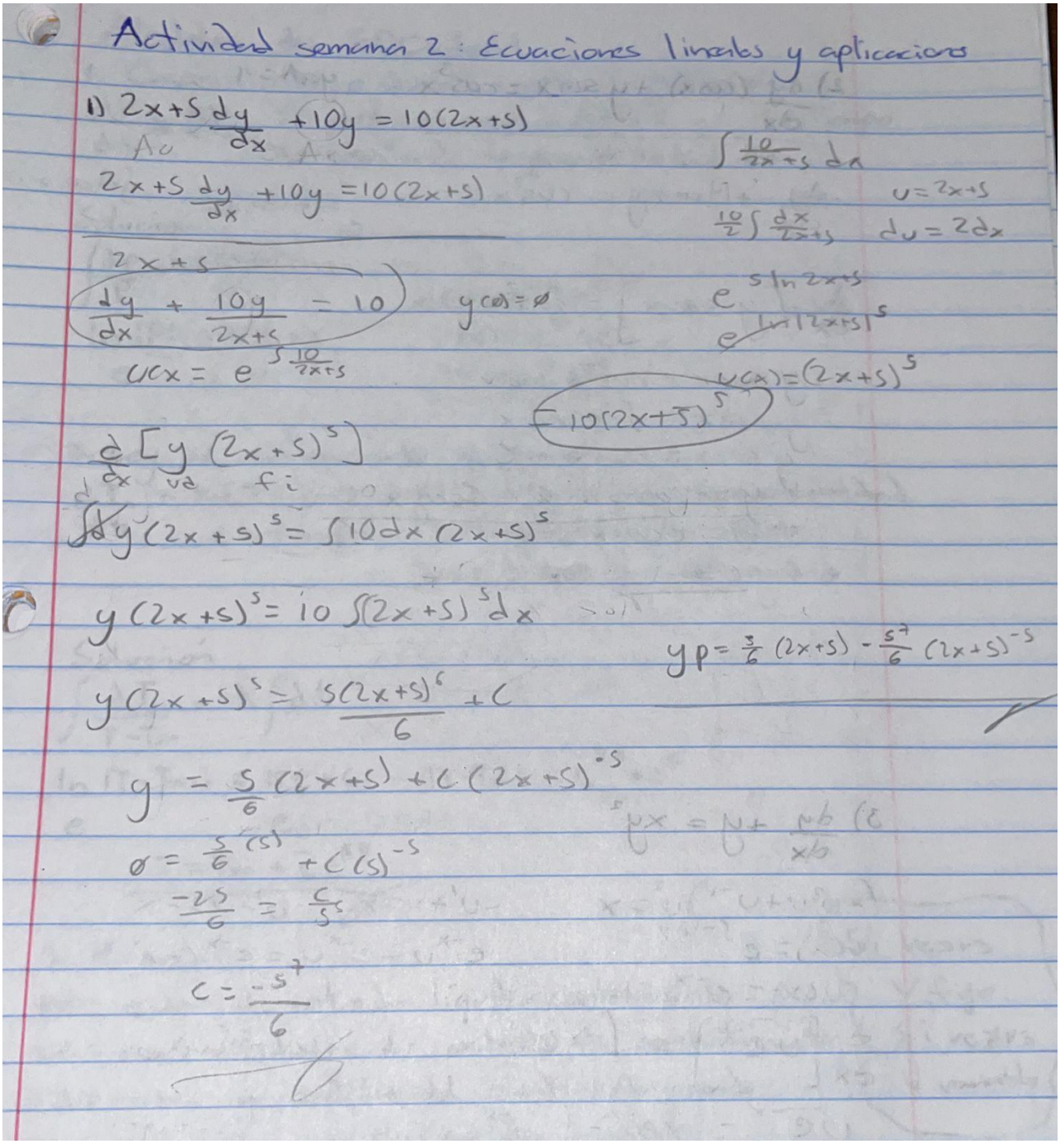
ACTIVIDAD SEMANA 2: ECUACIONES LINEALES Y APLICACIONES



2) $\frac{dy}{dx}(\cos x) + y\sin x = \cos^3 x$ $\quad y(0) = -1$

$\frac{dy}{dx} + (\tan x)y = \cos^2 x$

$u(x) = e^{\int \tan x\,dx}$

$e^{+\ln|\cos x|}$ $\quad \int \tan x\,dx$

$u(x) = \sec x$

$\frac{d}{dx}[y\sec x] = \cos^2 x \sec x$

$y\sec x = \cos^2 x \sec x$

$y\sec x = \int \cos x\,dx$

$y\sec x = \sin x + C$

$y = \cos x \sin x + C\cos x$

