

Homework 2

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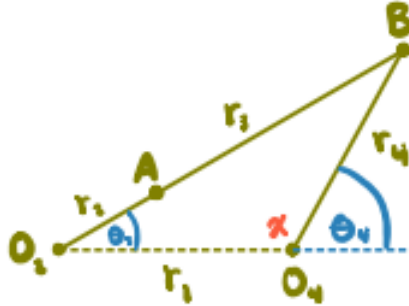
gnm54

```
[1]: # Importing packages that will be used throughout the document.  
import matplotlib.pyplot as plt  
from scipy.optimize import fsolve  
import sympy as sp  
import numpy as np  
from mech import * # This package was created by me for this class.  
plt.rcParams['figure.dpi'] = 300  
  
import warnings  
warnings.filterwarnings('ignore')
```

1 Problem 1

1.1 Part A

1.1.1 Limit Position 1



The value of x may be solved by the Law of Cosines. Then, the value of θ_4 may be solved by subtracting it from 180° . Finally, the value of θ_2 can be solved by the Law of Sines.

```
[2]: # Solving for the value of x
from sympy.solvers import solve
x = sp.Symbol('x')
solve(sp.Pow(180 + 520, 2) - sp.Pow(400, 2) - sp.Pow(400, 2) + 2*400*400*sp.
      ↪cos(x), x)[0]
```

```
[2]: -acos(-17/32) + 2*pi
```

```
[3]: # The value of x solved above is the conjugate. Here is the value of x:
x = sp.N(sp.acos(-17/32)*180/sp.pi)
x
```

```
[3]: 122.08995125628
```

```
[4]: # Calculating theta_4
th4_1 = 180 - x
th4_1
```

```
[4]: 57.9100487437197
```

```
[5]: # Calculating the value of theta_2
th2_1 = sp.N(sp.asin(sp.sin(x*sp.pi/180)/700*400)*180/sp.pi)
th2_1
```

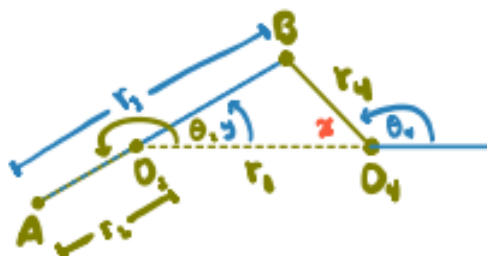
```
[5]: 28.9550243718598
```

1.1.2 Answers:

$$\theta_2 = 28.9550243718598^\circ$$

$$\theta_4 = 57.9100487437197^\circ$$

1.1.3 Limit Position 2



This is the same process as above except this time, the the side opposite to θ_4 is $r^3 - r^2$.

```
[6]: # Solving for the value of x
x2 = sp.N(sp.acos((340**2 - 400**2 - 400**2)/(-2*400*400))*180/sp.pi)
x2
```

```
[6]: 50.3013268250874
```

```
[7]: # Solving for theta_4
th4_2 = 180 - x2
th4_2
```

```
[7]: 129.698673174913
```

```
[8]: # Solving for theta_2
# You have to add 180 to get the value of the angle shown in the figure above.
th2_2 = sp.N(sp.asin(400*sp.sin(th4_2*sp.pi/180)/340)*180/sp.pi) + 180
th2_2
```

```
[8]: 244.849336587456
```

1.1.4 Answers:

$$\theta_2 = 244.849336587456^\circ$$

$$\theta_4 = 129.698673174913^\circ$$

1.2 Part B

The rocker angle is the maximum angle of θ_4 minus the minimum θ_4 .

```
[9]: th4_2 - th4_1
```

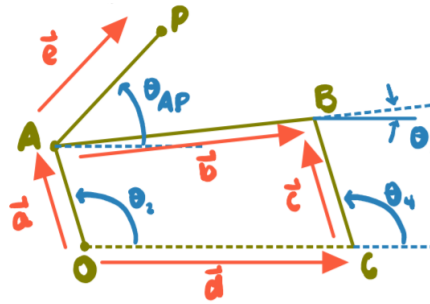
```
[9]: 71.7886244311929
```

1.2.1 Answer:

The rocker angle is 71.7886244311929°

2 Problem 2

Kinematic Diagram:



Defining Vectors and Joints for this mechanism:

```
[10]: O, A, B, C, P = get_joints('OABCP')
a = Vector((O, A), length=2)
b = Vector((A, B), length=4.1)
c = Vector((C, B), length=3)
d = Vector((O, C), length=4, angle=0, ls='--', color='black')
e = Vector((A, P), length=2.5)
# Vector f is not shown in the figure but is the vector from O to P
mech = Mechanism(vectors=(a, b, c, d, e), input_vector=a)
```

Position Loop Equation:

$$\begin{cases} 2\cos(\theta_2) + 4.1\cos(\theta_3) - 3\cos(\theta_4) - 4 = 0 \\ 2\sin(\theta_2) + 4.1\sin(\theta_3) - 3\sin(\theta_4) = 0 \end{cases}$$

Knowns: a, b, c, d, e, and θ_1 which is 0

Unknowns: θ_2 , θ_3 , and θ_4 , but θ_2 is the input angle

```
[11]: position_loop = lambda x, i: np.array((a.get_x(i) + b.get_x(x[0]) - c.
    ->get_x(x[1]) - d.length,
                                     a.get_y(i) + b.get_y(x[0]) - c.
    ->get_y(x[1])))
```

2.1 $\theta_2 = 45^\circ$

```
[12]: # Solving for the values of theta_3 and theta_4 when theta_2 is 45 degrees
guess = np.deg2rad([20, 60])
solution = fsolve(position_loop, guess, args=(np.pi/4, ))
np.rad2deg(solution)
```

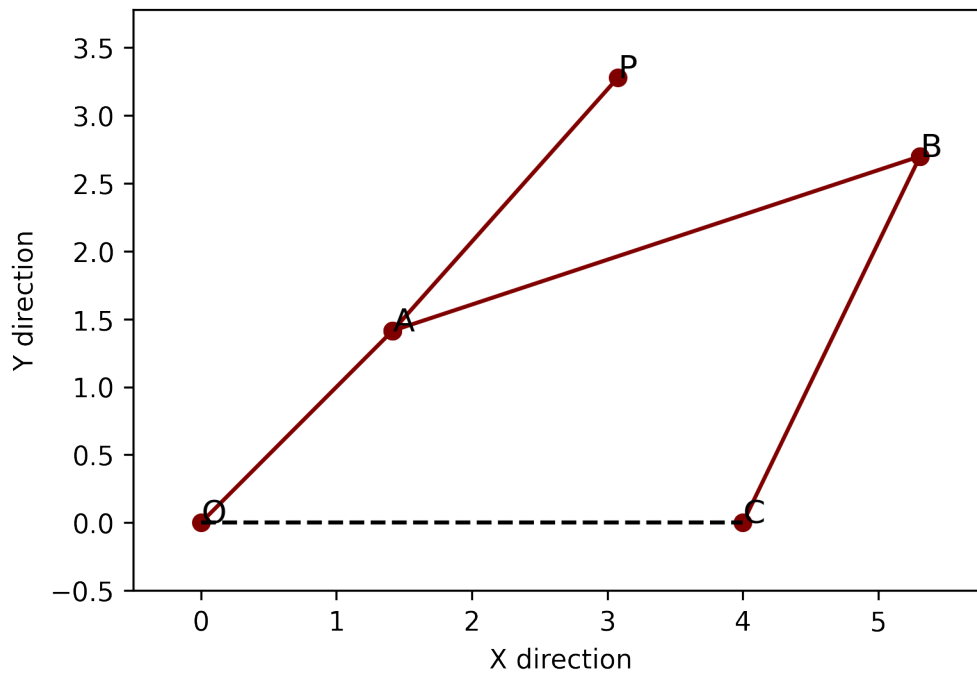
```
[12]: array([18.27915529, 64.16535766])
```

The position of point P can be determined by summing vectors a and e. The angle for vector e is $\theta_3 + 30^\circ$.

```
[13]: e.get_x(np.deg2rad(30) + solution[0])
      e.get_y(np.deg2rad(30) + solution[0])
```

```
[13]: 1.8659902906492987
```

```
[14]: mech.fix_position() # This fixes the positions of all the joints.
      mech.plot(cushion=0.5) # This produces a plot
      mech.tables(position=True) # Displays tables
```



POSITION

Vector	Length	Angle	x	y
R_OA	2	45.0	1.4142135623730951	1.4142135623730951
R_AB	4.1	18.27915529442809	3.8931125568365097	1.2859528062109784
R_CB	3	64.16535766360016	1.307326119209602	2.7001663685840476
R_OC	4	0.0	4.0	0.0
R_AP	2.5	48.27915529442809	1.6637548603092185	1.8659902906492987

Joint	x	y
A	1.4142135623730951	1.4142135623730951
B	5.307326119209602	2.7001663685840476
C	4.0	0.0

O	0	0
P	3.0779684226823134	3.280203853022394

To prove that this is the correct answer, this method will calculate the distances between all the points so that we can see that they match the lengths provided.

```
[15]: mech.calculate()
```

Distances:

```
- O to A: 2.0
- A to B: 4.099999999999999
- C to B: 3.0000000000000004
- O to C: 4.0
- A to P: 2.5
```

```
[16]: # Getting vector A to P
f = a + e
f.get_length()
f
```

```
[16]: Vector(joints=(Joint(name=O), Joint(name=P))), length=4.498180401941702,
angle=0.8171946078320964)
```

```
[17]: # Vector f angle in degrees
np.rad2deg(f.angle)
```

```
[17]: 46.82180206962757
```

2.1.1 Answers:

$$\theta_3 = 18.27915529442809^\circ$$

$$\theta_4 = 64.16535766360016^\circ$$

The angle of vector \vec{e} is $\theta_{AP} = 48.27915529442809^\circ$

The polar coordinates of the position vector from O to P is (4.498180401941702, 46.82180206962757°)

