d\vec{S} = -\frac{d}{dt}\iiint_V\rho dV$

$\nabla(\rho\vec{v}) = -\frac{\partial\rho}{\partial t}$

$\nabla\cdot\vec{v} = 0$

欧拉运动方程

$\vec{a} = \lim_{\Delta t\to 0}\frac{\vec{v}(x+\Delta x, y+\Delta y, z+\Delta z) - \vec{v}(x,y,z)}{\Delta t}$

$\vec{a} = (\vec{v}\cdot\nabla)\vec{v} + \frac{\partial\vec{v}}{\partial t}$

$\nabla = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$

$\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v} = -\frac{1}{\rho}\nabla P - \nabla\varphi$

定常 $\frac{\partial\vec{v}}{\partial t} = 0$

令 $\vec{\Omega} = \nabla\times\vec{v}$ 为涡度

$\frac{\partial\vec{v}}{\partial t} + \vec{\Omega}\times\vec{v} = -\frac{1}{\rho}\nabla P - \nabla\varphi - \frac{1}{2}\nabla(v^2)$

伯努利定理

$\frac{\partial\vec{v}}{\partial t} + \vec{\Omega}\times\vec{v} = -\frac{1}{\rho}\nabla P - \nabla\varphi - \frac{1}{2}\nabla(v^2)$

$\frac{\partial\vec{v}}{\partial t} = 0$ $\vec{v}\cdot(\vec{\Omega}\times\vec{v}) = 0$

$\vec{v}\cdot\nabla(\frac{P}{\rho} + \varphi + \frac{1}{2}v^2) = 0$

$\frac{P}{\rho} + \varphi + \frac{1}{2}v^2 = 常量$

$E = \frac{1}{2}v^2 + \varphi + U$

伯努利方程应用

①文丘里管

②球旋转与弧线球

③机翼升力

④水喷射吸力作用

$-\frac{dV}{dt} = S(-\frac{dh}{dt}) = S\sqrt{2gh}$ 积分

$\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v} = -\frac{1}{\rho}\nabla P - \nabla\varphi + \frac{1}{\rho}f_{粘}$

$\frac{F}{A} = \eta\frac{v_0}{d}$

$M = M(r) = \eta\cdot 2\pi r l\frac{d(\omega r)}{dr}\cdot r$

$= \eta\cdot 2\pi r l\cdot r\frac{d\omega}{dr}\cdot r$

$M(r) = M(r+\delta r)$

$r^3\frac{d\omega}{dr} = A$

$\omega = -\frac{A}{2r^2} + B$

$A = \frac{2a^2b^2\omega}{b^2-a^2}$

$B = \frac{b^2\omega}{b^2-a^2}$

$M = 2\pi\eta l r^3\frac{d\omega}{dr} = \frac{4\pi\eta l a^2b^2\omega}{b^2-a^2}$

$Re = \frac{\rho D v}{\eta}$

(1)

$\frac{\partial\vec{v}}{\partial t} + \nabla\times(\vec{v}\times\vec{v})$

$= \frac{\eta}{\rho}\nabla^2\vec{v}$

几种常见的摆

①单摆

②扭摆

$\tau = J\frac{d\omega}{dt} = J\frac{d^2\theta}{dt^2}$

$\tau = -k\theta$

③复摆

$\tau = -mgd\sin\theta$

$\tau = J\frac{d^2\theta}{dt^2}$

阻尼振动Q值

$Q = 2\pi E/(\Delta E)T$

$\frac{dE}{dt} = -\frac{\omega E}{Q}$

$x = Ae^{-\omega t/2Q}\cos(\omega_0 t)$

$m\ddot{x} + m\gamma\dot{x} + m\omega_0^2 x = 0$

$x = Ae^{i\omega t}$

$x_1 = Ae^{-\gamma t/2}\cos(\omega t + \varphi_0)$

$x_2 = Ae^{-(\frac{\gamma}{2} - \sqrt{\gamma^2/4 - \omega_0^2})t} + Be^{-(\frac{\gamma}{2} + \sqrt{\gamma^2/4 - \omega_0^2})t}$

$x_3 = (A + Bt)e^{-\gamma t/2}$

受迫振动与共振

设 $F(t) = F_0\cos(\omega t + \beta)$

$x = A\cos(\omega t + \varphi_0) + \frac{F_0}{m(\omega_0^2 - \omega^2)}\cos(\omega t + \beta)$

