Alán Tapia Parada A00227840

1. Dadas las ecuaciones diferenciales, resuelve cada una encontrando una solución explícita y clasifica. Resuelve la ecuación diferencial y encuentra el dominio de definición de la solución

a) $xy' = -3y + \frac{\operatorname{sen} x}{x^2}$

b) $\frac{dy}{dx} = (\cos x)(\cos 2y - \cos^2 y)$

a) $xy' = -3y + \frac{\operatorname{sen} x}{x^2}$

v. dep = y   orden = 1   ordinaria

v. ind = x   grado = 1   lineal

$$\frac{xy' + 3y = \frac{\operatorname{sen} x}{x^2}}{x}$$

$$\frac{\frac{\operatorname{sen} x}{x^2}}{\frac{x}{1}} = \frac{\operatorname{sen} x}{x^3}$$

$$y' + \frac{3y}{x} = \frac{\operatorname{sen} x}{x^3}$$

$$U(x) = e^{3\int \frac{1}{x}}$$

$$U(x) = e^{3\ln|x|}$$

$$U(x) = x^3$$

$$\int \frac{d}{dx} = \left[y\, x^3\right] = \int \frac{(\operatorname{sen} x)\cancel{x^3}}{\cancel{x^3}}$$

$$y x^3 = \int \operatorname{sen} x$$

$$y x^3 = -\cos x + C$$

$$y = \frac{-\cos x + C}{x^3}$$

b) $\frac{dy}{dx} = (\cos x)(\cos 2y - \cos^2 y)$

$\cos 2y = \cos^2 y - \text{sen}^2 y$

$\int \frac{dy}{(\cos 2y - \cos^2 y)} = \int (\cos x) dx$

$\int \frac{dy}{\cos^2 y - \text{sen}^2 y - \cos^2 y} = \int \cos x \, dx$

$\int -\frac{dy}{\text{sen}^2 y} = \int \cos x \, dx$

$\left(\frac{1}{\sec}\right)\left(\frac{1}{\sec}\right)$

$\int -\frac{dy}{\cos^2 x \tan^2 x}$

$\frac{\frac{1}{\frac{1}{\tan^2 y}}}{\sec^2 y} = \frac{\sec^2 y}{\tan^2 y} = 1$

$\int -\frac{dy}{\sec^{-2} y \tan^2 \cancel{x}}$

$\int -1 dy = \int \cos x dx$

$-y = (\text{sen} x + C) - 1$

$u = \cos y \quad v = \cos y$

$du = -\text{sen} y \quad dv = -\text{sen} y$

$y = -\text{sen} x + C$

$D(x,y) = \{(x,y) \in \mathbb{R}^2\}$

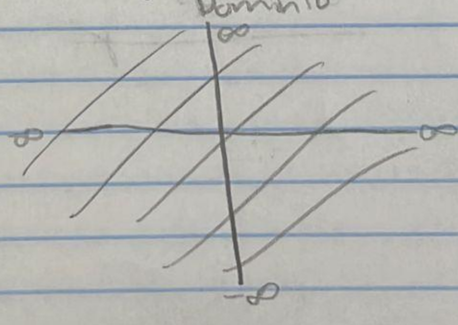
$D \frac{\partial f}{\partial y} = \cos x(-2\,\text{sen}\,2y - (\cos y\,\text{sen} y)) - (-\text{sen} y \cos y)$

$\cos x\,(-2\,\text{sen}\,2y + \cos y\,\text{sen} y + \text{sen} y \cos y)$

$\cos x\,(-2\,\text{sen}\,2y + 2\cos y\,\text{sen} y)$

$D \frac{\partial f}{\partial y} = \{x, y \in \mathbb{R}^2\}$

Dominio



2. Resuelve el problema considerando un lago con buena circulación...

$V = 1000\ kL$

$Re = 5\ kL/hr \quad rs = 2\ kL/hr$

$Ce = 7\ kg/kL \quad Cs = ?$

$Cs(0) = 2\ kg/kL$

$Cs = \frac{Q(t)}{V(t)}$

$\frac{dQ}{dt} = 5(7) - \frac{2Q}{(3)t + 1000}$

$\frac{dQ}{dt} = 35 - \frac{2Q}{3t+1000}$

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$U(x) = e^{\int -\frac{2}{3t+1000} dt}$

$-\frac{2}{3}\int \frac{1}{t+1000} dt$

$-\frac{2}{3} \ln|t+1000|^{-2/3}$

$U(x) = (t+1000)^{-2/3}$

$\frac{d}{dx}\left[Q(t+1000)^{-2/3}\right] = 35(t+1000)^{-2/3}$

$35\int (t+1000)^{-2/3} dt$

$35\left(\frac{(t+1000)^{1/3}}{\frac{1}{\frac{1}{3}}}\right.$

$35[3(t+1000)^{1/3}$

$Q(t+1000)^{-2/3} = 105(t+1000)^{1/3}$

$Q = \frac{105(t+1000)^{1/3}}{(t+1000)^{2/3}} + C$

$Q = 105(t+1000)$

$Q(t) = 105t + 105{,}000$

$Cs = \frac{105t + 105{,}000 + C}{3t + 1000}$

$2 = \frac{105{,}000 + C}{1000}$

$2000 = 105{,}000 + C$

$C = -103{,}000$

$1 = \frac{105t + 105{,}000 - 103{,}000}{3t+1000}$

$1 = \frac{105t + 2{,}000}{3t + 1000}$

$3t + 1000 = 105t + 2000$

$105t - 3t = -2000$

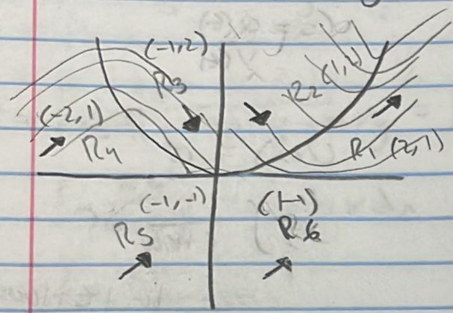
$102t = 2000$

$t = 19.6\ hr$

3. Sin resolver la ED, bosqueja algunas curvas solución

$\frac{dy}{dx} = x^2 - y \qquad x^2 - y = 0$

$y = x^2$



	$x^2 - y$
R1 (2,1)	+
R2 (1,2)	−
R3 (-1,2)	−
R4 (-2,1)	+
R5 (-1,-1)	+
R6 (1,-1)	+

$Df = \{(x,y) \in \mathbb{R}^2\}$

Si existe una solución única en P(2,1), ya que se encuentra dentro del dominio de la ecuación

Dominio