2.2 $\theta_2 = 87^\circ$

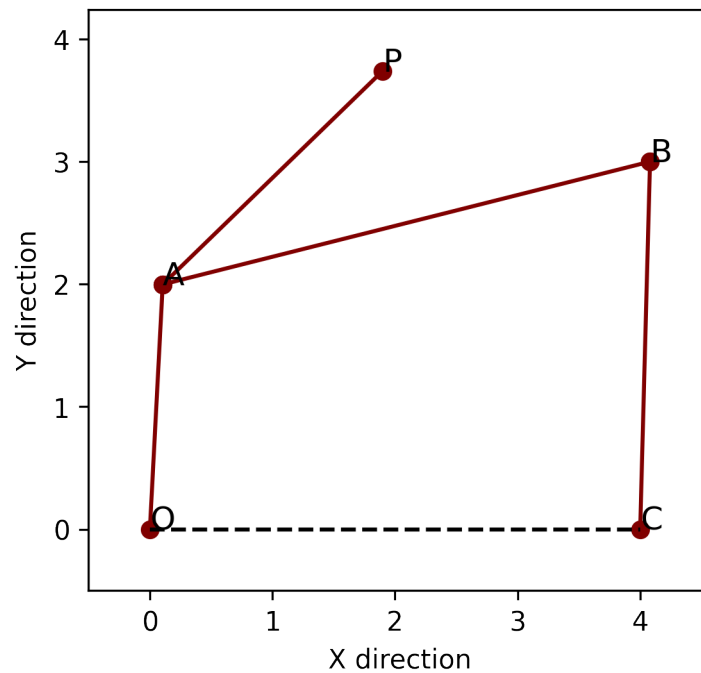
```
[18]: # Solving for the values of theta_3 and theta_4 when theta_2 is 87 degrees
mech.clear_joints()
guess = np.deg2rad([20, 60])
solution = fsolve(position_loop, guess, args=(np.deg2rad(87), ))
np.rad2deg(solution)
```

```
[18]: array([14.14093814, 88.46366837])
```

```
[19]: e.get_x(np.deg2rad(30) + solution[0])
      e.get_y(np.deg2rad(30) + solution[0])
```

```
[19]: 1.7410643097965348
```

```
[20]: mech.fix_position()
      mech.plot(cushion=0.5)
      mech.tables(position=True)
```



POSITION

Vector	Length	Angle	x	y
R_OA	2	87.0	0.10467191248588793	1.9972590695091477
R_AB	4.1	14.140938135101566	3.9757605844603874	1.0016625055632233
R_CB	3	88.46366836775566	0.08043249694627211	2.998921575072444
R_OC	4	0.0	4.0	0.0
R_AP	2.5	44.140938135101564	1.7940722028816778	1.7410643097965348

Joint	x	y
A	0.10467191248588793	1.9972590695091477
B	4.080432496946272	2.998921575072444
C	4.0	0.0

O		0		0
P		1.8987441153675657		3.7383233793056823

```
[21]: mech.calculate()
```

Distances:

- O to A: 1.9999999999999998
- A to B: 4.1000000000000014
- C to B: 3.0
- O to C: 4.0
- A to P: 2.5

```
[22]: f = a + e
      f.get_length()
      f
```

```
[22]: Vector(joints=(Joint(name=O), Joint(name=P))), length=4.192885748968891,
      angle=1.1008381471903976)
```

```
[23]: np.rad2deg(f.angle)
```

```
[23]: 63.07337976101109
```

2.2.1 Answers:

$$\theta_3 = 14.140938135101566^\circ$$

$$\theta_4 = 88.46366836775566^\circ$$

The angle of vector \vec{e} is $\theta_{AP} = 44.140938135101564^\circ$

The polar coordinates of the position vector from O to P is $(4.192885748968891, 63.07337976101109^\circ)$

2.3 $\theta_2 = 134^\circ$

```
[24]: # Solving for the values of theta_3 and theta_4 when theta_2 is 134 degrees
      mech.clear_joints()
      guess = np.deg2rad([20, 60])
      solution = fsolve(position_loop, guess, args=(np.deg2rad(134), ))
      np.rad2deg(solution)
```

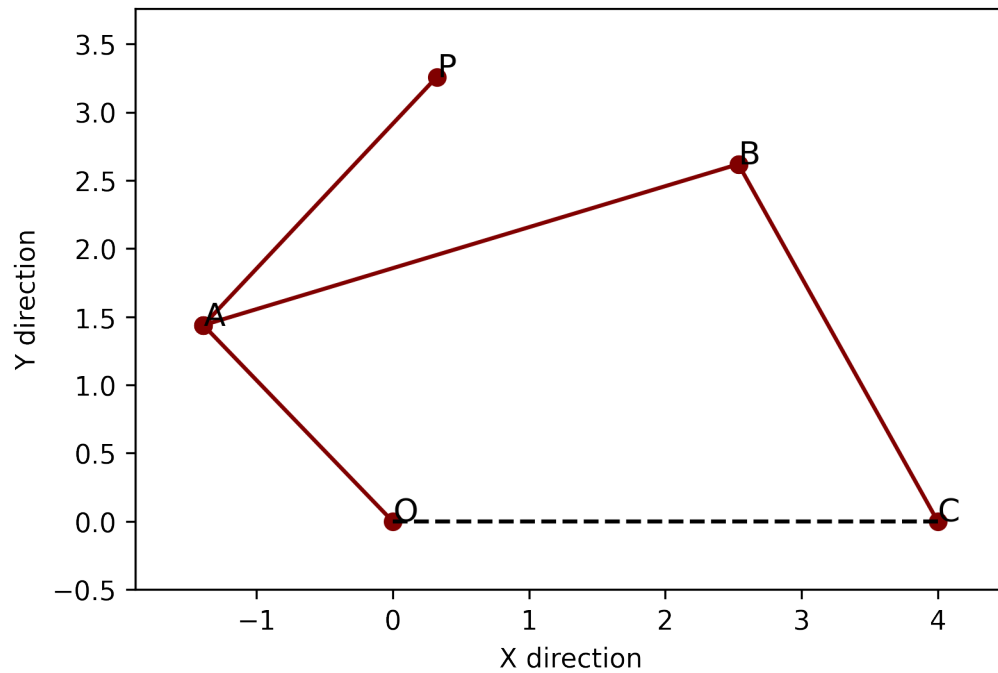
```
[24]: array([ 16.73316184, 119.18573817])
```

```
[25]: e.get_x(np.deg2rad(30) + solution[0])
      e.get_y(np.deg2rad(30) + solution[0])
```

```
[25]: 1.8204239399750564
```



```
[26]: mech.fix_position()
      mech.plot(cushion=0.5)
      mech.tables(position=True)
```



POSITION

Vector	Length	Angle	x	y
R_OA	2	134.0	-1.3893167409179947	1.4386796006773022
R_AB	4.1	16.733161844004133	3.9263896612363984	1.1804508579927901
R_CB	3	119.18573817282972	-1.4629270796817053	2.619130458670197
R_OC	4	0.0	4.0	0.0
R_AP	2.5	46.73316184400413	1.713492538287136	1.8204239399750564

Joint	x	y
A	-1.3893167409179947	1.4386796006773022
B	2.5370729203182947	2.619130458670197
C	4.0	0.0
O	0	0
P	0.32417579736914126	3.2591035406523585

```
[27]: mech.calculate()
```

Distances:

- O to A: 2.0
- A to B: 4.0999999999999925
- C to B: 3.0
- O to C: 4.0
- A to P: 2.5

```
[28]: f = a + e  
      f.get_length()  
      f
```

```
[28]: Vector(joints=(Joint(name=O), Joint(name=P))), length=3.275186381916708,  
      angle=1.4716546514890805)
```

```
[29]: np.rad2deg(f.angle)
```

```
[29]: 84.31960043112036
```

2.3.1 Answers:

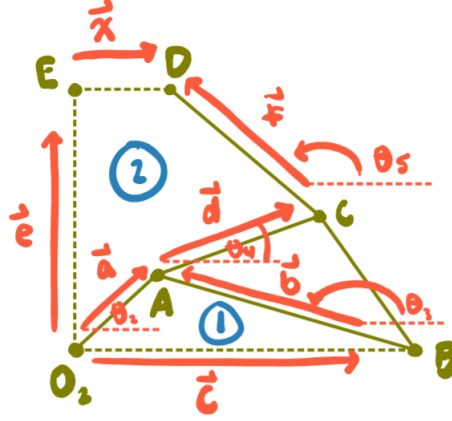
$$\theta_3 = 16.733161844004133^\circ$$

$$\theta_4 = 119.18573817282972^\circ$$

The angle of vector \vec{e} is $\theta_{AP} = 46.73316184400413^\circ$

The polar coordinates of the position vector from O to P is $(3.275186381916708, 84.31960043112036^\circ)$

3 Problem 3



The numbers in blue are the loop reference.

3.1 Knowns and Unknowns:

3.1.1 Answer:

A letter without the vector notation denotes a length. If the cell contains a variable then it is unknown.

Vector	Length	Angle
\vec{a}	2	θ_2
\vec{b}	10	θ_3
\vec{c}	c	0°
\vec{d}	4	θ_4
\vec{e}	e	90°
\vec{f}	8	θ_5
\vec{x}	3	0°

3.2 Vector Loops

3.2.1 Loop 1

$$\vec{a} - \vec{b} - \vec{c} = 0$$

3.2.2 Answer:

In the complex vector form:

$$ae^{i\theta_2} - be^{i\theta_3} - c = 0$$

Applying Euler's Identity and Magnitudes:

$$2\cos(\theta_2) + 2i\sin(\theta_2) - 10\cos(\theta_3) - 10i\sin(\theta_3) - c = 0$$

Collecting the real and imaginary terms, a system of equations may be defined:

$$\begin{cases} 2\cos(\theta_2) - 10\cos(\theta_3) - c = 0 \\ 2\sin(\theta_2) - 10\sin(\theta_3) = 0 \end{cases}$$

3.2.3 Loop 2

$$\vec{e} + \vec{x} - \vec{f} - \vec{d} - \vec{a} = 0$$

3.2.4 Answer:

In the complex vector form:

$$ee^{90i} + x - fe^{i\theta_5} - de^{i\theta_4} - ae^{i\theta_2} = 0$$

Applying Euler's Identity and Magnitudes:

$$ie + 3 - 8\cos(\theta_5) - 8i\sin(\theta_5) - 4\cos(\theta_4) - 4i\sin(\theta_4) - 2\cos(\theta_2) - 2i\sin(\theta_2) = 0$$

Collecting the real and imaginary terms, a system of equations may be defined:

$$\begin{cases} 3 - 8\cos(\theta_5) - 4\cos(\theta_4) - 2\cos(\theta_2) = 0 \\ e - 8\sin(\theta_5) - 4\sin(\theta_4) - 2\sin(\theta_2) = 0 \end{cases}$$

3.3 Finding the Stroke of Point B

The stroke of point B is calculated by finding the maximum length of \vec{c} and subtracting the its minimum length. Because points O_2 and B are in-line, the maximum value occurs when θ_2 is 0° and the minimum occurs when θ_2 is 180° . This means that the maximum value is $a + b = 2 + 10 = 12in$ and the minimum value is $b - a = 10 - 2 = 8in$.