稳态响应

$\omega \approx \omega_0$ 共振

$x = \frac{F_0}{2\omega_0 m}t\sin(\omega_0 t + \beta)$

有阻尼受迫振动

$m\ddot{x} + c\dot{x} + kx = F$

$c = m\gamma$ $k = m\omega_0^2$

$F = F_0\cos(\omega t + \beta)$

$= \hat{F_0}e^{i\omega t}$

$\hat{x} = \frac{\hat{F_0}}{m(\omega_0^2 - \omega^2 + i\gamma\omega)} = R\hat{F_0}$

$R = \frac{1}{m(\omega_0^2 - \omega^2 + i\gamma\omega)} = \rho e^{i\theta}$

ρ 为 R 振幅 放大因子

特解得出

$x = \rho F_0\cos(\omega t + \beta + \theta)$

$Q = \omega_0/\gamma$

$\gamma = \frac{c}{m}$

$m\ddot{x} + c\dot{x} + kx = F$

$x_1 = A\cos(\omega_1 t + \varphi)$ $x_2 = A\cos(\omega_2 t + \varphi)$

$x = x_1 + x_2 = 2A\cos(\frac{\omega_1 - \omega_2}{2}t)\cos(\frac{\omega_1 + \omega_2}{2}t + \varphi)$

$\approx 2A\cos(\frac{\omega_1 - \omega_2}{2}t)\cos(\omega_1 t + \varphi)$

钟摆的错误，周期中的错误

$\nu = |\frac{\omega_1 - \omega_2}{2\pi}| = |\nu_1 - \nu_2|$ 拍频

拍的应用：音叉校准乐器。

波动方程与声波在空气中传播

$\rho\frac{\partial^2\xi}{\partial t^2} - P\frac{\partial^2\xi}{\partial x^2} = 0$

$a = \frac{\partial^2\xi}{\partial t^2} = \frac{A[P(x,t) - P(x+\Delta x,t)]}{\Delta m} = \frac{1}{\rho}\times(-\frac{\partial P}{\partial x})$

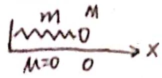
$PV = const$ $V = A[\xi(x+\Delta x) - \xi(x)] = A\Delta x[1 + \frac{\partial\xi}{\partial x}]$

$= A\Delta x[1 + \frac{\partial\xi}{\partial x}]$

$P[1 + \frac{\partial\xi}{\partial x}] = const$ $\Rightarrow$ $\frac{\partial P}{\partial x} \approx -P\frac{\partial^2\xi}{\partial x^2}$.

$\therefore \rho\frac{\partial^2\xi}{\partial t^2} - P\frac{\partial^2\xi}{\partial x^2} = 0$ $v = \sqrt{\frac{P}{\rho}}$

$x \pm vt$ + 负向 − 正向（沿 x 轴）



$F(x+\Delta x) - F(x) = [Y\frac{\partial y}{\partial x}|_{x+\Delta x} - Y\frac{\partial y}{\partial x}|_x] = Y\frac{\partial^2 y}{\partial x^2}\Delta x$

由牛二 $Y\frac{\partial^2 y}{\partial x^2}\Delta x = (\rho\Delta x)\frac{\partial^2 y}{\partial t^2}$

$\therefore \frac{\partial^2 y}{\partial t^2} - \frac{Y}{\rho}\frac{\partial^2 y}{\partial x^2} = 0$ $v = \sqrt{\frac{Y}{\rho}}$

波的能量与能流

$\Delta V\ (= S\Delta x)$ $\Delta m\ (= \rho\cdot S\cdot\Delta x)$

$\xi = A\cos\omega(t - \frac{x}{v})$

$W_k = \frac{1}{2}\Delta m\cdot\dot{\xi}^2 = \frac{1}{2}(\rho\Delta V)\omega^2A^2\sin^2\omega(t - \frac{x}{v})$

$W_p = \frac{1}{2}k(\Delta\xi)^2$

$\frac{f}{S} = P = Y\frac{\Delta\xi}{\Delta x}$, $f = \frac{SY}{\Delta x}\Delta\xi$ $\therefore k = \frac{SY}{\Delta x}$