$-1 = C\cos x$

$C = -1$

3) $\frac{dy}{dx} + y = xy^2$

$(-v^2 v + v^{-1})v^2 = x$ $\quad -v' + v = x$ $\quad v' - v = -x$

$u(x) = e^{\int -1\,dx}$ $\quad e^{-x}v - e^{-x}v = e^{-x}(-x)$

$u(x) = e^{-x}$

$\frac{d}{dx}[ve^{-x}] = \int -xe^{-x}$ $\quad uv - \int u\,dv$

$ve^{-x} = xe^{-x} - \int -e^{-x}(-1)\,dx$ $\quad v = y^{-1}$

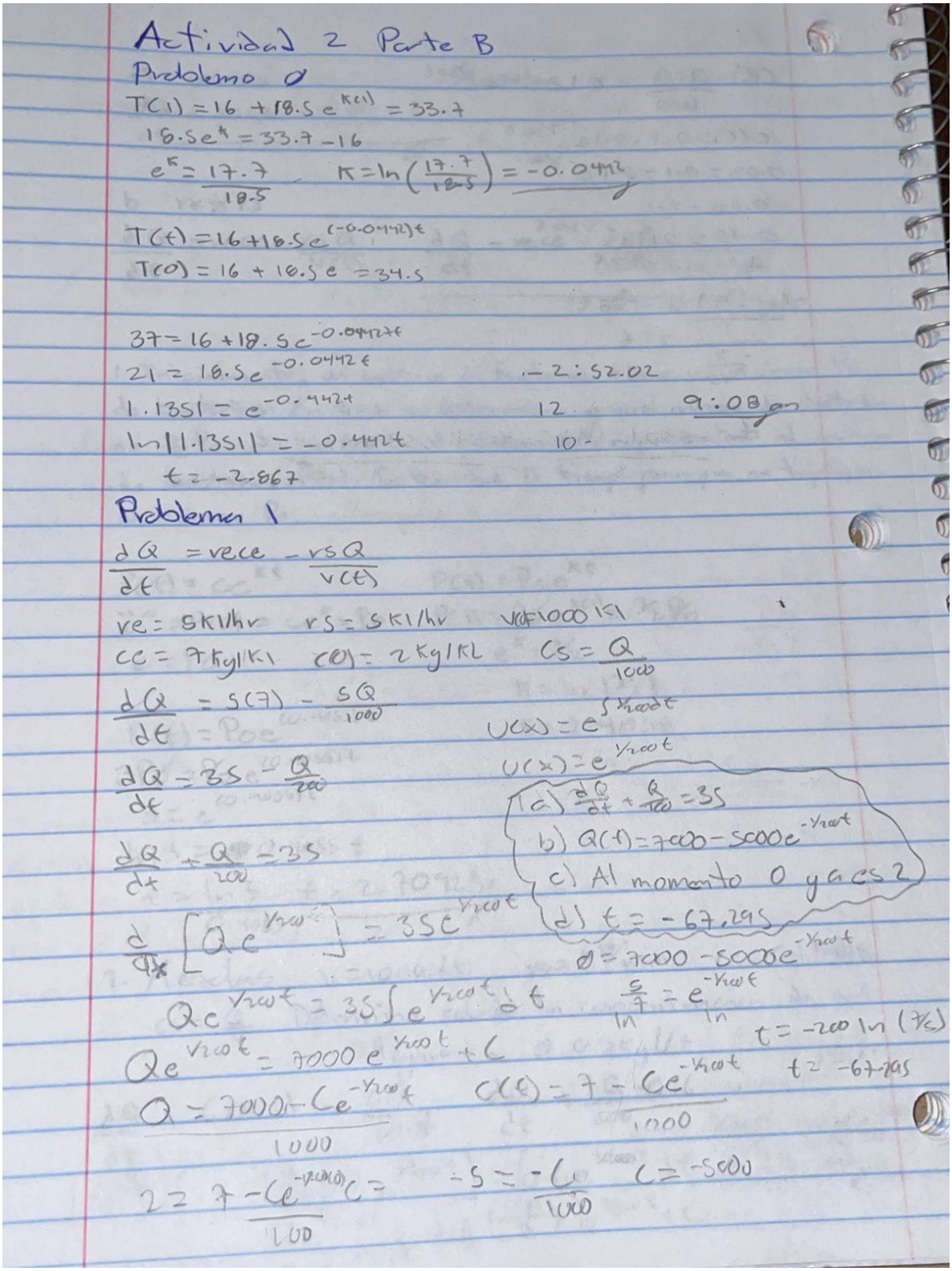
$ve^{-x} = xe^{-x} + e^{-x} + C$ $\quad v = \frac{1}{y}$