3.3.1 Answer:

- $B_{max} = 12in$
- $B_{min} = 8in$
- $stroke = B_{max} - B_{min} = 12 - 8 = 4in$

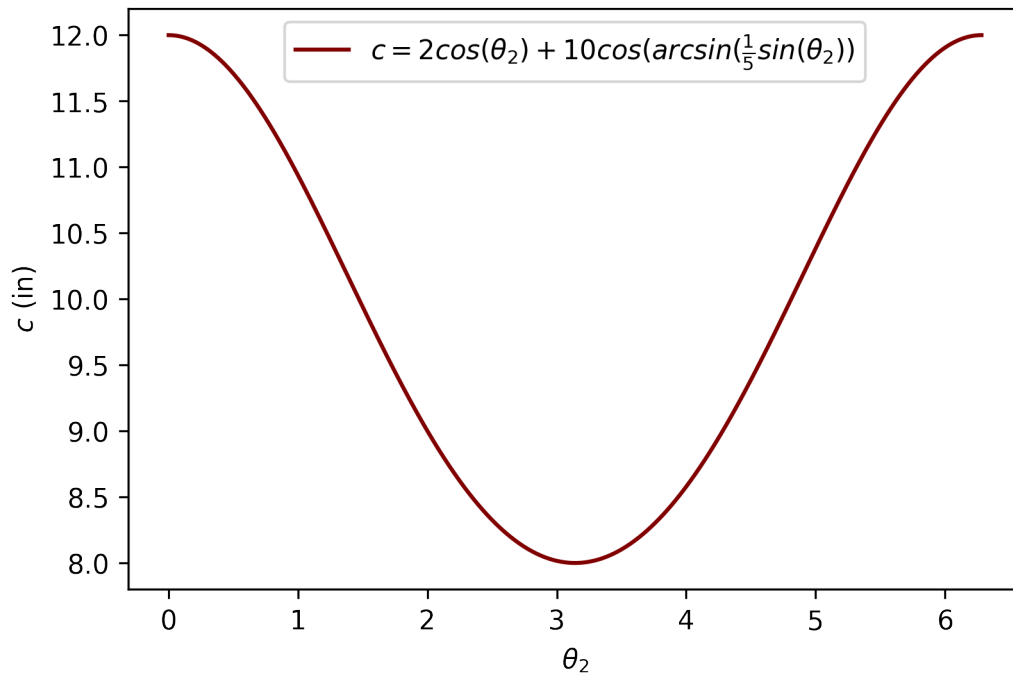
3.4 Finding B_x as a Function of θ_2

In order to obtain the length equation of \vec{c} in the x direction ($c_x = c = f(\theta_2)$), the position loop equation (loop 1) must be solved analytically. Here is the analytical solution: $c = 2\cos(\theta_2) - 10\cos(\arcsin(\frac{1}{5}\sin(\theta_2)))$.

3.4.1 Answer (Plotting the Function):

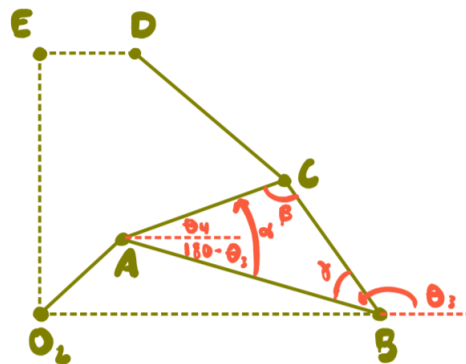
```
[30]: # Plotting on the interval from 0 to 2pi
f = lambda x: 2*np.cos(x)+10*np.cos(np.arcsin(0.2*np.sin(x)))
d = np.linspace(0, 2*np.pi, 1000)
plt.plot(d, f(d), color='maroon',_
↪label=r'$c=2\cos(\theta_2)+10\cos(\arcsin(\frac{1}{5}\sin(\theta_2)))$')
plt.legend()
plt.xlabel(r'$\theta_2$')
plt.ylabel(r'$c$ (in)')
```

```
plt.show()
# Note: I changed it to + 10cos(arcsin...) so that we wouldn't see the crossed
→ solution from page 186.
```



3.5 Finding D_y as a Function of θ_2 BONUS

In order to find $e_y(\theta_2) = e(\theta_2)$, the relationship between θ_4 and θ_3 must be established from the Law of Cosines in the figure below. Also, notice that there is also this relationship between θ_3 and θ_2 from the first position loop equation: $\theta_3 = \arcsin(\frac{1}{5}\sin(\theta_2))$



From the above figure: $\alpha = \theta_4 + 180^\circ - \theta_3$.

```
[31]: # Solving for alpha using the law of cosines.
alpha = np.arccos((7**2-10**2-4**2)/(-2*4*10))
```

```
alpha # This is in radians
```

```
[31]: 0.5781043645663436
```

Now we can begin by using the loop 2 equations in this form:

$$\begin{cases} 8\cos(\theta_5) = 3 - 4\cos(\theta_4) - 2\cos(\theta_2) \\ 8\sin(\theta_5) = e - 4\sin(\theta_4) - 2\sin(\theta_2) \end{cases}$$

Now if you square both sides of the equation then add, the θ_5 terms go away via the pythagorean identity.

$$64 = (3 - 4\cos(\theta_4) - 2\cos(\theta_2))^2 + (e - 4\sin(\theta_4) - 2\sin(\theta_2))^2$$

$$e = \sqrt{64 - (3 - 4\cos(\theta_4) - 2\cos(\theta_2))^2} + 4\sin(\theta_4) + 2\sin(\theta_2)$$

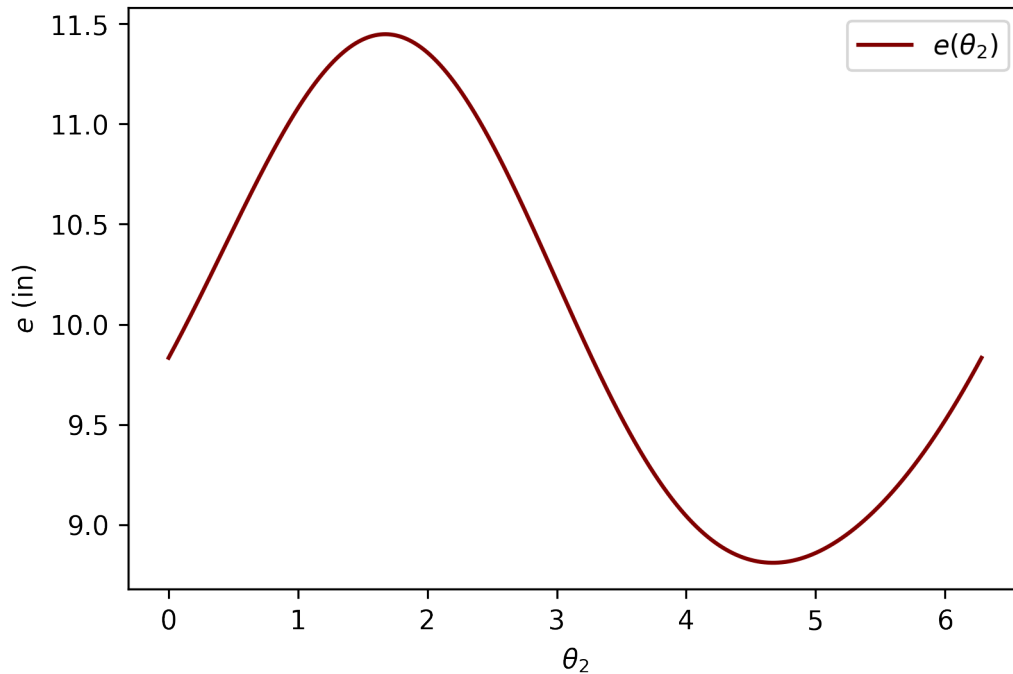
With the above relationships between θ_2 , θ_3 , and θ_4 , e can now be determined as a function of θ_2 alone by substituting $\theta_4 = \alpha - \pi + \arcsin(\frac{1}{5}\sin(\theta_2))$.

3.6 Finding the Stroke of Point D BONUS

The stroke can be found by determining the maximum and minimum value of $e(\theta_2)$ and taking the difference between the two. Note: The length of e is the y value of point F.

3.6.1 Plotting $e(\theta_2)$ BONUS

```
[32]: def e(t2):  
    # Because theta_3 is in the 2nd and 3rd quadrant, you have to do some hand_␣  
    ↪waving to get the real value of  
    # theta_3 which affects the real value of theta_4  
    t4 = alpha - np.arcsin(0.2*np.sin(t2))  
  
    return np.sqrt(64 - (3 - 4*np.cos(t4) - 2*np.cos(t2))**2) + 4*np.sin(t4) +_␣  
    ↪2*np.sin(t2)  
  
d = np.linspace(0, 2*np.pi, 10_000)  
e_values = e(d)  
plt.plot(d, e_values, label=r'$e(\theta_2)$', color='maroon')  
plt.legend()  
plt.xlabel(r'$\theta_2$')  
plt.ylabel(r'$e$ (in)')  
plt.show()
```



```
[33]: # Getting the maximum and minimum values
e_max = np.max(e_values)
e_max
```

```
[33]: 11.448596166062995
```

```
[34]: e_min = np.min(e_values)
e_min
```

```
[34]: 8.809289404889084
```

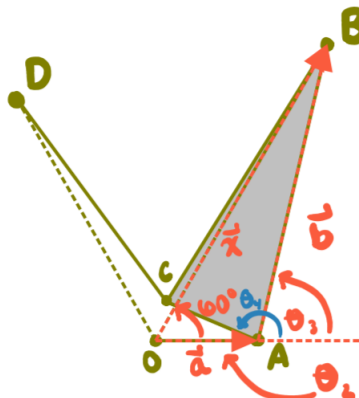
```
[35]: stroke = e_max - e_min
stroke
```

```
[35]: 2.639306761173911
```