$\therefore W_p = \frac{SY}{2}\Delta x(\frac{\Delta\xi}{\Delta x})^2$ $Y = \rho v^2$

$= \frac{1}{2}\rho v^2\cdot\Delta V\cdot(\frac{\partial\xi}{\partial x})^2$

$= \frac{1}{2}\rho\omega^2A^2\Delta V\sin^2\omega(t - \frac{x}{v})$

$\therefore W = \rho\omega^2A^2\Delta V\sin^2\omega(t - \frac{x}{v})$

振动系统不传能量

能量密度和能流密度

$w = \frac{W}{\Delta V} = \rho\omega^2A^2\sin^2\omega(t - \frac{x}{v})$

$\bar{w} = \frac{1}{2}\rho\omega^2A^2$

$P = \rho\omega AC$

平均能流 $\bar{w}\cdot\vec{v}\cdot\vec{S}$

能流密度 $\vec{I} = \bar{w}\vec{v} = \frac{1}{2}\rho\vec{v}\omega^2A^2$ 声强

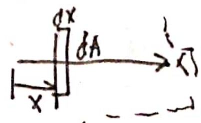
10dB(分贝) = 1贝尔 (Bel)

声强 $I.L = 10\lg(I/I_0)$dB $I_0 = 1.0\times 10^{-12}\,W\cdot m^{-2}$

I_0 为人类可听到最小平均强度

声压 $P.L = 20\lg(P/P_0)$dB

波的吸收



$-dA = \alpha A dx$

$A = A_0e^{-\alpha x}$

$I = I_0e^{-2\alpha x}$

$\vec{F} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$

$\int_1^2\vec{F}\cdot d\vec{s}$

$= \int_1^2 m\vec{v}\,d\vec{v} + \int_1^2 v^2 dm$

$= \frac{\int_1^2 m\,dv^2}{2} + \int v^2 dm$

$= \int v^2 dm$

在空间上干涉

$y_{10}=A_{10}\cos(\omega t+\varphi_1)$

$y_{20}=A_{20}\cos(\omega t+\varphi_2)$

$y_1=A_1\cos(\omega t+\varphi_1-kx_1)$

$y_2=A_2\cos(\omega t+\varphi_2-kx_2)$

$k=\frac{2\pi}{\lambda}$

$A=\sqrt{A_1^2+A_2^2+2A_1A_2\cos[(\varphi_1-\varphi_2)-(\varphi_2-kr_2)]}$

$\Delta=(\varphi_1-\varphi_2)+k(r_2-r_1)=2m\pi$ 时 $A=A_1+A_2$

$\Delta=(\varphi_1-\varphi_2)+k(r_2-r_1)=(2m+1)\pi$ 时 $A=|A_1-A_2|$

在时间上干涉

对于2个波源

若一个的振动相位可调

由 $0,\frac{\pi}{2},\pi,\dots 2\pi$

P 由最强→最弱→最强

这就是拍

群速度与相速度

$e^{i(\omega_1 t-k_1x)}$

$e^{i(\omega_2 t-k_2x)}$

$\frac{\omega_1}{k_1}=\frac{\omega_2}{k_2}=V$

$e^{i(\omega_1 t-k_1x)}+e^{i(\omega_2 t-k_2x)}$

$=e^{i\omega_1 t'}+e^{i\omega_2 t'}$

$t'=t-\frac{x}{V}$

无色散与色散

$V_1=\frac{\omega_1}{k_1}$ $V_2=\frac{\omega_2}{k_2}$ $V_1\neq V_2$

X光 有 反常色散

$k=\frac{\omega}{c}-\frac{a}{\omega c}$

$V_相=\frac{\omega}{k}$

{ 大于光速 / 与ω有关

叠加 $\{e^{i(\omega_1 t-k_1x)}, e^{i(\omega_2 t-k_2x)}$

$y_1+y_2=e^{\pm i[(\omega_1+\omega_2)t-(k_1+k_2)x]}$

$\{e^{\pm i[(\omega_1-\omega_2)t-(k_1-k_2)x]}+e^{-\pm i[(\omega_1-\omega_2)t-(k_1-k_2)x]}\}$