$v = \frac{xe^{-x} + e^{-x} + C}{e^{-x}}$ $\quad v = x + 1 + Ce^{x}$

$\frac{1}{y} = x + 1 + Ce^{x}$

$y = \frac{1}{x + 1 + Ce^{x}}$

Parte B



Actividad 2 Parte B

Problema 0

$T(1) = 16 + 18.5e^{k(1)} = 33.7$

$18.5e^{k} = 33.7 - 16$

$e^{k} = \frac{17.7}{18.5}$ $\quad k = \ln\left(\frac{17.7}{18.5}\right) = -0.0442$

$T(t) = 16 + 18.5e^{(-0.0442)t}$

$T(0) = 16 + 18.5e = 34.5$

$37 = 16 + 18.5e^{-0.0442t}$

$21 = 18.5e^{-0.0442t}$ $\quad -2:52.02$

$1.1351 = e^{-0.0442t}$ $\quad 12$ $\quad 9:08$ am

$\ln|1.1351| = -0.0442t$ $\quad 10$

$t = -2.867$

Problema 1

$\frac{dQ}{dt} = rece - \frac{rsQ}{v(t)}$

$re = 5 kl/hr$ $\quad rs = 5 kl/hr$ $\quad v_0 = 1000 kl$

$cc = 7 kg/kl$ $\quad c(0) = 2 kg/kl$ $\quad Cs = \frac{Q}{1000}$

$\frac{dQ}{dt} = 5(7) - \frac{5Q}{1000}$

$\frac{dQ}{dt} = 35 - \frac{Q}{200}$

$\frac{dQ}{dt} + \frac{Q}{200} = 35$

$\frac{d}{dt}\left[Qe^{\frac{1}{200}t}\right] = 35e^{\frac{1}{200}t}$

$Qe^{\frac{1}{200}t} = 35\int e^{\frac{1}{200}t}\,dt$

$Qe^{\frac{1}{200}t} = 7000e^{\frac{1}{200}t} + C$

$Q = 7000 - Ce^{-\frac{1}{200}t}$ $\quad c(t) = 7 - \frac{Ce^{-\frac{1}{200}t}}{1000}$

1000

$2 = 7 - \frac{Ce^{-\frac{1}{200}(0)}}{100}$ $\quad C = -5 = -\frac{C}{1000}$ $\quad C = -5000$