3.6.2 Answers BONUS:

- $D_{max} = 11.448596166062995in$
- $D_{min} = 8.809289404889084in$
- $stroke = 2.639306761173911in$

4 Problem 4



4.1 Loop 1 Known and Unknowns

Vector	Length	Angle
\vec{a}	2	$-50\pi t$
\vec{b}	6	$\theta_3(t)$
\vec{x}	$x(t)$	60°

The $\theta_2(t)$ comes from integrating $\omega_2(t) = -1500 \text{ rmp} = -50\pi \frac{\text{rad}}{\text{s}}$

```
[36]: t = sp.Symbol('t')
t2 = sp.Symbol('theta_2')
t2_t = -50*sp.pi*t
x = sp.Symbol('x')
t3 = sp.Symbol('theta_3')

a, b = 2, 6
x_angle = sp.pi/3
```

4.2 Position Loop 1

$$\vec{a} + \vec{b} - \vec{x} = 0$$

$$\begin{cases} 2\cos(\theta_2) + 6\cos(\theta_3) - x\cos(60) = 0 \\ 2\sin(\theta_2) + 6\sin(\theta_3) - x\sin(60) = 0 \end{cases}$$

```
[37]: # Solving for x and theta_3 as functions of theta_2
p1 = a*sp.cos(t2) + b*sp.cos(t3) - x*sp.cos(x_angle)
p2 = a*sp.sin(t2) + b*sp.sin(t3) - x*sp.sin(x_angle)

p_loop1 = solve([p1, p2], (x, t3), dict=True)
p_loop1
```



```
[37]: [{theta_3: -acos(-sqrt(34 - 2*cos(2*theta_2 + pi/3))/12 - sqrt(3)*cos(theta_2 +
pi/6)/6) + 2*pi,
      x: -sqrt(34 - 2*cos(2*theta_2 + pi/3)) + 2*sin(theta_2 + pi/6)},
      {theta_3: -acos(sqrt(34 - 2*cos(2*theta_2 + pi/3))/12 - sqrt(3)*cos(theta_2 +
pi/6)/6) + 2*pi,
      x: sqrt(34 - 2*cos(2*theta_2 + pi/3)) + 2*sin(theta_2 + pi/6)},
      {theta_3: acos(-sqrt(34 - 2*cos(2*theta_2 + pi/3))/12 - sqrt(3)*cos(theta_2 +
pi/6)/6),
      x: -sqrt(34 - 2*cos(2*theta_2 + pi/3)) + 2*sin(theta_2 + pi/6)},
      {theta_3: acos(sqrt(34 - 2*cos(2*theta_2 + pi/3))/12 - sqrt(3)*cos(theta_2 +
pi/6)/6),
      x: sqrt(34 - 2*cos(2*theta_2 + pi/3)) + 2*sin(theta_2 + pi/6)}]
```

I will chose the last solution because x and θ_3 are positive for all values of θ_2 .

```
[38]: x_t2 = p_loop1[-1][x] # x(theta_2)
      t3_t2 = p_loop1[-1][t3] # t3(theta_2)

      # Substituting -50pi*t in for theta_2
      x_t = x_t2.subs(t2, t2_t)
      t3_t = t3_t2.subs(t2, t2_t)
```

```
[39]: # x as a function of time
      x_t
```

```
[39]: 
$$\sqrt{34 - 2 \sin\left(100\pi t + \frac{\pi}{6}\right)} + 2 \cos\left(50\pi t + \frac{\pi}{3}\right)$$

```

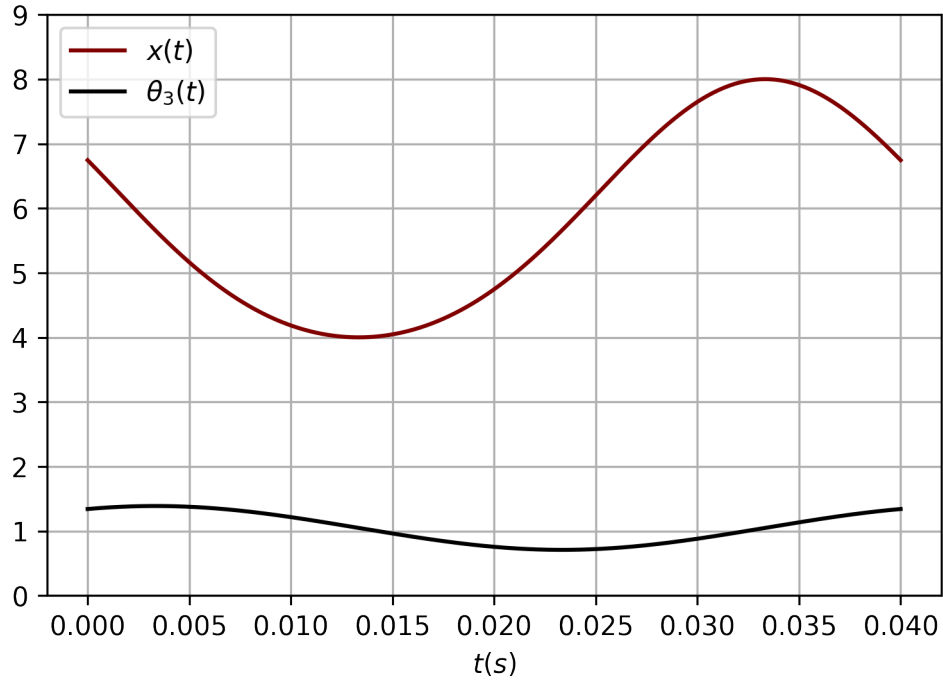
```
[40]: # Theta_3 as a function of time
      t3_t
```

```
[40]: 
$$\arccos\left(\frac{\sqrt{34 - 2 \sin\left(100\pi t + \frac{\pi}{6}\right)}}{12} - \frac{\sqrt{3} \sin\left(50\pi t + \frac{\pi}{3}\right)}{6}\right)$$

```

```
[41]: # Plotting x(t) and t3(t)
      x_t_lamb = sp.lambdify(t, x_t, modules=['numpy'])
      t3_t_lamb = sp.lambdify(t, t3_t, modules=['numpy'])

      d = np.linspace(0, 0.04, 1000)
      plt.plot(d, x_t_lamb(d), color='maroon', label=r'$x(t)$')
      plt.plot(d, t3_t_lamb(d), color='black', label=r'$\theta_3(t)$')
      plt.legend()
      plt.xlabel(r'$t(s)$')
      plt.ylim(0, 9)
      plt.grid()
      plt.show()
```



4.2.1 Acceleration at Point B Answer:

Just from this information alone, the acceleration of point B can now be determined.

$$\vec{a}_B = \vec{a}_{BO} + \vec{a}_O$$

The acceleration of point O is zero.

$$\vec{a}_B = \vec{a}_{BO}$$

Since the acceleration from O to B is purely translational, $\vec{a}_{BO} = \ddot{x}$. Therefore, $\vec{a}_B = \ddot{x}$.

[42]: *# Getting the 2nd order derivative of x with respect to time*
`x_t__ = x_t.diff(t, 2)` *# Two underscores at the end of the variable*
↪ declaration denotes a second order derivative
`x_t__`

[42]:
$$2500\pi^2 \left(-2 \cos \left(\pi \left(50t + \frac{1}{3} \right) \right) + \frac{2\sqrt{2} \sin \left(\pi \left(100t + \frac{1}{6} \right) \right)}{\sqrt{17 - \sin \left(\pi \left(100t + \frac{1}{6} \right) \right)}} - \frac{\sqrt{2} \cos^2 \left(\pi \left(100t + \frac{1}{6} \right) \right)}{\left(17 - \sin \left(\pi \left(100t + \frac{1}{6} \right) \right) \right)^{\frac{3}{2}}} \right)$$

4.2.2 Another Way to Get the Answer

The acceleration of B can also be found by using this path:

$$\vec{a}_B = \vec{a}_{AB} + \vec{a}_A$$

Now we must find \vec{a}_A which is equal to \vec{a}_{OA} . Because $\alpha_2 = 0$, the acceleration will only consist of the radial component.

$$\vec{a}_A = -2\omega_2^2 e^{i\theta_2}$$

```
[43]: # Getting acceleration of point A in the cartesian form.
e1 = lambda x: sp.Matrix([sp.cos(x), sp.sin(x)]) # Returns <sin(theta),
    ↪ cos(theta)>
e2 = lambda x: sp.Matrix([-sp.sin(x), sp.cos(x)]) # Returns <-sin(theta),
    ↪ cos(theta)>
mag = lambda m: sp.sqrt(m[0]**2 + m[1]**2) # Returns the magnitude of a vector

A_vector = -a*t2_t.diff(t)**2*e1(t2_t)
A_vector
```

```
[43]: 
$$\begin{bmatrix} -5000\pi^2 \cos(50\pi t) \\ 5000\pi^2 \sin(50\pi t) \end{bmatrix}$$

```

```
[44]: mag(A_vector).simplify() # Just making sure that this is working
```

```
[44]: 5000π2
```

Now the value of \vec{a}_{AB} can be considered. Since the vector from A to B is pure rotation, the following is true:

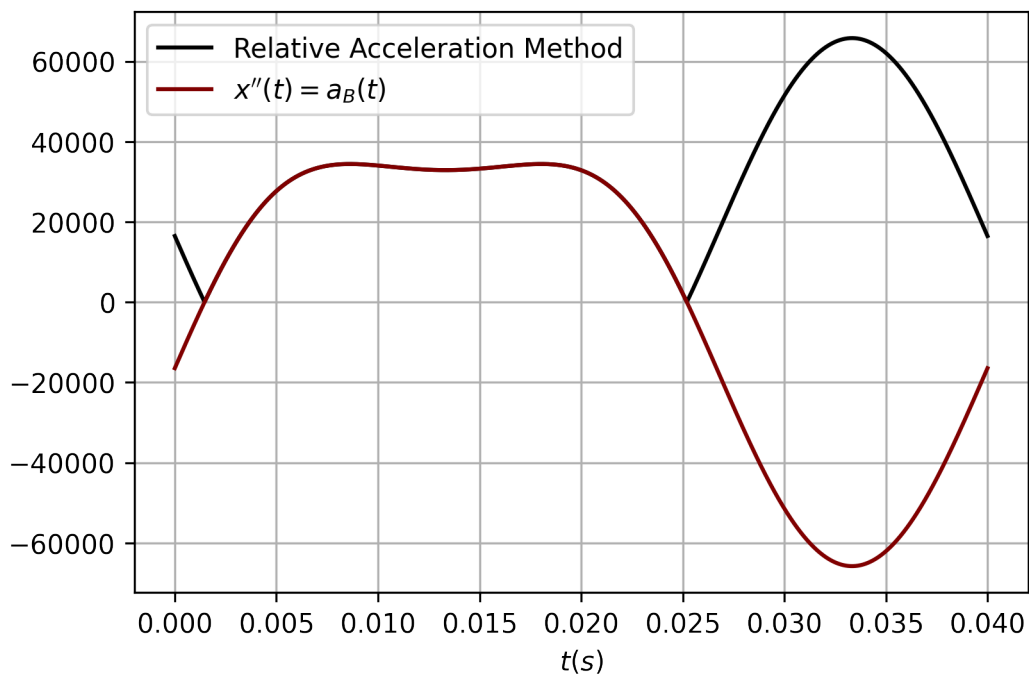
$$\vec{a}_{AB} = 6\alpha_3 i e^{i\theta_3} - 6\omega_3^2 e^{i\theta_3}$$

```
[45]: # Getting the acceleration vector from A to B
AB_vector = b*t3_t.diff(t, 2)*e2(t3_t) - b*t3_t.diff(t)**2*e1(t3_t)

# Summing the two vectors
B_vector = AB_vector + A_vector
B_other = mag(B_vector)

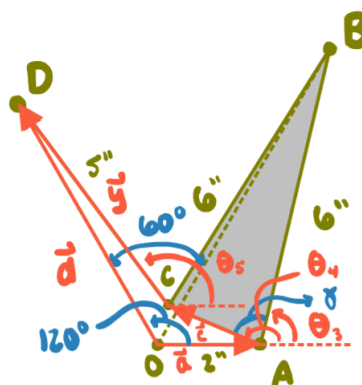
# Plotting the two functions to compare
B_other_lamb = sp.lambdify(t, B_other, modules=['numpy'])
x_t__lamb = sp.lambdify(t, x_t__, modules=['numpy'])

plt.plot(d, B_other_lamb(d), color='black', label='Relative Acceleration_
    ↪ Method')
plt.plot(d, x_t__lamb(d), color='maroon', label=r"$x'(t)=a_{B}(t)$")
plt.legend()
plt.xlabel(r"$t(s)$")
plt.grid()
plt.show()
```



As seen from above, the two methods agree with each other. The relative acceleration is a magnitude which is why all the values are above 0.

4.3 Loop 2 Known and Unknowns



Vector	Length	Angle
\vec{a}	2	$-50\pi t$
\vec{c}	2	$\theta_4(t)$
\vec{y}	5	$\theta_5(t)$
\vec{d}	$d(t)$	120°

```
[46]: # Solving for gamma via law of cosines
gamma = sp.acos((6**2 - 6**2 - 2**2)/(-2*6*2))
gamma
```

```
[46]: 1.40334824757521
```

```
[47]: t4_t = t3_t + gamma
t4, t5, d = sp.symbols('theta_4 theta_5 d')

d_angle = 2*sp.pi/3
c, y = 2, 5
```

4.4 Position Loop 2

$$\vec{a} + \vec{c} + \vec{y} - \vec{d} = 0$$

$$\begin{cases} 2\cos(\theta_2) + 2\cos(\theta_4) + 5\cos(\theta_5) - d\cos(120) = 0 \\ 2\sin(\theta_2) + 2\sin(\theta_4) + 5\sin(\theta_5) - d\sin(120) = 0 \end{cases}$$

```
[48]: # Solving the system for d and theta_5
p3 = a*sp.cos(t2_t) + c*sp.cos(t4_t) + y*sp.cos(t5) - d*sp.cos(d_angle)
p4 = a*sp.sin(t2_t) + c*sp.sin(t4_t) + y*sp.sin(t5) - d*sp.sin(d_angle)

# p_loop2 = solve([p3, p4], (d, t5), dict=True, quick=True)
# p_loop2
```

4.4.1 Acceleration of Point D Answer

I tried running the above cell, but sympy cannot handle solving this symbolically. Nevertheless, the solution of that system would get $d(t)$ then differentiating with respect to time twice would get the acceleration of point D because $\vec{a}_D = \vec{a}_{OD} + \vec{a}_O$. With \vec{a}_O being equal to zero, that would mean that $\vec{a}_{OD} = \vec{a}_D$ and \vec{a}_{OD} is purely translational. This is similar to the way the acceleration of point B was solved.

The above system can be solved for $d(t)$ by hand. First, place the system in this form:

$$\begin{cases} 2\cos(\theta_2) + 2\cos(\theta_4) - d\cos(120) = -5\cos(\theta_5) \\ 2\sin(\theta_2) + 2\sin(\theta_4) - d\sin(120) = -5\sin(\theta_5) \end{cases}$$

Now if you square both sides then add the two equations, the θ_5 can be eliminated via Pythagorean Identity. This results in an equation with d as the only unknown.

$$(2\cos(\theta_2) + 2\cos(\theta_4) - d\cos(120))^2 + (2\sin(\theta_2) + 2\sin(\theta_4) - d\sin(120))^2 = 25$$

Sympy will solve for d in the cell below. This will result in a function of time.

```
[49]: # Solving for d as a function of time
d_solutions = solve((2*sp.cos(t2_t) + 2*sp.cos(t4_t) - d*sp.cos(d_angle))**2 +
    ↪ (2*sp.sin(t2_t) + 2*sp.sin(t4_t) - d*sp.sin(d_angle))**2 - 25, d, dict=True)
# d_solutions Suppressing this output because it's too large
```

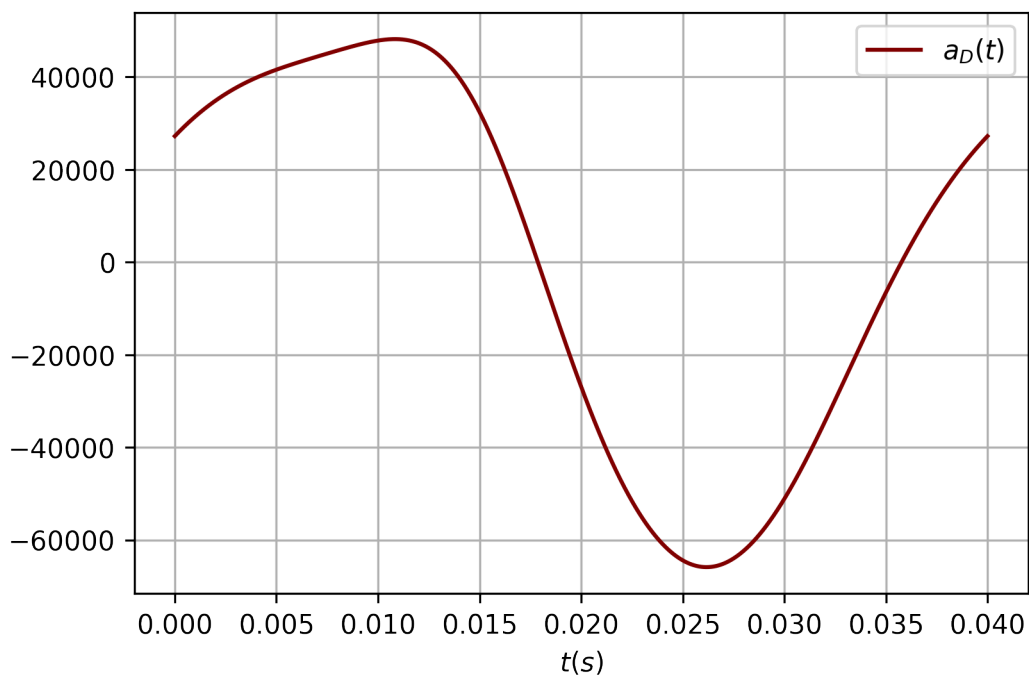
```
[50]: d_t = d_solutions[1][d]
      # d_t Suppressing this output because it's too large
```

The acceleration of d is \ddot{d} .

```
[51]: # getting the acceleration of point d as a function of time
      d_t__ = d_t.diff(t, 2)
      # d_t__ I am suppressing this output because it's too large
```

```
[52]: # Plotting the acceleration of point D
      d_t__lamb = sp.lambdify(t, d_t__, modules=['numpy'])
      period = np.linspace(0, 0.04, 1000)

      plt.plot(period, d_t__lamb(period), color='maroon', label=r'$a_{D}(t)$')
      plt.grid()
      plt.legend()
      plt.xlabel(r'$t(s)$')
      plt.show()
```



4.4.2 Acceleration of Point C Answer

The acceleration of point C, however, can be solved by using relative acceleration with the already solved information. Consider the following relative acceleration equation:

$$\vec{a}_C = \vec{a}_{AC} + \vec{a}_A$$

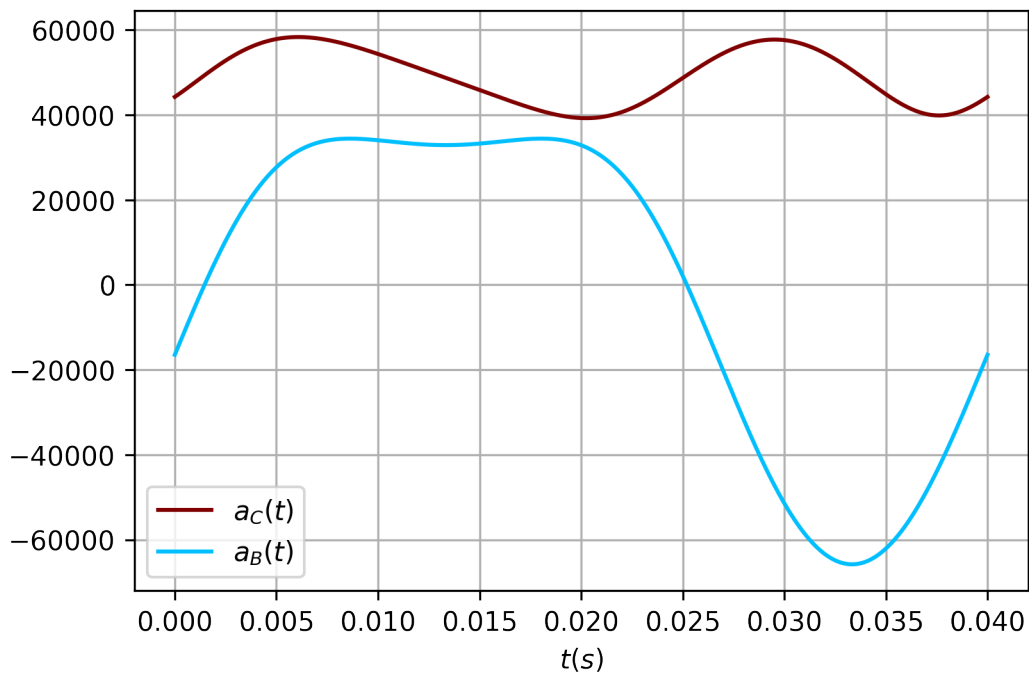
\vec{a}_A was solved earlier, but since \vec{a}_{AC} is purely rotational, the following is true:

$$\vec{a}_{AC} = 2\alpha_3 i e^{i(\theta_3 + \gamma)} - 2\omega_3^2 e^{i(\theta_3 + \gamma)}$$