$\omega_1\approx\omega_2$ $k_1\approx k_2$

波前速度仍为 ω/k

$V_{波}=\frac{\Delta\omega}{\Delta k}=\frac{\omega_1-\omega_2}{k_1-k_2}$

$\Delta\omega\to 0$ $V_g=\frac{d\omega}{dk}$

$V_g=\frac{d\omega}{dk}=\frac{c}{1+a/\omega^2}$

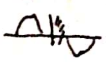
驻波与波腹

$y=f(x-vt)+g(x+vt)$

端处 $x=0$ $y=0$

$f(-vt)+g(vt)=0$

$\therefore y=f(x-vt)-f(-x-vt)$



$f(x-vt)=e^{-ik(x-vt)}=e^{i\omega(t-\frac{x}{v})}$

$f(-x-vt)=e^{i\omega(t+\frac{x}{v})}$

$y=-2ie^{i\omega t}\sin(\omega x/v)$

① x 满足 $\frac{\omega x}{v}=0$ $y=0$

② 任何确定 t，得到波形图。得到位移随 x 变化波形。

③ 对于 t_0，合成是驻波——同频反向两个叠加。波节处任何一个波 波长 $\lambda=\frac{2\pi}{k}=\frac{2\pi v}{\omega}$

y=0的点，波节 $x=n\frac{\pi v}{\omega}$

得到一个不随时间移动的驻波

④ 波节两侧质点振动方向相反

{ 固定端 → 波节 / 自由端 → 波腹

2端固定 $x=0$ $x=L$

$\lambda=2L$ $\lambda=L$ $\lambda=\frac{2}{3}L$

$\omega=\frac{\pi v}{L}$ $\frac{2\pi v}{L}$ $\frac{3\pi v}{L}$

对于两端固定的弦

最低频率叫基频

其它频率叫泛音

弦驻波基频与弦长、速度成正比，与弦长与L成反比

基频有最大振幅

它的频率一般决定音调

音色 泛音决定音色

多普勒效应

① 波源静止 $u=0$ $V\neq 0$

单位时间接受波数 $\frac{V+v}{\lambda}$

$\therefore \nu'=\nu(1+\frac{v}{V})$

（+v表示靠近波源）

② 人静止 $u=0$ $V\neq 0$

$\lambda'=\lambda-uT$

$\therefore \nu'=\frac{V}{\lambda'}=\frac{1}{1-\frac{u}{V}}\nu$

③ 同时运动

$\nu'=\frac{1+\frac{v}{V}}{1-\frac{u}{V}}\nu$

光多普勒 $\sim$ $\nu'=\nu\frac{1+\frac{u}{c}}{\sqrt{1-(\frac{u}{c})^2}}$

多普勒 流速计

v 与 x

$\nu'=\frac{u+V\cos\theta}{u}\nu_0$

$\nu''=\frac{u}{u-V\cos\theta}\nu'$

波数与马赫数

v 大于 u 超过 V

多 ~ 5.5

$\sin\alpha=\frac{V}{u}$

$\frac{u}{V}$ 叫马赫数

速度是进一步的推广

量子 相关

洛伦兹变换

$x^2+y^2+z^2-c^2t^2=x'^2+y'^2+z'^2-c^2t'^2$

$\Rightarrow y=y'$ $z=z'$

$x^2-c^2t^2=x'^2-c^2t'^2$

$x=\frac{x'+\beta ct'}{\sqrt{1-\beta^2}}$

$y=y'$

$z=z'$

$t=\frac{t'+\beta x'/c}{\sqrt{1-\beta^2}}$

K' 以 v 相对 K 运动

K中看 K' 原点 $\frac{dx}{dt}=V$

$x'=y'=z'=0$

$\therefore dx=\frac{\beta c dt'}{\sqrt{1-\beta^2}}$ $\beta=\frac{v}{c}$

$x'=\frac{x-vt}{\sqrt{1-v^2/c^2}}$

$y'=y$

$z'=z$

$t'=\frac{t-(v/c^2)x}{\sqrt{1-v^2/c^2}}$

$x=\frac{x'+vt'}{\sqrt{1-v^2/c^2}}$

$y=y'$

$z=z'$

$t=\frac{t'+(v/c^2)x'}{\sqrt{1-v^2/c^2}}$

时间间隔相对性