$u(x) = e^{5\frac{1}{200}dt}$

$u(x) = e^{\frac{1}{200}t}$

(a) $\frac{dQ}{dt} + \frac{Q}{200} = 35$

b) $Q(t) = 7000 - 5000e^{-\frac{1}{200}t}$

c) Al momento 0 gaces 2

d) $t = -67.29s$

$\emptyset = 7000 - 5000e^{-\frac{1}{200}t}$

$\frac{5}{7} = e^{-\frac{1}{200}t}$

$\ln\frac{5}{7} = \ln e^{-\frac{1}{200}t}$

$t = -200\ln(7/5)$

$t = -67.29s$

Problema 2

$T(0)=70°F \quad T(\tfrac{1}{2})=110°F \quad T(1)=145°F$

$T = T_m + Ce^{kt}$

$110 = T_m + (70 - T_m)e^{k/2} \qquad 145 = T_m + (70-T_m)e^{k}$

$\left(\frac{110-T_m}{70-T_m}\right)^2 = (e^{k/2})^2 \qquad e^{k} = \frac{145-T_m}{70-T_m}$

$\frac{(110-T_m)^2}{(70-T_m)^2} = \frac{145-T_m}{70-T_m}$

$(110-T_m)^2 = (145-T_m)(70-T_m)$

$12100 - 220T_m + T_m^2 = 10150 - 215T_m + T_m^2$

$12100 - 10150 + T_m^2 - T_m^2 = 220T_m - 215T_m$

$1950 = 5T_m$

$T_m = 390°F$

Problema 3

$\frac{dx}{dt} = Kx(1000-x), \quad x(0)=1 \quad x(4)=50 \quad x(6)=?$

$\int \frac{dx}{1000x - x^2} = K\int dt \qquad \int \frac{dx}{1000x - x^2} = Kt + C$

$\frac{A}{x} + \frac{B}{1000-x} = \frac{1}{1000x - x^2} \qquad A(1000-x) + B(x) = 1$

$\int \left(\frac{1}{1000x} + \frac{1}{1000(1000-x)}\right) dx = K\cdot t + C$

$\frac{1}{1000}\int \frac{1}{x} + \frac{1}{1000-x}\,dx = K\,dt$

$\frac{\ln x - \ln(1000-x)}{1000} = Kt + C$

$x = 1000\tilde{c}e^{1000Kt} - x\tilde{c}e^{1000Kt}$

$x + x\tilde{c}e^{1000Kt} = 1000\tilde{c}e^{1000Kt}$

$x(1+\tilde{c}e^{1000Kt}) = 1000\tilde{c}e^{1000Kt}$

$\frac{1}{\frac{1000}{x}} = \frac{1}{1000x}$

$\frac{1}{\frac{1000}{1000-x}} = \frac{1}{1000(1000-x)}$

$\ln\frac{x}{1000-x} = 1000Kt + 1000C$

$\frac{x}{1000-x} = \tilde{c}e^{1000Kt}$

$x = \frac{1000\tilde{c}e^{1000kt}}{1+\tilde{c}e^{1000kt}}$ Sustituir $x(0)=1$

$1 = \frac{1000\tilde{c}}{1+\tilde{c}}$ $1(1+\tilde{c}) = 1000\tilde{c}$ $1 = 999\tilde{c}$

$\tilde{c} = \frac{1}{999}$ $x = \frac{1000e^{1000kt}}{999+e^{1000kt}}$

$x = \frac{1000}{999e^{-1000kt}+1}$ Sustituir $x(4)=50$

$50 = \frac{1000}{999e^{-1000k(4)}}$ $50(999e^{-4000k}+1) = 1000$

$999e^{-4000k} = 1000/50 - 1$ $999e^{-4000k} = 19$

$e^{-4000k} = \frac{19}{999}$ $-4000k = \ln(19/999)$

$\ln \quad \ln$

$k = \frac{\ln(19/999)}{-4000} = 0.0009905$

Sustituir $x(6) = ?$

$x(6) = \frac{1000}{999e^{-1000(0.0009905)(6)}+1}$

$x(6) = 276.22$