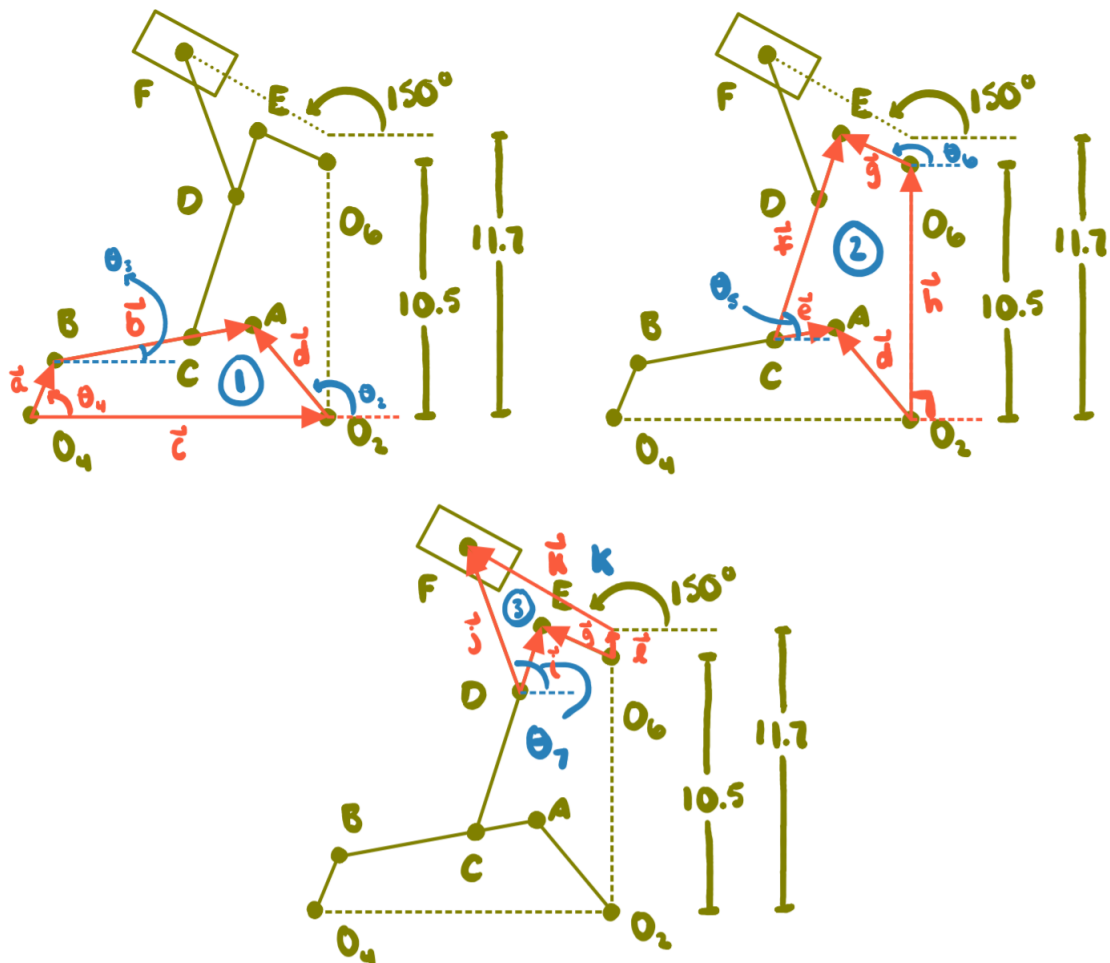
```
[53]: # Calculating the acceleration of point C
C_vector = c*t4_t.diff(t, 2)*e2(t4_t) - c*t4_t.diff(t)**2*e1(t4_t) + A_vector
# mag(C_vector) Suppressing this output because it's too large
```

This solution is ugly, but graphing it will show that it is correct.

```
[54]: # Plotting acceleration of point C
C_vector_lamb = sp.lambdify(t, mag(C_vector), modules=['numpy'])
time = np.linspace(0, 0.04, 1000) # This is the period
plt.plot(time, C_vector_lamb(time), label=r"$a_C(t)$", color='maroon')
plt.plot(time, x_t__lamb(time), label=r"$a_B(t)$", color='deepskyblue')
plt.legend()
plt.grid()
plt.xlabel(r'$t(s)$')
plt.show()
```



5 Problem 5



5.1 Known and Unkown Quantities

Vector	Length	Angle
\vec{a}	2.5	θ_4
\vec{b}	8.4	θ_3
\vec{c}	12.5	0°
\vec{d}	5	θ_2
\vec{e}	2.4	θ_3
\vec{f}	8.9	θ_5
\vec{g}	3.2	θ_6
\vec{h}	10.5	90°
\vec{i}	3	θ_5
\vec{j}	6.4	θ_7
\vec{k}	k	150°
\vec{l}	1.2	0°

There are 6 unknowns (excluding θ_4 because it's position is the input) and each loop provides 2 equations which means that there are 6 equations, making this a consistent system.

5.2 Finding the Limit Position of θ_4

The linkage system for loop 1 is not a Grashof linkage.

```
[55]: 2.5 + 12.5 < 8.4 + 5
```

```
[55]: False
```

The maximum value of θ_4 occurs when \vec{d} and \vec{b} are co-linear. The Law of Cosines could be used to determine this value, but a position loop could also solve it.

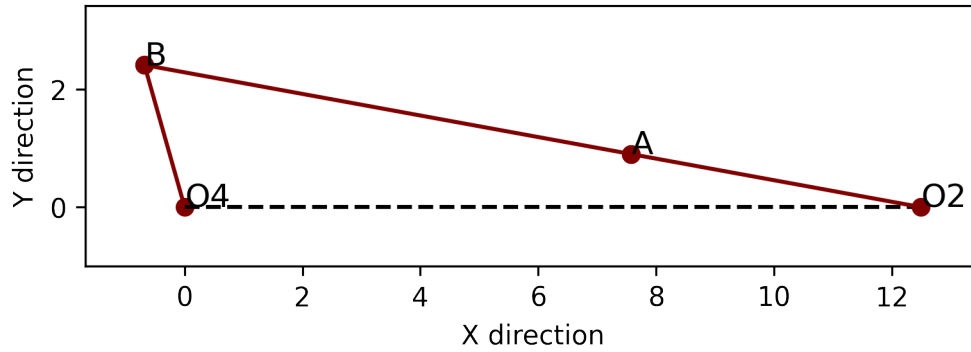
```
[56]: # Creating the limit position
A, B = get_joints('AB')
O2, O4 = Joint('O2'), Joint('O4')
a_limit = Vector((O4, B), length=2.5)
b_limit = Vector((A, B), length=8.4)
c_limit = Vector((O4, O2), length=12.5, angle=0, ls='--', color='black')
d_limit = Vector((O2, A), length=5)
limit = Mechanism(vectors=(a_limit, b_limit, c_limit, d_limit),
    ↳input_vector=a_limit)

# Implementing the position loop equation
limit_position_loop = lambda x: np.array([a_limit.get_x(x[0]) - b_limit.
    ↳get_x(x[1]) - d_limit.get_x(x[1]) - c_limit.length,
    a_limit.get_y(x[0]) - b_limit.
    ↳get_y(x[1]) - d_limit.get_y(x[1])])

solution = fsolve(limit_position_loop, np.deg2rad([120, 160]))
np.rad2deg(solution)
```

```
[56]: array([105.8404803 , 169.66039052])
```

```
[57]: limit.fix_position()
limit.plot()
limit.tables(position=True)
```



POSITION

Vector	Length	Angle	x	y
R_04B	2.5	105.84048029945892	-0.6823999999999741	2.4050634586222532
R_AB	8.4	169.66039051940945	-8.26359402985071	1.5076517203305604
R_04O2	12.5	0.0	12.5	0.0
R_02A	5	169.66039051940945	-4.918805970149232	0.8974117382920002

Joint	x	y
A	7.581194029850735	0.8974117382916929
B	-0.6823999999999741	2.4050634586222532
O2	12.5	0.0
O4	0	0

From the above data, $\theta_4 = 105.84048029945892^\circ$. Lets calculate the other limit position to check if the solution half way between 0° and θ_4 max exists.