K' 中 $A(x_1',y_1',z_1',t_1')$

$B(x_1',y_1',z_1',t_2')$

K中 $t_2-t_1=\frac{t_2'-t_1'}{\sqrt{1-v^2/c^2}}>t_2'-t_1'$

长度相对性

K' 中 x_1' x_2' $L'=|x_2'-x_1'|$

K 中 $L=|x_2-x_1|=L'\sqrt{1-v^2/c^2}$

测量形象 VS 视觉形象

同时相对性

K' 中 $A(x_1',y_1',z_1',t')$

$B(x_2',y_2',z_2',t')$

$\Delta t=t_2-t_1=\frac{v}{c^2}\frac{x_2'-x_1'}{\sqrt{1-v^2/c^2}}$

时序与因果关系

$t_1-t_2=\frac{t_1'-t_2'+\frac{v}{c^2}(x_1'-x_2')}{\sqrt{1-v^2/c^2}}$

$V_i=\left|\frac{x_1'-x_2'}{t_1'-t_2'}\right|<c$ $v<c$

相互作用传递 因果律不倒

时空间隔不变

$s=[c^2(t_1-t_2)^2-(x_1-x_2)^2-(y_1-y_2)^2-(z_1-z_2)^2]^{\frac{1}{2}}$

$\tau=\frac{1}{c}s$ $d\tau=\frac{1}{c}ds$

τ 原时间间隔

$d\tau=dt\sqrt{1-\frac{u^2}{c^2}}$

$=dt'\sqrt{1-\frac{u'^2}{c^2}}$

爱因斯坦利用闵可夫斯基四维张量语言，引入时空"四维坐标"，且动量能量构成四维动量

物理规律四维协变

庞加莱 菲兹杰拉德 洛伦兹 运动物体收缩

庞加莱 单独提出洛伦兹变换

速度合成律

$dx=\frac{dx'+vdt'}{\sqrt{1-\frac{v^2}{c^2}}}$

$dy=dy'$

$dz=dz'$

$dt=\frac{dt'+\frac{v}{c^2}dx'}{\sqrt{1-\frac{v^2}{c^2}}}$

$\Rightarrow$ $u_x=\frac{u_x'+v}{1+\frac{vu_x'}{c^2}}$

$u_y=u_y'\cdot\frac{\sqrt{1-v^2/c^2}}{1+\frac{u_x'v}{c^2}}$

$u_z=u_z'\cdot\frac{\sqrt{1-v^2/c^2}}{1+\frac{u_x'v}{c^2}}$

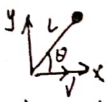
光行差问题

$x'=x\,ch\varphi-ct\,sh\varphi$

$ct'=-x\,sh\varphi+ct\,ch\varphi$

取 c=1 $\{x'=x\,ch\varphi-t\,sh\varphi$, $y'=y$, $z'=z$, $t'=t\,ch\varphi-x\,sh\varphi$

$th^{-1}v=\varphi$ $v=1=c$ $th\varphi=1$



K: $\{x=l\cos\theta$, $y=l\sin\theta$, $t=-l$

K': $\{x'=l\cos\theta\,ch\varphi-(-l)sh\varphi$, $y'=l\sin\theta$, $t'=(-l)ch\varphi-(l\cos\theta)sh\varphi$

$tg\theta'=\frac{l\sin\theta}{l\cos\theta\,ch\varphi+l\,sh\varphi}$

$\theta'<\frac{\pi}{2}$ 能看到

即 $tg\theta'<\infty$

or $l\cos\theta\,ch\varphi+l\,sh\varphi>0$

$\cos\theta>-th\varphi$

$v=1$ 时 都能看到

$du_x=\frac{1-\frac{v^2}{c^2}}{(1+\frac{vu_x'}{c^2})^2}du_x'$

$du_y=\frac{du_y'\sqrt{1-v^2/c^2}}{1+u_x'\frac{v}{c^2}}-\frac{u_y'\sqrt{1-v^2/c^2}}{(1+\frac{u_x'v}{c^2})^2}\frac{vdu_x'}{c^2}$

$du_z=\frac{du_z'\sqrt{1-v^2/c^2}}{1+u_x'\frac{v}{c^2}}-\frac{u_z'\sqrt{1-v^2/c^2}}{(1+\frac{u_x'v}{c^2})^2}\frac{vdu_x'}{c^2}$