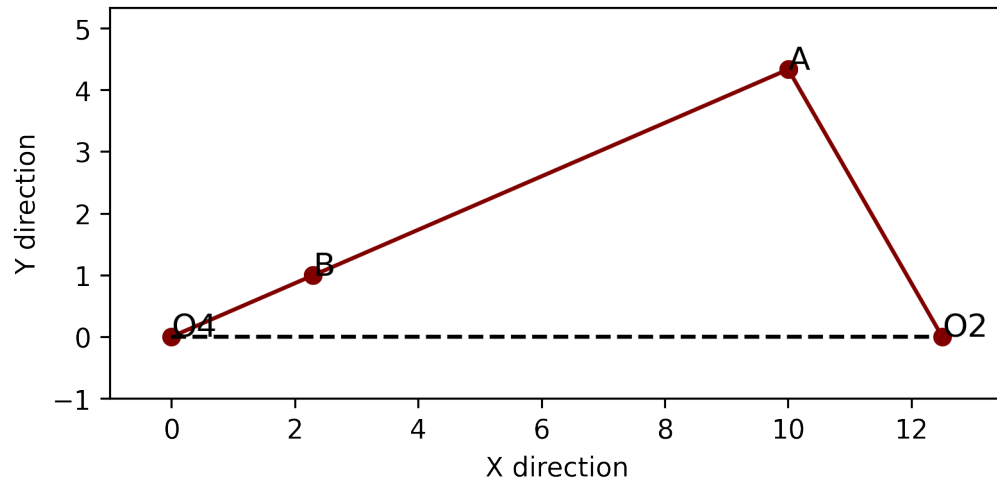
```
[58]: A, B = get_joints('AB')
O2, O4 = Joint('O2'), Joint('O4')
a_limit = Vector((O4, B), length=2.5)
b_limit = Vector((B, A), length=8.4)
c_limit = Vector((O4, O2), length=12.5, angle=0, ls='--', color='black')
d_limit = Vector((O2, A), length=5)
limit2 = Mechanism(vectors=(a_limit, b_limit, c_limit, d_limit),
    ↪input_vector=a_limit)

limit_2 = lambda x: np.array([a_limit.get_x(x[0]) + b_limit.get_x(x[0]) -
    ↪d_limit.get_x(x[1]) - c_limit.length,
                                a_limit.get_y(x[0]) + b_limit.get_y(x[0]) -
    ↪d_limit.get_y(x[1])])
```

```
solution = fsolve(limit_2, np.deg2rad([50, 40]))
np.rad2deg(solution)
```

```
[58]: array([ 23.41489115, 119.96824853])
```

```
[59]: # The solution is possible because theta_4 is 23.41 degrees which is less than
      ↪ half of its max
      limit2.fix_position()
      limit2.plot()
```



```
[60]: # Getting the half way point between 0 and theta_4's limit position
      t4 = 105.84048029945892/2
      t4
```

```
[60]: 52.92024014972946
```

5.3 Position Analysis

Position Loop Equations:

$$\begin{cases} 2\cos(52.92^\circ) + 8.4\cos(\theta_3) - 5\cos(\theta_2) - 12.5 = 0 \\ 2\sin(52.92^\circ) + 8.4\sin(\theta_3) - 5\sin(\theta_2) = 0 \\ 8.9\cos(\theta_5) - 3.2\cos(\theta_6) + 5\cos(\theta_2) - 2.4\cos(\theta_3) = 0 \\ 8.9\sin(\theta_5) - 3.2\sin(\theta_6) - 10.5 + 5\sin(\theta_2) - 2.4\sin(\theta_3) = 0 \\ 6.4\cos(\theta_7) - k\cos(150^\circ) + 3.2\cos(\theta_6) - 3\cos(\theta_5) = 0 \\ 6.4\sin(\theta_7) - k\sin(150^\circ) - 1.2 + 3.2\sin(\theta_6) - 3\sin(\theta_5) = 0 \end{cases}$$

```
[61]: # Defining the mechanism
A, B, C, D, E, F, G = get_joints('ABCDEFG')
O2, O4, O6 = Joint('O2'), Joint('O4'), Joint('O6')
a = Vector((O4, B), length=2.5)
b = Vector((B, A), length=8.4)
c = Vector((O4, O2), length=12.5, angle=0, ls='--', color='black')
d = Vector((O2, A), length=5)
e = Vector((C, A), length=2.4, show=False)
f = Vector((C, E), length=8.9)
g = Vector((O6, E), length=3.2)
h = Vector((O2, O6), length=10.5, angle=np.pi/2, ls='--', color='black')
i = Vector((D, E), length=3, show=False)
j = Vector((D, F), length=6.4)
k = Vector((G, F), angle=np.deg2rad(150), ls=':', color='black')
l = Vector((O6, G), length=1.2, angle=np.pi/2, ls=':', color='black')
mechanism = Mechanism(vectors=(a, b, c, d, e, f, g, h, i, j, k, l),
    ↪input_vector=a, omega=-30, alpha=0)

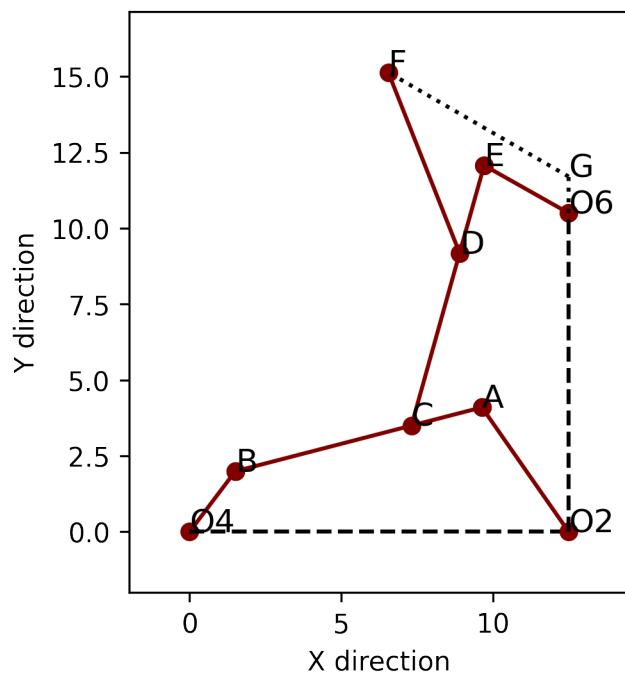
order = ('Theta 2', 'Theta 3', 'Theta 5', 'Theta 6', 'Theta 7', 'k')

position_loops = lambda t: np.array([
    a.get_x(np.deg2rad(t[4])) + b.get_x(t[1]) - d.get_x(t[0]) - c.length,
    a.get_y(np.deg2rad(t[4])) + b.get_y(t[1]) - d.get_y(t[0]),
    f.get_x(t[2]) - g.get_x(t[3]) + d.get_x(t[0]) - e.get_x(t[1]),
    f.get_y(t[2]) - g.get_y(t[3]) - h.length + d.get_y(t[0]) - e.get_y(t[1]),
    j.get_x(t[4]) - k.get_unknown_length(t[5])*np.cos(k.angle) + g.get_x(t[3]),
    ↪- i.get_x(t[2]),
    j.get_y(t[4]) - k.get_unknown_length(t[5])*np.sin(k.angle) - l.length + g.
    ↪get_y(t[3]) - i.get_y(t[2]),
])

guess = np.concatenate((np.deg2rad([120, 20, 70, 170, 120]), np.array([7])))
p_loop = fsolve(position_loops, guess)
p_loop
```

```
[61]: array([2.17998043, 0.25342828, 1.29821101, 2.62870235, 1.94696281,
        6.86701739])
```

```
[62]: mechanism.fix_position()
mechanism.plot(cushion=2)
mechanism.tables(position=True, to_five=True)
```



POSITION

Vector	Length	Angle	x	y
R_O4B	2.50000	52.92024	1.50732	1.99449
R_BA	8.40000	14.52037	8.13169	2.10608
R_O4O2	12.50000	0.00000	12.50000	0.00000
R_O2A	5.00000	124.90368	-2.86099	4.10058
R_CA	2.40000	14.52037	2.32334	0.60174
R_CE	8.90000	74.38201	2.39608	8.57140
R_O6E	3.20000	150.61355	-2.78826	1.57023
R_O2O6	10.50000	90.00000	0.00000	10.50000
R_DE	3.00000	74.38201	0.80767	2.88923
R_DF	6.40000	111.55275	-2.35109	5.95251
R_GF	6.86702	150.00000	-5.94701	3.43351
R_O6G	1.20000	90.00000	0.00000	1.20000

Joint	x	y
A	9.63900738965331	4.100575725863581
B	1.5073154945133524	1.9944924166313514
C	7.315666849322855	3.4988376360150624
D	8.904077792318697	9.180998376100744
E	9.711744373503024	12.070232650720582

F	6.552988491303877	15.133508695172324
G	12.499999999972339	11.6999999999768787
02	12.5	0.0
04	0	0
06	12.5	10.5

```
[63]: # Checking the distances
mechanism.calculate()
```

Distances:

- 04 to B: 2.5
- B to A: 8.3999999998982823
- 04 to 02: 12.5
- 02 to A: 5.0
- C to A: 2.3999999998982824
- C to E: 8.9
- 06 to E: 3.2000000025126614
- 02 to 06: 10.5
- D to E: 3.0
- D to F: 6.3999999999999995
- G to F: 6.867017390807077
- 06 to G: 1.199999999768787

5.4 Velocity Analysis

The velocity analysis will consist of the same loops defined earlier. Every unknown angle defined will now have an unknown angular velocity corresponding to the same number. \vec{k} will only have an unknown slip velocity.

From the position analysis, all the values of θ_n are known.