$a_x=\frac{(1-\frac{v^2}{c^2})^{3/2}}{(1+\frac{vu_x'}{c^2})^3}a_x'$

$a_y=\frac{1-\frac{v^2}{c^2}}{(1+\frac{vu_x'}{c^2})^2}a_y'-\frac{\frac{vu_y'}{c^2}(1-\frac{v^2}{c^2})}{(1+\frac{vu_x'}{c^2})^3}a_x'$

a_z 同

宇航员以 0.8c 飞向一个8光年远天体，返回。以地球K系，去时K'系，回来K''系。地球与天体各有一个K钟，$t=t'=0$ 对准。

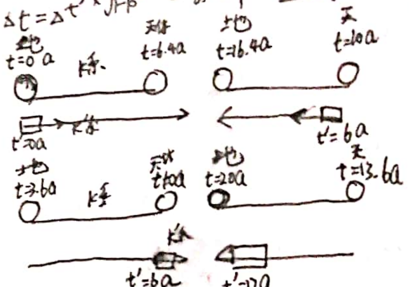
Q1：宇航员看 起飞，到天体，回地球 时刻所有钟读数。

因 $\beta=0.8$ $\gamma^{-1}=0.6$

相对K' 到达K钟指零发生在 $-\frac{32}{3}$ 年

在宇航员看来，K'系中 $t'=\frac{32}{3}$ 的事件

$\Delta t=\Delta t'\times\sqrt{1-\beta^2}=6.4$年（示数）



Q2：假定 2010年元旦起 每年拍电报

t 0 1 2 3 4 5 6 7 8 9 10 …… 20

t' 0 3 6 6 $\frac{20}{3}$ 7 $\frac{22}{3}$ $\frac{23}{3}$ 8 $\frac{25}{3}$ $\frac{26}{3}$ … 12

t''

t' 0 1 2 3 4 5 6

6 7 8 9 10 11 12

t''

t 0 3 6 9 12 15 18 时 …… 20

相对论力学

$\frac{d}{dt}(mv_x)=F_x$ ……

考虑一个体系，相对K系 m_1, m_2 速度X方向上 u_1, u_2

$P=m_1u_1+m_2u_2=0$ 质心系 $M=m_1+m_2$

∴ 整个体系对K $V=\frac{p}{M}=0$

K'系中，为满足 $P'=m_1'u_1'+m_2'u_2'$

$M'=m_1'+m_2'$

K'系相对K 以V运动，对K静止体系，对K' 有 $-V_0$

$\therefore P'=-M'V$ $\frac{m_1}{m_2}=-\frac{u_2}{u_1}$

$m_1'u_1'+m_2'u_2'=-(m_1'+m_2')V\Rightarrow\frac{m_1'}{m_2'}=-\frac{u_2'+V}{u_1'+V}$

由相对论速度公式

$u_1=\frac{u_1'+V}{1+\frac{Vu_1'}{c^2}}$ $u_2=\frac{u_2'+V}{1+\frac{Vu_2'}{c^2}}$ 得 $\frac{m_1'}{m_2'}$

$\frac{m_1'/m_1}{m_2'/m_2}=\frac{1+Vu_1'/c^2}{1+Vu_2'/c^2}$

$1+\frac{Vu_1'}{c^2}=\frac{\sqrt{1-u_1'^2/c^2}}{\sqrt{1-u_1^2/c^2}}\cdot\sqrt{1-\frac{V^2}{c^2}}$ 同 $1+\frac{Vu_2'}{c^2}$

$\therefore\frac{m_1'/m_1}{m_2'/m_2}=\frac{\sqrt{1-u_1^2/c^2}/\sqrt{1-u_1'^2/c^2}}{\sqrt{1-u_2^2/c^2}/\sqrt{1-u_2'^2/c^2}}$

$m=\frac{a}{\sqrt{1-u^2/c^2}}$ $a=m_0$

$\therefore p=mv/\sqrt{1-\frac{v^2}{c^2}}$ $T=m_0c^2\left[\frac{1}{\sqrt{1-v^2/c^2}}-1\right]$

$E_0=m_0c^2$ $E=mc^2$ $E^2=p^2c^2+m_0^2c^4=p^2c^2+E_0^2$