$$\omega_4 = -30 \frac{rad}{s}$$

$$\begin{cases} 2.5(-30)(-\sin(52.92^\circ)) + 8.4\omega_3(-\sin(\theta_3)) - 5\omega_2(-\sin(\theta_3)) = 0 \\ 2.5(-30)(\cos(52.92^\circ)) + 8.4\omega_3(\cos(\theta_3)) - 5\omega_2(\cos(\theta_3)) = 0 \\ 8.9\omega_5(-\sin(\theta_5)) - 3.2\omega_6(-\sin(\theta_6)) + 5\omega_2(-\sin(\theta_2)) - 2.4\omega_3(-\sin(\theta_3)) = 0 \\ 8.9\omega_5(\cos(\theta_5)) - 3.2\omega_6(\cos(\theta_6)) + 5\omega_2(\cos(\theta_2)) - 2.4\omega_3(\cos(\theta_3)) = 0 \\ 6.4\omega_7(-\sin(\theta_7)) - \dot{k}\cos(150^\circ) + 3.2\omega_6(-\sin(\theta_6)) - 3\omega_5(-\sin(\theta_5)) = 0 \\ 6.4\omega_7(\cos(\theta_7)) - \dot{k}\sin(150^\circ) + 3.2\omega_6(\cos(\theta_6)) - 3\omega_5(\cos(\theta_5)) = 0 \end{cases}$$

```
[64]: a_v, b_v, c_v, d_v, e_v, f_v, g_v, h_v, i_v, j_v, k_v, l_v = mechanism.
      ↪ get_velocities()

order2 = ('Omega 2', 'Omega 3', 'Omega 5', 'Omega 6', 'Omega 7', 'k_dot')

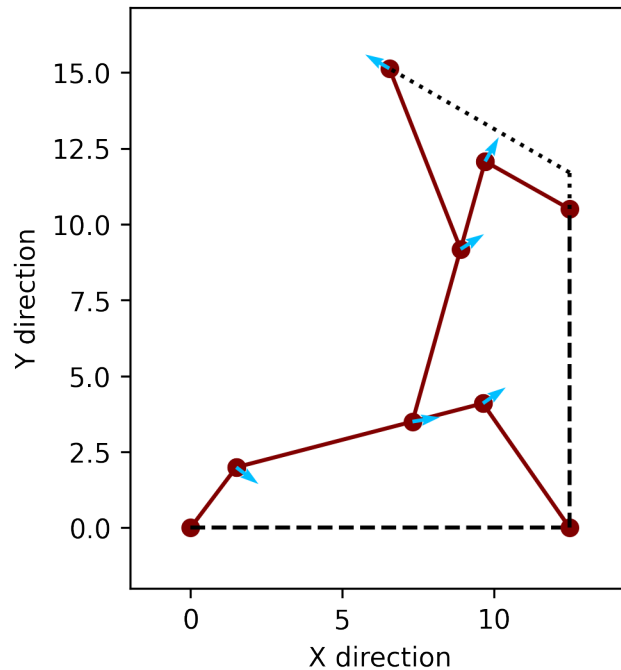
velocity_loops = lambda x: np.array([
    a_v.tan_x(a_v.omega) + b_v.tan_x(x[1]) - d_v.tan_x(x[0]),
```

```

a_v.tan_y(a_v.omega) + b_v.tan_y(x[1]) - d_v.tan_y(x[0]),
f_v.tan_x(x[2]) - g_v.tan_x(x[3]) + d_v.tan_x(x[0]) - e_v.tan_x(x[1]),
f_v.tan_y(x[2]) - g_v.tan_y(x[3]) + d_v.tan_y(x[0]) - e_v.tan_y(x[1]),
j_v.tan_x(x[4]) - k_v.slip_x(x[5]) + g_v.tan_x(x[3]) - i_v.tan_x(x[2]),
j_v.tan_y(x[4]) - k_v.slip_y(x[5]) + g_v.tan_y(x[3]) - i_v.tan_y(x[2]),
])

guess = np.array([15, 15, 30, 12, 30, 3])
v_loop = fsolve(velocity_loops, guess)
mechanism.fix_velocity()
mechanism.plot(velocity=True, cushion=2)
mechanism.tables(velocity=True, to_five=True)

```



VELOCITY

Vector	Mag	Angle	x	y
V_04B	75.00000	322.92024	59.83477	-45.21946
V_BA	76.08683	104.52037	-19.07681	73.65650
V_0402	0.00000	90.00000	0.00000	0.00000
V_02A	49.69785	34.90368	40.75796	28.43704
V_CA	21.73909	104.52037	-5.45052	21.04472
V_CE	37.71928	164.38201	-36.32661	10.15487
V_06E	20.13840	60.61355	9.88187	17.54719

V_02D6		0.00000		90.00000		0.00000		0.00000
V_DE		12.71436		164.38201		-12.24493		3.42299
V_DF		29.74448		201.55275		-27.66474		-10.92687
V_GF		6.39467		150.00000		-5.53795		3.19734
V_06G		0.00000		90.00000		0.00000		0.00000

Vector		Omega		Slip Vel
-----+-----+-----				
V_04B		-30.00000		0.00000
V_BA		9.05796		0.00000
V_0402		0.00000		0.00000
V_02A		-9.93957		0.00000
V_CA		9.05796		0.00000
V_CE		4.23812		0.00000
V_06E		-6.29325		0.00000
V_0206		0.00000		0.00000
V_DE		4.23812		0.00000
V_DF		4.64758		0.00000
V_GF		0.00000		6.39467
V_06G		0.00000		0.00000

Joint		Mag		Angle		x		y
-----+-----+-----+-----+-----								
A		49.69785		34.90368		40.75796		28.43704
B		75.00000		322.92024		59.83477		-45.21946
C		46.79605		9.08903		46.20848		7.39232
D		26.25049		32.55132		22.12679		14.12420
E		20.13840		60.61355		9.88187		17.54719
F		6.39467		150.00000		-5.53795		3.19734
G		0.00000		90.00000		0.00000		0.00000
02		0.00000		90.00000		0.00000		0.00000
04		0.00000		90.00000		0.00000		0.00000
06		0.00000		90.00000		0.00000		0.00000

5.4.1 Velocity of Link 8 Answer

The velocity of link 8 is $6.39467 \frac{units}{s}$

5.5 Acceleration Analysis

Similarly to the velocity analysis, only the angular accelerations for all the vectors are unknown except for \vec{k} which has an unknown slip acceleration \ddot{k} . The value of \ddot{k} is the acceleration of link 8.

$$\alpha_4 = 0 \frac{rad}{s^2}$$

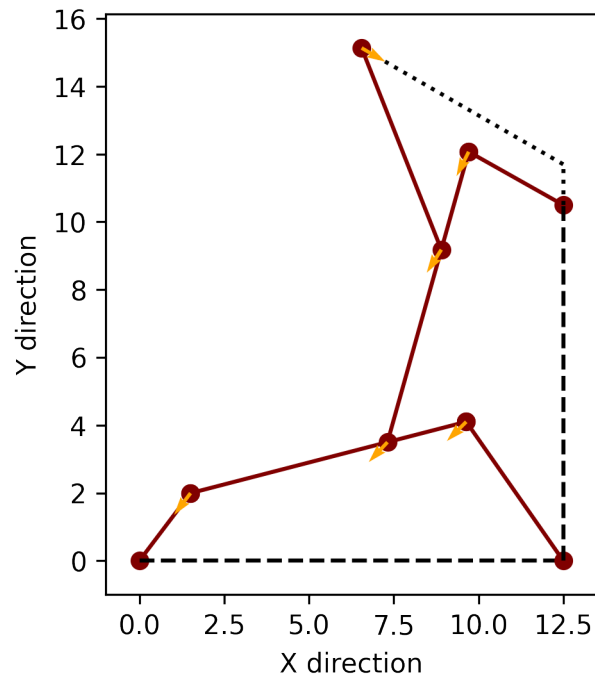
Look at the set up for the acceleration loop equation in the code below. All vectors will only have a normal and tangential component of acceleration with the exception of \vec{k} .


```
[65]: a_a, b_a, c_a, d_a, e_a, f_a, g_a, h_a, i_a, j_a, k_a, l_a = mechanism.
      ↪get_accelerations()

order3 = ('Alpha 2', 'Alpha 3', 'Alpha 5', 'Alpha 6', 'Alpha 7', 'k_ddot')

acceleration_loops = lambda x: np.array([
    a_a.normal_tan_x(a_a.alpha) + b_a.normal_tan_x(x[1]) - d_a.
    ↪normal_tan_x(x[0]),
    a_a.normal_tan_y(a_a.alpha) + b_a.normal_tan_y(x[1]) - d_a.
    ↪normal_tan_y(x[0]),
    f_a.normal_tan_x(x[2]) - g_a.normal_tan_x(x[3]) + d_a.normal_tan_x(x[0]) -
    ↪e_a.normal_tan_x(x[1]),
    f_a.normal_tan_y(x[2]) - g_a.normal_tan_y(x[3]) + d_a.normal_tan_y(x[0]) -
    ↪e_a.normal_tan_y(x[1]),
    j_a.normal_tan_x(x[4]) - k_a.slip_x(x[5]) + g_a.normal_tan_x(x[3]) - i_a.
    ↪normal_tan_x(x[2]),
    j_a.normal_tan_y(x[4]) - k_a.slip_y(x[5]) + g_a.normal_tan_y(x[3]) - i_a.
    ↪normal_tan_y(x[2])
])

guess = np.array([10, 10, 30, -30, 20, 10])
a_loop = fsolve(acceleration_loops, guess)
mechanism.fix_acceleration()
mechanism.plot(acceleration=True)
mechanism.tables(acceleration=True, to_five=True)
```



ACCELERATION

Vector	Mag	Angle	x	y
-----+	-----+	-----+	-----+	-----+
A_04B	2250.00000	232.92024	-1356.58395	-1795.04317
A_BA	690.38940	197.89664	-656.98313	-212.15724
A_0402	0.00000	90.00000	0.00000	0.00000
A_02A	2843.11552	224.90928	-2013.56707	-2007.20042
A_CA	197.25412	197.89664	-187.70947	-60.61635
A_CE	762.80051	332.28495	675.28555	-354.75913
A_06E	2572.93540	243.43693	-1150.57205	-2301.34320
A_0206	0.00000	90.00000	0.00000	0.00000
A_DE	257.12377	332.28495	227.62434	-119.58173
A_DF	3404.97848	19.22594	3215.07384	1121.23978
A_GF	2121.04337	330.00000	1836.87744	-1060.52169
A_06G	0.00000	90.00000	0.00000	0.00000

Vector	Alpha	Slip Acc
-----+	-----+	-----+
A_04B	0.00000	0.00000
A_BA	-4.84036	0.00000
A_0402	0.00000	0.00000
A_02A	559.97479	0.00000
A_CA	-4.84036	0.00000
A_CE	-83.80469	0.00000
A_06E	803.06630	0.00000
A_0206	0.00000	0.00000
A_DE	-83.80469	0.00000
A_DF	-531.58924	0.00000
A_GF	0.00000	-2121.04337
A_06G	0.00000	0.00000

Joint	Mag	Angle	x	y
-----+	-----+	-----+	-----+	-----+
A	2843.11552	224.90928	-2013.56707	-2007.20042
B	2250.00000	232.92024	-1356.58395	-1795.04317
C	2668.88470	226.83297	-1825.85761	-1946.58406
D	2580.60233	237.71982	-1378.19640	-2181.76146
E	2572.93540	243.43693	-1150.57205	-2301.34320
F	2121.04337	330.00000	1836.87744	-1060.52169
G	0.00000	90.00000	0.00000	0.00000
02	0.00000	90.00000	0.00000	0.00000
04	0.00000	90.00000	0.00000	0.00000
06	0.00000	90.00000	0.00000	0.00000

5.5.1 Acceleration of Link 8 Answer

The magnitude of the acceleration of link 8 is $2121.04337 \frac{units}{s^2}$